Problem Set #1

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Problem 1

Part (a). The Lagrangian is:

$$\max_{c_{1,t},c_{2,t+1}} L = (1-\beta)ln(c_{1,t}) + \beta ln(c_{2,t+1}) - \lambda(p_t c_{1,t} + p_{t+1}c_{2,t+1} - p_t e_1 - p_{t+1}e_2)$$

To satisfy first order condition:

$$\frac{\partial L}{\partial c_{1,t}} = \frac{1-\beta}{c_{1,t}} - \lambda p_t = 0$$

$$\Rightarrow \lambda = \frac{1-\beta}{c_{1,t}p_t} \tag{1}$$

$$\frac{\partial L}{\partial c_{2,t+1}} = \frac{\beta}{c_{2,t+1}} - \lambda p_{t+1} = 0$$

$$\Rightarrow \lambda = \frac{\beta}{c_{2,t+1}p_{t+1}} \tag{2}$$

from (1) and (2) we can get the Euler equation:

$$c_{2,t+1} = \frac{\beta p_t}{(1-\beta)p_{t+1}} c_{1,t} \tag{3}$$

To satisfy complementary slackness condition:

$$\lambda(p_t c_{1,t} + p_{t+1}c_{2,t+1} - p_t e_1 - p_{t+1}e_2) = 0$$

However, here $\lambda \neq 0$ because $\beta \neq 1$ and $c_{1,t} \neq 0$. Thus,

$$p_t c_{1,t} + p_{t+1}c_{2,t+1} - p_t e_1 - p_{t+1}e_2 = 0 \tag{4}$$

Substituting (3) into (4)

$$\Rightarrow c^*_{1,t} = \frac{(p_t e_1 + p_{t+1}e_2)(1-\beta)}{p_t} \tag{5}$$

$$\Rightarrow c^*_{2,t+1} = \frac{(p_t e_1 + p_{t+1}e_2)\beta}{p_{t+1}} \tag{6}$$

Part (b). The Lagrangian is:

$$\max_{c_{2,1}} L = \beta ln(c_{2,1}) - \lambda(p_1 c_{2,1} - p_1 e_2)$$

To satisfy first order condition:

$$\frac{\partial L}{\partial c_{2,1}} = \frac{\beta}{c_{2,1}} - \lambda p1 = 0$$

To satisfy complementary slackness condition:

$$\lambda(p_1 c_{2,1} - p_1 e_2) = 0$$

However, here $\lambda \neq 0$ because $\beta \neq 1$ and $c_{2,1} \neq 0$. Thus,

$$\Rightarrow c^*_{2,1} = \frac{p_1 e_2}{p_1} = e_2$$

Part (c). Plugging our answer in part (b) into the CE budget constraint we have:

$$\begin{aligned}
c^*_{1,1} + c^*_{2,1} &= e_1 + e_2 \\
c^*_{1,1} + e_2 &= e_1 + e_2 \\
\Rightarrow c^*_{1,1} &= e_1
\end{aligned}$$

Here, both the young and old consume all their repective endowments. There is no inter-generational exchange happening. These answers are different from the answers in part (a).

$$\{c_{1,t}, c_{2,t}\}_{t=1}^{\infty} = \{e_1, e_2\}$$

Since there is no inter-generational exchange happening, the prices can be arbitrary.