Problem Set 1

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Problem 2(a)
The Lagrange function is:

$$(1-\beta)ln(c_{1,t}) + \beta ln(c_{2,t+1}) - \lambda[p_t e_1 + p_{t+1}e_2 - p_t c_{1,t} - p_{t+1}c_{2,t+1}]$$

The first order conditions are,

$$\frac{1-\beta}{c_{1,t}} = \lambda p_t$$
$$\frac{\beta}{c_{2,t+1}} = \lambda p_{t+1}$$

Combining the two equations above, we get

$$c_{2,t+1} = \frac{\beta}{1-\beta}\frac{p_t}{p_{t+1}}c_{1,t}$$

Plugging it into the budget constraint, we get

$$c_{1,t} = \frac{(p_t e_1 + p_{t+1}e_2)(1-\beta)}{p_t}$$
$$c_{2,t+1} = \frac{(p_t e_1 + p_{t+1}e_2)\beta}{p_{t+1}}$$

Problem 2(b)
Since the utility of initial old increases as her consumption in the second period increases, the initial old would want to exhaust her resource in the last period of her life, so $c_{2,1} = \frac{p_1 e_2}{p_1} = e_2$.

Problem 2(c)
At competitve equilibrium, the amount of borrowing and lending must be equal for each period. In period 1, the initial old would want to consume all her endowment e_2, leaving the young born in period 1 with nothing to borrow and her own endowment e_1 to consume. Going into the next period, the individual born in period 1 consumes her period-2 endowment e_2, leaving the one born in period 2 with e_1 to consume. This reasoning applies to every period–the old in that period consumes all her endowment e_2, and the young could only consume all the nonstorable e_1. $\{c_{1,t}, c_{2,t}\}_{t=1}^{\infty} = \{e_1, e_2\}$.

Since there is no intergenerational transfer going on, prices are indeterminate. The competitive equilibrium is not necessarily equal to my anwer in part (a). Plugging e_1, e_2 into the intertemporal euler equation in part (a), we get $\frac{e_2}{e_1} = \frac{\beta}{1-\beta}\frac{p_t}{p_{t+1}}$. When normalizing $p_1 = 1$, we get $p_t = (\frac{\beta}{1-\beta}\frac{e_1}{e_2})^{t-1}$. Only when imposing this vector of prices could we obtain a competitive equilibrium that is same to the answer in part (a).