

## Problem Set #1

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### Problem 1

**Part (a).** The Lagrangian is:

$$\max_{c_{1,t}, c_{2,t+1}} L = (1 - \beta) \ln(c_{1,t}) + \beta \ln(c_{2,t+1}) - \lambda(p_t c_{1,t} + p_{t+1} c_{2,t+1} - p_t e_1 - p_{t+1} e_2)$$

To satisfy first order condition:

$$\begin{aligned} \frac{\partial L}{\partial c_{1,t}} &= \frac{1 - \beta}{c_{1,t}} - \lambda p_t = 0 \\ \Rightarrow \lambda &= \frac{1 - \beta}{c_{1,t} p_t} \end{aligned} \tag{1}$$

$$\begin{aligned} \frac{\partial L}{\partial c_{2,t+1}} &= \frac{\beta}{c_{2,t+1}} - \lambda p_{t+1} = 0 \\ \Rightarrow \lambda &= \frac{\beta}{c_{2,t+1} p_{t+1}} \end{aligned} \tag{2}$$

from (1) and (2) we can get the Euler equation:

$$c_{2,t+1} = \frac{\beta p_t}{(1 - \beta) p_{t+1}} c_{1,t} \tag{3}$$

To satisfy complementary slackness condition:

$$\lambda(p_t c_{1,t} + p_{t+1} c_{2,t+1} - p_t e_1 - p_{t+1} e_2) = 0$$

However, here  $\lambda \neq 0$  because  $\beta \neq 1$  and  $c_{1,t} \neq 0$ . Thus,

$$p_t c_{1,t} + p_{t+1} c_{2,t+1} - p_t e_1 - p_{t+1} e_2 = 0 \tag{4}$$

Substituting (3) into (4)

$$\Rightarrow c_{1,t}^* = \frac{(p_t e_1 + p_{t+1} e_2)(1 - \beta)}{p_t} \tag{5}$$

$$\Rightarrow c_{2,t+1}^* = \frac{(p_t e_1 + p_{t+1} e_2)\beta}{p_{t+1}} \tag{6}$$

**Part (b).** The Lagrangian is:

$$\max_{c_{2,1}} L = \beta \ln(c_{2,1}) - \lambda(p_1 c_{2,1} - p_1 e_2)$$

To satisfy first order condition:

$$\frac{\partial L}{\partial c_{2,1}} = \frac{\beta}{c_{2,1}} - \lambda p_1 = 0$$

To satisfy complementary slackness condition:

$$\lambda(p_1 c_{2,1} - p_1 e_2) = 0$$

However, here  $\lambda \neq 0$  because  $\beta \neq 1$  and  $c_{2,1} \neq 0$ . Thus,

$$\Rightarrow c_{2,1}^* = \frac{p_1 e_2}{p_1} = e_2$$

**Part (c).** Plugging our answer in part (b) into the CE budget constraint we have:

$$\begin{aligned} c_{1,1}^* + c_{2,1}^* &= e_1 + e_2 \\ c_{1,1}^* + e_2 &= e_1 + e_2 \\ \Rightarrow c_{1,1}^* &= e_1 \end{aligned}$$

Here, both the young and old consume all their respective endowments. There is no inter-generational exchange happening. These answers are different from the answers in part (a).

$$\{c_{1,t}, c_{2,t}\}_{t=1}^{\infty} = \{e_1, e_2\}$$

Since there is no inter-generational exchange happening, the prices can be arbitrary.