Consumption Taxes and Economic Efficiency with Idiosyncratic Wage Shocks
Author(s): Shinichi Nishiyama and Kent Smetters
Source: *Journal of Political Economy*, Vol. 113, No. 5 (October 2005), pp. 1088-1115
Published by: University of Chicago Press
Stable URL: http://www.jstor.org/stable/10.1086/432137

Consumption Taxes and Economic Efficiency with Idiosyncratic Wage Shocks

Shinichi Nishiyama

Congressional Budget Office and Georgia State University

Kent Smetters

University of Pennsylvania and National Bureau of Economic Research

Fundamental tax reform is examined in an overlapping-generations model in which heterogeneous agents face idiosyncratic wage shocks and longevity uncertainty. A progressive income tax is replaced with a flat consumption tax. If idiosyncratic wage shocks are insurable (i.e., no risk), this reform improves (interim) efficiency, a result consistent with the previous literature. But if, more realistically, wage shocks are uninsurable, this reform reduces efficiency, even though national wealth and output increase over the entire transition path. This efficiency loss, in large part, stems from reduced intragenerational risk sharing that was previously provided by the progressive tax system.

I. Introduction

The potential economic gain from replacing the current progressive income tax system with a flat (proportional) consumption tax has generated a considerable amount of attention in recent years. Examples of a flat consumption tax include a value-added tax, which is used in many

Helpful comments were received from Kenneth L. Judd, José-Víctor Ríos-Rull, the editor (Nancy L. Stokey), three referees, and seminar participants at the Congressional Budget Office, the Econometrics Society summer meeting, Harvard University, the Koç-Sabancı Conference in Macroeconomics, the NBER Public Economics meeting, the Stanford Institute for Theoretical Economics, University of Wisconsin, and University of Southern California. The analysis and conclusions expressed herein are those of the authors and should not be interpreted as those of the Congressional Budget Office.

[*Journal of Political Economy,* 2005, vol. 113, no. 5]
 0022-3808/2005/11305-0006$10.00

European nations, and a national retail sales tax, which is gaining some attention in the United States as a possible substitute tax base.

Previous analyses have shown that this reform could significantly increase national saving and output in the long run. But these long-run macroeconomic gains might reflect, in part, losses to earlier generations that suffer implicit wealth levies on their assets accumulated before the adoption of the consumption tax. These losses are effectively captured by future generations through a reduction in their tax rates. Some of these long-run gains might also stem from redistributing resources from poorer households, which see their effective tax rates increase, toward richer households, which see their tax rates decline.

A highly relevant question, therefore, is whether adopting a consumption tax can generate a positive amount of new resources even after all households have been compensated for their potential losses, that is, whether this reform improves efficiency in a Pareto sense. We examine this question using an overlapping-generations (OLG) model in which heterogeneous agents with elastic labor supply face idiosyncratic wage shocks and uncertainty about longevity. By construction, the model's prereform neoclassical economy is stationary, and so economic growth in the initial steady state is driven by exogenous (labor-augmenting) constant technological change as well as by constant increases in the exogenous size of the population. Stokey and Rebelo (1995) have shown that adopting flat taxes would likely have little impact on the long-run growth rates in a model incorporating endogenous growth.

In our model, lump-sum transfers are given to current and future households that would otherwise lose from the reform. Households that would otherwise gain from the reform, however, face a lump-sum tax. These lump-sum transfers/taxes are chosen to return each household's expected remaining lifetime utility exactly to its prereform level, conditional on the household's state at the time of the reform or, in the case of future households, the state into which they are born. If there is a positive level of new net resources remaining after these transfers/taxes are made across all households, then we say that the reform has "increased efficiency" on an interim basis; conversely, the reform has "reduced efficiency" if the new net resources are negative. Following Auerbach and Kotlikoff (1987), who solved the first large-scale deterministic OLG model, we distribute these new net resources, whether positive or negative, evenly to all future generations (growth adjusted over time).

Our analysis reveals that if idiosyncratic wage shocks are insurable (i.e., no risk), then adopting a consumption tax improves efficiency by about \$154,000 per future household. A positive gain is consistent with the previous literature's use of deterministic models. But if, more realistically, wage shocks are uninsurable, this reform *reduces* efficiency by about $86,000 per future household, even though national wealth and output increase over the entire transition path. Since both experiments control for the *inter*generational transfer of resources, the difference in efficiency outcomes mostly stems from the reduced *intra*generational risk sharing in the second experiment following the replacement of the progressive tax system with a flat tax. Household saving is also less interest elastic in the presence of precautionary savings when wage shocks are uninsurable.

Section II briefly outlines the sources of efficiency changes when a flat consumption tax is adopted. Section III presents the model that we use to simulate the introduction of a revenue-neutral flat consumption tax. Section IV summarizes the calibration of the baseline economy. Section V reports simulation evidence for various experiments. Section VI presents concluding remarks. Appendices A and B describe the computational algorithm and some additional calibration information in detail.

II. Sources of Efficiency Gains

Replacing the current income tax system with a revenue-neutral flat consumption tax would have two main effects: It would tax consumption rather than income, and it would flatten tax rates. This section summarizes the impact of these two effects.[1]

A. *Taxing Consumption Rather than Wage and Capital Income*

In order to isolate the first effect (taxing consumption instead of income) from the second effect (flattening tax rates), suppose that the income tax were initially flat. Switching the tax base to consumption alone would produce two subeffects: It would eliminate the tax on interest income and levy an implicit lump-sum tax on existing capital.

Eliminating the tax on interest income removes the distortion to the relative price of consumption over time, as emphasized by Judd (1985, 1999) and Chamley (1986). But removing this intertemporal distortion does not come free. If the new tax were revenue neutral, the effective tax rate on labor income would increase, producing a labor supply distortion that increases more than linearly in the tax rate. Eliminating the tax on interest income also gives households alive at the time of the reform an implicit lump-sum transfer equal to the amount that they

[1] An expanded discussion of the sources of efficiency gains and losses can be found in our earlier draft (Nishiyama and Smetters 2003). See also Eaton and Rosen (1980*a*, 1980*b*), Judd (1985, 1999), Bradford (1986), Chamley (1986), Auerbach and Kotlikoff (1987), Fullerton and Rogers (1993), Hall and Rabushka (1995), Engen, Gravelle, and Smetters (1997), and Auerbach and Hines (2001).

would have paid in *capital income* taxes on their existing capital; this intergenerational transfer must be financed with an even larger distorting effective tax on labor. In the OLG economy with deterministic wages, Auerbach and Kotlikoff (1987) find that replacing a linear income tax with a linear *wage* tax, thereby eliminating the interest tax, *reduces* long-run output and welfare.[2] Even larger losses would likely emerge in a model with uncertain wages in which a precautionary savings motive reduces the interest elasticity of saving (Engen and Gale 1996; Engen et al. 1997).

The second subeffect of switching to a consumption tax base is that it would impose a lump-sum tax on the *capital stock* held by households alive at the time of the reform since the consumption of those assets is now taxed. Under revenue neutrality, this lump-sum tax would reduce the distorting taxes on households alive at the time of the reform as well as on all future households (Summers 1981). Auerbach and Kotlikoff (1987) and Altig et al. (2001) find that replacing a linear income tax with a linear *consumption* tax substantially *increases* long-run output and welfare, just the opposite qualitative result from adopting a wage tax. The key difference is that a consumption tax also imposes a lump-sum tax on existing wealth. In fact, *over* 100 percent of the long-run gain in the Auerbach and Kotlikoff model stems from this wealth levy (Engen et al. 1997).

B. *Flattening Tax Rates*

Adding progressive tax rates to the picture tends to magnify the intertemporal price distortions caused by the income tax since capital accumulation now has the added disadvantage of increasing the household's future marginal tax rate by increasing its future income. Flattening tax rates, therefore, would eliminate this extra distortion. Flattening tax rates would also eliminate the incentive of households to intertemporally substitute their labor supply toward lower-taxed ages in order to avoid the high tax rates during their middle-age years when their labor productivity is high and after assets have been accumulated for retirement. Not surprisingly, flattening tax rates tends to produce sizable long-run welfare and efficiency gains across a range of models with deterministic wages (Auerbach and Kotlikoff 1987; Altig et al. 2001; Jorgenson and Yun 2001). We verify these results with a version of our model with deterministic wages.

But flattening tax rates also reduces risk sharing when households

[2] In contrast, as shown by Judd (1985), Chamley (1986), and subsequent papers, the optimal long-run tax rate on capital income is zero in the Ramsey model with infinite horizons.

face uninsurable earnings risks. A literature using static two-period models has shown that a *proportional* income tax can potentially be more efficient than a *lump-sum tax* in the presence of idiosyncratic wages (see Eaton and Rosen 1980*a*, 1980*b*, 1980*c*; Varian 1980; Persson 1983; Tuomala 1984; Hamilton 1987; Kaplow 1994). Whereas a lump-sum tax collects the same amount of tax from each person, a proportional income tax collects relatively less revenue from households with smaller income realizations, thereby providing some insurance against wage risk. Relative to a proportional tax, though, a *progressive* income tax provides even more insurance. Flattening tax rates, therefore, could actually reduce efficiency by decreasing the insurance provided by the tax system. This effect is central to our results reported below.

III. Model

The model we use to analyze tax reform has three sectors: heterogeneous households with elastic labor supply, a competitive representative firm with constant returns to scale production technology, and a government with a full commitment technology.

A. *The Household Sector*

Households are heterogeneous with respect to age i, working ability e_i (hourly wage), and beginning-of-period wealth holding a_i. New households of age 20 enter the economy each year. A household of age i observes its idiosyncratic working ability shock, e_i, at the beginning of each year and chooses its optimal consumption c_i, working hours h_i, and end-of-period wealth holding a_{i+1}, with the government's policy schedule and future factor prices taken as given. Because there are no aggregate shocks, agents have perfect foresight of the government's policy variables, interest rates, and the cross-sectional average wage. But agents do not know their own future wages and life spans.

Let $\boldsymbol{s}_i$ denote the individual state vector of an age i household,

$$\boldsymbol{s}_i = (i,\ e_i,\ a_i), \tag{1}$$

where $i \in I = \{20,\ \ldots,\ 109\}$ is the household's age and $e_i \in E = [e^{\min}, e^{\max}]$ is the household's working ability (the hourly wage), which follows a Markov process described in more detail below. The term $a_i \in A = [a^{\min},\ a^{\max}]$ is the household's beginning-of-period wealth.

Let $\boldsymbol{S}_t$ denote the aggregate state of the economy at the beginning of year t,

$$\boldsymbol{S}_t = (x_t(\boldsymbol{s}_i),\ W_{LS,t},\ W_{G,t}), \tag{2}$$

where $x_t(\boldsymbol{s}_i)$ is the joint distribution of households with $\boldsymbol{s}_i \in I \times E \times A$.

The term $W_{LS,t}$ is the net wealth held by a hypothetical lump-sum redistribution authority (LSRA) that transfers to each household alive at the time of the reform just enough resources (possibly a negative amount) to return its expected remaining lifetime utility to its prereform level; the LSRA is described more below. The term $W_{G,t}$ is the beginning-of-period net government wealth not including the wealth held by the LSRA.

Let Ψ_t denote the government policy schedule known at the beginning of year t,

$$\Psi_t = \{W_{LS,s+1},\ W_{G,s+1},\ C_{G,s},\ \tau_{I,s}(\cdot),\ \tau_{C,s},\ tr_{LS,s}(\boldsymbol{s}_i)\}_{s=t}^{\infty}, \tag{3}$$

where $C_{G,s}$ is government consumption, $\tau_{I,s}(\cdot)$ is a progressive income tax function, $\tau_{C,s}$ is a consumption tax rate, and $tr_{LS,s}(\boldsymbol{s}_i)$ is a lump-sum transfer schedule. When we denote the expectation operator as $E[\cdot]$, the value function of a household is then

$$v(\boldsymbol{s}_i, \boldsymbol{S}_t;\ \Psi_t) = \max_{c_i, h_i} u_i(c_i, h_i) + \beta(1+\mu)^{\alpha(1-\gamma)}\phi_i E[v(\boldsymbol{s}_{i+1}, \boldsymbol{S}_{t+1};\ \Psi_{t+1})|e_i] \tag{4}$$

subject to

$$\begin{aligned} a_{i+1} &= \frac{1}{(1+\mu)\phi_i}\{w_t e_i h_i + (1+r_t)[a_i + tr_{LS,t}(\boldsymbol{s}_i)] \\ &\quad - \tau_{I,t}(w_t e_i h_i,\ r_t[a_i + tr_{LS,t}(\boldsymbol{s}_i)]) - (1+\tau_{C,t})c_i\} \\ &\geq a_{i+1}^{\min}(\boldsymbol{s}_i), \\ a_{20} &= 0, \quad a_{110} \geq 0, \end{aligned} \tag{5}$$

where the utility function, $u_i(\cdot)$, takes the Cobb-Douglas form nested within a time-separable isoelastic specification,

$$u(c_i,\ h_i) = \frac{(\{[1 + (n_i/2)]^{-\zeta}c_i\}^{\alpha}(h_i^{\max} - h_i)^{1-\alpha})^{1-\gamma}}{1-\gamma};$$

γ is the coefficient of relative risk aversion; n_i is the number of dependent children at the parents' age i; ζ is the "adult equivalency scale" that converts the consumption by children into their adult-equivalent amounts; and $h_i^{\max}$ is the maximum working hours.[3]

The constant β is the rate of time preference, ϕ_i is the survival rate at the end of age i, w_t is the wage rate per efficiency unit of labor (accordingly, $w_t e_i h_i$ is total labor compensation at age i at time t), and r_t is the rate of return to capital. Individual variables of the model are

[3] When $\gamma = 1$, we use the "pure" isoelastic form, $u(\cdot,\ \cdot) = (\{\cdot\}^{1-\gamma} - 1)/(1-\gamma)$, which subtracts unity in the numerator. (This form is not compatible with the existence of a steady state iff $\gamma \neq 1$.) By L'Hopital's rule, this function approaches log as $\gamma \to 1$.

normalized by the exogenous rate of labor-augmenting technological change, μ. Our choice for $u_i(\cdot)$ is consistent with the conditions that are necessary for the existence of a long-run steady state in the presence of constant population growth (King, Plosser, and Rebelo 1988). Hence, μ is also equal to the per capita growth rate of output and capital in steady state (i.e., $\boldsymbol{S}_{t+1} = \boldsymbol{S}_t$ for all t and $\boldsymbol{s}_i \in I \times E \times A$). The term $\beta(1+\mu)^{\alpha(1-\gamma)}$, therefore, is the *growth-adjusted* rate of time preference.

To avoid potentially biasing the results against a consumption tax, we assume that annuity markets are perfect so that the progressive income tax provides some insurance only against wage fluctuations and not against uncertainty about longevity.[4] This assumption also allows us to more directly compare the results produced by a version of our model with deterministic wages against the results produced by previous models with deterministic wages. Assets at age $i+1$, therefore, are scaled up by the factor $1/\phi_i$ to account for the income provided by the annuity above the rate of return to capital.

The term $a_{i+1}^{\min}(\boldsymbol{s}_i)$ is the state-contingent minimum level of end-of-period wealth that is sustainable, that is, even if the household receives the worst possible shocks in future working abilities. Since the worst possible wage is positive at most ages in the Markov transition process described below, younger households have some borrowing capacity. As shown by Hubbard and Judd (1986), imposing a strict constraint against borrowing (i.e., $a_{i+1}^{\min}(\boldsymbol{s}_i) \geq 0$) could bias the results against a consumption tax by reducing the importance of the intertemporal substitution margin that is distorted by the prereform income tax.

The decision rule of an age i household in year t is a function of its individual state $\boldsymbol{s}_i$, the aggregate state $\boldsymbol{S}_t$, and the government policy schedule Ψ_t, and it is summarized as

$$d(\boldsymbol{s}_i, \boldsymbol{S}_t; \Psi_t) = \{c_i(\boldsymbol{s}_i, \boldsymbol{S}_t; \Psi_t), h_i(\boldsymbol{s}_i, \boldsymbol{S}_t; \Psi_t), a_{i+1}(\boldsymbol{s}_i, \boldsymbol{S}_t; \Psi_t)\}.$$

B. *The Measure of Households*

Let $x_t(\boldsymbol{s}_i)$ denote the measure of households, and let $X_t(\boldsymbol{s}_i)$ be the corresponding cumulative measure. The measure of households is normalized by the population growth rate, ν. The population of age 20

[4] A previous draft of our paper (Nishiyama and Smetters 2003) reports simulations in which households do not have access to annuities, thereby leaving accidental bequests; the qualitative results are similar.

households, therefore, is normalized to be unity in the baseline economy along the balanced growth path, that is,

$$\int_E dX_t(20,\ e_{20},\ 0) = 1.$$

Let $\mathbf{1}_{[a=y]}$ be an indicator function that returns unity if $a = y$ and zero if $a \neq y$. Then, for $i \in I$, the law of motion of the measure of households is

$$x_{t+1}(\boldsymbol{s}_{i+1}) = \frac{\phi_i}{1+\nu}\int_{E\times A} \mathbf{1}_{[a_{i+1}=a_{i+1}(\boldsymbol{s}_i,\boldsymbol{S}_t;\Psi_t)]}\pi_{i,i+1}(e_{i+1}|e_i)dX_t(\boldsymbol{s}_i),$$

where $\pi_{i,i+1}(e_{i+1}|e_i)$ is the conditional probability that the age $i+1$ working ability is e_{i+1} when the age i working ability is e_i.

C. *The Firm's Problem*

National wealth W_t is the sum of total private wealth, LSRA net wealth $W_{LS,t}$, and the remaining government net wealth $W_{G,t}$. Total labor supply L_t is measured in efficiency units:

$$W_t = \sum_{i=20}^{109}\int_{E\times A} a_i dX_t(\boldsymbol{s}_i) + W_{LS,t} + W_{G,t} \tag{6}$$

and

$$L_t = \sum_{i=20}^{109}\int_{E\times A} e_i h_i(\boldsymbol{s}_i,\ \boldsymbol{S}_t;\ \Psi_t)dX_t(\boldsymbol{s}_i). \tag{7}$$

Our simulations consider two boundary cases in terms of international factor mobility: a closed economy (no mobility) and a small open economy (perfect mobility).

In the case of a closed economy, the capital stock is equal to national wealth,

$$K_t = W_t, \tag{8}$$

and gross national product Y_t is determined by constant returns to scale production,

$$Y_t = F(K_t,\ L_t). \tag{9}$$

The profit-maximizing condition of the firm is

$$r_t = F_K(K_t,\ L_t) - \delta \tag{10}$$

and

$$w_t = F_L(K_t, L_t), \tag{11}$$

where δ is the depreciation rate of capital.

In the case of the small open economy, factor prices r_t^* and w_t^* are fixed at international levels, and the domestic capital stock $K_{D,t}$ and labor supply L_t are determined so that the firm's profit-maximizing conditions satisfy the following equalities:

$$r_t^* = F_K(K_{D,t}, L_t) - \delta,$$

$$w_t^* = F_L(K_{D,t}, L_t).$$

Gross domestic product $Y_{D,t}$ is then determined by the production function

$$Y_{D,t} = F(K_{D,t}, L_t),$$

and gross national product Y_t is equal to

$$Y_t = (r_t^* + \delta)W_t + w_t^* L_t.$$

Net foreign investment is shown by the difference between national wealth and domestic capital stock, that is, $W_t - K_{D,t}$.

D. *The Government's Policy*

We follow Auerbach and Kotlikoff (1987) by measuring the pure efficiency gains from a policy change using an LSRA, but we extend their approach to a heterogeneous-agent OLG model. To see how the LSRA works, suppose that a new policy is announced at the beginning of period 1. The LSRA first makes a lump-sum transfer (tax if negative), $tr_{CV,1}(\boldsymbol{s}_i)$, to *each living* household of age i to bring its expected remaining lifetime utility at state $\boldsymbol{s}_i$ back to its prereform level in the baseline economy. Next, the LSRA makes a lump-sum transfer (or tax), $tr_{CV,t}(\boldsymbol{s}_{20})$, to *each future* household (i.e., each newborn household in periods 2, 3, and so on) to make it as well off in the baseline economy, conditional on its initial state at age 20. Thus far, however, the net present value of these transfers at the beginning of period 1 across living and future households will generally not sum to zero. So, finally, the LSRA makes an additional lump-sum transfer (tax), Δtr, to each future household so that the net present value across all transfers is zero. For illustrative purposes, we assume that these additional transfers are uniform across future generations on a growth-adjusted basis. The lump-sum transfers made by the LSRA, therefore, are

$$tr_{LS,t}(\boldsymbol{s}_i) = \begin{cases} tr_{CV,t}(\boldsymbol{s}_i) & \text{if } t = 1 \\ tr_{CV,t}(\boldsymbol{s}_i) + \Delta tr & \text{if } t > 1 \text{ and } i = 20 \\ 0 & \text{otherwise.} \end{cases}$$

If $\Delta tr > 0$, then tax reform has produced extra resources after the expected remaining lifetime utility of each household has been restored to its prereform level. In this case, we say that tax reform has generated efficiency gains. If, however, $\Delta tr < 0$, then tax reform reduces efficiency. The total net lump-sum transfer to living households at time t, $Tr_{LS,t}$, is

$$Tr_{LS,t} = \sum_{i=20}^{109} \int_{E\times A} tr_{LS,t}(\boldsymbol{s}_i) dX_t(\boldsymbol{s}_i). \tag{12}$$

Tax revenue in the rest of the government comes from federal income taxes $T_{I,t}$ and consumption taxes $T_{C,t}$:[5]

$$T_{I,t} = \sum_{i=20}^{109} \int_{E\times A} \tau_{I,t}(w_t e_i h_i(\boldsymbol{s}_i, \boldsymbol{S}_t; \Psi_t), r_t[a_i + tr_{LS,t}(\boldsymbol{s}_i)]) dX_t(\boldsymbol{s}_i) \tag{13}$$

and

$$T_{C,t} = \sum_{i=20}^{109} \int_{E\times A} \tau_{C,t} c_i(\boldsymbol{s}_i, \boldsymbol{S}_t; \Psi_t) dX_t(\boldsymbol{s}_i). \tag{14}$$

The laws of motion of the government wealth and the LSRA wealth, after being normalized by exogenous productivity growth and population growth, are

$$W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)}[(1+r_t)W_{G,t} + (T_{I,t} + T_{C,t}) - C_{G,t}] \tag{15}$$

and

$$W_{LS,t+1} = \frac{1}{(1+\mu)(1+\nu)}(1+r_t)(W_{LS,t} - Tr_{LS,t}). \tag{16}$$

The normalizations imply that $W_{G,t+1}$ and $W_{LS,t+1}$ remain constant in the steady state.

DEFINITION. *Recursive competitive equilibrium.*—The time series of factor prices and government policy variables,

$$\Omega_t = \{r_s,\ w_s,\ W_{LS,s+1},\ W_{G,s+1},\ C_{G,s},\ \tau_{C,s},\ tr_{LS,s}(\boldsymbol{s}_i)\}_{s=t}^{\infty};$$

[5] Our previous draft (Nishiyama and Smetters 2003) includes a detailed progressive Social Security system as well. The key qualitative results are the same with both models.

TABLE 1
Parameters and Initial Values

		Run 1: No Wage Shocks	Runs 2–5: "Main"	Run 6: Low γ	Run 7: One-Half Transitory Wage Shocks
Time preference parameter	β	1.006	.968	.963	.970
Share parameter for consumption	α	.416	.470	.472	.450
Coefficient of relative risk aversion	γ	2.0	2.0	1.0	2.0
Capital share of output	θ	.30	.30	.30	.30
Depreciation rate of capital stock	δ	.047	.047	.047	.047
Long-term real growth rate	μ	.018	.018	.018	.018
Population growth rate	ν	.010	.010	.010	.010
Total factor productivity	A	.949	.949	.949	.949
Interest rate	r	.062	.062	.062	.062
Wage rate	w	1.0	1.0	1.0	1.0

the value function of households, $\{v(\boldsymbol{s}_i, \boldsymbol{S}_s; \Psi_s)\}_{s=t}^{\infty}$; the decision rules of households,

$$\{d(\boldsymbol{s}_i, \boldsymbol{S}_s; \Psi_s)\}_{s=t}^{\infty} = \{c_i(\boldsymbol{s}_i, \boldsymbol{S}_s; \Psi_s), h_i(\boldsymbol{s}_i, \boldsymbol{S}_s; \Psi_s), a_{i+1}(\boldsymbol{s}_i, \boldsymbol{S}_s; \Psi_s)\}_{s=t}^{\infty};$$

and the measure of households, $\{x_s(\boldsymbol{s}_i)\}_{s=t}^{\infty}$, are in a recursive *competitive equilibrium* if, in every period $s = t, \ldots, \infty$, each household solves the utility maximization problem (1)–(5) taking Ψ_t as given; the firm solves the profit maximization problem, and the capital and labor markets clear, that is, equations (6)–(11) hold; and the government policy rules satisfy (12)–(16). The economy is in a *steady-state* recursive competitive equilibrium if the aggregate state of the economy is constant, that is, $\boldsymbol{S}_{t+1} = \boldsymbol{S}_t$ for all t and $\boldsymbol{s}_i \in I \times E \times A$.

IV. Calibration

This section outlines how we calibrate the model to the U.S. economy. The calibration is similar to that of the four-period model in Nishiyama (2002) but extends it significantly to a 90-period setting, the maximum "economic length of life" possible by nonchildren economic agents. Table 1 summarizes the key parameters used in each of our experiments or "runs."

TABLE 2
NUMBER OF PEOPLE UNDER 18 YEARS OF AGE IN A MARRIED HOUSEHOLD

Age Cohorts	Number of Children
20–24	.642
25–29	1.167
30–34	1.451
35–39	1.755
40–44	1.753
45–49	1.439
50–54	.908
55–59	.562
60–64	.231
65–69	.156
70–74	.055
75 plus	.000

SOURCE.—Authors' calculations from the 2001 SCF.

A. *Households*

1. Utility Function Parameters

The coefficient of relative risk aversion, γ, is assumed to be 2.0 under our "main" set of parameter assumptions shown in table 1, although we also consider a value of unity under the "low-γ" set of assumptions. The number of dependent children at the parents' age i, n_i, is calculated using the 2001 Survey of Consumer Finances (SCF) as shown in table 2. The "adult equivalency scale," ζ, is set at 0.6.[6] As discussed later, β is chosen to hit a target capital-output ratio, producing an interest rate of 6.2 percent in the initial steady state.

The annual working hours in the model are the sum of the working hours of a husband and a wife. The maximum working hours, $h_i^{\max}$, are set at 8,760, which equals 12 hours per day per person $\times$ 365 days $\times$ two persons.[7] Using a smaller value for maximum hours would reduce the effective labor supply elasticity, thereby reducing some of the distortions in the prereform tax system (Engen et al. 1997). The parameter α is chosen to be 0.470 in the main version of our model so that the average working hours of households between age 20 and age 64 equal 3,414 hours in the initial steady-state economy, the average number of hours supplied by married households in the 2001 SCF. As shown in table 1, the values of α and β are modified under the different model

[6] Hence, a married couple with two dependent children must consume about 52 percent (i.e., $2^{0.6} = 1.517$) more than a married couple with no children to attain the same level of utility, ceteris paribus.

[7] The ninety-fifth and ninety-ninth percentiles of the working hours per married couple aged 20–64 in the 2001 SCF are 5,280 and 6,375, respectively.

specifications in order to achieve the same average working hours and interest rate in the initial steady state as just discussed.

2. Working Ability

The working ability in this calibration corresponds to the hourly wage (labor income per hour) of each household in the 2001 SCF.[8] The average hourly wage of a married couple (family members #1 and #2 in the SCF) used in the calibration is calculated by

$$\text{hourly wage} = \frac{\text{regular and additional salaries } (\#1 + \#2)}{\max\{\text{working hours } (\#1 + \#2),\ 2{,}080\}}.$$

The max operator in the denominator is used to adjust the hourly wage for a small fraction of households with large reported salaries but few reported working hours, such as executives and the self-employed. In these few cases, they are assigned 2,080 working hours, which is the number of hours that would be worked by a single person in a year under the assumption of a normal 40-hour workweek. The qualitative results herein, however, would remain the same if the max operator were replaced with the simple sum of working hours between both partners (i.e., working hours #1 + #2) and observations with near-zero joint working hours were dropped (see Nishiyama and Smetters 2003).

Table 3 shows the eight discrete levels of working abilities of five-year age cohorts. We use a shape-preserving cubic spline interpolation between each five-year age cohort to obtain the working ability for each age cohort.[9] In the version of our model in which we "turn off" the idiosyncratic wage shocks (run 1), the hourly wages of the representative household are assumed to be the weighted averages of the values shown in table 3.

Table 3 shows only the different potential "wage buckets" by age and the proportion of households in each bucket. It does not show the uncertainty over wages. Using the Panel Study of Income Dynamics (PSID), we construct Markov transition matrices that specify the probabilities that a household's wage will move from one wage to a different wage the next year. Separate transition matrices were constructed for four age ranges—20–29, 30–39, 40–49, and 50–59—in order to capture the possibility that the probabilities themselves might change over the

[8] According to Bureau of Labor Statistics data, the average hourly earnings of production workers have increased by 3.8 percent from 2000 to 2001. Since the 2001 SCF wages correspond to year 2000 whereas our tax function introduced below is calibrated to the year 2001, we multiply the SCF wages shown in table 3 by 1.038 to convert the hourly wages in 2000 into growth-adjusted wages in 2001.

[9] Estimating eight different wage rates for each age would have relied on too few observations.

TABLE 3
Working Abilities of a Household (in U.S. Dollars per Hour)

Age Cohort	e^1	e^2	e^3	e^4	e^5	e^6	e^7	e^8
20–24	3.78	8.12	10.01	12.02	16.96	21.88	28.32	40.35
25–29	5.31	9.85	13.72	16.83	20.70	26.70	34.65	60.17
30–34	6.67	12.07	16.01	21.14	28.40	32.94	41.38	178.40
35–39	5.84	11.96	16.49	21.94	28.85	44.46	78.62	321.10
40–44	7.22	13.58	18.32	24.85	33.63	45.84	93.28	256.38
45–49	5.58	12.76	17.97	24.66	33.83	52.03	94.26	278.18
50–54	4.82	13.64	20.36	27.94	38.43	51.56	88.50	276.52
55–59	2.34	10.31	15.17	23.78	32.90	48.95	95.64	326.66
60–64	.00	3.23	10.67	17.85	27.90	41.96	88.07	545.99
65–69	.00	.00	1.84	10.69	18.59	28.89	53.44	239.25
70–74	.00	.00	.00	1.73	10.68	20.10	36.52	188.00
75–79	.00	.00	.00	1.26	9.86	20.29	26.40	83.88
Percentile	0–20th	20th–40th	40th–60th	60th–80th	80th–90th	90th–95th	95th–99th	99th–100th

Source.—Authors' calculations from the 2001 SCF.

life cycle. (For households aged 60 and older, we used the matrix for ages 50–59.) Appendix A reports the estimation procedure and the matrices in more detail.

3. Population Growth and Mortality

The population growth rate ν is set to 1 percent per year, which is consistent with the average growth rate since 1965. The survival rates ϕ_i at the end of age $i = \{20, \ldots, 109\}$ are the weighted averages of the male and female survival rates calculated by the Social Security Administration (table 4.C6). The survival rates at the end of age 109 are replaced by zero, thereby capping the maximum length of life. See Appendix A for more details.

B. Production

1. Capital and Private Wealth

Capital K is the sum of private fixed assets and government fixed assets. In 2000, private fixed assets were \$21,165 billion, government fixed assets were \$5,743 billion, and the public held about \$3,410 billion of government debt (source: Department of Commerce, Bureau of Economic Analysis [BEA]). Government net wealth, therefore, is set equal to 9.5 percent of total private wealth in the initial steady-state economy in our model. Moreover, the time preference parameter β is chosen in each version of our model so that the capital-GDP ratio in the initial steady-state economy is 2.74, the empirical value in 2000 (source: BEA).

2. Production Technology

Production takes the Cobb-Douglas form, $F(K_t, L_t) = A_t K_t^{\theta} L_t^{1-\theta}$, where L_t is the sum of working hours in efficiency units. The capital share of output is

$$\theta = 1 - \frac{\text{compensation of employees} + (1-\theta) \times \text{proprietors' income}}{\text{national income} + \text{consumption of fixed capital}}.$$

The value of θ in 2000 was 0.30 (source: BEA).[10] The annual per capita growth rate μ is assumed to be 1.8 percent, the average rate between 1869 and 1996 (Barro 1997). Total factor productivity A is set at 0.949, which normalizes the wage (per efficient labor unit) to unity.

3. The Depreciation Rate of Fixed Capital

The depreciation rate of fixed capital δ is chosen by the following steady-state condition:

$$\delta = \frac{\text{total gross investment}}{\text{fixed capital}} - \mu - \nu.$$

In 2000, private gross fixed investment accounted for 17.2 percent of GDP, and government (federal and state) gross investment accounted for 3.3 percent of GDP (source: BEA). With a capital-output ratio of 2.74, the ratio of gross investment to fixed capital is 7.5 percent. When productivity and population growth rates are subtracted, the annual depreciation rate is 4.7 percent.

C. *Prereform Income Taxes*

Federal income tax and state and local taxes are assumed to be at the level in year 2001 before the passage of the Economic Growth and Tax Relief Reconciliation Act of 2001 (EGTRRA). Since households in our model are assumed to be married, we use a standard deduction of \$7,600. However, following Altig et al. (2001), we allow higher-income households to itemize deductions when it is more valuable to do so, and we assume that the value of the itemized deduction increases linearly in the adjusted gross income (AGI).[11] The additional exemption per dependent person is \$2,900 where the number of dependent chil-

[10] The average of θ in years between 1996 and 2000 is 0.31.

[11] In particular, the deduction taken by a household is the greater of the standard deduction and $0.0755 \times$ AGI, or $\max\{\$7,600, 0.0755 \times \text{AGI}\}$.

TABLE 4
MARGINAL INDIVIDUAL INCOME TAX RATES IN 2001 (Married Household, Filed Jointly)

Taxable Income	Marginal Income Tax Rate (%)
$0–$45,200	15.0
$45,200–$109,250	28.0
$109,250–$166,500	31.0
$166,500–$297,350	36.0
$297,350–	39.6

dren is consistent with table 2. Table 4 shows the statutory marginal tax rates before EGTRRA.[12]

The standard deduction, the personal exemption, and all tax brackets grow with productivity over time so that there is no real bracket creep; this indexing is also needed for the initial economy to be in steady state. Many tax base reductions (e.g., economic depreciation allowances, the near nontaxation of housing, etc.), though, do not appear in the deductions taken by households. Because a household's economic income exceeds taxable income, effective tax rates are lower than the statutory rates. In 2000, the ratio of total individual income tax revenue (not including Social Security and Medicare taxes) to nominal GDP was 0.102 and the ratio of corporate income tax to GDP was 0.021. Each statutory federal income tax rate shown in table 4, therefore, is multiplied by 0.76 so that income tax revenue (including corporate income tax) totals 12.3 percent of GDP in the initial steady state under our "main" calibration.[13] Also, since the effective tax rate on capital income is reduced by investment tax incentives, accelerated depreciation, and other factors (Auerbach 1996), the tax function is further adjusted so that the cross-sectional average tax rate on capital income is about 25 percent lower than the average tax on labor income.[14] State and local income taxes are modeled parsimoniously with a 4.0 percent flat tax on income above the deduction and exemption levels used at the federal level.

V. Policy Experiments

This section reports simulation evidence from replacing the current progressive income tax with a flat consumption tax. Seven different runs

[12] The key qualitative results reported herein would be unaffected if the tax function were instead modeled as net taxes, i.e., after transfers indicated in the Statistics of Income were subtracted.

[13] This adjustment factor is 1.10 for run 1, 0.737 for run 6, and 0.796 for run 7.

[14] This relative reduction to the tax rate on capital is commonly used by the Congressional Budget Office, and it balances the legal tax preferences given to capital vs. the legal tax benefits given to labor, including tax-preferred fringe benefits.

TABLE 5
Percentage Change in Selected Macro Variables Relative to Baseline

Run and Year	Without LSRA: GNP	Without LSRA: National Wealth	Without LSRA: Labor Supply	With LSRA: GNP	With LSRA: National Wealth	With LSRA: Labor Supply
1 (no wage shocks, closed economy):						
2001	9.8	.0	14.4	10.6	.0	15.5
2010	14.1	20.3	11.5	15.1	20.8	12.8
2150	16.6	29.4	11.5	18.3	35.9	11.4
2 (wage shocks, closed economy):						
2001	5.4	.0	7.7	4.2	.0	6.0
2010	9.9	21.3	5.4	6.1	9.7	4.6
2150	12.9	33.1	5.2	8.0	15.0	5.1
	Run 2 with Small Open Economy Plus Any Additional Modification Shown					
3:						
2001	6.9	.0	9.9	5.8	.0	8.4
2010	11.3	26.6	4.8	8.0	15.8	4.7
2150	16.0	47.4	2.5	11.1	28.0	3.9
4 ($20,000 deduction):						
2001	2.8	.0	4.0	2.3	.0	3.3
2010	6.8	20.6	.9	4.3	11.9	1.0
2150	11.2	38.4	−.4	6.9	21.5	.7
5 ($40,000 deduction):						
2001	−7.9	.0	−11.3	−6.1	.0	−8.7
2010	−4.4	3.6	−7.9	−6.3	−7.1	−5.9
2150	.2	14.9	−6.1	−6.3	−10.7	−4.5
6 ($\gamma = 1.0$):						
2001	8.2	.0	11.8	8.0	.0	11.5
2010	14.2	35.5	5.1	12.2	28.5	5.2
2150	19.8	61.0	2.2	16.8	49.2	2.9
7 (1/2 transitory wage shocks):						
2001	8.5	.0	12.2	7.9	.0	11.2
2010	12.6	28.0	6.0	9.9	19.3	5.8
2150	17.4	50.1	3.4	13.1	32.8	4.6

are presented that vary in the assumed economic environment (e.g., a closed vs. an open economy) as well as the nature of the reform (e.g., incorporating a deduction into the consumption tax). We first discuss the impact that each experiment has on aggregate macroeconomic variables before turning to the impact on household welfare (without the LSRA) and efficiency (with the LRSA).

A. *Impact on Macroeconomic Variables*

Run 1 in table 5 considers an economy with a representative household facing deterministic (equivalently, fully insurable) wages over the life cycle, with the wage-age profile set equal to the weighted averages of the values shown in table 3. Notice that national income, labor supply, and wealth increase over the entire transition path. These results are broadly consistent with the simulations using deterministic models reported by Auerbach and Kotlikoff (1987) and Altig et al. (2001), although our calibration is, if anything, more favorable to the adoption of a consumption tax relative to those papers (see the discussion in Nishiyama and Smetters [2003]). Notice that the long-run gains to output and wealth in run 1 are larger *with* the LRSA than without. As discussed below, run 1 produces large welfare gains to many younger households alive at the time of the reform. As the LSRA returns these households' expected remaining lifetime utilities to their prereform level, these additional resources are netted by the LSRA and distributed to future households. The LSRA, therefore, carries positive wealth that adds to national wealth.

Run 2 in table 5 then considers the impact of including uninsurable wage shocks. National wealth, labor, and output again increase sharply throughout the transition path. Although household saving is less interest elastic in the presence of precautionary savings, notice that national wealth actually increases by even more than in run 1 when the LSRA is turned off. This strong gain in long-run capital stock reflects an increase in precautionary saving in response to a lower level of risk sharing provided by the new tax system. In contrast to run 1, though, the gains are smaller with the LSRA than without; as shown below, tax reform now leads to resource losses to households alive at the time of the reform. The LSRA must, therefore, carry debt that, in turn, decreases national wealth.

Runs 3–7 then attempt to "stress-test" the results from run 2 by changing the economic setting or reform in ways that are even more favorable to the adoption of a consumption tax. In each of these runs, the small-open-economy case is used because the capital stock is relatively more responsive to the elimination of the tax on interest income since interest rates do not decline as the capital intensity increases. This assumption, therefore, gives tax reform the best chance of success. Run 4 also allows the consumption tax to be somewhat progressive by not taxing the first $20,000 of consumption; run 5 does not tax the first $40,000 of consumption. Run 6 decreases the risk aversion to unity, which also increases the intertemporal substitution elasticity to unity. Finally, run 7 considers the effect of reducing the transitory shocks in our model by one-half of their baseline values in order to deal with potential measurement error in the PSID, as identified by Floden and Lindé (2001) and others. (The modifications to the Markov transition matrices are discussed in App. A.) Except in the case of a larger $40,000 deduction after the consumption tax is introduced (run 5), the macro gains are fairly strong and positive over the transition path. The relatively mediocre performance of run 5 reflects the high tax rates that are required to afford the large deduction. The long-run gains are especially large in run 7 as the economy gets closer to a *small-open-economy* version of run 1. Although it is not shown to conserve on space, repeating run 1 under the assumption of a small open economy would produce even larger gains.

B. *Welfare and Efficiency Calculations*

Table 6 shows the welfare change faced by a household in select birth cohorts and income classes at the time of the reform in 2001 without the LSRA. For households alive at the time of the reform, these calculations are conditional on their current state; for future households, the calculations are conditional on the state into which they are born. Since showing the welfare changes conditional on all three state variables (age, income class, and wealth) would be cumbersome, table 6 shows the population-weighted average of welfare change across all households in just two states—age and income class. All households over age 80 in year 1, though, are in the lowest productivity group, e^1, and so their values are omitted to save space since they correspond closely to those shown for agents aged 79 in the lowest productivity group in year 1. All the values in table 6 are expressed in units of $1,000 and are adjusted (reduced) by exogenous technological change over time.[15]

Table 6 also shows the gains to each household under the LSRA. By construction, agents with ages 20 and older represent current economic households that are held harmless by the LSRA after the reform; the gains to them, therefore, are always zero. Similarly, agents with ages below 20 represent future economic households and therefore receive any surplus (or deficit) lump-sum transfer, Δtr, at age 20 (when they become separate economic agents). Each future household also receives the same Δtr regardless of its productivity at age 20, and so only one column is needed under the LSRA.

Let us first consider run 1. Recall that in this run, a representative household faces a deterministic wage-age profile equal to the weighted averages of the values shown in table 3, and so only one productivity group is shown in table 6. Not surprisingly, without the LRSA, older households are harmed by adopting a consumption tax since they now pay taxes on their accumulated assets when they consume. For example,

[15] In other words, a household that turns 20 in year $t \geq 2001$ actually gains $s \times (1 + \mu)^{t-2001}$, where s is the amount shown in the table and μ is the per capita growth rate in the baseline economy. In contrast to the Auerbach and Kotlikoff model, our reported values cannot be expressed as a share of remaining full lifetime income since that value is generally stochastic in our model.

TABLE 6

CHANGE IN WELFARE PER HOUSEHOLD ($1,000 in 2001)

Run and Age in 2001	Without LSRA:* Select Productivities e^1	e^3	e^5	e^8	With LSRA:† For all Productivities
1 (no wage shocks, closed economy):					
79	−39.4				.0
60	−45.8	. . .	. . .	. . .	.0
40	92.0	. . .	. . .	. . .	.0
20	73.5	. . .	. . .	. . .	.0
0	96.2	. . .	. . .	. . .	154.0
−20	96.8	. . .	. . .	. . .	154.0
2 (wage shocks, closed economy):					
79	−47.2	−50.6	−64.6	−117.0	.0
60	−79.5	−104.4	−139.1	197.2	.0
40	−48.7	−58.3	−75.9	−41.3	.0
20	−12.6	−22.7	−28.4	−22.0	.0
0	12.0	5.4	3.1	12.8	−85.9
−20	13.9	7.6	5.5	15.7	−85.9
3 (wage shocks, open economy):					
79	−42.6	−46.0	−59.9	−107.9	.0
60	−56.8	−72.8	−82.3	663.5	.0
40	−30.0	−24.6	.8	458.9	.0
20	−41.6	−45.1	−38.3	1.4	.0
0	−35.1	−36.9	−28.4	14.2	−90.2
−20	−34.0	−35.6	−26.7	16.4	−90.2
4 (wage shocks, open economy, $20,000 deduction):					
79	−32.6	−36.7	−53.0	−126.6	.0
60	−39.2	−60.8	−104.1	181.0	.0
40	−4.6	−11.7	−19.6	180.2	.0
20	−8.3	−16.3	−16.9	7.2	.0
0	−3.0	−8.8	−6.7	22.2	−52.3
−20	−2.0	−7.5	−5.0	24.7	−52.3
5 (wage shocks, open economy, $40,000 deduction):					
79	−34.5	−39.8	−60.1	−176.9	.0
60	−40.2	−69.1	−175.2	−959.4	.0
40	1.2	−20.1	−97.3	−490.0	.0
20	13.2	7.7	1.4	6.8	.0
0	18.0	14.6	11.4	21.6	−54.7
−20	19.4	15.9	13.4	24.7	−54.7
6 (wage shocks, open economy, γ = 1.0):					
79	−42.2	−45.9	−61.6	−118.8	.0
60	−60.3	−76.7	−85.7	788.9	.0
40	−15.4	.1	38.8	582.8	.0
20	−31.9	−31.7	−20.3	27.7	.0
0	−21.6	−19.8	−6.4	44.9	−47.9
−20	−20.2	−18.1	−4.4	47.3	−47.9
7 (1/2 wage shocks, open economy):					
79	−31.0	−36.4	−57.8	−153.4	.0
60	−41.5	−65.2	−72.8	1,044.8	.0
40	−30.8	−23.9	18.0	723.8	.0
20	−43.6	−46.7	−32.4	37.9	.0
0	−38.3	−39.3	−22.8	51.4	−76.1
−20	−37.3	−37.9	−21.0	53.9	−76.1

* Standard equivalent variations measures.
† Value of Δtr.

a person aged 79 at the time of the reform would be willing to pay \$39,400 in order to *avoid* the reform's adoption. Younger households, however, gain from the reform. For example, the reform is worth about \$92,000 to 40-year-olds alive at the beginning of the reform. These large gains are consistent with the efficiency gains reported in previous models with deterministic wages. With the LSRA, a considerable amount of revenue—about \$154,000 for each future household—is left over after all would-be losing households are fully compensated.

Runs 2–7 allow for uninsurable stochastic wages and paint a very different picture. Each run produces considerable efficiency *losses*, that is, losses to future households after all would-be losing households have been compensated by the LSRA. Run 2 produces a loss of \$85,900 to each future household, largely driven by the loss in risk sharing provided by the consumption tax relative to the previous progressive income tax system. The effective savings elasticity is also smaller in the presence of precautionary savings.

Interestingly, run 3 produces a slightly *larger* efficiency loss of \$90,200, even though output and wealth increase more in the small-open-economy model in run 3 relative to the closed-economy model in run 2 (table 5). Intuitively, the LSRA can borrow from the capital market at a relatively lower interest rate in run 2 as the capital intensity increases along the transition path; in contrast, the interest rate is fixed in run 3. This difference in efficiency gains emphasizes that efficiency outcomes are not necessarily proportional to gains in macroeconomic variables.

In run 4, some progressivity is introduced by levying a flat consumption tax only on consumption above \$20,000, a deduction that is more generous (after adjusting for real growth) than the amount recommended in the 1995 "flat tax" plan by Hall and Rabushka (1995). The efficiency loss now drops from \$90,200 to \$52,300 per future household. This deduction generates some risk sharing but is still not as powerful as the insurance provided by the progressive marginal tax rate schedule found in the prereform tax system. Unlike the deduction under the flat tax, progressive taxation provides risk sharing with changes in income at higher levels. In the presence of precautionary savings, consumption is also less volatile than wage shocks.

Since the deduction in run 4 was successful in reducing some of the efficiency losses, run 5 doubles the deduction to \$40,000. Tax reform now produces about \$54,700 in efficiency losses, or about \$2,400 more in losses than with a smaller deduction. The reason is that the larger deduction now requires an even larger consumption tax rate, which reduces labor supply (table 5). In general, the efficiency gains are nonmonotonic in the level of the deduction. A small deduction improves risk sharing. But, at a large deduction, the labor supply distortions, which increase more than linearly in the tax rate, begin to take over. We found that a deduction around $30,000 (not shown) performed the best, producing an efficiency loss of about $39,000.

In run 6, the level of risk aversion is reduced from a value of two to unity, which also increases the intertemporal substitution elasticity from one-half to unity. The efficiency loss is reduced from $90,200 in run 3 to $47,900 per future household. A smaller level of risk aversion reduces the value of risk sharing provided by the prereform progressive tax system. The larger intertemporal substitution elasticity also increases the costs of the distortions in the prereform tax system. Still, the insurance provided by the progressive tax is sufficiently valuable and outweighs its costs.

Finally, run 7 reduces the transitory shocks by one-half, which reduces the efficiency losses to $76,100 per household. The reduction in efficiency losses is not larger because a decline in transitory shocks also implies that any given shock will be more permanent over time. As a result, the insurance provided by the former progressive income tax system is still quite valuable. Only in the limit, as the shocks approach zero, will the tax system have no insurance value.

VI. Concluding Remarks

Tax reform was analyzed within an OLG model with uninsurable wage shocks and longevity risks using a calibration that was otherwise quite favorable to the adoption of a flat consumption tax. Without wage shocks (or, equivalently, with fully insurable shocks), adopting a flat consumption tax produces a positive amount of extra resources—that is, after all the would-be losers are fully compensated—equal to $154,000 per each future household (corresponding to agents below age 20 at the time of the reform as well as the unborn). This large gain is consistent with previous studies using models with deterministic wages. However, more realistically, with uninsurable wage shocks, adopting a flat consumption tax produces an efficiency loss of $85,900 to each future household, even though the capital stock and output increase over the entire transition path. This result is qualitatively robust to variations in the model and calibration that are even more favorable to the adoption of a flat consumption tax.

These results point to the importance of incorporating the risk-sharing aspects of the prereform progressive income tax system into the analysis of tax reform. While adopting a flat consumption tax would eliminate numerous distortions contained within the current progressive income tax system, it would also reduce the amount of risk sharing provided by the tax system when wages are stochastic.

Appendix A

Transition Matrices and Survival Rates

A. *Markov Transition Matrices*

The Markov transition matrices of working ability are constructed for four age groups—20–29, 30–39, 40–49, and 50–59—from the hourly wages in the PSID individual data for 1990, 1991, 1992, and 1993. The transition matrix of each age group is the average of three transition matrices, for 1989–90, 1990–91, and 1991–92 (for households aged 60 or older, we used the matrix for ages 50–59):

$$\Gamma_{i\in\{20,\ldots,29\}} = \begin{pmatrix} .5964 & .2499 & .0875 & .0464 & .0118 & .0048 & .0029 & .0003 \\ .2093 & .4594 & .2322 & .0756 & .0104 & .0088 & .0042 & .0001 \\ .1044 & .1902 & .4084 & .2385 & .0342 & .0153 & .0048 & .0042 \\ .0642 & .0831 & .2016 & .4576 & .1314 & .0380 & .0241 & .0000 \\ .0313 & .0202 & .0784 & .2947 & .4285 & .0882 & .0408 & .0179 \\ .0246 & .0005 & .0898 & .1084 & .2462 & .3216 & .1862 & .0227 \\ .0108 & .0248 & .0432 & .0373 & .1163 & .2858 & .3923 & .0895 \\ .0376 & .0440 & .0000 & .0012 & .2615 & .0291 & .3714 & .2552 \end{pmatrix},$$

$$\Gamma_{i\in\{30,\ldots,39\}} = \begin{pmatrix} .6936 & .2078 & .0546 & .0330 & .0031 & .0018 & .0061 & .0000 \\ .1972 & .5587 & .2001 & .0341 & .0077 & .0006 & .0000 & .0016 \\ .0620 & .1796 & .5233 & .2018 & .0154 & .0110 & .0069 & .0000 \\ .0214 & .0413 & .2024 & .5411 & .1526 & .0281 & .0116 & .0015 \\ .0272 & .0068 & .0348 & .3065 & .4581 & .1182 & .0484 & .0000 \\ .0163 & .0309 & .0084 & .0907 & .2946 & .3798 & .1512 & .0281 \\ .0404 & .0000 & .0007 & .0621 & .0830 & .2624 & .4869 & .0645 \\ .0000 & .0302 & .0000 & .0334 & .0379 & .0384 & .3209 & .5392 \end{pmatrix},$$

$$\Gamma_{i\in\{40,\ldots,49\}} = \begin{pmatrix} .7111 & .2340 & .0352 & .0110 & .0070 & .0017 & .0000 & .0000 \\ .1847 & .5571 & .2078 & .0261 & .0142 & .0052 & .0020 & .0029 \\ .0579 & .1520 & .5429 & .1996 & .0339 & .0117 & .0020 & .0000 \\ .0214 & .0430 & .1833 & .5587 & .1576 & .0311 & .0027 & .0022 \\ .0191 & .0145 & .0217 & .3155 & .4644 & .1055 & .0593 & .0000 \\ .0416 & .0089 & .0512 & .1385 & .1427 & .3653 & .2094 & .0424 \\ .0247 & .0086 & .0354 & .0493 & .0777 & .2486 & .4942 & .0615 \\ .0000 & .0543 & .0000 & .0475 & .0786 & .1300 & .2502 & .4394 \end{pmatrix},$$

$$\Gamma_{i\in\{50,\ldots,78\}} = \begin{pmatrix} .7000 & .2164 & .0514 & .0121 & .0110 & .0015 & .0076 & .0000 \\ .2215 & .5452 & .2117 & .0189 & .0027 & .0000 & .0000 & .0000 \\ .0439 & .1743 & .5353 & .2043 & .0197 & .0123 & .0102 & .0000 \\ .0170 & .0525 & .1651 & .6075 & .1220 & .0190 & .0169 & .0000 \\ .0276 & .0085 & .0352 & .2608 & .4774 & .1690 & .0215 & .0000 \\ .0002 & .0127 & .0429 & .0898 & .2605 & .3345 & .2444 & .0150 \\ .0189 & .0210 & .0409 & .0213 & .2008 & .2026 & .4079 & .0866 \\ .0000 & .0000 & .0000 & .0000 & .0127 & .1701 & .2386 & .5786 \end{pmatrix},$$

where $\Gamma_i(j, k) = \pi(e_{i+1} = e^k_{i+1} | e_i = e^j_i)$. For run 7 (the one-half wage shock case)

TABLE A1
Survival Rates in 1998 in the United States (Weighted Average of Men and Women)

Age	Survival Rate	Age	Survival Rate	Age	Survival Rate
20	.999104	50	.995603	80	.937788
21	.999057	51	.995222	81	.931527
22	.999027	52	.994797	82	.924684
23	.999018	53	.994324	83	.917252
24	.999023	54	.993795	84	.909150
25	.999034	55	.993198	85	.900275
26	.999040	56	.992534	86	.890541
27	.999033	57	.991818	87	.879882
28	.999006	58	.991051	88	.868264
29	.998962	59	.990216	89	.855676
30	.998911	60	.989315	90	.842119
31	.998857	61	.988305	91	.827606
32	.998796	62	.987134	92	.812154
33	.998727	63	.985773	93	.795784
34	.998651	64	.984249	94	.778522
35	.998564	65	.982548	95	.761075
36	.998466	66	.980759	96	.743640
37	.998358	67	.979000	97	.726432
38	.998240	68	.977325	98	.709688
39	.998111	69	.975647	99	.693653
40	.997966	70	.973769	100	.676630
41	.997807	71	.971613	101	.658554
42	.997640	72	.969264	102	.639355
43	.997451	73	.966703	103	.618962
44	.997252	74	.963868	104	.597297
45	.997027	75	.960661	105	.574281
46	.996778	76	.957027	106	.549828
47	.996514	77	.952967	107	.523850
48	.996237	78	.948449	108	.496251
49	.995938	79	.943423	109	.000000

Source.—Authors' calculations from the Social Security Administration (2001, table 4.C6).

in Section V, the transition matrices are modified to $\Gamma_{1/2,i} = (\Gamma_i + \mathbf{I}_8)/2$, where $\mathbf{I}_8$ is the 8×8 identity matrix.

B. Survival Rates of Households

The survival rates ϕ_i at the end of age $i = \{20, \ldots, 109\}$, as shown in table A1, are the weighted averages of men and women calculated from the period life table (table 4.C6) in Social Security Administration (2001). The survival rates at the end of age 109 are replaced by zero.

Appendix B

The Computation of Equilibria

The state of a household is $\boldsymbol{s}_i = (i, e_i, a_i) \in I \times E \times A$, where $I = \{20, \ldots, 109\}$, $E = [e^{\min}, e^{\max}]$, and $A = [a^{\min}, a^{\max}]$. To compute an equilibrium, we discretize the state space of a household as $\hat{s}_i \in I \times \hat{E}_i \times \hat{A}$, where $\hat{E}_i = \{e_i^1, e_i^2, \dots, e_i^{N_e}\}$ and $\hat{A} = \{a^1, a^2, \dots, a^{N_a}\}$.[16] Then, the state of the economy is $\hat{S}_t = (x_t(\hat{s}_i), W_{LS,t}, W_{G,t})$. For all these discrete points, we compute the optimal decision of households

$$d(\hat{s}_i, \hat{S}_t; \Psi_t) = (c_i(\cdot), h_i(\cdot), a_{i+1}(\cdot)) \in (0, c^{\max}] \times [0, h_i^{\max}] \times A,$$

the marginal value with respect to wealth $\partial v(\hat{s}_i, \hat{S}_t; \Psi_t)/\partial a$, and the values $v(\hat{s}_i, \hat{S}_t; \Psi_t)$, given the expected factor prices and policy variables. We use the marginal values (with log linear interpolation) in the Euler equation to obtain optimal savings and the values to calculate welfare changes measured by equivalent variations in wealth.

A. *A Steady-State Equilibrium*

The algorithm to compute a steady-state equilibrium is as follows. Let Ψ denote the time-invariant government policy rules $\Psi = (W_{LS}, W_G, C_G, \tau_I(\cdot), \tau_C, tr_{LS}(\hat{s}_i))$.

1. Set the initial values of factor prices (r^0, w^0) and the policy variables (W_{LS}^0, τ_C^0).[17]
2. Given $\Omega^0 = (r^0, w^0, W_{LS}^0, \tau_C^0)$, find the decision rule of a household $d(\hat{s}_i; \Psi, \Omega^0)$ for all $\hat{s}_i \in I \times \hat{E}_i \times \hat{A}$.
   a. For age $i = 109$, find the decision rule $d(\hat{s}_{109}; \cdot)$. Since the survival rate $\phi_{109} = 0$, the end-of-period wealth $a_{i+1}(\hat{s}_{109}; \cdot) = 0$ for all $\hat{s}_{109}$. Compute consumption and working hours $(c_i(\hat{s}_{109}; \cdot), h_i(\hat{s}_{109}; \cdot))$, the marginal value $\partial v(\hat{s}_{109}; \cdot)/\partial a$, and the value $v(\hat{s}_{109}; \cdot)$ for all $\hat{s}_{109}$.
   b. For age $i = 108, \dots, 20$, find the decision rule $d(\hat{s}_i; \cdot)$, the marginal value $\partial v(\hat{s}_i; \cdot)/\partial a$, and the value $v(\hat{s}_i; \cdot)$ for all $\hat{s}_i$, using $\partial v(\hat{s}_{i+1}; \cdot)/\partial a$ and $v(\hat{s}_{i+1}; \cdot)$ recursively. (i) Set the initial guess of $a_{i+1}^0(\hat{s}_i; \cdot)$. (ii) Given $a_{i+1}^0(\hat{s}_i; \cdot)$, compute $(c_i(\hat{s}_i; \cdot), h_i(\hat{s}_i; \cdot))$. Plug these into the Euler equation with $\partial v(\hat{s}_i; \cdot)/\partial a$. (iii) If the Euler error is sufficiently small, then stop.[18] Otherwise, update $a_{i+1}^0(\hat{s}_i; \cdot)$ and return to step ii.
3. Find the steady-state measure of households $x(\hat{s}_i; \Psi, \Omega^0)$ using the decision rule obtained in step 2. This computation is done forward from age 20 to age 109.
4. Compute new factor prices (r^1, w^1) and the policy variables (W_{LS}^1, τ_C^1) using the steady-state measure of households obtained in step 3.
5. Compare $\Omega^1 = (r^1, w^1, W_{LS}^1, \tau_C^1)$ with Ω^0. If the difference is sufficiently small, then stop.[19] Otherwise update Ω^0 and return to step 2.

[16] In this paper, $N_e = 8$ and $N_a = 100$. Since there are 90 different ages, the total number of discrete states is 72,000. The grid points are not equally spaced.

[17] If we find the capital-labor ratio, both r and w are calculated from the given production function and depreciation rate.

[18] The program uses a bisection search. The iteration for a_{i+1} stops when the Euler error in terms of $\partial u(c, h)/\partial c < 10^{-12}$ or $|a_{i+1}^1 - a_{i+1}^0| < \$5 \times 10^{-3}$.

[19] The computation is stopped if

$$\max\left\{\left|\frac{\Delta(K/L)}{K/L}\right|, \left|\frac{\Delta W_{LS}}{W_{LS}}\right|, \left|\frac{\Delta(1+\tau_C)}{1+\tau_C}\right|\right\} < 10^{-3}.$$

B. An Equilibrium Transition Path

Assume that the economy is in the initial steady state in period 0 and that the new policy schedule Ψ_1, which was not expected in period 0, is announced at the beginning of period 1, where

$$\Psi_1 = \{W_{LS,t+1},\ W_{G,t+1},\ C_{G,t},\ \tau_{I,t}(\cdot),\ \tau_{C,t},\ tr_{LS,t}(\hat{\mathbf{s}}_i)\}_{t=1}^{\infty}.$$

Let $\hat{\mathbf{S}}_1 = (x_1(\hat{\mathbf{s}}_i),\ W_{LS,1},\ W_{G,1})$ be the aggregate state of the economy at the beginning of period 1. The state of the economy $\hat{\mathbf{S}}_1$ is usually equal to that of the initial steady state. The algorithm to compute a transition path to a new steady-state equilibrium is as follows.

1. Choose a sufficiently large number, T, such that the economy is said to reach the new steady state within T periods.[20] Set the initial guess, $\Omega_1^0 = \{r_s^0,\ w_s^0,\ W_{LS,s}^0,\ \tau_{C,s}^0\}_{s=1}^{T}$, on factor prices and the policy variables for $t = 1, 2, \ldots, T$. Because there are no aggregate productivity shocks in this model, a time series $\Omega_t = \{r_s,\ w_s,\ W_{LS,s},\ \tau_{C,s}\}_{s=t}^{T}$ is deterministic and each household perfectly foresees Ω_t on the basis of the information $\hat{\mathbf{S}}_t$ in an equilibrium. Since $\hat{\mathbf{S}}_t$ is in a household decision rule only to make the household expect Ω_t rationally, in the computation below, we can avoid the "curse of dimensionality" by rewriting $d(\hat{\mathbf{s}}_i,\ \hat{\mathbf{S}}_t;\ \Psi_t)$ as $d(\hat{\mathbf{s}}_i;\ \Psi_t,\ \Omega_t)$.
2. Given $W_{LS,T}^0$, find the final steady-state decision rule $d(\hat{\mathbf{s}}_i;\ \Psi_T,\ \Omega_T^0)$, the marginal value $\partial v(\hat{\mathbf{s}}_i;\ \cdot)/\partial a$, the value $v(\hat{\mathbf{s}}_i;\ \cdot)$, and the measure of households $x_T(\hat{\mathbf{s}}_i)$ for all $\hat{\mathbf{s}}_i \in I \times \hat{E}_i \times \hat{A}$, and update $\Omega_T^0 = (r_T^0,\ w_T^0,\ W_{LS,T}^0,\ \tau_{C,T}^0)$. (See the algorithm for a steady-state equilibrium.)
3. For period $t = T-1,\ T-2,\ \ldots,\ 1$, on the basis of the guess Ω_t^0, find backward the decision rule $d(\hat{\mathbf{s}}_i;\ \Psi_t,\ \Omega_t^0)$, the marginal value $\partial v(\hat{\mathbf{s}}_i;\ \cdot)/\partial a$, and the value $v(\hat{\mathbf{s}}_i;\ \cdot)$ for all $\hat{\mathbf{s}}_i$, using $\partial v(\hat{\mathbf{s}}_{i+1};\ \Psi_{t+1},\ \Omega_{t+1}^0)/\partial a$ and $v(\hat{\mathbf{s}}_{i+1};\ \cdot)$ recursively.
4. For period $t = 1,\ 2,\ \ldots,\ T-1$, compute forward $(r_t^1,\ w_t^1,\ W_{LS,t+1}^1,\ \tau_{C,t}^1)$ and the measure of households $x_{t+1}(\hat{\mathbf{s}}_i)$, using the decision rule $d(\hat{\mathbf{s}}_i;\ \Psi_t;\ \Omega_t^0)$ obtained in step 3 and the state of economy $\hat{\mathbf{S}}_t = (x_t(\hat{\mathbf{s}}_i),\ W_{LS,t},\ W_{G,t})$ recursively.
5. Compare $\Omega_1^1 = \{r_s^1,\ w_s^1,\ W_{LS,s}^1,\ \tau_{C,s}^1\}_{s=1}^{T}$ with Ω_1^0. If the difference is sufficiently small, then stop. Otherwise, update Ω_1^0 and return to step 2.

C. The Lump-Sum Redistribution Authority

When the LSRA is assumed, the following computation is added to the iteration process. Set the initial value of an additional lump-sum transfer $\Delta tr = 0$.

1. For period $t = T,\ T-1,\ \ldots,\ 2$, compute the lump-sum transfers to newborn households $tr_{CV}(\hat{\mathbf{s}}_{20};\ \Psi_t,\ \Omega_t^0)$ to make those households as well off as under the prereform economy.

   a. Set the initial value of lump-sum transfers $tr_{CV}(\hat{\mathbf{s}}_{20};\ \Psi_t,\ \Omega_t^0)$ to newborn households.

   b. Given $tr_{CV}(\hat{\mathbf{s}}_{20};\ \Psi_t,\ \Omega_t^0)$, find the decision rule of newborn households $d(\hat{\mathbf{s}}_{20};\ \cdot)$ and values $v(\hat{\mathbf{s}}_{20};\ \cdot)$.

[20] As in Auerbach and Kotlikoff (1987), we found setting T at 150 to be sufficient.

c. Find the compensating variation in wealth $\Delta tr_{CV}(\hat{s}_{20};\ \Psi_t,\ \Omega_t^0)$ to make those households indifferent from the baseline economy. (The initial wealth of newborn households is assumed to be zero since they do not receive any bequests.) If the absolute value of $\Delta tr_{CV}(\hat{s}_{20};\ \cdot)$ is sufficiently small, then go to step *d*. Otherwise update $tr_{CV}(\hat{s}_{20};\ \cdot)$ by adding $\Delta tr_{CV}(\hat{s}_{20};\ \cdot)$ and return to step *b*.

d. Set the lump-sum transfers $tr_{LS,t}(\hat{s}_{20}) = tr_{CV}(\hat{s}_{20};\ \Psi_t,\ \Omega_t^0) + \Delta tr$ and find the decision rule of newborn households $d(\hat{s}_{20};\ \cdot)$.

2. For period $t = 1$, compute the lump-sum transfers to all current households $tr_{CV}(\hat{s}_i;\ \Psi_1,\ \Omega_1^0)$ to make those households as well off as in the prereform economy. The procedure is similar to step 1. Set the lump-sum transfers $tr_{LS,1}(\hat{s}_i) = tr_{CV}(\hat{s}_i;\ \cdot)$.
3. Compute an additional lump-sum transfer Δtr to newborn households so that the net present value of all transfers becomes zero. Compute the LSRA wealth, $\{W_{LS,s}^1\}_{s=1}^T$, which will be used to calculate national wealth. Recompute Δtr and $\{W_{LS,s}^1\}_{s=1}^T$ using new interest rates $\{r_s\}_{s=1}^T$.

References

Altig, David, Alan J. Auerbach, Laurence J. Kotlikoff, Kent A. Smetters, and Jan Walliser. 2001. "Simulating Fundamental Tax Reform in the United States." *A.E.R.* 91 (June): 574–95.

Auerbach, Alan J. 1996. "Tax Reform, Capital Allocation, Efficiency, and Growth." In *Economic Effects of Fundamental Tax Reform*, edited by Henry J. Aaron and William G. Gale. Washington, DC: Brookings Inst. Press.

Auerbach, Alan J., and James R. Hines Jr. 2001. "Taxation and Economic Efficiency." Working Paper no. 8181 (March), NBER, Cambridge, MA.

Auerbach, Alan J., and Laurence J. Kotlikoff. 1987. *Dynamic Fiscal Policy*. Cambridge: Cambridge Univ. Press.

Barro, Robert J. 1997. *Macroeconomics*. 5th ed. Cambridge, MA: MIT Press.

Bradford, David F. 1986. *Untangling the Income Tax*. Cambridge, MA: Harvard Univ. Press.

Chamley, Christophe. 1986. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives." *Econometrica* 54 (May): 607–22.

Eaton, Jonathan, and Harvey S. Rosen. 1980*a*. "Labor Supply, Uncertainty, and Efficient Taxation." *J. Public Econ.* 14 (December): 365–74.

———. 1980*b*. "Optimal Redistributive Taxation and Uncertainty." *Q.J.E.* 95 (September): 357–64.

———. 1980*c*. "Taxation, Human Capital, and Uncertainty." *A.E.R.* 70 (September): 705–15.

Engen, Eric M., and William G. Gale. 1996. "The Effects of Fundamental Tax Reform on Savings." In *Economic Effects of Fundamental Tax Reform*, edited by Henry J. Aaron and William G. Gale. Washington, DC: Brookings Inst. Press.

Engen, Eric M., Jane Gravelle, and Kent A. Smetters. 1997. "Dynamic Tax Models: Why They Do the Things They Do." *Nat. Tax J.* 50 (September): 657–82.

Floden, Martin, and Jesper Lindé. 2001. "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?" *Rev. Econ. Dynamics* 4 (April): 406–37.

Fullerton, Don, and Diane Lim Rogers. 1993. *Who Bears the Lifetime Tax Burden?* Washington, DC: Brookings Inst. Press.

Hall, Robert E., and Alvin Rabushka. 1995. *The Flat Tax.* 2nd ed. Stanford, CA: Hoover Inst. Press.

Hamilton, Jonathan H. 1987. "Optimal Wage and Income Taxation with Wage Uncertainty." *Internat. Econ. Rev.* 28 (June): 373–88.

Hubbard, R. Glenn, and Kenneth L. Judd. 1986. "Liquidity Constraints, Fiscal Policy, and Consumption." *Brookings Papers Econ. Activity,* no. 1: 1–50.

Jorgenson, Dale W., and Kun-Young Yun. 2001. *Lifting the Burden: Tax Reform, the Cost of Capital, and U.S. Economic Growth.* Cambridge, MA: MIT Press.

Judd, Kenneth L. 1985. "Redistributive Taxation in a Simple Perfect Foresight Model." *J. Public Econ.* 28 (October): 59–83.

———. 1999. "Optimal Taxation and Spending in General Competitive Growth Models." *J. Public Econ.* 71 (January): 1–26.

Kaplow, Louis. 1994. "A Note on Taxation as Social Insurance for Uncertain Labor Income." *Public Finance* 49 (2): 244–56.

King, Robert G., Charles I. Plosser, and Sergio Rebelo. 1988. "Production, Growth and Business Cycles: I. The Basic Neoclassical Model." *J. Monetary Econ.* 21 (March/May): 195–232.

Nishiyama, Shinichi. 2002. "Bequests, Inter Vivos Transfers, and Wealth Distribution." *Rev. Econ. Dynamics* 5 (October): 892–931.

Nishiyama, Shinichi, and Kent Smetters. 2003. "Consumption Taxes and Economic Efficiency in a Stochastic OLG Economy." Working Paper no. 9492 (February), NBER, Cambridge, MA.

Persson, Mats. 1983. "The Distribution of Abilities and the Progressive Income Tax." *J. Public Econ.* 22 (October): 73–88.

Social Security Administration. 2001. *Social Security Bulletin: Statistical Supplement.* Washington, DC: Soc. Security Admin.

Stokey, Nancy L., and Sergio Rebelo. 1995. "Growth Effects of Flat-Rate Taxes." *J.P.E.* 103 (June): 519–50.

Summers, Lawrence H. 1981. "Capital Taxation and Accumulation in a Life Cycle Growth Model." *A.E.R.* 71 (September): 533–44.

Tuomala, Matti. 1984. "Optimal Degree of Progressivity under Income Uncertainty." *Scandinavian J. Econ.* 86 (2): 184–93.

Varian, Hal R. 1980. "Redistributive Taxation as Social Insurance." *J. Public Econ.* 14 (August): 49–68.