Testing Alternative Multi-Factor Models

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Abstract

A GMM-based system for two alternative linear factor models can be used to test if the pricing errors (the intercepts) differ, with a bootstrap approach to find the appropriate critical values in finite samples. As an illustration, the test is applied to the Fama-French model.

Keywords: GMM, pricing errors, bootstrap

JEL: G12, C33, G10

1 Introduction

A linear factor model is often expressed as a GMM system. Combining two such systems (with different factors) into one provides a simple way of testing whether their pricing errors differ.

2 Testing with Excess Return Factors

2.1 Two Factor Models

Consider a factor model for the excess return of an $n \times 1$ vector of test assets (R_t^e)

$$R_t^e = \alpha + \beta f_t + \varepsilon_t, \tag{1}$$

where f_t is a $K \times 1$ vector of excess return factors and β is an $n \times K$ matrix. As usual the $n \times 1$ vector α measures the pricing errors. Clearly, the equation for test asset i is found

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in row i of this system. See Campbell, Lo, and MacKinlay (1997) 6 and Cochrane (2005) 12 for details on such models.

An alternative model is

$$R_t^e = \delta + \gamma h_t + u_t, \tag{2}$$

where h_t is an $L \times 1$ vector of excess return factors, δ is $n \times 1$ and γ is $n \times L$. In some cases h_t is a superset of f_t, but that is not necessary. We are interested in comparing the fit of the two models, the null hypothesis being that they are equal ($\alpha = \delta$).

2.2 Moment Conditions and Test

To estimate (1) and (2) as a joint system with GMM, use the moment conditions

$$\frac{1}{T}\sum\nolimits_{t=1}^{T}\left(\begin{bmatrix} \tilde{f}_t \otimes (R_t^e - \alpha - \beta f_t) \\ \tilde{h}_t \otimes (R_t^e - \delta - \gamma h_t) \end{bmatrix}\right) = \mathbf{0}_{n(1+K+1+L)\times 1}, \tag{3}$$

where $\tilde{f}_t$ stacks 1 and f_t and $\tilde{h}_t$ stacks 1 and h_t. The symbol $\otimes$ denotes a Kronecker product. There are $n(1+K)$ moment conditions in the first set and as many parameters. Similarly, there are $n(1+L)$ moment conditions in the second set and again as many parameters. The system is thus exactly identified and will give the OLS estimates of the parameters. (An alternative approach would be to impose $\delta = \alpha$ and get a system with n overidentifying restrictions.)

Let $\hat{\theta} = [\hat{\alpha}, \operatorname{vec}\hat{\beta}, \hat{\delta}, \operatorname{vec}\hat{\gamma}]$ denote the GMM estimate of the true parameter vector θ_0. From the general properties of GMM, we know that $\sqrt{T}(\hat{\theta} - \theta_0)$ typically converges (as T increases) to a normal distribution

$$\sqrt{T}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0, V), \text{ with } V = D^{-1}S(D^{-1})'. \tag{4}$$

In this expression D is the (probability limit of the) Jacobian matrix of the moment conditions. The inverse has a very simple structure (see the appendix for details). Also, S is the covariance matrix of ($\sqrt{T}\times$) the sample moment conditions, which can can be estimated by, for instance, a standard approach (which produces a White (1980) covariance matrix) or the Newey and West (1987) method.

To test if the two models have the same pricing errors, write the restrictions as

$$P\theta = \mathbf{0}_{n\times 1}, \text{ where } P = \begin{bmatrix} I_n & \mathbf{0}_{n\times nK} & -I_n & \mathbf{0}_{n\times nL} \end{bmatrix}. \tag{5}$$

Testing can be done by a straightforward chi-square test

$$T\hat{\theta}'P'(PVP')^{-1}P\hat{\theta} \xrightarrow{d} \chi^2_q. \tag{6}$$

Calculating the inverse (middle term) is is readily done, since PVP' is small ($n \times n$). It is intuitive that the degrees of freedom, denoted q in (6), equals n, since (5) has one restriction for each test assets. However, as will be seen later, this is not always true.

2.3 A Special Case: Nested Models

An important special case is where the models are nested and the aim is to ascertain whether the extra factors affect the average returns of the test assets. In this case, we can think of equation (1) as the smaller model and equation (2) as a larger model, where h_t includes the original factors (f_t) as well as, some extra factors (g_t)

$$R^e_t = \gamma_f f_t + \gamma_g g_t + u_t, \tag{7}$$

where γ_f and γ_g are matrices of coefficients.

It is clear that some of the slopes on the extra factors (γ_g) must be non-zero for those extra factors to matter for the average return of the test assets. However, it is well known that this is not sufficient. To see this, regress the extra factors on the original factors

$$g_t = \mu + \kappa f_t + v_t, \tag{8}$$

where μ is a vector of pricing errors (intercepts), κ is a matrix of coefficients and v_t is a vector of residuals.

Using this to substitute for g_t in (7) gives

$$R^e_t = (\gamma_f + \gamma_g\kappa)f_t + \gamma_g(\mu + v_t) + u_t. \tag{9}$$

The term $(\mu + v_t)$ can be thought of as an orthogonalized (with respect to f_t) version of the extra factors g_t, calculated from the estimated intercept and fitted residuals from (8). Notice that estimating (9) gives the same estimate (and t-stats) of γ_g as from estimating (7). In addition, the estimate of $\gamma_f + \gamma_g\kappa$ will be the same as the estimate of β obtained from the small model (1).[1]

[1]The first result is related to the Frisch-Waugh theorem on the properties of linear regressions, see, for instance, Davidson and MacKinnon (2004) p 67. The second result follows directly from the fact that f_t and v_t are uncorrelated.

Suppose we know the coefficients in (9). The expected excess returns are then

$$\mathrm{E}\, R_t^e = \beta\, \mathrm{E}\, f_t + \gamma_g \mu, \tag{10}$$

since the residuals (u_t and v_t) have zero means and $\gamma_f + \gamma_g \kappa = \beta$. This highlights that $\gamma_g \mu \neq \mathbf{0}$ is needed for the extra factors to matter for the pricing of the test assets. Testing $\alpha = \delta$ is the same as testing $\gamma_g \mu = \mathbf{0}$.

3 Testing with General Factors

With general (not excess return) factors, the two models imply that

$$\mathrm{E}\, R_t^e = \beta\lambda \text{ and } \mathrm{E}\, R_t^e = \gamma\psi, \tag{11}$$

where β and γ are the coefficients from the time series regressions (1) and (2) and where λ are the risk premia for the f_t factors and ψ for the h_t factors.

To estimate the factor risk premia (λ and ψ) from the cross-section of test assets, we set up a joint GMM system as to mimic the traditional two-step approach (see Cochrane (2005) 12 for the case of one model). That is, we combine the moment conditions for the time series regressions (3) with the sample analogues of the cross-sectional equations (11). The latter add $2n$ moment conditions, but only $K + L$ parameters (in λ and ψ), so there are overidentifying restrictions.

To easily estimate the parameters, we follow the standard approach of forming linear combinations of the cross-sectional moment conditions to basically get an exactly identified system. In practice, this means that the time series parameters ($\alpha, \beta, \delta, \gamma$) are still estimated by OLS and that the factor risk premia are estimated by

$$\lambda = (B\beta)^{-1} B \overline{R_t^e} \text{ and } \psi = (G\gamma)^{-1} G \overline{R_t^e}, \tag{12}$$

where $\overline{R_t^e}$ is the $n \times 1$ vector of average excess returns, while B and G are matrices that define the linear combinations discussed above. For instance, $B = \beta'$ and $G = \gamma'$ give the same estimates as a traditional cross-sectional regression (see Cochrane (2005)).

With these point estimates we can test if the pricing errors are the same ($\overline{R_t^e} - \beta\lambda = \overline{R_t^e} - \gamma\psi$) by testing the cross-sectional moment conditions directly (see the appendix). The sampling uncertainty in this test is driven by the uncertainty about the estimated parameters ($\beta, \lambda, \gamma, \psi$), since the average excess returns of the test assets cancel out.[2]

[2] A way to double check the logic of this approach is apply it on excess return factors: include the excess

4 Simulation Results

To explore the properties of the test, this section reports simulation results for testing if some *extra factors* matter for the cross-section of excess returns, that is, the setting in section 2.3.

The *small model* (1) has only one factor, which has properties similar to monthly U.S. equity market excess returns. The number of test assets (n) is initially 5, but I will also comment on results for a larger cross-section. In the simulations, we test against a *larger model* (7) where h_t includes the single factor f_t and two extra factors. The extra factors (g_t) are generated from (8), so as to be similar to the SMB and HML portfolios.[3]

The simulation results below will relate the properties of the test (6) to *four other tests*. *First*, individual (for each test asset) tests of $\alpha_i = \delta_i$, which is easily implemented by just using row i of the P matrix in (5). With this change, the formula (6) still applies, but we now expect the degrees of freedom (q) to be one (the 5% critical value of a χ^2_1 distribution is 3.84). This is equivalent to doing a t-test.

Second, the maximum (across the n test assets) of the individual test statistics. In a Bonferroni correction, this maximum value should exceed the $1 - p/n$ critical value to reject the null hypothesis on the significance level p. With five test assets ($n = 5$) and a 5% significance level ($p = 0.05$), we compare the maximum with the 1% critical value from a χ^2_1 distribution which is 6.63.

Third, a test of whether the original factors (f_t) can "price" the extra factors (g_t). This can be implemented by estimating (8) as a system and testing whether all elements in μ are zero. If that is the case, then the extra factors cannot matter for the average returns of the test assets as discussed above. The degrees of freedom is expected to equal the number of extra factors (here 2).

Fourth, a test of whether all the slopes of all the extra factors (γ_g) are zero, also discussed above. This is a traditional test of linear restrictions, but can also be implemented by redefining the P matrix. The degrees of freedom is expected to be the number of elements in γ_g (here 10).

return factors also as test assets and define the B and G matrices in (12) so that we effectively do cross-sectional GLS. In this setting, the test of $\beta\lambda = \gamma\psi$ should coincide with the joint test in (6), which indeed is the case.

[3] In the simulations, the f_t values are drawn from a normal distribution with the same mean and variance as the monthly U.S. equity market excess returns 1979–2014. The extra factors are generated from (8) where v_t are iid normally distributed with zero means and variances of $1/2\,\mathrm{Var}(f_t)$. With κ values around 0.7, the extra factors have the same volatility as the original factor (f_t).

4.1 Simulating the Size: When the Extra Factors are NOT Priced

To *simulate the size* (probability of rejecting a true null hypothesis), $\gamma_g \mu = 0$ must be imposed on the data generating process.

In the first set of simulations the loadings on the extra factors are zero ($\gamma_g = 0$), but the *extra factors are not priced* by the original factors ($\mu = 0.3$ in (8)). The choice of μ is around half of the mean of f_t and similar to the pricing errors for SMB and HML.

The n return series of the test assets are generated from the small model (1) by setting the betas to values between 0.5 and 1.5 (on average 1). The errors (ε_t) are iid normally distributed with zero means and variances similar to monthly returns on the Fama and French (1993) size/value portfolios.

	Same pricing errors:			Extra factors:	
T			max	pricing	
	joint	indiv.	indiv.	errors	coeffs
250	4.92	2.12	3.33	15.08	21.28
500	5.87	2.57	4.05	22.15	19.99
1000	6.84	2.97	4.74	34.02	19.14
50000	11.05	3.83	6.58	937.38	18.80
Asymptotic	11.07	3.84	6.63	5.99	18.31
df	5.00	1.00	1.00	2.00	10.00
p-value (%)	5.00	5.00	1.00	5.00	5.00

Table 1: **Simulation results under H_0: same pricing errors, version I, $n = 5$.** The table shows the 95th percentiles of test statistics for five different tests. In these simulations, the extra factors are not priced by the original factors ($\mu \neq 0$), but the slopes on the extra factors are zero ($\gamma_g = 0$). The number of simulations is 5000.

The simulation results are reported in *Table 1.* The first column shows the simulated 95th percentiles of the test statistics for the joint test of $\alpha = \delta$ (for all assets). Each row of the table is for a different sample size (the calibration is done on monthly data). The results indicate that the simulated 95th percentiles in small samples are considerably lower than the asymptotic 5% critical value (11.07 from a χ^2_5). The second and third columns show that the individual tests and the Bonferroni correction have similar properties.

Recall that $\gamma_g \mu \neq 0$ is needed for the extra factors to matter. The remaining columns therefore study the properties of tests of μ and γ_g. The fourth column shows high test statistics for the test of whether the extra factors are priced by the original factors. This is to be expected, since the simulations use a value of $\mu = 0.3$. The fifth column shows results for the joint test of all the loadings on the extra factors (γ_g). In this case, the small sample percentiles are fairly close to the asymptotic values.

The last observation suggests using an ad hoc decision rule that involves two tests: reject the null of same pricing errors if $\mu = 0$ *and* $\gamma_g = 0$ are both rejected at traditional critical values. In the simulations, this rule actually has a rejection rate close to 5% for all considered sample sizes.

Simulations with a larger cross section ($n = 25$) mostly show similar patterns, except that the joint tests of $\gamma_g = 0$ now has too high values. (See the appendix for details.) As a consequence, the ad hoc rule discussed above show too many rejections.

4.2 Simulating the Size: When the Extra Factors ARE Priced

The second set of simulations is also done under the null hypothesis ($\gamma_g \mu = 0$), but in the opposite way. Here, the pricing errors μ in (8) are set to zero, but the loadings on the extra factors (γ_g) are non-zero and vary across assets (but with an average magnitude of one for the first extra factor and of minus one for the second extra factor).

The results are shown in *Table 2*. All three tests of the pricing errors (columns 1–3) have fairly stable values (across sample sizes) of the 95th percentiles. However, both the joint test and the Bonferroni corrected test appear to converge (as the sample size increases) to "wrong" values—see the row for $T = 50,000$.

In fact, the joint test does not converge to a χ^2_5 variable, but to a χ^2_2. (This is true for all percentiles, not just the 95th.) The reason is straightforward: when the loadings on the extra factors are clearly non-zero ($\gamma_g \neq 0$), then the pricing errors depend only on whether the extra factors are priced by the original factors or not—and this is a test with 2 degrees of freedom. In fact, the first and fourth column (test of $\mu = 0$) are very similar—and their rank correlation is close to unity. This affects also the Bonferroni corrected test in column 3.

5 How to Use the Tests in Empirical Applications?

As always, it is crucial to use appropriate critical values. But, the previous simulations show that those values depend on the sample size and the data generating process (for instance, the value of μ). One way out of this dilemma is to set up simulations that approximate the generating process of available data—while making sure that we are testing under the null hypothesis.

	Same pricing errors:			Extra factors:	
T			max	pricing	
	joint	indiv.	indiv.	errors	coeffs
250	6.25	3.96	4.95	6.10	6959.97
500	6.13	3.94	4.80	6.11	12161.68
1000	5.89	3.69	4.52	5.90	22501.61
50000	5.88	3.72	4.61	5.88	971187.90
Asymptotic	11.07	3.84	6.63	5.99	18.31
df	5.00	1.00	1.00	2.00	10.00
p-value (%)	5.00	5.00	1.00	5.00	5.00

Table 2: **Simulation results under H_0: same pricing errors, version II, $n = 5$.** The table shows 95th percentiles of test statistics for five different tests. In these simulations, the extra factors are priced by the original factors ($\mu = 0$), but the slopes on the extra factors are non-zero ($\gamma_g \neq 0$). The number of simulations is 5000.

One approach could be to redo the Monte Carlo simulations presented above, but where we use parameter values estimated on an available sample (including the covariance matrix of the residuals). Alternatively, we could apply a bootstrap—which is done below.

In this bootstrap, we first draw (with replacement) T values of the time index (integer values between 1 and T): $s_1, s_2..., s_T$. The new sample of factors is then $[f_{s_t}, h_{s_t}]$ for observation t. Notice, that we draw then entire vector of factors (in a given period) together, so the covariance structure of them is preserved. Second, we draw a new set of time indices for the fitted residuals of the small model (1) to create a new sample of residuals. Combining with the point estimates of β, we generate a new sample of returns of the test assets.[4] Each such sample gives a point estimate of the parameter vector θ_i^* from the GMM system (3). To construct appropriate critical values, we test (in each simulated sample) the hypothesis

$$P(\theta_i^* - \overline{\theta_i^*}) = \mathbf{0}_{n\times 1}, \tag{13}$$

where P is defined in (5) and where $\overline{\theta_i^*}$ denotes the average value of θ_i^* (across the simulations, perhaps 3000).

Applying this approach on the five FF portfolios on the "diagonal" of the 5×5 matrix (that is, small growth to large value) for the sample Jan 1979 to Dec 2014 ($T = 432$) gives a simulated 95th percentile of 6.53. The test statistic from the sample is 9.05, so the

[4]Drawing the regressors (factors) and the residuals independently from each other assumes that they are not related (eg. no heteroskedasticity related to the squares of the regressors). This is a reasonable assumption on monthly return data.

null hypothesis that HML and SMB do not matter for the pricing errors can be rejected on the 5% significance level (actually, even on the 1% level).[5]

6 Concluding Remarks

A GMM-based system for two alternative linear factor models can be used to test if the pricing errors (the intercepts) differ. A bootstrap approach can be used to find the appropriate critical values in finite samples.

A Appendix: Technical Details

A.1 Details for Section 2

It is straightforward to demonstrate that the inverse of the Jacobian matrix for the moment conditions in (3) is

$$D^{-1} = -\begin{bmatrix} [\mathrm{E}(\tilde{f}_t \tilde{f}_t')]^{-1} \otimes I_n & \mathbf{0}_{n\tilde{K}\times n\tilde{L}} \\ \mathbf{0}_{n\tilde{L}\times n\tilde{K}} & [\mathrm{E}(\tilde{h}_t \tilde{h}_t')]^{-1} \otimes I_n \end{bmatrix}, \tag{14}$$

where $\tilde{K} = K + 1$ and $\tilde{L} = L + 1$. This structure of the inverse Jacobian facilitates the computations, especially in large systems. In practice, we replace the expectations with a sample average.

A.2 Details for Section 3

The moment conditions for the case of general factors are

$$\bar{m} = \frac{1}{T}\sum\nolimits_{t=1}^{T} \left(\begin{bmatrix} \tilde{f}_t \otimes (R_t^e - \alpha - \beta f_t) \\ R_t^e - \beta\lambda \\ \tilde{h}_t \otimes (R_t^e - \delta - \gamma h_t) \\ R_t^e - \gamma\psi \end{bmatrix} \right) = \mathbf{0}_{n(\tilde{K}+1+\tilde{L}+1)\times 1}. \tag{15}$$

To estimate the parameters, premultiply (15) by $A = \mathrm{diag}(I_{n\tilde{K}}, B, I_{n\tilde{L}}, G)$, where B

[5] If the test is applied to all 25 FF portfolios, then bootstrapped 95th percentile is 16.50, while the test statistic from the sample is 13.67. In this case, we cannot reject the null hypothesis on the 5% significance level (only on the 13% level).

is $K \times n$ and G is $L \times n$. This matrix can also be written

$$A = \begin{bmatrix} I_{n\tilde{K}} & \mathbf{0}_{n\tilde{K}\times n} & \mathbf{0}_{n\tilde{K}\times n\tilde{L}} & \mathbf{0}_{n\tilde{K}\times n} \\ \mathbf{0}_{K\times n\tilde{K}} & B_{K\times n} & \mathbf{0}_{K\times n\tilde{L}} & \mathbf{0}_{K\times n} \\ \mathbf{0}_{n\tilde{L}\times n\tilde{K}} & \mathbf{0}_{n\tilde{L}\times n} & I_{n\tilde{L}} & \mathbf{0}_{n\tilde{L}\times n} \\ \mathbf{0}_{L\times n\tilde{K}} & \mathbf{0}_{L\times n} & \mathbf{0}_{L\times n\tilde{L}} & G_{L\times n} \end{bmatrix}.$$

$A\bar{m}$ has as many elements as there are parameters, so calculating the point estimates is straightforward.

The Jacobian of the moment conditions (15) with respect to the parameters $\theta = [\alpha, \operatorname{vec}\beta, \lambda, \delta, \operatorname{vec}\gamma, \psi]$ is $D = \operatorname{diag}(D_f, D_h)$ where

$$D_f = -\begin{bmatrix} \mathrm{E}(\tilde{f}_t\tilde{f}_t') \otimes I_n & \mathbf{0}_{n\tilde{K}\times K} \\ \begin{bmatrix} 0 & \lambda' \end{bmatrix} \otimes I_n & \beta_{n\times K} \end{bmatrix} \text{ and } D_h = -\begin{bmatrix} \mathrm{E}(\tilde{h}_t\tilde{h}_t') \otimes I_n & \mathbf{0}_{n\tilde{L}\times L} \\ \begin{bmatrix} 0 & \psi' \end{bmatrix} \otimes I_n & \gamma_{n\times L} \end{bmatrix}.$$

The D matrix can also be written

$$D = \begin{bmatrix} D_f & \mathbf{0}_{n(\tilde{K}+1)\times n\tilde{L}+L} \\ \mathbf{0}_{n(\tilde{L}+1)\times n\tilde{K}+K} & D_h \end{bmatrix}.$$

Let $\bar{m}$ denote the moment conditions (15) evaluated at the point estimates. It is well known (see, for instance, Cochrane (2005)) that $\sqrt{T}\bar{m}$ has an asymptotic normal distribution with zero means (under the null hypothesis) and a (reduced rank) covariance matrix equal to

$$\Psi = \tilde{\Psi} S \tilde{\Psi}', \text{ where } \tilde{\Psi} = I - D\,(AD)^{-1}\,A.$$

To test if the pricing errors are the same, we formulate the linear restrictions

$$P\bar{m} = \mathbf{0}_{n\times 1}, \text{ where } P = \begin{bmatrix} \mathbf{0}_{n\times n\tilde{K}} & I_n & \mathbf{0}_{n\times n\tilde{L}} & -I_n \end{bmatrix}$$

and calculate the test statistics

$$T\bar{m}'P'(P\Psi P')^{-1}P\bar{m} \xrightarrow{d} \chi^2_q,$$

where we expect the degrees of freedom to equal $\max(n-K, n-L)$, since a linear combination does not reduce the rank of the covariance matrix.

To test coefficients, we use the fact that $\sqrt{T}(\theta - \theta_0)$ also has an asymptotic normal distribution with the covariance matrix

$$V = \tilde{V} S \tilde{V}', \text{ where } \tilde{V} = (AD)^{-1}\,A.$$

B Appendix: Additional Simulations

For simulations using a larger cross-section on test assets ($n = 25$), see Tables 3–4.

T	Same pricing errors: joint	indiv.	max indiv.	Extra factors: pricing errors	coeffs
250	11.86	2.14	4.62	15.39	112.76
500	13.33	2.53	5.24	22.13	86.68
1000	16.20	2.97	6.30	34.39	76.08
50000	35.60	3.84	9.31	937.63	67.66
Asymptotic	37.65	3.84	9.55	5.99	67.50
df	25.00	1.00	1.00	2.00	50.00
p-value (%)	5.00	5.00	0.20	5.00	5.00

Table 3: **Simulation results under H_0: same pricing errors, version I, $n = 25$.** The table shows 95th percentiles of test statistics for five different tests. In these simulations, the extra factors are not priced by the original factors ($\mu \neq 0$), but the slopes on the extra factors are zero ($\gamma_g = 0$). The number of simulations is 5000.

T	Same pricing errors: joint	indiv.	max indiv.	Extra factors: pricing errors	coeffs
250	7.20	3.72	4.70	6.09	47761.24
500	6.64	3.83	4.74	6.05	69405.41
1000	6.41	3.84	4.81	6.11	117959.33
50000	6.00	3.87	4.74	6.00	4718219.76
Asymptotic	37.65	3.84	9.55	5.99	67.50
df	25.00	1.00	1.00	2.00	50.00
p-value (%)	5.00	5.00	0.20	5.00	5.00

Table 4: **Simulation results under H_0: same pricing errors, version II, $n = 25$.** The table shows 95th percentiles of test statistics for five different tests. In these simulations, the extra factors are priced by the original factors ($\mu = 0$), but the slopes on the extra factors are non-zero ($\gamma_g \neq 0$). The number of simulations is 5000.

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