Measuring the Dividend Price Ratio with Reinvested Dividends

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First define gross cum- and ex-dividend returns

$$R_{t+1}^{x} = \frac{P_{t+1}}{P_{t}}$$

$$R_{t+1}^{d} = \frac{P_{t+1} + D_{t+1}}{P_{t}}.$$

We obtain the ratios after substracting one from the other to get the dividends:

$$R_{t+1}^d - R_{t+1}^x = \frac{D_{t+1}}{P_t}$$
$$\frac{R_{t+1}^d - R_{t+1}^x}{R_{t+1}^x} = \frac{D_{t+1}}{P_{t+1}}.$$

Our formula to account for reinvested dividend is then:

$$\frac{\tilde{D}_{t+1}}{P_{t+1}} = \frac{1}{P_{t+1}} \sum_{i=0}^{-11} D_{t+1+i} \prod_{k=0}^{i-1} R_{t+i-k}^d$$

$$= \sum_{i=0}^{-11} \frac{D_{t+1+i}}{P_{t+1+i}} \frac{P_{t+1+i}}{P_{t+1}} \prod_{k=0}^{i-1} R_{t+i-k}^d.$$

Or in recursive form:

$$\begin{split} \frac{\tilde{D}_{t+1}}{P_{t+1}} &= \sum_{i=0}^{-11} \frac{D_{t+1+i}}{P_{t+1+i}} \prod_{k=0}^{i-1} \frac{R_{t+i-k}^d}{R_{t+i-k}^x} \\ &= \left(\frac{D_{t+1-10}}{P_{t+1-10}} + \frac{D_{t+1-11}}{P_{t+1-11}} \frac{R_{t+1-10}^d}{R_{t+1-10}^x} \right) \frac{R_{t+1-9}^d}{R_{t+1-9}^x} \dots \end{split}$$