ECOM027 – Research Methods

Methods and tools for Empirical Macroeconomics

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Introduction

The goal of applied macroeconomics is to design models useful for policy analysis and forecasting.

The main methods are:

- Time series econometrics, e.g. Vector Autoregressions (VAR)
- Dynamic stochastic general equilibrium (DSGE) models, e.g. Real Business Cycles (RBC) and New Keynesian (NK) models

Today:

- Vector Autoregression (VAR) and Structural Vector Autoregression (SVAR) models
- VAR toolbox in MATLAB

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Trend and cycles

Trend and cycles

Most macroeconomic time series exhibit a trend.

For instance, Gross Domestic Product (GDP):

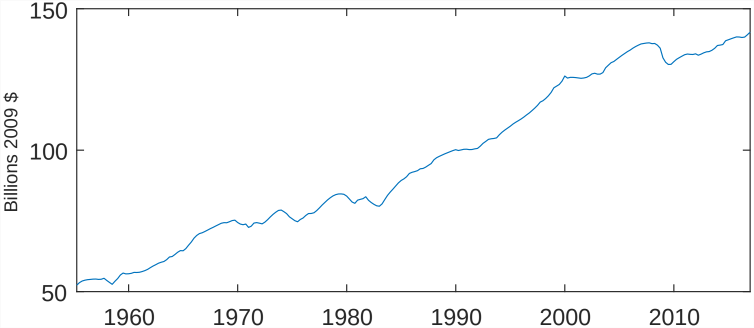

Figure 1: Real GDP per capita.

Trend and cycles

In applied macro, we are often interested in cyclical fluctuations of GDP.

To obtain the cyclical component of the time series we have two options:

- Estimate and remove the time trend.
- Filter out the lower frequency components.

Log-linear trend

We assume that the variable of interest grows linearly with time:

$$y_t = c + \delta t + \varepsilon_t$$

Steps:

1. Estimate the δ coefficient to get the trend.
2. Remove the trend component to obtain the cycle.

Log-linear trend

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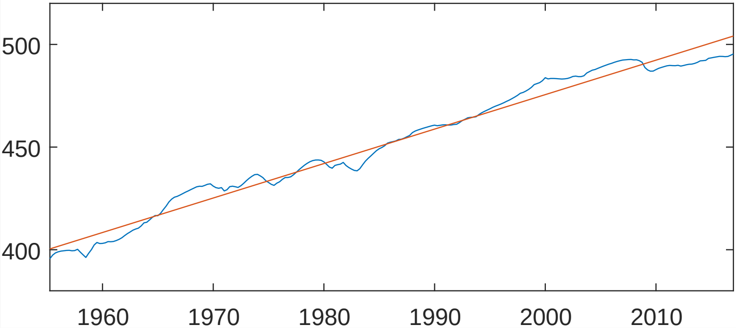

Figure 2: Log real GDP per capita with linear trend.

Log-linear trend

2. Remove the trend component to obtain the cycle:

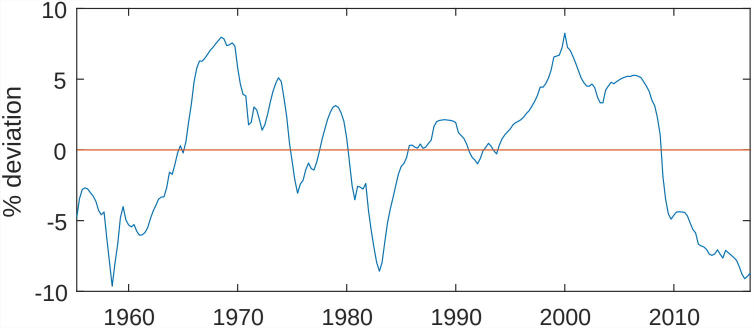

Figure 3: % deviations of GDP per capita around linear trend.

Filtering

The time series moves at different frequencies, higher frequencies and lower frequencies.

We use the Hodrick–Prescott filter to separate the higher and lower frequencies of the time series

Steps:

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2. Remove the lower frequency to get the cycle.

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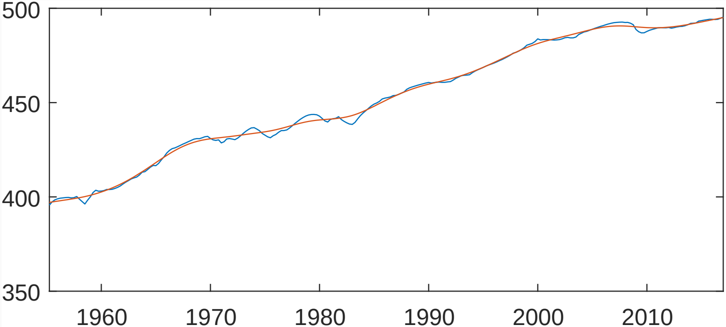

Figure 4: Log real GDP per capita with HP filtered trend.

2. Remove the lower frequency to get the cycle:

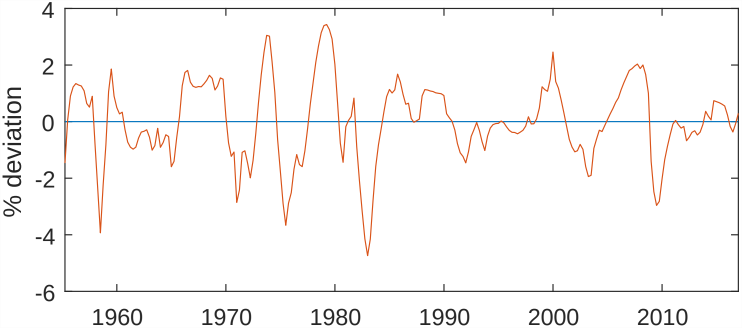

Figure 5: HP filtered real GDP per capita.

Comparison: Linear trend vs HP filter

Both approaches can successfully identify recessions:

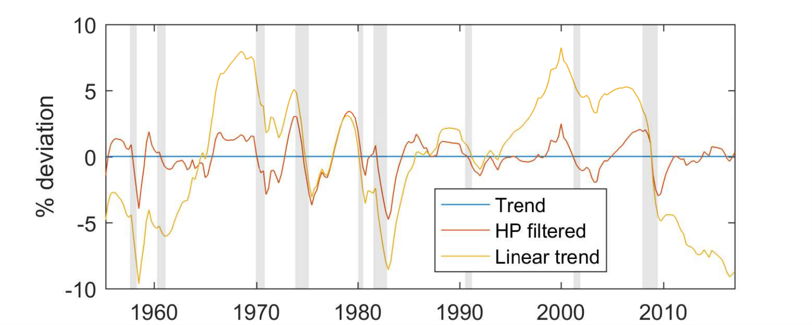

Figure 6: Linear de-trended vs HP filtered with NBER recessions.

Vector Autoregression models

Vector Autoregressions (VARs)

Vector Autoregressions are the multivariate equivalent of an Autoregressive (AR) process.

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Recall an AR process of order one (AR1). This process is a sequence $\{y_t\}_{t=0}^{\infty}$ defined by:

$$y_t = \rho y_{t-1} + e_t \tag{1}$$

where:

- Initial value y_0 is known
- e_t is an *i.i.d.* innovation that follows $e_t \sim \mathcal{N}(0, \sigma_e^2)$

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Stability condition: $|\rho| < 1$

Vector Autoregressions (VARs)

A VAR is a model with ***n*** variables with ***k*** lags.

Example: **two** variables $\{y_1, y_2\}$, **one** lag

$$y_{1,t} = \phi_{11} y_{1,t-1} + \phi_{12} y_{2,t-1} + e_{1,t}$$
$$y_{2,t} = \phi_{21} y_{1,t-1} + \phi_{22} y_{2,t-1} + e_{2,t}$$

Group elements as follows

$$Y_t = \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} \quad \Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \quad e_t = \begin{pmatrix} e_{1,t} \\ e_{2,t} \end{pmatrix}$$

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Vector Autoregressions (VARs)

VAR models of this form

$$Y_t = \Phi Y_{t-1} + e_t \tag{3}$$

are often referred to as ***reduced-form*** models:

- Parameters Φ do not have any intrinsic economic meaning.
- Innovations e_t do not have an economic or ***structural*** interpretation.

Reduced-form models are useful for forecasting:

- **One**-period ahead: $\mathbb{E}_t\left[Y_{t+1}\right] = \Phi \mathbb{E}_t\left[Y_t\right] + \mathbb{E}_t\left[e_{t+1}\right] = \Phi Y_t$
- **Two**-periods ahead: $\mathbb{E}_t\left[Y_{t+2}\right] = \Phi \mathbb{E}_t\left[Y_{t+1}\right] = \Phi\Phi Y_t$
- ***k***-periods ahead: $\mathbb{E}_t\left[Y_{t+k}\right] = \Phi \cdots \Phi Y_t = \Phi^k Y_t$

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Identification and causality

Innovations and dynamic responses

VAR models are useful to predict the dynamic response of a set of macroeconomic variables to a sudden change in one of the variables.

Thus, we compute **Impulse Responses Functions** (IRFs):

1. The innovation $e_{1,t}$ increases by one percent in period, $t = 1$.
2. The variable $y_{1,t}$ immediately responds in the first period, $t = 1$.
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This gives us a clear picture of the dynamic response of variables …

… but what **caused** the innovation $e_{1,t}$ to jump in the first place?

Structural shocks and contemporaneous responses

If we want some economic insight on the response of variables, we need to identify ***structural*** shocks $\varepsilon_{i,t}$.

- A structural shock $\varepsilon_{i,t}$ is likely to impact more than one variable $y_{i,t}$ contemporaneously!
- The effect of the shock will show up in the residuals $e_{i,t}$ but we cannot say if this is coming from $\varepsilon_{1,t}$ or $\varepsilon_{2,t}$

This is why movements in the residuals $e_{i,t}$ are without economic meaning:

- A movement in $e_{1,t}$ is not necessarily due to a shock to the $y_{1,t}$ variable
- It might be the contemporaneous by-product of a shock to $y_{2,t}$ or to both variables $y_{1,t}$ and $y_{2,t}$

To resolve this issue we need to impose some structure on the model.

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Structural Vector Autoregressions (SVARs)

The standard VAR does not allow for contemporaneous interactions. SVARs augment VARs model by introducing them.

Model becomes:

$$y_{1,t} = a_{12}y_{2,t} + b_{11}y_{1,t-1} + b_{12}y_{2,t-1} + c_1\varepsilon_{1,t}$$
$$y_{2,t} = a_{21}y_{1,t} + b_{21}y_{1,t-1} + b_{22}y_{2,t-1} + c_2\varepsilon_{2,t}$$

which can be written as

$$AY_t = BY_{t-1} + C\varepsilon_t \qquad (4)$$

where

$$A = \begin{pmatrix} 1 & -a_{12} \\ -a_{21} & 1 \end{pmatrix}, C = \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

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SVARs: reduced form and identification problems

Note the SVAR can be written in reduced-form:

$$Y_t = \Phi Y_{t-1} + e_t \tag{5}$$

where

$$\Phi = A^{-1}B \tag{6}$$

$$e_t = A^{-1}C\varepsilon_t \tag{7}$$

No problem estimating Φ from (5), but impossible to disentangle (6) and (7) to recover estimates ***structural matrices*** A, B and C.

In words, you cannot tell if a movement in Y_t is caused by $\varepsilon_{1,t}$ or $\varepsilon_{2,t}$ because what you ”observe” is only a movement in e_t.

Therefore, we must impose further structure to fully identify the model.

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Identification of SVARs

Common (and easiest) approach is to assume a ***recursive*** structure:

- $y_{1,t}$ only affected by $\varepsilon_{1,t}$,
- $y_{2,t}$ only affected by $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$,
- $y_{3,t}$ only affected by $\varepsilon_{1,t}$, $\varepsilon_{2,t}$ and $\varepsilon_{3,t}$,
- And so on.

This translates into saying that the last variable can be contemporaneously influenced by all variables, while the first one only by itself.

Usually this is justified by assuming that some variables move faster than others.

Example

Structural VAR with three variables:

$$Y_t = \begin{pmatrix} \Pi_t \\ GDP_t \\ FFR_t \end{pmatrix} = \begin{pmatrix} inflation_t \\ output_t \\ policyrate_t \end{pmatrix} \tag{8}$$

and four lags. Thus model is:

$$AY_t = \sum_{s=1}^{4} B_s Y_{t-s} + C\varepsilon_t \tag{9}$$

where the imposed form of the structural matrix A is:

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{10}$$

Example

The restrictions on the structural matrix A and the ordering of the variables

$$Y_t = \begin{pmatrix} \Pi_t \\ GDP_t \\ FFR_t \end{pmatrix}, \quad A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (8), (10)$$

imply that:

- The policy rate (FFR_t) responds to contemporaneous changes to other variables
- Output (GDP_t) responds current inflation but not interest rate
- Inflation (Π_t) is **not** contemporaneously affected by GDP_t or FFR_t and only responds to these variables with a lag

Example: practical implementation

We use a VAR toolbox designed by Ambrogio Cesa-Bianchi.

The Live Script `SVAR_demo.mlx`:

1. Downloads data from Federal Reserve data server (FRED)
2. Cleans raw data and creates final time–series for estimation
3. Estimates the model parameters of the reduced–form VAR
4. Applies a recursive identification scheme (short–run restriction) via a Cholesky decomposition
5. Plots impulse responses with confidence intervals
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Material

You can have access to the material in many ways.

Replication codes and slides:

1. Direct download (`zip version`)
2. Github repository (`view`)
3. If you are a **git** user, clone the repository with

```
git clone --recurse-submodules https://github.com/LRondina/SVARs-Intro
```

Official version of the toolkit: `VAR Toolbox 2.0`

Data: `https://fred.stlouisfed.org/`

Applications and Further Reading

Example questions:

- What is the response of output to monetary policy shock?
- What is the response of labour supply to a productivity shock?
- How do oil price shocks affect U.K. economy?

Some further reading:

- ”Vector autoregressions” by Stock J. H., Watson M. W., *Journal of Economic Perspectives* (2001)
- ”Macroeconomic Shocks and Their Propagation” by Ramey V. A., *Handbook of Macroeconomics Vol 2*, (2016)
- ”Identification in Macroeconomics” by Nakamura E., Steinsson J., *Journal of Economic Perspectives* (2018)