PUBPOL 529: REVIEW OF SIGNIFICANCE TESTS

University of Michigan
Fall 2010

1. One-sample Test of Means

1.1 Large Samples (One-sample *z*-test)

Purpose: To compare a population mean, μ, to some hypothesized value μ_0.

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) Random sample
2.) Quantitative Variable with Interval Scale
3.) Large Sample ($n \geq 25$ or 30)

Procedure:

1.) State the null hypothesis: $H_0: \mu = \mu_0$

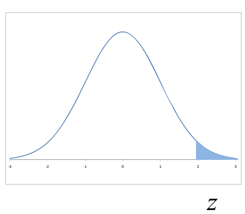
Choose an alternative hypothesis from one of the following.
(i) $H_a: \mu > \mu_0$ (ii) $H_a: \mu < \mu_0$ (iii) $H_a: \mu \neq \mu_0$

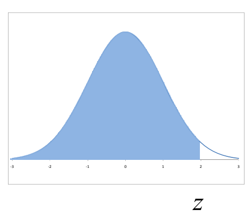
2.) Calculate the test statistic:

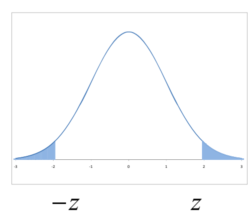
$$z = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}}$$

3.) Find the *P*-value using a standard normal distribution and the appropriate alternative hypothesis listed in (1).

(i)$P = P(Z > z)$ (ii) $P = P(Z < z)$ (iii) $P = 2 \times P(Z > |z|)$







4.) Conclusion: Reject the null hypothesis H_0 if the *P*-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

Confidence Interval: $\bar{Y} \pm z_{\alpha/2}\hat{\sigma}_{\bar{Y}}$ where $z_{\alpha/2} = z$-score with probability $\alpha/2$ in each tail.

$$\hat{\sigma}_{\bar{Y}} = \frac{s}{\sqrt{n}}$$

STATA Commands:

ttest *varname* = μ_0 [, level(#)]
ttesti *n* $\bar{Y}$ *s* μ_0 [, level(#)]

Note: Commands inside brackets are optional.

1.2 Small Samples (One-sample *t*-test)

Purpose: To compare a population mean, μ, to some hypothesized value μ_0 for small samples ($n < 25$ or 30).

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) Random sample
2.) Quantitative Variable with Interval Scale
3.) Population Distribution is Normal

Procedure:

1.) State the null hypothesis: $H_0: \mu = \mu_0$

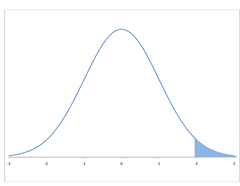
Choose an alternative hypothesis from one of the following.
(i) $H_a: \mu > \mu_0$ (ii) $H_a: \mu < \mu_0$ (iii) $H_a: \mu \neq \mu_0$

2.) Calculate the test statistic and the degrees of freedom:

$$t = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}} \qquad d.f. = n - 1$$

3.) Find the *P*-value using the <u>*t*-distribution</u>, *d.f.*, and the appropriate alternative hypothesis listed in (1).

(i) $P = P(T > t)$ (ii) $P = P(T < t)$ (iii) $P = 2 \times P(T > |t|)$



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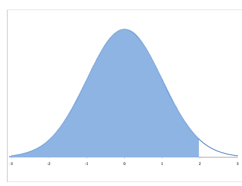
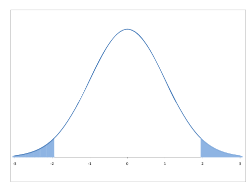


t



$-t$ t

4.) Conclusion: Reject the null hypothesis H_0 if the *P*-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

Confidence Interval: $\bar{Y} \pm t_{\alpha/2}\hat{\sigma}_{\bar{Y}}$ where $t_{\alpha/2}$ = *t*-score with probability $\alpha/2$ in each tail with *d.f.* = *n – 1*

$$\hat{\sigma}_{\bar{Y}} = \frac{s}{\sqrt{n}}$$

STATA Commands: `ttest` *`varname`* `=` μ_0 `[, level(#)]`
`ttesti` *`n`* $\bar{Y}$ *`s`* μ_0 `[, level(#)]`

Note: Commands inside brackets are optional.

2. Two-sample Test of Means

2.1 Large Samples (Two-sample *z*-test)

Purpose: To compare two population means, μ_1 and μ_2, to the hypothesized difference of 0.

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) The Two Samples are Independent
2.) Both Samples are Random samples
3.) Quantitative Variable with Interval Scale
4.) Large Sample ($n_1 \geq 20$ and $n_2 \geq 20$)

Procedure:

1.) State the null hypothesis: $H_0: \mu_2 - \mu_1 = 0$

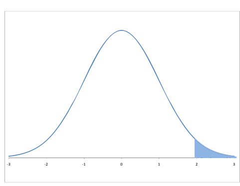
Choose an alternative hypothesis from one of the following.
(i) $H_a: \mu_2 - \mu_1 > 0$ (ii) $H_a: \mu_2 - \mu_1 < 0$ (iii) $H_a: \mu_2 - \mu_1 \neq 0$

2.) Calculate the test statistic:

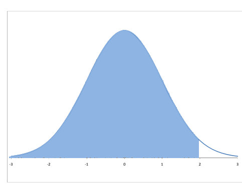
$$z = \frac{\left(\bar{Y}_2 - \bar{Y}_1\right) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

3.) Find the *P*-value using a standard normal distribution and the appropriate alternative hypothesis listed in (1).

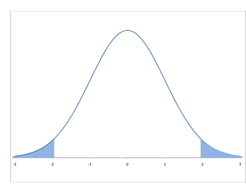
(i) $P = P(Z > z)$ (ii) $P = P(Z < z)$ (iii) $P = 2 \times P(Z > |z|)$



z



z



$-z$ z

4.) Conclusion: Reject the null hypothesis H_0 if the *P*-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

Confidence Interval: $\left(\bar{Y}_2 - \bar{Y}_1\right) \pm z_{\alpha/2}\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ where $z_{\alpha/2}$ = *z*-score with probability $\alpha/2$ in each tail.

STATA Commands:

```
ttest varname1 = varname2, unpaired [level(#)]
ttest varname, by(groupvar) [,level(#)]
ttesti n1 Ȳ1 s1 n2 Ȳ2 s2 [, level(#)]
```

Note: Commands inside brackets are optional.

2.2 Small Samples (Two-sample *t*-test)

Purpose: To compare two population means, μ_1 and μ_2, to the hypothesized difference of 0 for small samples ($n_1 < 20$ or $n_2 < 20$).

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) The Two Samples are Independent
2.) Both Samples are Random Samples
3.) Quantitative Variable with Interval Scale
4.) Population Distributions are Normal for Both Samples

Procedure:

1.) State the null hypothesis: $H_0\colon \mu_2 - \mu_1 = 0$

Choose an alternative hypothesis from one of the following.
(i) $H_a\colon \mu_2 - \mu_1 > 0$ (ii) $H_a\colon \mu_2 - \mu_1 < 0$ (iii) $H_a\colon \mu_2 - \mu_1 \neq 0$

2.) Choose one of the following assumptions:
   (i) Populations have Equal Variances
   (ii) Populations have Unequal Variances

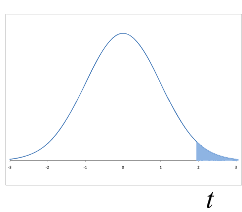
3.) Calculate the test statistic and the degrees of freedom:
   (i) Populations have Equal Variances

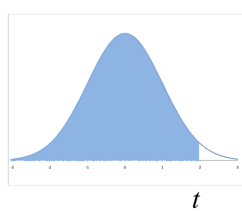
$$t = \frac{\left(\bar{Y}_2 - \bar{Y}_1\right) - 0}{\hat{\sigma}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \quad \text{where} \quad \hat{\sigma} = \sqrt{\frac{\left(n_1 - 1\right)s_1^2 + \left(n_2 - 1\right)s_2^2}{n_1 + n_2 - 2}}, \quad d.f. = n_1 + n_2 - 2$$

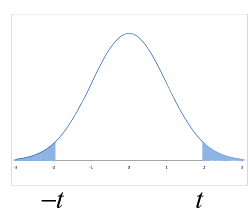
   (ii) Populations have Unequal Variances

$$t = \frac{\left(\bar{Y}_2 - \bar{Y}_1\right) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} \quad \text{where} \quad d.f. = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)^2}$$

4.) Find the *P*-value using the <u>*t*-distribution</u>, *d.f.*, and the appropriate alternative hypothesis listed in (1).
(i) $P = P(T > t)$ (ii) $P = P(T < t)$ (iii) $P = 2 \times P(T > |t|)$







5.) Conclusion: Reject the null hypothesis H_0 if the *P*-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

Confidence Interval: Choose one of the following assumptions

(i) Populations have Equal Variances

$$\left(\bar{Y}_2 - \bar{Y}_1\right) \pm t_{\alpha/2}\hat{\sigma}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

where $t_{\alpha/2} =$ t-score with probability $\alpha/2$ in each tail

with $d.f. = n_1 + n_2 - 2$

$$\hat{\sigma} = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

(ii) Populations have Unequal Variances

$$\left(\bar{Y}_2 - \bar{Y}_1\right) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

where $t_{\alpha/2} =$ t-score with probability $\alpha/2$ in each tail

with $d.f. = \dfrac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$

STATA Commands:

```
ttest varname1 = varname2, unpaired [level(#)]
ttest varname, by(groupvar) [level(#)]
```

`ttesti` n_1 $\bar{Y}_1$ s_1 n_2 $\bar{Y}_2$ s_2 `[, level(#)]`

Note: Commands inside brackets are optional.

3. One-sample Test of Proportions

3.1 Large Samples (One-sample z-test)

Purpose: To compare a population proportion, π, to some hypothesized value π_0.

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) Random sample
2.) Qualitative Variable
3.) Large Sample ($n > 10/\min(\pi_0, 1- \pi_0)$)

Procedure:

1.) State the null hypothesis: $H_0 : \pi = \pi_0$

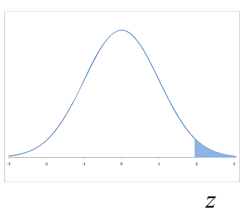
Choose an alternative hypothesis from one of the following.
(i) $H_a : \pi > \pi_0$ (ii) $H_a : \pi < \pi_0$ (iii) $H_a : \pi \neq \pi_0$

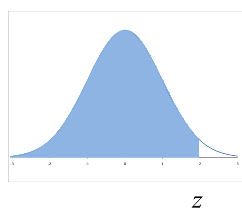
2.) Calculate the test statistic:

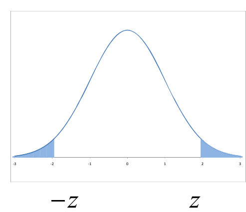
$$z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0 \left(1 - \pi_0\right)}{n}}}$$

3.) Find the P-value using a <u>standard normal distribution</u> and the appropriate alternative hypothesis listed in (1).

(i) $P = P(Z > z)$ (ii) $P = P(Z < z)$ (iii) $P = 2 \times P(Z > |z|)$







4.) Conclusion: Reject the null hypothesis H_0 if the P-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

Confidence Interval: $\hat{\pi} \pm z_{\alpha/2} \sqrt{\frac{\hat{\pi}\left(1 - \hat{\pi}\right)}{n}}$ where $z_{\alpha/2} = z$-score with probability $\alpha/2$ in each tail.

STATA Commands:

```
prtest varname = π0 [, level(#)]
prtesti n π̂ π0 [, level(#)]
```

Note: Commands inside brackets are optional.

3.2 Small Samples (One-sample binomial test)

Purpose: To compare a population proportion, π, to some hypothesized value π_0 for small samples ($n \leq 10/\min(\pi_0, 1-\pi_0)$).

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) Random sample
2.) Each observation falls into one of two categories (dichotomous variable)
3.) The probability of falling in each category is the same for every observation
4.) The outcomes of successive observations are independent

Procedure:

1.) State the null hypothesis: $H_0: \pi = \pi_0$
Choose an alternative hypothesis from one of the following.
(i) $H_a: \pi > \pi_0$ (ii) $H_a: \pi < \pi_0$ (iii) $H_a: \pi \neq \pi_0$

2.) There is *no* test statistic to calculate. We need to calculate the *P*-value directly.

3.) To find the *P*-value, we first need to find the actual observed X in a given category out of the sample n.
Ex.) Suppose we want to test whether there is a bias against females in hiring by a company. Suppose we observed that out of 10 people hired, 30% were females. Then $X = 10 \times 0.3 = 3$ females were hired.

Then we need to find the *P*-value for the appropriate alternative hypothesis listed in (1) and using the formula:

$$P(X) = \frac{n!}{X!(n-X)!}\pi^X (1-\pi)^{n-X}$$

For alternative hypothesis (i) – (iii) the *P*-values are:

(i) $P = P(X \geq x)$
We need to add all the probabilities that are greater than or equal to what we have observed.
Ex.) $P = P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10)$

(ii) $P = P(X \leq x)$
We need to add all the probabilities that are less than or equal to what we have observed.
Ex.) $P = P(0) + P(1) + P(2) + P(3)$

(iii) $P = P(X \leq x \text{ or } X \geq (n-x))$
We need to add all the probabilities that are as extreme as the value that we have observed.
Ex.) $P = P(0) + P(1) + P(2) + P(3) + P(7) + P(8) + P(9) + P(10)$

4.) Conclusion: Reject the null hypothesis H_0 if the *P*-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

Confidence Interval: Several different methods are available, but we have not covered any in class.

STATA Commands:

```
bitest varname = π0
bitesti n  X  π0
```

4. Two-sample Test of Proportions

4.1 Large Samples (Two-sample *z*-test)

Purpose: To compare two population proportions, π_1 and π_2, to the hypothesized difference of 0.

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) The Two Samples are Independent
2.) Both Samples are Random samples
3.) Qualitative Variable
4.) Large Sample (At least five observations fall in each category in each sample)

Procedure:

1.) State the null hypothesis: $H_0: \pi_2 - \pi_1 = 0$

Choose an alternative hypothesis from one of the following.

(i) $H_\mathrm{a}: \pi_2 - \pi_1 > 0$ (ii) $H_\mathrm{a}: \pi_2 - \pi_1 < 0$ (iii) $H_\mathrm{a}: \pi_2 - \pi_1 \neq 0$

2.) To calculate the test statistic:

First, find the pooled estimate proportion $\hat{\pi}$.

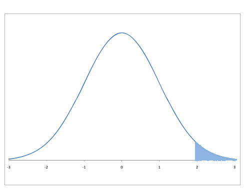
$$\hat{\pi} = \frac{x_1 + x_2}{n_1 + n_2} \quad \text{where} \quad x_1 = \hat{\pi}_1 \times n_1 \quad \text{and} \quad x_2 = \hat{\pi}_2 \times n_2$$

Then calculate the test statistic.

$$z = \frac{\left(\hat{\pi}_2 - \hat{\pi}_1\right) - 0}{\sqrt{\hat{\pi}\left(1 - \hat{\pi}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

3.) Find the *P*-value using a standard normal distribution and the appropriate alternative hypothesis listed in (1).

(i) $P = P(Z > z)$ (ii) $P = P(Z < z)$ (iii) $P = 2 \times P(Z > |z|)$



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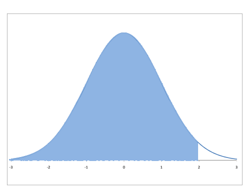

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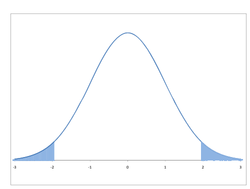


$-z$ z

4.) Conclusion: Reject the null hypothesis H_0 if the *P*-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

Confidence Interval: $$\left(\hat{\pi}_2 - \hat{\pi}_1\right) \pm z_{\alpha/2}\sqrt{\frac{\hat{\pi}_1\left(1-\hat{\pi}_1\right)}{n_1} + \frac{\hat{\pi}_2\left(1-\hat{\pi}_2\right)}{n_2}}$$

where $z_{\alpha/2} =$ *z*-score with probability $\alpha/2$ in each tail.

STATA Commands:

```
prtest varname1 = varname2 [, level(#)]
prtest varname, by(groupvar) [level(#)]
prtesti n1 π̂1 n2 π̂1 [, level(#)]
```

Note: Commands inside brackets are optional.

4.2 Small Samples (Fisher's Exact Test)

Purpose: To compare two population proportions, π_1 and π_2, to the hypothesized difference of 0 for small samples (Less than five observations fall in at one or more of the categories in each sample)

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) The Two Samples are Independent
2.) Observations are taken from random samples
3.) Qualitative Variable
4.) Observations fall into one of two categories, but not both (Mutual Exclusivity)

Procedure:

1.) State the null hypothesis: $H_0: \pi_2 - \pi_1 = 0$

Choose an alternative hypothesis from one of the following.
(i) $H_a: \pi_2 - \pi_1 > 0$ (ii) $H_a: \pi_2 - \pi_1 < 0$ (iii) $H_a: \pi_2 - \pi_1 \neq 0$

2.) There is *no* test statistic to calculate. We need to calculate the *P*-value directly.

3.) We did not cover how to find the *P*-value by hand. Use STATA to find the *P*-value.

4.) Conclusion: Reject the null hypothesis H_0 if the *P*-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

Confidence Interval: Several different methods are available, but we have not covered any in class.

STATA Commands: **tabulate** *varname1 varname2*, exact
tabi x_{11} x_{12} … \ x_{21} x_{22} … \ …

```
. tabulate var1 var2, exact

           |         var2
      var1 |         1          2 |     Total
-----------+----------------------+----------
         1 |         2         23 |        25
         2 |         0         20 |        20
-----------+----------------------+----------
     Total |         2         43 |        45

           Fisher's exact =                 0.495
   1-sided Fisher's exact =                 0.303

. tabi 2 23 \ 0 20

           |          col
       row |         1          2 |     Total
-----------+----------------------+----------
         1 |         2         23 |        25
         2 |         0         20 |        20
-----------+----------------------+----------
     Total |         2         43 |        45

           Fisher's exact =                 0.495
   1-sided Fisher's exact =                 0.303
```

5. Test for Independence

5.1 Large Samples (Chi-squared Test)

Purpose: To determine whether two categorical variables are statistically independent.

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) Random Sample
2.) Two Categorical Variables
3.) Large Sample (*Expected* frequencies are at least 5 for all cells)

Procedure:

1.) Null hypothesis: H_0: The Variables are Statistically Independent
Alternative hypothesis: H_a: The Variables are Statistically Dependent

2.) Calculate the test statistic and the degrees of freedom:

First, find the expected cell frequencies: $$f_e = \frac{(\text{Row total}) \times (\text{Column total})}{\text{Total sample size}}$$

Test statistic: $$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

Degrees of freedom: $$d.f. = (\text{Number of rows} - 1)(\text{Number of columns} - 1)$$

3.) Find the *P*-value using the χ^2-distribution, and the *d.f.*

The *P*-value: P = Right-tail probability above the observed χ^2-value

4.) Conclusion: Reject the null hypothesis H_0 if the *P*-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

STATA Commands:

```
tabulate varname1 varname2, chi2 [expected]
tabi x11 x12 … \ x21 x22 … \ … , chi2 [expected]
```

The **`expected`** option displays the expected cell frequencies.

5.2 Small Samples (Fisher's Exact Test)

Purpose: To determine whether two categorical variables are statistically independent for small samples (*Expected* cell frequencies are less than 5)

The Fisher's Exact Test as described in section 4.2 can be used.

5.3 Ordinal Variables (z-test using Gamma)

Purpose: To determine whether two categorical ordinal variables are independent.

Assumptions: The following assumptions must be satisfied for the test to be accurate and valid.

1.) Random Sample
2.) Ordinal Categories
3.) Large Sample (C and D should exceed 50)

Procedure:

1.) State the null hypothesis: $H_0: \gamma = 0$

Choose an alternative hypothesis from one of the following.

(i) $H_a: \gamma > 0$ (ii) $H_a: \gamma < 0$ (iii) $H_a: \gamma \neq 0$

2.) Calculate the test statistic:

First, find: C = the total number of concordant pairs of observations
D = the total number of discordant pairs of observations

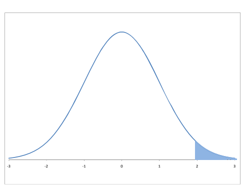
Gamma: $$\hat{\gamma} = \frac{C - D}{C + D}$$

Test statistic: $$z = \frac{\hat{\gamma} - 0}{\hat{\sigma}_{\hat{\gamma}}}$$

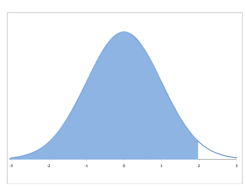
Note: We did not cover how to find the asymptotic standard error (ASE) for gamma $\hat{\sigma}_{\hat{\gamma}}$. It can be calculated using STATA.

3.) Find the P-value using a standard normal distribution and the appropriate alternative hypothesis listed in (1).

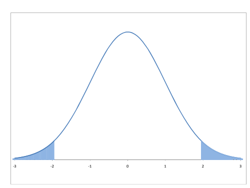
(i) $P = P(Z > z)$ (ii) $P = P(Z < z)$ (iii) $P = 2 \times P(Z > |z|)$



z



z



$-z$ z

4.) Conclusion: Reject the null hypothesis H_0 if the P-value is less than a pre-specified α-level. Otherwise, cannot reject H_0.

STATA Commands:

```
tabulate varname1 varname2, gamma
tabi x11 x12 … \ x21 x22 … \ … , gamma
```

or to use the Kendall's tau-b test,

```
tabulate varname1 varname2, taub
tabi x11 x12 … \ x21 x22 … \ … , taub
```