DYNAMIC NELSON SIEGEL YIELD CURVE ESTIMATION FOR INDIAN G-SEC MARKET

SAYAN MAITY

A project report submitted in partial fulfilment of the requirement for the award of the degree of MASTER OF ARTS (Actuarial Economics)

MADRAS SCHOOL OF ECONOMICS
and
CENTRAL UNIVERSITY OF TAMIL NADU

MAY, 2019

MADRAS SCHOOL OF ECONOMICS

Chennai: 600025

Degree and Branch	**:**	**MASTER OF ARTS** **(ACTUARIAL ECONOMICS)**
Month and Year of Submission	**:**	**May 2019**
Title of the Project Work	**:**	**DYNAMIC YIELD CURVE ESTIMATION FOR INDIAN G-SEC MARKET**
Name of the Student	**:**	**SAYAN MAITY**
Registration Number	**:**	**P1703113**
Name and Designation of the Supervisor	**:**	**Dr. SRIKANT P.** **Professor** MADRAS SCHOOL OF ECONOMICS Chennai: 600025

BONA FIDE CERTIFICATE

Certified that this Project Report titled **DYNAMIC YIELD CURVE ESTIMATION FOR INDIAN G-SEC MARKET** is the Bona fide work of Mr. Sayan Maity who carried out the project under my supervision. Certified further, that to the best of my knowledge the work reported herein does not form part of any other Project Report of the basis of which a degree or award was conferred on an earlier occasion on this or any other candidate

Dr. K. R. Shanmugam

Director

Madras School of Economics

Chennai: 600025

Dr. Srikant P.

Professor

Madras School of Economics

Chennai: 600025

ABSTRATCT

The aim of this paper is to estimate dynamic Nelson Siegel yield curve to model term structure for Indian G-Sec market. The movements of the yield curve depend upon three latent variable factors. This paper models this in sample factors and forecasts the yield out of the sample. Also, this paper looks for any structural break in the latent variable over time.

We have two models to estimate the dynamic yield curve: two-step process and one-step process using Kalman filter and compared the results including estimated coefficients and error variances We use Minimum Mean Square Error and Monte Carlo simulation for out of sample fore casting. To detect structural break use Bai-Perron (2003)'s test for L+1 vs L sequentially determined structural breaks.

ACKNOWLEDGEMENT

First and foremost, I extend my gratitude towards my thesis mentor Professor Srikant P. for giving me the opportunity to carry out this study. His constant guidance and insights shaped this project to where it is. I would also like to thank Professor Naveen Srinivasan who offered me his full support and co-operation.

Subha Pratik Satapathy generously provided me with his Python codes. Thank you for being such a great classmate.

Abhiraj Anand provided me with various important information which helped me to successfully complete my research. Thank you Abhiraj.

Special thanks to Dr Parthajit Kayal for inspiring me to do this project.

I would like to thank Sayantan Roy, Avishek Rakshit, Agnij Sur, Devesh Verma, Sumanjay Dutta, Torsha Chakravorty for giving me their helping hand to complete my dissertation.

I would also like to thank Rupkatha Das, Anuj Talwar, Visesh Goel, Anal Deb Mitra for supporting me to balance out my academic and cultural activity.

Moreover, I like to thank Anshul Gupta for being such a great roommate and letting me work in any situation.

I end by thanking my parents and family for always standing by me.

TABLE OF CONTENTS

LIST OF TABLES **7**
LIST OF FIGURES **8**

CHAPTER 1. INTRODUCTION **9-10**

CHAPTER 2. LITERATURE REVIEW **11-12**

CHAPTER 3. MODEL **13-16**

3.1 Nelson Siegel Model 13

3.2 Dynamic Nelson Siegel Model 13-16

CHAPTER 4. ESTIMATION **17-19**

3.1 Nelson Siegel IN Cross Section 17

3.2 Two-step Model 17

3.3 One Step Model 18-19

CHAPTER 5. DATA **20**

CHAPTER 6. ESTIMATION **21-28**

6.1 Two-step DNS model 21

6.2 One step state space model 22

6.3 Comparison of two models 23

6.4 Test for Structural Break 24-26

6.5 Forecast 26-28

CHAPTER 7. CONCLUSION **29**

APPENDIX **30-36**

REFERENCES **37-39**

LIST OF TABLES

TABLE 1: VAR coefficient regression **21**

TABLE 2: Two-step state disturbance covariance matrix **21**

TABLE 3: One-step State transition matrix **22**

TABLE 4: One-state SSM State disturbance covariance matrix **22**

TABLE 5: Structural Break test beta0 **25**

TABLE 6: Structural Break test beta1 **25**

TABLE 7: Structural Break test beta2 **25**

TABLE 8: Summary Statistics **30-31**

TABLE 9: Residual mean and standard deviation **32**

TABLE 10: Forecast Yield MMSE and Monte Carlo **33-37**

LIST OF FIGURES

Figure 1: DNS factor loading **14**

Figure 2: Mesh plot of yield curve over time **20**

Figure 3: Level Factor **23**

Figure 4: Slope Factor **23**

Figure 5: Curvature Factor **23**

Figure 6: Loading on curvature **24**

Figure 7: Forecast on yield **27**

Figure 8: Forecast of standard error **27**

Figure 9: Forecasted yield frequency distribution **28**

1. Introduction

The term structure of interest rate, or the yield curve is a function that relates the time to maturity to the yield to maturity for a sample of bonds at a given period. The term structure is conventionally measured by means of spot rate curve or yield curve on zero coupon bonds. As this entire term structure is not directly observable, it becomes necessary to build certain models for estimating the term structure and to forecast future behavior of yield curve as close as possible. Among all those models, Nelson-Siegel (1987) model (NS) and its extensions have proven to be most effective in case of in-sample fitting and out of sample forecasting of the term structure. In case of India, The Clearing Corporation of India Limited (CCIL) & National Stock Exchange (NSE) are tasked to estimate the daily zero-coupon yield curve for Government Securities market in India. The CCIL uses extended Nelson-Siegel-Svensson (1994) model while NSE uses standard NS Model for yield curve estimation.

Macroeconomists, financial Economists and market participants all have attempted to build good models of the yield curve. But the resulting models are very different in form and fit which reflects the modelling demands of researchers from different field and their motives for modelling the yield curve. The yield curve models developed by macroeconomists focuses on the role of expectations of inflation and future real economic activity in the determination of the yields while financial economist focuses on any explicit role of such determinants.

At any time, many of different yields may be observed. The yield curves evolve dynamically; hence they have not only a cross sectional, but also a temporal dimension. In standard NS setup, the parameters in the yield equations are static/invariant in time. This eases the process of estimation but doesn't fully capture the underlying dynamics of the yield curve. In the dynamic NS, these parameters are fixed in cross section but varied in time.

Financial asset returns typically conform to a certain type of restricted vector auto-regression, displaying factor structure. This factor structure is said to be operative when a high dimensional object (such as daily bond yields) is driven by underlying low dimensional set of objects. In real world, term structure data –and correspondingly, modern empirical term structure models – involves multiple factors. These factors are the principal components that explains most of the variations in the yield curve model. The main three factors that explains most the variations are Level, Slope and Curvature. These factors have different and specific macroeconomic variants. Such as Inflation is clearly related to yield curve level and the stages of business cycle are clearly related to slope.

The original Nelson-Siegel model is a static model where the parameters are don't change over time. This can provide a good fit to the cross-section of yields at any given point of time. But to understand the evolution of term structure over time a dynamic representation is required.

Diebold and Li (2006) supply such a model by replacing the parameters of NS by time-varying factors. Given the associated Nelson-Siegel factor loadings, Diebold and Li show that these factors can also be interpreted as level, slope and curvature factors. Once the model is viewed as a factor model, a dynamic structure can be postulated for all the three factors.

The modelling of interest rates has long been a primary issue of the disconnect between macroeconomics and finance literatures. In the standard finance models, the short-term interest rate is a linear function of a few unobserved factors. Movements in long term yields are importantly determined by changing the risk premiums, which in turn also depends on the latent factors. According to the macro literature, the short-term interest rate is set by the central bank according to macroeconomic stabilization goals. Also, the macro literature commonly views the long term yields largely determined by the expectations of future short-term interest rates, which in turn depends on expectations of the macro variables. So, possible changes in risk premium are often ignored and expectation hypothesis of the term structure is employed.

In the paper, we are estimating yield curve based on using on Indian G-sec data for last 10 years. We apply two methods of estimation. One is two step method where we estimate the factors from the static NS model and then fit a dynamic model (VAR) to those factors.

Another one is one step method where exact maximum likelihood estimations may be done using the Kalman filter. Both models are formalized by Diebold and Li(2006a) and Diebold(2006b).

2. Literature Review

There had been many theories to model the term structure of interest rate. One of the earlier approaches was by McCulloch (1971, 1975), who models discount curve using polynomial spines. Due to its polynomial structure the fitted curve diverges at longer maturity. This was improved by Vasichek and Fong (1982), who models the yield curve using exponential splines. They used a negative transformation of maturity which ensures the convergence of zero-coupon yield to a fixed limit with increasing maturity. But overall this process remained very problematic as the implied forward rates are not necessarily positive. One of the alternatives was proposed by Fama and Bliss (1987), who construct yields from estimated forward rates, instead of the estimated discount curve. The forward rates are called 'unsmoothed Fama Bliss' forward rates and yield is calculated by integration the forward rate for the given maturity period and averaging. These are often used as the raw yields for modelling the yield curve.

The advantage of Nelson-Siegel (1987) comes from the fact that it can adapt any shape of yield curve using a flexible smooth and parametric function. Since then various extensions or restrictions have been proposed to incorporate additional flexibility. Litterman and Scheinkman (1991) models yield curve by only considering the first 3 components of principal component analysis as they intuitively explain the level, slope, and curvature factor. Svensson (1994) introduces additional parameters that allows yield curve to have an additional hump. These two models are very popular among the central banks to construct zero coupon yield curve. Apart from that Bjork and Christensen (1999) proposed a similar model to Svensson by introducing a second slope factor to the three factor NS model. On the other hand, Bliss (1997) proposed a second opinion to make NS model more flexible by relaxing the restriction that the slope and curvature should governed by the same decay component and allowing for two decay components.

All these models are static in nature i.e. parameters don't vary over time. This helps to provide a good fit to the cross-section of yields at a given point of time. But to understand the evolution of bond market a dynamic model is required. Diebold, Piazzesi, and Rudebusch (2005) proposed a two-factor dynamic model based on Litterman and Scheinkman (1991) variation of NS model. They argued that since the first two principal components explain nearly all variation in interest rates, a two-factor model may suffice to forecast the term structure. Though, two factors will not be enough to fit the entire yield curve. The first three factor dynamic variation of NS model was proposed in Diebold & Li (2002) This model is based on original NS model and thus have the substantial flexibility to match the changing shapes of yield curve. The dynamic framework was proposed in the Diebold & Li (2006) and Diebold, Rudebusch and Aruoba, (2006b). Both the papers assume the state space representation of the dynamic NS model by specifying the 1st order autoregressive process through state equations which can be either individual AR (1) or single multivariable VAR (1) process. This paper follows these two papers closely to estimate and forecast the yield curve dynamics.

For Indian context there has not been many works regarding the dynamics of yield curve. One of the main works is done by Sowmya, Prasanna, Bhaduri (2016) which estimates the dynamic latent factor NS model using the NSE daily data which was averaged for a monthly basis. But there have been a few problems regarding the NSE data which was addressed by Virmani (2006). Where NSE has been using the standard NS model to fit the daily yield, there could be improvement by using different models such as NSS. Moreover, NSE has stopped publishing daily zero-coupon yield data since 2016 onwards. On this basis this paper provides an improvement on the previous estimation and could be a refresher on dynamic NS estimation for Indian market.

3. Model

3.1 The Nelson Siegel Model[1]

Consider a cross sectional environment for fixed t. Then according to Nelson Siegel Model (1987)

$$y(\tau) = \beta_0 + \beta_1\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$

This is a cross section representation of yield given maturity τ on explanatory variables 1, $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right), \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$ with parameters $\beta_0, \beta_1, \beta_2$

3.2 The Dynamic Nelson Siegel Model

Following Diebold & Li (2006), The NS parameters are recognized as time varying for the yield curve to be time varying. This changes some perspective associated with static Nelson Siegel.

In a time-series environment, the static NS becomes-

$$y(\tau) = \beta_{0t} + \beta_{1t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$

This is a time-series representation of yield given maturity τ on explanatory variables 1, $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right), \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$ with parameters $\beta_{0t}, \beta_{1t}, \beta_{2t}$. For a cross-section time these betas can be considered as parameters but over the time their value changes. As a result, it behaves like a NS yield curve with time varying parameters. But the DNS separates the yield curve into 3 dynamic factors namely $\beta_{0t}, \beta_{1t}, \beta_{2t}$.

DNS is an ideal example of 'dynamic factor model'. In such models a high-dimensional set of variables is driven by much lower dimensional set of dynamics. This is very convenient because it converts seeming incomprehensive high-dimensional situation, such a daily yield across maturities into easily manageable lowdimensional factors, like these latent yield factors. To understand these factors, we must analyze the behavior of the factor loadings.

[1] Source: Diebold, Francis X., and Glenn D. Rudebusch. *Yield curve modeling and forecasting: the dynamic Nelson-Siegel approach*. Princeton University Press, 2013

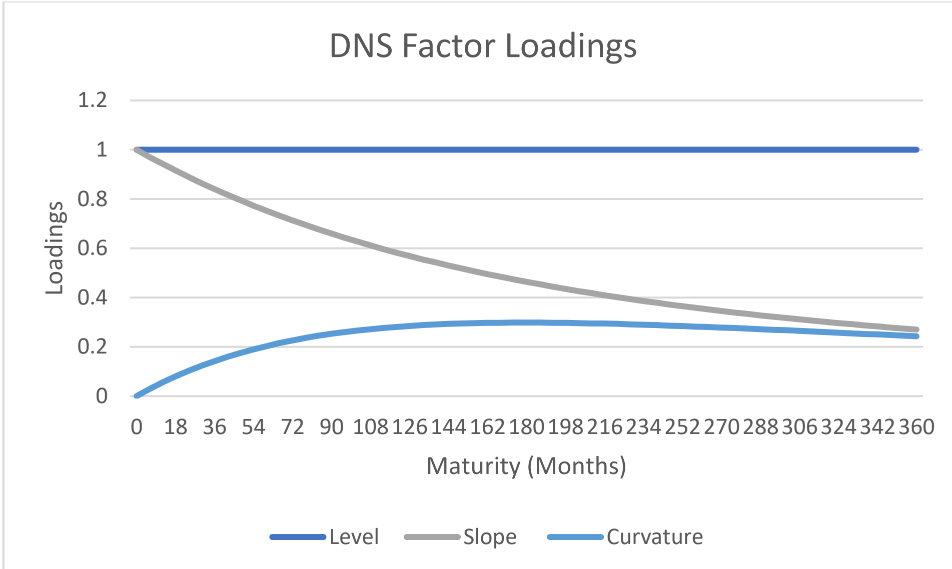

Figure 1 DNS Factor Loading

First look at the factor loading of β_{1t} i. e $\frac{1-e^{-\lambda\tau}}{\lambda\tau}$. It begins at 1 but monotonically decays down to 0. This is often called “short term factor” and it mostly affects the short-term yields. Next, we have the factor loading of β_{2t} i.e. $\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}$. It begins at 0, increases, and then again decay down to 0. This is called “medium term factor” and it affects the medium-term yields. Lastly, we have β_{0t} which is constant at 1 and unlike other two factors, doesn’t decay down to 0. It is called “long term factor” as it affects the long yields.

These factors also determine the shape of yield curve. Such as β_{0t} determine the level of yield curve: a shift in β_{0t}shifts the yield curve parallelly. Similarly, β_{1t} determines the slope of the yield curve: an increase in β_{1t} increase the short run yield but the long run yield remains unchanged. Finally, β_{2t} determines the curvature of the yield curve: an increase in β_{2t} increase medium yield leaving the short and long run yields.

To emphasize the “level, slope & curvature” factor we will rewrite DNS model as

$$y_t(\tau) = l_t + s_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + c_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$

where $t = 1, \dots, T$; $\tau = 1, \dots, N$. Diebold (2006b) provides a state-space representation of model to work with.

$$y_t = \Lambda f_t + \varepsilon_t$$

Where the variables are

$$y_t = \begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{pmatrix}, \qquad f_t = \begin{pmatrix} l_t \\ s_t \\ c_t \end{pmatrix}, \qquad \varepsilon_t = \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{pmatrix}$$

& the parameter matrix Λ

$$\Lambda = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{pmatrix}$$

where, $t = 1, \dots, T$. The stochastic errors $\varepsilon_t(\tau)$ are idiosyncratic or depends on the maturity τ. Hence each yield is driven by both common and idiosyncratic factors.

Next, we have the transition equation to describe the common factor dynamics. We use VAR (1) model such as

$$(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t$$

where the variables are

$$f_t = \begin{pmatrix} l_t \\ s_t \\ c_t \end{pmatrix}, \qquad \eta_t = \begin{pmatrix} \eta_t^l \\ \eta_t^s \\ \eta_t^c \end{pmatrix}$$

and the parameter vector & matrices are

$$\mu_t = \begin{pmatrix} \mu^l \\ \mu^s \\ \mu^c \end{pmatrix}, \qquad A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Here μ is the factor mean and A governs factor dynamics.

We make standard assumptions such as white noise transition and factor disturbances are orthogonal to each other and to the initial state:

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right),$$

$$E(f_0 \acute{\eta}_t) = 0$$

$$E(f_0 \acute{\varepsilon}_t) = 0$$

Moreover, the Diebold-Li model is formulated such that the state equation factor disturbances η_t are correlated, and therefore the corresponding covariance matrix Q is non-diagonal. However, the model imposes diagonality on the covariance matrix H of the observation equation disturbances ε_t such that deviations of observed yields at various maturities are uncorrelated.

4.Estimation

Several procedures have been proposed to estimate the DNS model. We only look at two of the most important of them proposed by Diebold and Li (2006a) and Diebold, Rudebusch and Aruoba (2006b). First one is a two-step model involving VAR estimations. And the other one is a one-step approach involving maximum likelihood estimation using state-space representation in conjunction with Kalman filter approach. But at first, we like to discuss about the estimation of static NS approach.

4.1 Nelson-Siegel in cross section

Though the four-parameter Nelson Siegel curve is essentially non-linear, it may be estimated by iteratively numerical minimization of the sum of square function or the nonlinear least square method. Value of λ is fixed or can be calibrated. $y(\tau)$ is regressed on variables 1, $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right), \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$.

We know that λ determines where the loading factor of l_t is maximized. Now as a medium-term factor l_t maximizes at τ_m or at the medium maturity. Given τ_m, one can identify the corresponding λ_m.

$$\max_{\lambda} \frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}$$

$$Given\ \tau = \tau_m\,, \qquad we\ get\ \lambda = \lambda_m$$

4.2 Two-step DNS

First, we fit the static NS yield curve by OLS for each period. This gives a 3-dimensional time series of estimated coefficients $\{\hat{l}_t, \hat{s}_t, \hat{c}_t\}_{t=1}^{T}$ and corresponding n dimensional series of residual pricing errors. $\{\hat{\varepsilon}_t(\tau_1), \hat{\varepsilon}_t(\tau_2), \dots, \hat{\varepsilon}_t(\tau_N)\}_{t=1}^{T}$. Thus, according to the standard state-space model fashion a N-dimensional time series of yields boils down to a 3-dimensional time series of yield factors.

In the next step we fit a dynamic model to $\{\hat{l}_t, \hat{s}_t, \hat{c}_t\}_{t=1}^{T}$. We use VAR (1) model to fit the dynamic model. This yields the estimates of the dynamic parameters governing the evolution of yield factor or transition equation parameters along with the transition disturbances or the estimates of the factor innovation.

We use the `fitlm` function in MATLAB to get the OLS regression parameters and then ran a VAR (1) regression using the `varm` function to obtain the estimates.

4.3 One-step DNS

The basic insight of these approach is that exploitation to the static state space structure of DNS allows to run all the estimation simultaneously. The One approached and achieved in several ways.

Using Kalman Filter, exact maximum likelihood estimation can be done which delivers the innovations needed for evaluation of Gaussian Pseudo Likelihood. Alternatively, maximum likelihood can be estimated using data augmentation method like expectation maximization (EM) algorithm still using Kalman filter. One can also do a full Bayesian Analysis by exploiting the state space structure of DNS by specifying prior distribution for all coefficients.

In our approach we specify the state space model as

$$x_t = Ax_{t\,\_\,1} + \eta_t$$

$$y_{'t} = y_t - \Lambda\mu = \Lambda x_t + \varepsilon_t$$

where, $x_t = f_t - \mu$

and, $y_{'t} = y_t - \Lambda\mu$

This resembles the form of the state space model

$$x_t = Ax_{t\,\_\,1} + Bu_t$$

$$y_t = Cx_t + D\epsilon_t$$

where, $A = A$, $C = \Lambda$ and we get the error terms as

$\eta_t = Bu_t$

$\varepsilon_t = D\epsilon_t$

Since the disturbances in each model must be the same, the covariance of ηt in the Diebold-Li formulation must equal the covariance of the scaled white noise SSM process But. Similarly, the covariance of et must equal that of the process $D\epsilon_t$. Moreover, since the ut and ϵt disturbances in the SSM formulation are defined as uncorrelated, unit-variance white noise vector processes, their covariance matrices are identity matrices.

Therefore, in an application of the linear transformation property of Gaussian random vectors, the covariances of the Diebold-Li formulation are related to the parameters of the SSM formulation such that

$Q=BB'$

$H=DD'$

Thus, to formulate a state space model all these 4 matrices must be needed. To specify these factors implicitly we must specify a mapping function which maps form input parameter to the model parameters

Since the Diebold-Li model includes a non-zero offset (mean) for each of the three factors, which represents a simple yet common regression component, this example uses a mapping function. Moreover, the mapping function also imposes a symmetry constraint on the covariance matrix $Q=BB'$ and a diagonality constraint of the covariance matrix $H=DD'$, both of which are particularly well-suited to an implicit approach. In addition, the mapping function also allows us to estimate the λ decay rate parameter as well

We use the `ssm` function in the MATLAB to create a state space model.

5.Data

We use daily zero coupon yields for Indian G-Sec and T Bills from Jan 2011 to Mar 2019 estimated by Nelson-Siegel-Svensson Model for maturities of 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120, 126, 132, 138, 144, 150, 156, 162, 168, 174, 180, 186, 192, 198, 204, 210, 216, 222, 228, 234, 240, 246, 252, 258, 264, 270, 276, 282, 288, 294, 300, 306, 312, 318, 324, 330, 336, 342, 348, 354 & 360 months. Fig. 2 gives a 3-dimensional mesh plot of term structure and maturities

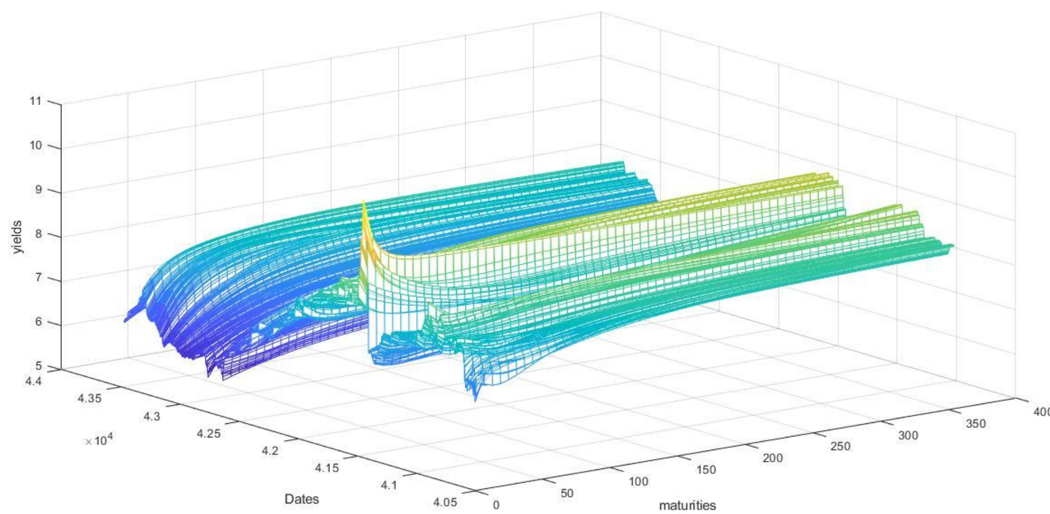

Figure 2 Mesh plot of yield curve over time

From the mesh plot we can see that the shape of yield curve had been changed over the time.

Average Yield seems to increase with the increasing maturity. Also, standard deviation seems to fall and then stay in a same level for rest of the maturities. Summary statistics are reported in table (See Table 9)

6.Empirical Result

In this section we present the results for both the two step and one step DNS model. Also, we are going to present the result of Bai-Perron test. to detect any structural break.

6.1 Two-step DNS

- We fixed the value of λ at 0.01 which maximizes the curvature factor loading at maturity of 180 months or 15 years.
- Given the value of λ we calculate the values of factor loadings for level slope and curvature factors. We run an ordinary least-square regression with these factors on yield or $y(\tau)$.
- This process is repeated for all observed yield curves. This gives a three-dimensional time series of estimates of level, slope & curvature factor.
- We fit a first order auto regressive model to these time series factor obtained. We fit a VAR (1) model, including an additive constant which accounts the mean value of each factors, to maintain the consistency with the state-space model factors.

We get the var coefficient matrix or transition matrix as follows

	(1)	(2)	(3)
VARIABLES	Beta_0	Beta_1	Beta_2
L.Beta_0	1.003***	-0.00365	-0.0206
	(0.00455)	(0.00406)	(0.0134)
L.Beta_1	0.00318	0.987***	0.00465
	(0.00524)	(0.00467)	(0.0154)
L.Beta_2	0.00493**	-0.00143	0.977***
	(0.00231)	(0.00206)	(0.00677)
Constant	-0.0280	0.0193	0.193*
	(0.0375)	(0.0334)	(0.110)

Table 1 VAR coefficient regression

These coefficients are obtained by running a first order vector autoregression model for all three coefficients. It turns out that all the coefficients for own terms are significant.

Two-Step State Disturbance Covariance Matrix (Q)		
0.0086	-0.0049	-0.0202
-0.0049	0.0066	0.0083
-0.0202	0.0083	0.0745

Table 2 Twostep state disturbance covariance matrix

6.2 One step state-space model

- A parameter mapping function is specified to map the parameter vector to the state space model. This mapping function maps a parameter vector to SSM model parameters, deflates the observations to account for the means of each factor, and imposes constraints on the covariance matrices.
- The maximum likelihood estimation (MLE) of SSM models via the Kalman filter is notoriously sensitive to the initial parameter values. So, we use the results of the two-step approach to initialize the estimation. the matrix *A* of the SSM model is set to the estimated 3-by-3 AR coefficient matrix of the VAR (1) model stacked in a column-wise manner into the first 9 elements of the column vector.
- The matrix *B* of the SSM model is a 3-by-3 matrix constrained such that $Q=BB'$, and in what follows the estimate of *B* is the lower Cholesky factor of *Q*. Therefore, to ensure that *Q* is symmetric, positive definite, and allows for non-zero off-diagonal covariances, 6 elements associated with the lower Cholesky factor of *Q* must be allocated in the initial parameter column vector. We initialize the elements of the initial parameter vector with the square root of the estimated innovation variances of the VAR (1) model. the initial parameter vector is arranged such that the elements of *B* along and below the main diagonal are stacked in a column-wise manner.
- Since the covariance matrix *H* in the Diebold-Li formulation is diagonal, the matrix *D* of the SSM model is also constrained to be diagonal such that $H=DD'$. Therefore, the elements of the initial parameter vector associated with *D* are set to the square root of the diagonal elements of the sample covariance matrix of the residuals of the VAR (1) model, one such element for each maturity of the input yield data again stacked in a column-wise manner.
- Finally, the elements of the initial parameter vector associated with the factor means are simply set to the sample averages of the regression coefficients obtained by OLS in the first step of the original two-step approach.

	SSM State Transition Matrix	
1.0017	0.0040	0.0059
-0.0016	0.9836	0.0027
-0.0235	-0.0073	0.9739

Table 3 One-Step state transition Matrix

We also get the disturbance matrix as follows

	SSM State Disturbance Covariance Matrix	
0.0100	-0.0008	-0.0042
-0.0008	0.0118	-0.0026
-0.0042	-0.0026	0.0810

Table 4 One-state SSM State disturbance covariance matrix

6.3 Comparison of two models

From the previous two analysis we find that the estimated covariance matrices are in relatively close agreement, and that the estimated variance increases as we proceed from level to slope to curvature along the main diagonal. The factor means obtained from two step model are 8.07494, -0.7796, 0.9875 for level, slope and curvature factor respectively. This almost accurately resembles the state space factor means 8.07494, -0.7797, 0.9874 respectively for the factors.

Now we want to see the movement of the factors in these two models. We plotted estimated factors for both the models and plotted them against time. For all the three factors: level, slope and curvature, we get a very similar movements from both the model estimations.

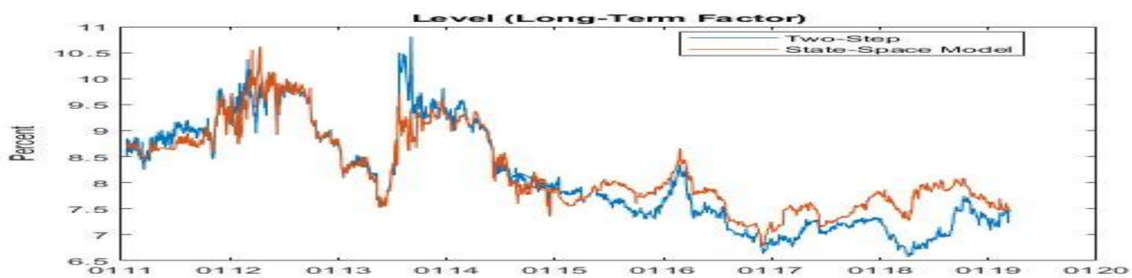

Figure 3 Level Factor

Figure 4 Slope Factor

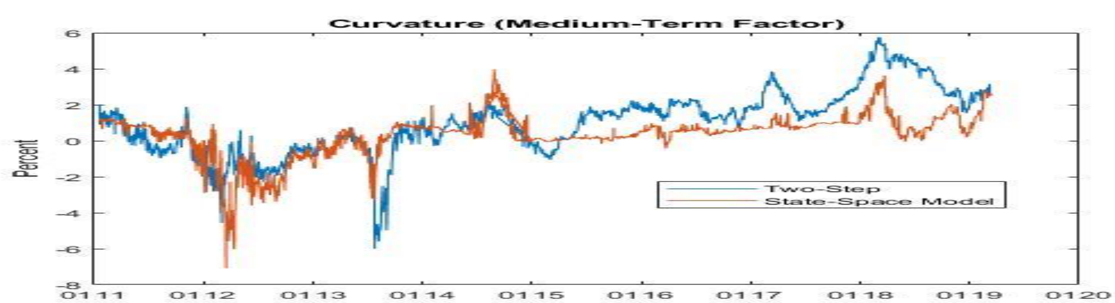

Figure 5 Curvature Factor

From the state space model, we get the estimated decay rate $\lambda = 0.0110$ which translates to 13.5 years of maturity and very closely resembles to the fixed $\lambda = 0.01$. λ determines at which point the curvature of yield curve is maximized. From two step model we see that curvature is maximized at the maturity of 180 months or 15 years. From state space model it is maximized at 162 months or 13.5 years of maturity. Here we calculated the curvature factor loading and plotted them against maturity.

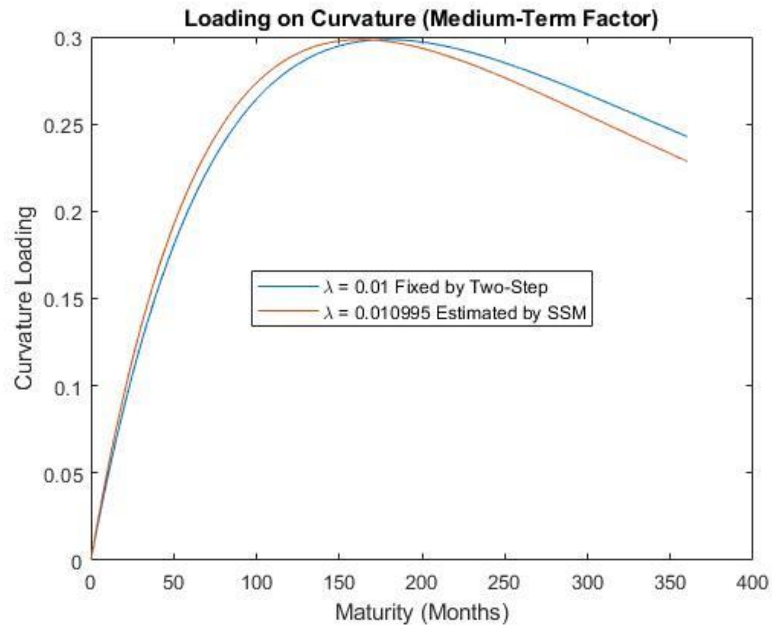

Figure 6 Loading on curvature

As a final in-sample performance comparison, we now compare the means and standard deviations of observation equation residuals of the two approaches. The results are expressed in basis points (bps).

6.4 Test for structural break

From the visual inspection of the mesh plot of yields, there is a clear shift in the shape of yield curve over a certain period. This is also clearly visible in the of long-run intercept and short run slope factor. All these clearly indicates towards structural break present in the time series data.

To find and to test the structural break for individual factors we regress each factor over time. We then use Bai-Perron (2003)'s test for L+1 vs L sequentially determined structural breaks. For L structural breaks there should be L+1 regime exists. The null hypothesis would be the factor equals to the factor value of regression. Here the value of F- statistic is much higher than the critical values of F at 0.05 level. So, the null hypothesis is rejected, and factor values are not equal to the estimated value.

For β_0 we can find only 3 structural breaks to be significant. These 3 structural breaks are 17 October,2012, 10 June 2014, 27 June 2016.

Sequential F-statistic determined breaks:			3
Break Test	F-statistic	Scaled F-statistic	Critical Value**
0 vs. 1 *	5798.218	5798.218	8.58
1 vs. 2 *	596.9579	596.9579	10.13
2 vs. 3 *	200.2666	200.2666	11.14
3 vs. 4	7.698911	7.698911	11.83

* Significant at the 0.05 level.
** Bai-Perron (Econometric Journal, 2003) critical values.

Table 5 Structural Break test beta0

For β_1 we can find 4 structural breaks to be significant. These 4 structural breaks are 19 October,2012, 5 June 2014, 1 January 2016, 22 December 2017.

Sequential F-statistic determined breaks:			4
Break Test	F-statistic	Scaled F-statistic	Critical Value**
0 vs. 1 *	1134.428	1134.428	8.58
1 vs. 2 *	201.3246	201.3246	10.13
2 vs. 3 *	882.7196	882.7196	11.14
3 vs. 4 *	557.1956	557.1956	11.83
4 vs. 5	0.000000	0.000000	12.25

* Significant at the 0.05 level.
** Bai-Perron (Econometric Journal, 2003) critical values.

Table 6 Structural break test beta1

For β_2 we can find 4 structural breaks to be significant. These 4 structural breaks are 12 April,2012, 24 September 2014, 11 June 2015 & 22 November 2017.

Sequential F-statistic determined breaks:			4
Break Test	F-statistic	Scaled F-statistic	Critical Value**
0 vs. 1 *	2415.795	2415.795	8.58
1 vs. 2 *	567.4833	567.4833	10.13
2 vs. 3 *	435.2959	435.2959	11.14
3 vs. 4 *	139.2807	139.2807	11.83
4 vs. 5	9.102824	9.102824	12.25

* Significant at the 0.05 level.
** Bai-Perron (Econometric Journal, 2003) critical values.

Table 7 Structural Break test beta1

Thus, the level and slope factor have similar patterns in structural break and their breaks occur a nearly in the same dates. These points can be identified as a change I regime or policy.

6.5 Forecast

To forecast the yields and standard errors, we use two methods: the minimum mean square error (MMSE) forecasting and Monte Carlo simulation capabilities of the SSM functionality.

Since the Diebold-Li model depends only on the estimated factors, the yield curve is forecasted by forecasting the factors. However, as discussed above, when forecasting yields we must compensate for the offset adjustment made during SSM estimation, and so must use the deflated yields upon which the estimation is based.

Using the deflated yields, we now call the forecast function to compute the MMSE forecasts of the deflated yields 1,2, ..., 12 months into the future. The actual forecasted yields are then computed by adding the estimated offset $C\mu$ to the deflated counterparts.

Now that the deterministic MMSE forecasts have been computed using the forecast function, we now illustrate how the same results may be approximated using the simulate function.

Before performing Monte Carlo simulation, however, we must first initialize the mean vector and covariance matrix of the initial states (factors) of the fitted SSM model to ensure the simulation begins with the most recent information available. To do this, the following code segment calls the smooth function to obtain the smoothed states obtained by backward recursion.

Since the following step initializes the fitted mean and covariance to that available at the very end of the historical data set, state smoothing obtained from the smooth function, using information from the entire data set, is equivalent to state filtering obtained from the filter function, using only information which precedes the last observation

Now that the initial mean and covariance of the states have been set, compute out-of-sample forecasts via Monte Carlo simulation.

In the following code segment, each sample path represents the future evolution of a simulated yield curve over a 12-month forecast horizon. The simulation is repeated 100,000 times.

In a manner like the forecasts computed previously, the simulated yield curve matrix has a row for each future period in the forecast horizon (12 in this example), and a column for each maturity. However, in contrast to the MMSE forecast matrix, the simulated yield curve matrix has a third dimension to store the 100,000 simulated paths.

Again, notice that the deflated yields are simulated, and then post-processed to account for the factor offsets.

Now that the yields have been simulated, compute the sample mean and standard deviation of the 100,000 trials. These statistics are the sample analog of the MMSE forecasts and standard errors. To facilitate the calculation of sample means and standard deviations

Now visually compare the MMSE forecasts and corresponding standard errors obtained from the forecast function along with those obtained from the simulate function via Monte Carlo. The results are virtually identical. Also, the forecasted yields seem to be normally distributed.

Of course, the additional benefit of Monte Carlo simulation is that it allows for a more detailed analysis of the distribution of yields beyond the mean and standard error, and in turn provides additional insight into how that distribution affects the distribution of other variables dependent upon it. For example, in the insurance industry the simulation of yield curves is commonly used to assess the distribution of profits and losses associated with annuities and pension contracts.

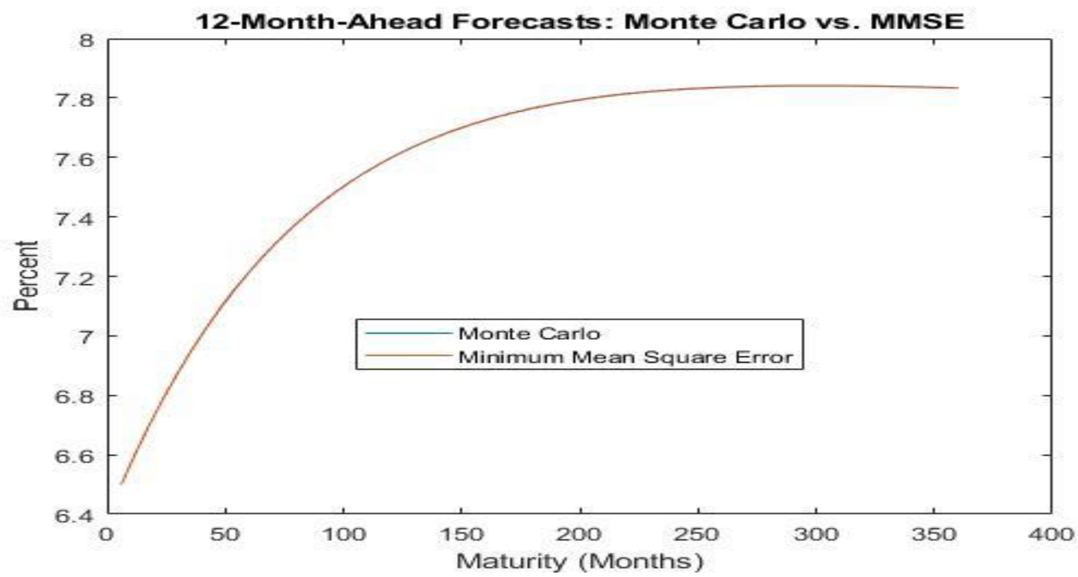

Figure 7 Forecast of yield

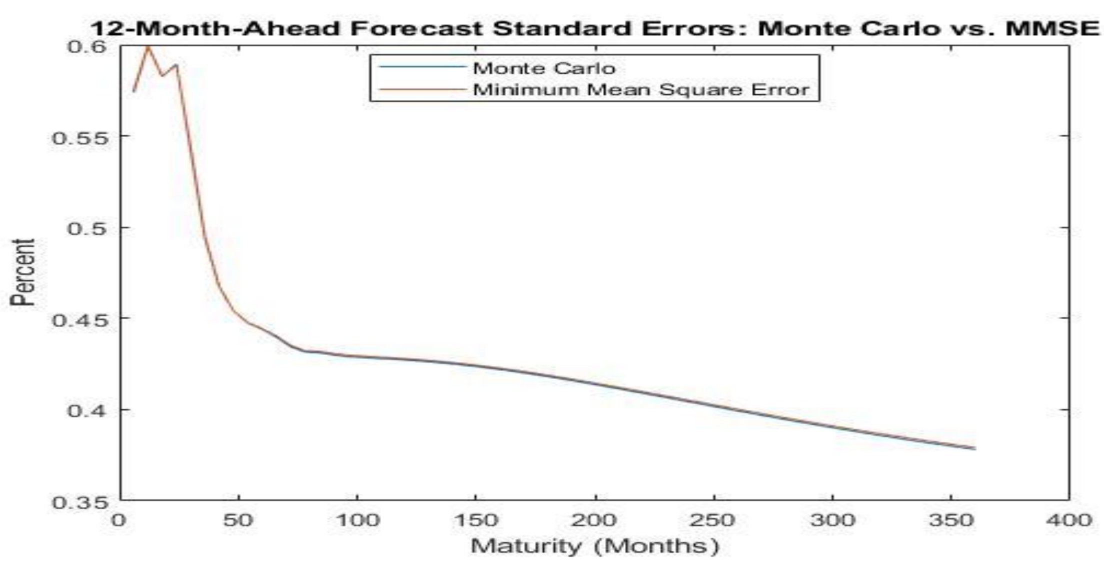

Figure 8 Forecast of standard error

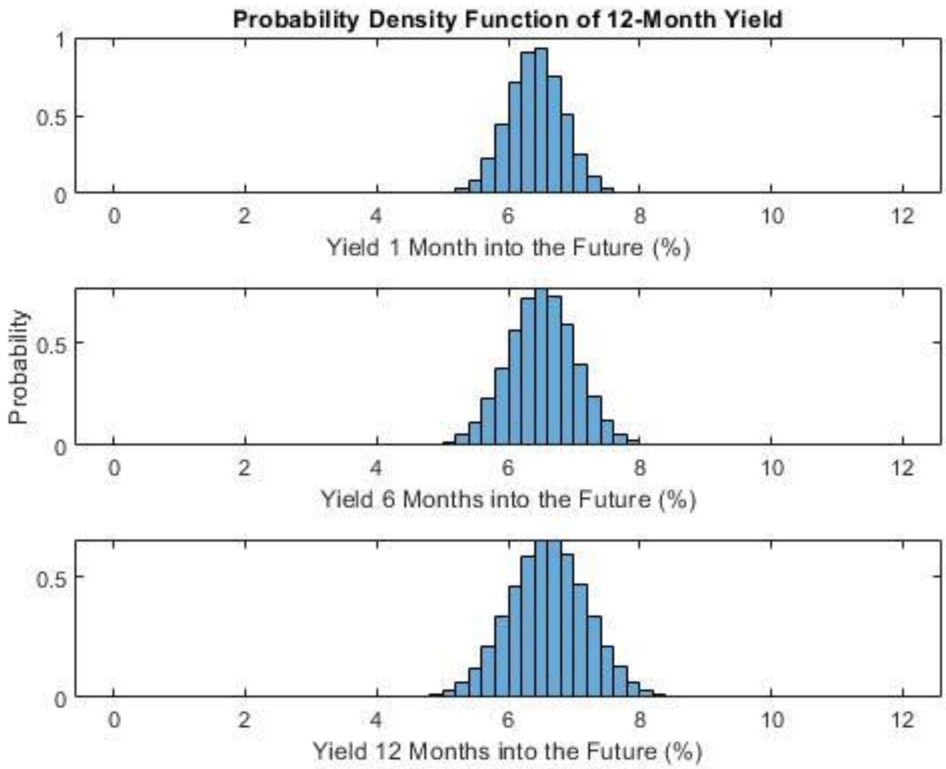

Figure 9 Forecasted yields' frequency distribution

Conclusion

In this paper we estimated the dynamic yield curve for Indian market and forecast based on that dynamic model. We found the in-sample dynamics through the transition matrix from both the two step and one step state-space model. We also calculated the out of sample forecast through minimum mean square error and Monte Carlo simulation of both the yields and the standard errors. These are the standard purpose for a dynamic model to be used instead of a cross sectional model

Also, we tried to find the possible structural breaks in the model parameters. Given the different parameter dynamics we get different structural break for different parameters. But this break points are gathered around a specific period suggesting regime switching.

This gives us a better model to analyze the dynamics of term structure. With high frequency daily data this gives us better accuracy than monthly data.

As we find some structural breaks in the model we could further test for regime switching model to account for the policy changes in RBI.

Appendix

Maturities	mean	sd	min	max	ACF (1)
0.5	7.411	0.936	5.710	10.78	0.996582
1.0	7.415	0.818	5.803	9.899	0.99613
1.5	7.440	0.749	5.859	9.613	0.995687
2.0	7.478	0.704	5.912	9.475	0.995658
2.5	7.520	0.672	5.960	9.399	0.995867
3.0	7.564	0.648	6.005	9.354	0.996109
3.5	7.607	0.630	6.048	9.326	0.996285
4.0	7.647	0.616	6.087	9.307	0.996378
4.5	7.684	0.605	6.124	9.294	0.996406
5.0	7.718	0.596	6.158	9.284	0.996391
5.5	7.750	0.589	6.190	9.277	0.996356
6.0	7.778	0.583	6.220	9.271	0.996314
6.5	7.804	0.578	6.248	9.267	0.996274
7.0	7.827	0.575	6.274	9.263	0.996239
7.5	7.849	0.572	6.299	9.260	0.996213
8.0	7.868	0.569	6.322	9.258	0.996194
8.5	7.886	0.567	6.344	9.255	0.996183
9.0	7.902	0.566	6.364	9.253	0.996178
9.5	7.917	0.565	6.383	9.252	0.996179
10.0	7.930	0.564	6.401	9.250	0.996184
10.5	7.943	0.563	6.418	9.249	0.996193
11.0	7.955	0.562	6.434	9.247	0.996205
11.5	7.966	0.562	6.449	9.246	0.996221
12.0	7.976	0.562	6.464	9.245	0.996239
12.5	7.985	0.562	6.477	9.244	0.996259
13.0	7.994	0.562	6.490	9.243	0.996281
13.5	8.002	0.562	6.502	9.242	0.996305
14.0	8.010	0.563	6.514	9.241	0.99633
14.5	8.017	0.563	6.524	9.245	0.996357

15.0	8.024	0.563	6.535	9.256	0.996386
15.5	8.031	0.564	6.545	9.267	0.996415
16.0	8.037	0.565	6.554	9.278	0.996446
16.5	8.043	0.565	6.563	9.287	0.996478
17.0	8.048	0.566	6.571	9.297	0.99651
17.5	8.054	0.567	6.579	9.305	0.996543
18.0	8.059	0.568	6.587	9.314	0.996577
18.5	8.064	0.569	6.594	9.321	0.996611
19.0	8.069	0.569	6.601	9.329	0.996645
19.5	8.073	0.570	6.608	9.336	0.996679
20.0	8.077	0.572	6.614	9.343	0.996712
20.5	8.082	0.573	6.620	9.349	0.996746
21.0	8.086	0.574	6.626	9.355	0.996778
21.5	8.090	0.575	6.632	9.361	0.99681
22.0	8.093	0.576	6.637	9.366	0.996842
22.5	8.097	0.577	6.642	9.372	0.996872
23.0	8.101	0.579	6.647	9.377	0.996901
23.5	8.104	0.580	6.652	9.382	0.996928
24.0	8.107	0.581	6.657	9.386	0.996954
24.5	8.111	0.583	6.661	9.391	0.996979
25.0	8.114	0.584	6.665	9.395	0.997001
25.5	8.117	0.585	6.669	9.399	0.997022
26.0	8.120	0.587	6.673	9.403	0.997041
26.5	8.123	0.588	6.677	9.407	0.997058
27.0	8.126	0.590	6.681	9.411	0.997073
27.5	8.128	0.591	6.684	9.414	0.997086
28.0	8.131	0.593	6.687	9.417	0.997096
28.5	8.134	0.594	6.691	9.421	0.997104
29.0	8.136	0.596	6.694	9.424	0.99711
29.5	8.139	0.597	6.697	9.427	0.997113
30.0	8.141	0.599	6.700	9.430	0.997114

Table 8 Summary Statistics

Maturity (Months)	State Space Model Mean (bps)	State Space Model SD (bps)	Two Step model Mean (bps)	Two Step model SD (bps)
6.0000	-15.1180	52.5232	4.2136	20.4631
12.0000	-17.8747	39.9011	-0.2704	8.2027
18.0000	-18.2714	31.8988	-2.2737	3.7401
24.0000	-17.2942	26.0154	-2.7897	4.5296
30.0000	-15.6462	21.3690	-2.5284	5.8172
36.0000	-13.7440	17.5828	-1.9127	6.5076
42.0000	-11.8237	14.4565	-1.1850	6.7061
48.0000	-10.0123	11.8611	-0.4777	6.5664
54.0000	-8.3711	9.7022	0.1424	6.2024
60.0000	-6.9229	7.9060	0.6473	5.6946
66.0000	-5.6689	6.4125	1.0311	5.1001
72.0000	-4.5983	5.1726	1.3000	4.4608
78.0000	-3.6947	4.1455	1.4662	3.8078
84.0000	-2.9393	3.2971	1.5445	3.1658
90.0000	-2.3133	2.5991	1.5498	2.5569
96.0000	-1.7990	2.0272	1.4964	2.0045
102.0000	-1.3800	1.5614	1.3973	1.5422
108.0000	-1.0418	1.1846	1.2638	1.2243
114.0000	-0.7716	0.8822	1.1057	1.1161
120.0000	-0.5581	0.6421	0.9314	1.2174
126.0000	-0.3919	0.4538	0.7480	1.4412
132.0000	-0.2646	0.3085	0.5611	1.7065
138.0000	-0.1692	0.1986	0.3754	1.9704
144.0000	-0.0997	0.1179	0.1948	2.2133
150.0000	-0.0511	0.0609	0.0222	2.4267
156.0000	-0.0191	0.0229	-0.1401	2.6067
162.0000	-0.0000	0.0000	-0.2902	2.7520
168.0000	0.0091	0.0111	-0.4268	2.8628
174.0000	0.0108	0.0133	-0.5491	2.9397
180.0000	0.0072	0.0089	-0.6564	2.9839
186.0000	0.0000	0.0000	-0.7485	2.9970
192.0000	-0.0093	0.0117	-0.8251	2.9803
198.0000	-0.0195	0.0247	-0.8865	2.9357
204.0000	-0.0296	0.0378	-0.9327	2.8647
210.0000	-0.0390	0.0500	-0.9641	2.7690
216.0000	-0.0468	0.0606	-0.9812	2.6504
222.0000	-0.0527	0.0687	-0.9843	2.5104
228.0000	-0.0563	0.0739	-0.9741	2.3507
234.0000	-0.0572	0.0756	-0.9511	2.1728
240.0000	-0.0553	0.0736	-0.9159	1.9783
246.0000	-0.0505	0.0676	-0.8691	1.7688
252.0000	-0.0425	0.0573	-0.8114	1.5460
258.0000	-0.0315	0.0427	-0.7432	1.3118
264.0000	-0.0173	0.0236	-0.6653	1.0690
270.0000	0.0000	0.0000	-0.5783	0.8220
276.0000	0.0203	0.0281	-0.4827	0.5814
282.0000	0.0436	0.0608	-0.3791	0.3829
288.0000	0.0698	0.0979	-0.2681	0.3416
294.0000	0.0987	0.1393	-0.1502	0.5114
300.0000	0.1304	0.1851	-0.0259	0.7700
306.0000	0.1645	0.2351	0.1042	1.0588
312.0000	0.2012	0.2891	0.2397	1.3618
318.0000	0.2401	0.3471	0.3802	1.6734
324.0000	0.2813	0.4090	0.5251	1.9910
330.0000	0.3245	0.4745	0.6742	2.3131
336.0000	0.3696	0.5436	0.8269	2.6387
342.0000	0.4166	0.6162	0.9829	2.9669
348.0000	0.4652	0.6919	1.1419	3.2970
354.0000	0.5155	0.7708	1.3036	3.6286
360.0000	0.5672	0.8527	1.4676	3.9612

Table 9 Residual mean and standard deviation

MMSE	1	2	3	4	5	6	7	8	9	10	11	12
0.5	6.301348	6.324755	6.340611	6.360731	6.378363	6.395749	6.416581	6.4308	6.446681	6.465628	6.48399	6.498768
1	6.418767	6.437146	6.454965	6.472381	6.48964	6.507855	6.524643	6.54238	6.556929	6.573412	6.590608	6.603814
1.5	6.527852	6.543777	6.561261	6.57736	6.593422	6.609567	6.627265	6.640982	6.654726	6.670464	6.6864	6.702317
2	6.629132	6.644295	6.66026	6.675646	6.69258	6.707347	6.722118	6.737277	6.748701	6.762245	6.778734	6.792534
2.5	6.719785	6.736298	6.751906	6.76902	6.781427	6.794942	6.811808	6.825584	6.835625	6.848469	6.862798	6.87619
3	6.806718	6.82228	6.835115	6.849972	6.865328	6.879396	6.891001	6.904202	6.91492	6.928911	6.940669	6.954289
3.5	6.888092	6.901648	6.914805	6.928292	6.940477	6.954439	6.967068	6.979166	6.991108	7.002014	7.015584	7.026294
4	6.961472	6.976031	6.98759	7.000998	7.012098	7.024929	7.037213	7.048371	7.05963	7.071909	7.08273	7.094916
4.5	7.030878	7.044609	7.055109	7.067798	7.07911	7.091675	7.102368	7.113267	7.124404	7.13399	7.14587	7.15672
5	7.095128	7.108562	7.119229	7.130071	7.140851	7.151601	7.163151	7.173349	7.18249	7.19343	7.203606	7.213294
5.5	7.154813	7.166091	7.17679	7.187869	7.197642	7.208679	7.219059	7.228642	7.237983	7.247728	7.257804	7.267478
6	7.209715	7.220623	7.230544	7.241308	7.25044	7.261193	7.270811	7.280132	7.289143	7.297487	7.307957	7.31672
6.5	7.260394	7.27119	7.280522	7.290547	7.299557	7.309411	7.318651	7.327521	7.336043	7.344559	7.353897	7.362305
7	7.307395	7.317512	7.326644	7.336404	7.344986	7.354088	7.362943	7.371406	7.379604	7.387514	7.396797	7.405073
7.5	7.351013	7.360723	7.369249	7.378508	7.386686	7.395657	7.404246	7.412082	7.419849	7.427494	7.43605	7.443945
8	7.391187	7.400463	7.408512	7.417415	7.4253	7.433785	7.442014	7.449599	7.45679	7.464224	7.472348	7.480057
8.5	7.42838	7.437511	7.444952	7.453382	7.460925	7.469145	7.476908	7.484375	7.491229	7.49835	7.506059	7.513272
9	7.462677	7.47125	7.478549	7.486741	7.49381	7.501659	7.509194	7.516266	7.522906	7.529504	7.537077	7.544189
9.5	7.494251	7.502541	7.509522	7.517434	7.524118	7.531704	7.53881	7.545715	7.552041	7.558379	7.565588	7.572507
10	7.523384	7.531289	7.537993	7.545597	7.552067	7.559373	7.566201	7.572793	7.578848	7.584949	7.592025	7.598626
10.5	7.550151	7.55779	7.564265	7.571582	7.577725	7.584829	7.591399	7.597763	7.603601	7.60941	7.616246	7.622611
11	7.574728	7.582181	7.588389	7.595436	7.601367	7.608235	7.614577	7.62067	7.626282	7.631932	7.638508	7.644677
11.5	7.597357	7.604546	7.610532	7.617353	7.623068	7.629706	7.635831	7.641731	7.647121	7.652583	7.658967	7.664942
12	7.618118	7.625081	7.630858	7.637466	7.642981	7.649412	7.655328	7.661033	7.666241	7.671501	7.677699	7.683497
12.5	7.637127	7.643893	7.649479	7.655896	7.661222	7.667465	7.673199	7.678725	7.683758	7.688849	7.694875	7.700508
13	7.654527	7.661106	7.666517	7.672749	7.677911	7.683976	7.689539	7.694899	7.699781	7.704716	7.710581	7.716065
13.5	7.67043	7.676835	7.682085	7.688153	7.693156	7.699062	7.70447	7.709674	7.714413	7.719203	7.724922	7.730273
14	7.684941	7.691185	7.696288	7.702205	7.707062	7.712821	7.718085	7.723149	7.727756	7.732413	7.737998	7.743226
14.5	7.698159	7.704254	7.709222	7.714999	7.719724	7.725346	7.730481	7.735413	7.7399	7.744438	7.749899	7.755012
15	7.710174	7.716134	7.720978	7.726628	7.731231	7.736729	7.741741	7.746557	7.750933	7.755361	7.760707	7.765718

15.5	7.721074	7.726909	7.731641	7.737174	7.741665	7.74705	7.751951	7.756657	7.760933	7.765261	7.770503	7.775419
16	7.73094	7.73666	7.741288	7.746714	7.751104	7.756384	7.761186	7.765791	7.769977	7.774213	7.779359	7.784189
16.5	7.739847	7.745458	7.749996	7.755325	7.759623	7.764806	7.769516	7.774029	7.778131	7.782283	7.787343	7.792093
17	7.74786	7.753377	7.75783	7.76307	7.767284	7.772378	7.777002	7.781435	7.785462	7.789541	7.794517	7.7992
17.5	7.755051	7.760476	7.764852	7.770011	7.774146	7.779165	7.783717	7.788071	7.792028	7.796042	7.800944	7.805561
18	7.761478	7.766828	7.771129	7.776218	7.780281	7.785225	7.789707	7.793991	7.797892	7.801838	7.806678	7.811235
18.5	7.767192	7.772465	7.776707	7.781729	7.785738	7.790612	7.795033	7.799252	7.803096	7.806988	7.811767	7.816274
19	7.772253	7.777468	7.781654	7.786604	7.790565	7.79537	7.799741	7.803896	7.8077	7.811539	7.816265	7.820726
19.5	7.776718	7.781856	7.785995	7.790899	7.7948	7.799564	7.803869	7.807983	7.811737	7.815536	7.820211	7.824629
20	7.780613	7.785702	7.789791	7.794643	7.798505	7.803218	7.807486	7.811548	7.815258	7.819024	7.823651	7.828031
20.5	7.783982	7.789025	7.793078	7.797892	7.80171	7.806371	7.810599	7.814625	7.8183	7.822025	7.826623	7.830965
21	7.786884	7.791879	7.795898	7.800672	7.804453	7.80908	7.813272	7.817257	7.820901	7.824594	7.829155	7.833473
21.5	7.789338	7.794297	7.798282	7.803022	7.806775	7.81136	7.815528	7.819477	7.823099	7.826767	7.831289	7.835583
22	7.79139	7.796311	7.800268	7.804979	7.808705	7.81326	7.817397	7.821318	7.824915	7.828559	7.833056	7.837327
22.5	7.793065	7.797955	7.801888	7.806571	7.810274	7.814801	7.818914	7.82281	7.826388	7.830011	7.834483	7.838735
23	7.794393	7.799254	7.803168	7.80783	7.811511	7.816013	7.820108	7.823979	7.827543	7.831148	7.835597	7.839834
23.5	7.795407	7.800245	7.804137	7.80878	7.812448	7.816921	7.820997	7.824853	7.828402	7.831995	7.836428	7.840648
24	7.796126	7.800942	7.804821	7.809447	7.813099	7.817555	7.821613	7.825461	7.828991	7.83258	7.836987	7.841198
24.5	7.796575	7.801366	7.805243	7.809858	7.813494	7.817931	7.821985	7.825805	7.829326	7.832905	7.837322	7.841513
25	7.79678	7.801574	7.805428	7.810014	7.813652	7.818094	7.822125	7.825935	7.829454	7.833012	7.837407	7.841593
25.5	7.796755	7.801513	7.805367	7.809976	7.813615	7.818024	7.822053	7.825839	7.829339	7.832905	7.837303	7.84149
26	7.796544	7.801279	7.805149	7.809702	7.813348	7.817746	7.821769	7.825568	7.829091	7.832648	7.837021	7.841193
26.5	7.796101	7.800826	7.804714	7.809281	7.812897	7.817306	7.82135	7.825107	7.828596	7.832174	7.836564	7.840739
27	7.795518	7.800273	7.804121	7.808692	7.812348	7.816702	7.820728	7.824515	7.828001	7.831582	7.835952	7.840127
27.5	7.794799	7.79949	7.803381	7.80794	7.811569	7.815972	7.819972	7.823763	7.827256	7.830828	7.835186	7.839374
28	7.793878	7.79861	7.802463	7.807014	7.810666	7.815048	7.819094	7.822807	7.826333	7.829928	7.834313	7.838482
28.5	7.792855	7.797588	7.801486	7.806062	7.809658	7.814069	7.818094	7.82188	7.825362	7.828916	7.833292	7.837521
29	7.791749	7.796475	7.800323	7.804956	7.808536	7.812952	7.816974	7.820659	7.824244	7.827833	7.832217	7.836422
29.5	7.79049	7.795198	7.799099	7.80365	7.807322	7.811723	7.815668	7.819499	7.823045	7.826658	7.830995	7.83516
30	7.789118	7.793978	7.797725	7.802329	7.806057	7.810359	7.814435	7.818158	7.821761	7.825358	7.829727	7.833861

M C	1	2	3	4	5	6	7	8	9	10	11	12
0.5	6.30235	6.321928	6.34114	6.359993	6.378494	6.396649	6.414465	6.431947	6.449102	6.465936	6.482455	6.498665
1	6.418655	6.437088	6.455177	6.472926	6.490344	6.507434	6.524204	6.54066	6.556807	6.572651	6.588198	6.603453
1.5	6.526738	6.544106	6.561149	6.577871	6.59428	6.61038	6.626178	6.641678	6.656887	6.67181	6.686453	6.700819
2	6.62715	6.643527	6.659596	6.675363	6.690833	6.706012	6.720905	6.735517	6.749853	6.76392	6.777721	6.791262
2.5	6.720404	6.73586	6.751024	6.765901	6.780498	6.79482	6.808871	6.822657	6.836182	6.849452	6.862471	6.875244
3	6.806983	6.82158	6.835902	6.849953	6.863738	6.877263	6.890531	6.903549	6.91632	6.928849	6.941142	6.953201
3.5	6.887333	6.901133	6.914672	6.927954	6.940984	6.953768	6.966309	6.978613	6.990683	7.002525	7.014141	7.025538
4	6.961875	6.974934	6.987744	7.000312	7.012641	7.024736	7.036602	7.048242	7.059661	7.070863	7.081852	7.092633
4.5	7.031001	7.04337	7.055504	7.067408	7.079086	7.090541	7.101779	7.112803	7.123617	7.134225	7.144632	7.15484
5	7.095076	7.106805	7.118311	7.129598	7.14067	7.151531	7.162186	7.172637	7.18289	7.192947	7.202813	7.212491
5.5	7.154442	7.165576	7.176499	7.187213	7.197723	7.208033	7.218146	7.228067	7.237798	7.247344	7.256708	7.265893
6	7.209419	7.220001	7.230381	7.240564	7.250553	7.260351	7.269962	7.27939	7.288637	7.297709	7.306607	7.315336
6.5	7.260304	7.270374	7.280252	7.289941	7.299446	7.308769	7.317914	7.326885	7.335684	7.344316	7.352783	7.361088
7	7.307377	7.316972	7.326383	7.335615	7.344671	7.353553	7.362267	7.370814	7.379198	7.387421	7.395488	7.403401
7.5	7.350897	7.360051	7.36903	7.377838	7.386477	7.394952	7.403265	7.41142	7.419418	7.427264	7.43496	7.44251
8	7.391108	7.399853	7.408431	7.416846	7.4251	7.433197	7.441139	7.44893	7.456572	7.464068	7.471421	7.478634
8.5	7.428236	7.436602	7.444809	7.45286	7.460757	7.468503	7.476102	7.483556	7.490868	7.498041	7.505077	7.511978
9	7.462493	7.470509	7.478372	7.486085	7.493652	7.501074	7.508356	7.515498	7.522505	7.529378	7.53612	7.542734
9.5	7.494076	7.501767	7.509313	7.516714	7.523975	7.531098	7.538085	7.54494	7.551664	7.55826	7.564731	7.571079
10	7.523171	7.530562	7.537813	7.544926	7.551904	7.55875	7.565466	7.572054	7.578518	7.584858	7.591079	7.597181
10.5	7.549949	7.557063	7.564042	7.570888	7.577605	7.584195	7.59066	7.597003	7.603225	7.60933	7.615319	7.621195
11	7.574571	7.581429	7.588157	7.594757	7.601233	7.607587	7.61382	7.619936	7.625936	7.631823	7.637599	7.643266
11.5	7.597188	7.603809	7.610305	7.616679	7.622932	7.629068	7.635088	7.640994	7.64679	7.652476	7.658055	7.66353
12	7.61794	7.624342	7.630624	7.636788	7.642837	7.648771	7.654595	7.660309	7.665915	7.671417	7.676815	7.682112
12.5	7.636956	7.643158	7.649243	7.655214	7.661073	7.666823	7.672465	7.678002	7.683435	7.688766	7.693998	7.699132
13	7.65436	7.660376	7.66628	7.672073	7.677759	7.683338	7.688814	7.694187	7.699461	7.704636	7.709715	7.714699
13.5	7.670265	7.676111	7.681847	7.687478	7.693003	7.698426	7.703748	7.708972	7.714099	7.719131	7.724069	7.728916
14	7.684777	7.690466	7.696049	7.701529	7.706908	7.712188	7.717369	7.722456	7.727448	7.732348	7.737158	7.741879
14.5	7.697996	7.70354	7.708983	7.714325	7.719569	7.724717	7.72977	7.73473	7.739599	7.744379	7.749071	7.753677
15	7.710013	7.715425	7.720739	7.725955	7.731076	7.736102	7.741037	7.745882	7.750639	7.755308	7.759892	7.764393

15.5	7.720914	7.726206	7.731401	7.736502	7.74151	7.746426	7.751253	7.755993	7.760646	7.765215	7.769701	7.774105
16	7.730781	7.735962	7.741049	7.746044	7.750949	7.755765	7.760493	7.765136	7.769696	7.774173	7.778569	7.782886
16.5	7.739687	7.744768	7.749756	7.754655	7.759465	7.764189	7.768828	7.773384	7.777857	7.782251	7.786566	7.790803
17	7.747704	7.752692	7.75759	7.762401	7.767126	7.771766	7.776323	7.780799	7.785195	7.789513	7.793754	7.797919
17.5	7.754895	7.759799	7.764616	7.769347	7.773994	7.778558	7.783041	7.787445	7.79177	7.796019	7.800193	7.804293
18	7.761321	7.766149	7.770892	7.77555	7.780127	7.784622	7.789038	7.793377	7.797639	7.801825	7.805939	7.80998
18.5	7.76704	7.771799	7.776474	7.781067	7.78558	7.790013	7.794368	7.798648	7.802852	7.806983	7.811042	7.815029
19	7.772103	7.776799	7.781414	7.785948	7.790403	7.79478	7.799081	7.803307	7.80746	7.81154	7.81555	7.81949
19.5	7.776559	7.7812	7.78576	7.790241	7.794644	7.79897	7.803222	7.807401	7.811507	7.815543	7.819509	7.823406
20	7.780456	7.785046	7.789556	7.793989	7.798346	7.802628	7.806836	7.810972	7.815037	7.819032	7.822959	7.826819
20.5	7.783834	7.788378	7.792845	7.797235	7.801551	7.805792	7.809961	7.814059	7.818088	7.822047	7.82594	7.829766
21	7.786733	7.791238	7.795665	7.800018	7.804296	7.808502	7.812636	7.816701	7.820697	7.824625	7.828487	7.832283
21.5	7.789191	7.79366	7.798053	7.802372	7.806617	7.810792	7.814896	7.818931	7.822898	7.826799	7.830634	7.834405
22	7.791242	7.795679	7.800041	7.804331	7.808548	7.812695	7.816772	7.820781	7.824723	7.8286	7.832412	7.836161
22.5	7.792917	7.797326	7.801662	7.805926	7.810118	7.814241	7.818295	7.822282	7.826203	7.830058	7.833851	7.83758
23	7.794247	7.798632	7.802945	7.807186	7.811357	7.815459	7.819493	7.823461	7.827363	7.831201	7.834976	7.83869
23.5	7.795259	7.799623	7.803916	7.808138	7.81229	7.816374	7.820392	7.824343	7.82823	7.832054	7.835815	7.839514
24	7.795978	7.800325	7.804601	7.808806	7.812943	7.817013	7.821016	7.824954	7.828828	7.832639	7.836389	7.840077
24.5	7.79643	7.800761	7.805023	7.809215	7.813339	7.817396	7.821388	7.825315	7.829179	7.83298	7.83672	7.8404
25	7.796635	7.800954	7.805204	7.809385	7.813499	7.817546	7.821529	7.825447	7.829303	7.833096	7.836829	7.840502
25.5	7.796614	7.800924	7.805165	7.809337	7.813443	7.817482	7.821458	7.825369	7.829219	7.833007	7.836735	7.840403
26	7.796388	7.80069	7.804923	7.809089	7.813189	7.817223	7.821193	7.8251	7.828945	7.83273	7.836454	7.84012
26.5	7.795973	7.800269	7.804497	7.808659	7.812754	7.816785	7.820751	7.824656	7.828498	7.832281	7.836004	7.839668
27	7.795386	7.799678	7.803903	7.808062	7.812155	7.816183	7.820149	7.824052	7.827894	7.831675	7.835398	7.839063
27.5	7.794642	7.798933	7.803156	7.807313	7.811405	7.815434	7.819399	7.823302	7.827145	7.830928	7.834652	7.838318
28	7.793756	7.798046	7.802269	7.806427	7.81052	7.814549	7.818516	7.822421	7.826265	7.83005	7.833777	7.837446
28.5	7.79274	7.797032	7.801257	7.805416	7.809511	7.813542	7.817512	7.82142	7.825267	7.829056	7.832786	7.836459
29	7.791608	7.795902	7.800129	7.804292	7.80839	7.812425	7.816398	7.82031	7.824162	7.827956	7.831691	7.835369
29.5	7.79037	7.794668	7.798899	7.803066	7.807168	7.811208	7.815186	7.819103	7.822961	7.826759	7.8305	7.834185
30	7.789038	7.79334	7.797576	7.801748	7.805856	7.809901	7.813885	7.817808	7.821672	7.825477	7.829225	7.832916

Table 10 Forecast Yield MMSE and Monte Carlo

References

- Aljinović, Zdravka, Tea Poklepović, and Kristina Katalinić. "Best fit model for yield curve estimation." *Croatian Operational Research Review* 3.1 (2012): 28-40.

- Bai, Jushan, and Pierre Perron. "Critical values for multiple structural change tests." *The Econometrics Journal* 6.1 (2003): 72-78.

- Bandara, Wachi, and Richard Munclinger. "Term structure modeling with structural breaks: a simple arbitrage-free approach." *Available at SSRN 1974033* (2011).

- Björk, Tomas, and Bent Jesper Christensen. "Interest rate dynamics and consistent forward rate curves." *Mathematical Finance* 9.4 (1999): 323-348.

- Bliss, Robert Russell. "Testing term structure estimation methods." *Advances in Futures and Options research* 9 (1997): 197-232.

- Caldeira, João F., et al. "Forecasting the yield curve with the arbitrage-free dynamic Nelson–Siegel model: Brazilian evidence." *EconomiA* 17.2 (2016): 221-237.v

- Choudhry, Moorad, and Michele Lizzio. *Advanced fixed income analysis*. Elsevier, 2015

- Christensen, Jens HE, Francis X. Diebold, and Glenn D. Rudebusch. "The affine arbitrage-free class of Nelson–Siegel term structure models." *Journal of Econometrics* 164.1 (2011): 4-20.

- Diebold, F. X. "Li., C. (2002), "Forecasting the Term Structure of Government Bond Yields,"." *Manuscript, Department of Economics, University of Pennsylvania*.

- Diebold, Francis X., and Canlin Li. "Forecasting the term structure of government bond yields." *Journal of econometrics*130.2 (2006): 337-364.

- Diebold, Francis X., and Glenn D. Rudebusch. *Yield curve modeling and forecasting: the dynamic Nelson-Siegel approach*. Princeton University Press, 2013.

- Diebold, Francis X., Canlin Li, and Vivian Z. Yue. "Global yield curve dynamics and interactions: a dynamic Nelson–Siegel approach." *Journal of Econometrics* 146.2 (2008): 351-363.

- Diebold, Francis X., Glenn D. Rudebusch, and S. Boragan Aruoba. "The macroeconomy and the yield curve: a dynamic latent factor approach." *Journal of econometrics* 131.1-2 (2006): 309-338.

- Diebold, Francis X., Lei Ji, and Canlin Li. "A three-factor yield curve model: non-affine structure, systematic risk sources, and generalized duration." *Macroeconomics, Finance and Econometrics: Essays in Memory of Albert Ando* 1 (2006): 240-274.

- Diebold, Francis X., Monika Piazzesi, and Glenn D. Rudebusch. "Modeling bond yields in finance and macroeconomics." *American Economic Review* 95.2 (2005): 415-420.

- Fama, Eugene F., and Robert R. Bliss. "The information in long-maturity forward rates." *The American Economic Review* (1987): 680-692.
- Filipović, Damir. "A note on the Nelson–Siegel family." *Mathematical finance* 9.4 (1999): 349-359.
- Geyer, Charles J. "Practical markov chain monte carlo." *Statistical science* (1992): 473-483.
- Levant, Jared, and Jun Ma. "Investigating United Kingdom's monetary policy with Macro-Factor Augmented Dynamic Nelson–Siegel models." *Journal of Empirical Finance* 37 (2016): 117-127.
- Litterman, Robert, and Jose Scheinkman. "Common factors affecting bond returns." *Journal of fixed income* 1.1 (1991): 54-61.
- McCulloch, J. Huston. "An estimate of the liquidity premium." *Journal of Political Economy* 83.1 (1975): 95-119.
- Nath, Golaka C. "Indian corporate bonds market–an analytical prospective." *Available at SSRN 2062269* (2012).
- Prasanna, Krishna, and Subramaniam Sowmya. "Yield curve in India and its interactions with the US bond market." *International Economics and Economic Policy* 14.2 (2017): 353-375.
- Sahoo, Satyananda, and Indranil Bhattacharyya. "Yield curve dynamics of the Indian g-sec market: A macro-finance approach." *Indian Economic Review* (2012): 157-182.
- Sinha, Arunima, and Rajnish Mehra. "The Term Structure of Interest Rates in India." *Available at SSRN 2735925* (2016).
- Sowmya, Subramaniam, Krishna Prasanna, and Saumitra Bhaduri. "Linkages in the term structure of interest rates across sovereign bond markets." *Emerging Markets Review*27 (2016): 118-139.
- Svensson, Lars EO. *Estimating and interpreting forward interest rates: Sweden 1992-1994*. No. w4871. National bureau of economic research, 1994.
- Tsui, A., J. X. Wu, and Zhaoyong Zhang. "Modelling the term structure of Japanese bond yields with the Nelson-Siegel model." (2013).
- Vasicek, Oldrich A., and H. Gifford Fong. "Term structure modeling using exponential splines." *The Journal of Finance*37.2 (1982): 339-348.
- Virmani, Vineet. "Assessing NSE's Daily Zero-Coupon Yield Curve Estimates: A Comparison with Few Competing Alternatives." (2006).

- Yu, Wei-Choun, and Donald M. Salyards. "Parsimonious modeling and forecasting of corporate yield curve." *Journal of Forecasting* 28.1 (2009): 73-88.
- Yu, Wei-Choun, and Eric Zivot. "Forecasting the term structures of Treasury and corporate yields using dynamic Nelson-Siegel models." *International Journal of Forecasting*27.2 (2011): 579-591.