APPENDIX F OF THE SUPPLEMENTAL MATERIAL: SOME VARIATIONS IN THE MODEL, THE DATA, AND THE ESTIMATION APPROACH (*NOT INTENDED FOR PUBLICATION*)

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In this appendix, we discuss a few variations in our model setup, the data that we use, and our estimation approach.

I. Model with Working capital and growth of land supply

This section presents a variation of our benchmark model by incorporating working capital and growth in land supply.

I.1. **The Model.** As in the benchmark model, the economy consists of two types of agents—a representative household and a representative entrepreneur. There are four types of commodities: labor, goods, land, and loanable bonds. The representative household's utility depends on consumption goods, land services (housing), and leisure; the representative entrepreneur's utility depends on consumption goods only. Goods production requires labor, capital, and land as inputs. The entrepreneur needs external financing for investment spending. Imperfect contract enforcement implies that the entrepreneur's borrowing capacity is constrained by the value of collateral assets, consisting of land and capital stocks.

We add two variations here. First, we assume that a fraction ϕ_w of firms' wage payments needs to be financed by working capital, which is repaid within the period and carries no interest. This modification implies that the total amount of debt, including the intertemporal loans and the working capital, is limited by the value of the firms' collateral assets (land and capital). Second, we assume that aggregate land endowment grows at a constant rate of $\bar{\lambda}_l$. To maintain balanced growth, we assume that existing land holdings in each sector also grow at the same rate absent any shocks.

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Shocks that drive land reallocation would lead to different growth rates of land held in the two sectors.

We obtain 2 main results. First, absence working capital (i.e., $\phi_w = 0$), the model with exogenous land growth has an identical steady-state equilibrium and fluctuations around the steady state as in the benchmark model. Second, the parameter estimates in the model with working capital are very similar to those in the benchmark model.

I.1.1. *The representative household.* Similar to Iacoviello (2005), the household has the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t A_t\left\{\log(C_{ht}-\gamma_h C_{h,t-1})+\varphi_t\log L_{ht}-\psi_t N_{ht}\right\}, \tag{1}$$

where C_{ht} denotes consumption, L_{ht} denotes land holdings, and N_{ht} denotes labor hours. The parameter $\beta \in (0,1)$ is a subjective discount factor, the parameter γ_h measures the degree of habit persistence, and the term E is a mathematical expectation operator. The terms A_t, φ_t, and ψ_t are preference shocks. We assume that the intertemporal preference shock A_t follows the stochastic process

$$A_t = A_{t-1}(1+\lambda_{at}), \quad \ln\lambda_{at} = (1-\rho_a)\ln\bar{\lambda}_a + \rho_a\ln\lambda_{a,t-1} + \varepsilon_{at}, \tag{2}$$

where $\bar{\lambda}_a > 0$ is a constant, $\rho_a \in (-1,1)$ is the persistence parameter, and ε_{at} is an identically and independently distributed (i.i.d.) white noise process with mean zero and variance σ_a^2. The housing preference shock φ_t follows the stationary process

$$\ln\varphi_t = (1-\rho_\varphi)\ln\bar{\varphi} + \rho_\varphi\ln\varphi_{t-1} + \varepsilon_{\varphi t}, \tag{3}$$

where $\bar{\varphi} > 0$ is a constant, $\rho_\varphi \in (-1,1)$ measures the persistence of the shock, and $\varepsilon_{\varphi t}$ is a white noise process with mean zero and variance σ_φ^2. The labor supply shock ψ_t follows the stationary process

$$\ln\psi_t = (1-\rho_\psi)\ln\bar{\psi} + \rho_\psi\ln\psi_{t-1} + \varepsilon_{\psi t}, \tag{4}$$

where $\bar{\varphi} > 0$ is a constant, $\rho_\psi \in (-1,1)$ measures the persistence, and $\varepsilon_{\psi t}$) is a white noise process with mean zero and variance σ_ψ^2.

Denote by q_{lt} the relative price of housing (in consumption units), R_t the gross real loan rate, and w_t the real wage; denote by S_t the household's purchase in period t of the loanable bond that pays off one unit of consumption good in all states of nature in period $t+1$. In period 0, the household begins with $L_{h,-1} > 0$ units of housing and

$S_{-1} > 0$ units of the loanable bond. The flow of funds constraint for the household is given by

$$C_{ht} + q_{lt}(L_{ht} - \bar{\lambda}_l L_{h,t-1}) + \frac{S_t}{R_t} \leq w_t N_{ht} + S_{t-1}, \tag{5}$$

where we have imposed the implicit assumption that the household's land holding grows "naturally" at the constant rate $\bar{\lambda}_l$.

The household chooses C_{ht}, $L_{h,t}$, N_{ht}, and S_t to maximize (1) subject to (2)-(5) and the borrowing constraint $S_t \geq -\bar{S}$ for some large number $\bar{S}$.

I.1.2. *The representative entrepreneur.* The entrepreneur has the utility function

$$\mathrm{E}\sum_{t=0}^{\infty} \beta^t \left[\log(C_{et} - \gamma_e C_{e,t-1})\right], \tag{6}$$

where C_{et} denotes the entrepreneur's consumption and γ_e is the habit persistence parameter.

The entrepreneur produces goods using capital, labor, and land as inputs. The production function is given by

$$Y_t = Z_t [L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}]^{\alpha} N_{et}^{1-\alpha}, \tag{7}$$

where Y_t denotes output, K_{t-1}, N_{et}, and $L_{e,t-1}$ denote the inputs capital, labor, and land, respectively, and the parameters $\alpha \in (0,1)$ and $\phi \in (0,1)$ measure the output elasticities of these production factors. We assume that the total factor productivity Z_t is composed of a permanent component Z_t^p and a transitory component ν_t such that $Z_t = Z_t^p \nu_{zt}$, where the permanent component Z_t^p follows the stochastic process

$$Z_t^p = Z_{t-1}^p \lambda_{zt}, \quad \ln \lambda_{zt} = (1-\rho_z)\ln \bar{\lambda}_z + \rho_z \ln \lambda_{z,t-1} + \varepsilon_{zt}, \tag{8}$$

and the transitory component follows the stochastic process

$$\ln \nu_{zt} = \rho_{\nu_z} \ln \nu_{z,t-1} + \varepsilon_{\nu_z t}. \tag{9}$$

The parameter $\bar{\lambda}_z$ is the steady-state growth rate of Z_t^p; the parameters ρ_z and ρ_{ν_z} measure the degree of persistence. The innovations ε_{zt} and $\varepsilon_{\nu_z t}$ are i.i.d. white noise processes that are mutually independent with mean zero and variances given by σ_z^2 and $\sigma_{\nu_z}^2$, respectively.

The entrepreneur is endowed with K_{-1} units of initial capital stock and $L_{-1,e}$ units of initial land. Capital accumulation follows the law of motion

$$K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2\right] I_t, \tag{10}$$

where I_t denotes investment, $\bar{\lambda}_I$ denotes the steady-state growth rate of investment, and $\Omega > 0$ is the adjustment cost parameter.

The entrepreneur faces the flow of funds constraint

$$C_{et} + q_{lt}(L_{et} - \bar{\lambda}_l L_{e,t-1}) + B_{t-1} = Z_t[L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}]^{\alpha} N_{et}^{1-\alpha} - \frac{I_t}{Q_t} - w_t N_{et} + \frac{B_t}{R_t}, \tag{11}$$

where B_{t-1} is the amount of matured debt and B_t/R_t is the value of new debt. Following Greenwood, Hercowitz, and Krusell (1997), we interpret Q_t as the investment-specific technological change. Specifically, we assume that $Q_t = Q_t^p \nu_{qt}$, where the permanent component Q_t^p follows the stochastic process

$$Q_t^p = Q_{t-1}^p \lambda_{qt}, \quad \ln \lambda_{qt} = (1 - \rho_q) \ln \bar{\lambda}_q + \rho_q \ln \lambda_{q,t-1} + \varepsilon_{qt}, \tag{12}$$

and the transitory component μ_t follows the stochastic process

$$\ln \nu_{qt} = \rho_{\nu_q} \ln \nu_{q,t-1} + \varepsilon_{\nu_q t}. \tag{13}$$

The parameter $\bar{\lambda}_q$ is the steady-state growth rate of Q_t^p; the parameters ρ_q and ρ_{ν_q} measure the degree of persistence. The innovations ε_{qt} and $\varepsilon_{\nu_q t}$ are i.i.d. white noise processes that are mutually independent with mean zero and variances given by σ_q^2 and $\sigma_{\nu_q}^2$, respectively.

The entrepreneur's consumption, investment, and production can be partly financed externally. In addition, a fraction ϕ_w of the wage payments need to be financed externally. In particular, the entrepreneur faces the borrowing constraint

$$B_t \leq \theta_t E_t[\phi_k q_{k,t+1} K_t + \phi_l \bar{\lambda}_l q_{l,t+1} L_{et}] - \phi_w w_t N_{et} R_t, \tag{14}$$

where $q_{k,t+1}$ is the shadow price of capital in consumption units.[1] Under this credit constraint, the amount that the entrepreneur can borrow is limited by a fraction of the value of the collateral assets—land and capital—net of the required repayment of working capital. The constants ϕ_k and ϕ_l represent the fractions of capital and land that can be pledged as collateral. Note that the collateral value of land grows at the rate $\bar{\lambda}_l$, as does the entrepreneur's land holdings.

Following Kiyotaki and Moore (1997), we interpret this type of credit constraints as reflecting the problem of costly contract enforcement: if the entrepreneur fails to pay the debt, the creditor can seize the land and the accumulated capital; since it is costly to liquidate the seized land and capital stock, the creditor can recoup up to a fraction θ_t of the total value of the collateral assets. We interpret θ_t as a "collateral shock" that

[1]Since the price of new capital is $1/Q_t$, Tobin's q in this model is given by $q_{kt}Q_t$, which is the ratio of the value of installed capital to the price of new capital.

reflects the tightness of the credit market. We assume that θ_t follows the stochastic process

$$\ln \theta_t = (1 - \rho_\theta) \ln \bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}, \tag{15}$$

where $\bar{\theta}$ is the steady-state value of θ_t, $\rho_\theta \in (0, 1)$ is the persistence parameter, and $\varepsilon_{\theta t}$ is an i.i.d. white noise process with mean zero and variance σ_θ^2.

The entrepreneur chooses C_{et}, N_{et}, I_t, $L_{e,t}$, K_t, and B_t to maximize (6) subject to (7) through (15).

I.1.3. *Market clearing conditions and equilibrium.* In a competitive equilibrium, the markets for goods, labor, land, and loanable bonds all clear. The goods market clearing condition implies that

$$C_t + \frac{I_t}{Q_t} = Y_t, \tag{16}$$

where $C_t = C_{ht} + C_{et}$ denotes aggregate consumption. The labor market clearing condition implies that labor demand equals labor supply:

$$N_{et} = N_{ht} \equiv N_t. \tag{17}$$

The land market clearing condition implies that

$$L_{ht} + L_{et} = \bar{\lambda}_l^t \bar{L}, \tag{18}$$

where $\bar{L}$ is the fixed aggregate land endowment. Finally, the bond market clearing condition implies that

$$S_t = B_t. \tag{19}$$

A competitive equilibrium consists of sequences of prices $\{w_t, q_{lt}, R_t\}_{t=0}^{\infty}$ and allocations $\{C_{ht}, C_{et}, I_t, N_{ht}, N_{et}, L_{ht}, L_{et}, S_t, B_t, K_t, Y_t\}_{t=0}^{\infty}$ such that (i) taking the prices as given, the allocations solve the optimizing problems for the household and the entrepreneur and (ii) all markets clear.

I.2. **Derivations.**

I.2.1. *Euler equations.* Denote by μ_{ht} the Lagrangian multiplier for the flow of funds constraint (5). The first-order conditions for the household's optimizing problem are given by

$$\mu_{ht} = A_t \left[\frac{1}{C_{ht} - \gamma_h C_{h,t-1}} - \mathrm{E}_t \frac{\beta \gamma_h}{C_{h,t+1} - \gamma_h C_{ht}} (1 + \lambda_{a,t+1}) \right], \tag{20}$$

$$w_t = \frac{A_t}{\mu_{ht}} \psi_t, \tag{21}$$

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}} \bar{\lambda}_l q_{l,t+1} + \frac{A_t \varphi_t}{\mu_{ht} L_{ht}}, \tag{22}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}}. \tag{23}$$

Equation (20) equates the marginal utility of income and of consumption; equation (21) equates the real wage and the marginal rate of substitution (MRS) between leisure and income; equation (22) equates the current relative price of land to the marginal benefit of purchasing an extra unit of land, which consists of the current utility benefits (i.e., the MRS between housing and consumption) and the land's discounted future resale value; and equation (23) is the standard Euler equation for the loanable bond.

Denote by μ_{et} the Lagrangian multiplier for the flow of funds constraint (11), μ_{kt} the multiplier for the capital accumulation equation (10), and μ_{bt} the multiplier for the borrowing constraint (14). With these notations, the shadow price of capital in consumption units is given by

$$q_{kt} = \frac{\mu_{kt}}{\mu_{et}}. \tag{24}$$

The first-order conditions for the entrepreneur's optimizing problem are given by

$$\mu_{et} = \frac{1}{C_{et} - \gamma_e C_{e,t-1}} - \mathrm{E}_t \frac{\beta \gamma_e}{C_{e,t+1} - \gamma_e C_{et}}, \tag{25}$$

$$(1-\alpha) \frac{Y_t}{N_{et}} = \left[1 + \phi_w \frac{\mu_{bt}}{\mu_{et}} R_t \right] w_t, \tag{26}$$

$$\begin{aligned} \frac{1}{Q_t} &= q_{kt} \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 - \Omega \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right) \frac{I_t}{I_{t-1}} \right] \\ &\quad + \beta \Omega \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} q_{k,t+1} \left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I \right) \left(\frac{I_{t+1}}{I_t} \right)^2, \end{aligned} \tag{27}$$

$$q_{kt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} \left[\alpha \frac{Y_{t+1}}{K_t} + q_{k,t+1}(1-\delta) \right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \phi_k \mathrm{E}_t q_{k,t+1}, \tag{28}$$

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} \left[\alpha \phi \frac{Y_{t+1}}{L_{et}} + \bar{\lambda}_l q_{l,t+1} \right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \phi_l \bar{\lambda}_l \mathrm{E}_t q_{l,t+1}, \tag{29}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} + \frac{\mu_{bt}}{\mu_{et}}. \tag{30}$$

Equation (25) equates the marginal utility of income to the marginal utility of consumption since consumption is the numéraire. Equation (26) is the labor demand equation. Since part of the wage payment needs to be externally financed by working capital ($\phi_w > 0$), the effective cost of labor exceeds the market wage rate if the borrowing constraint is binding. Equation (27) is the investment Euler equation, which equates the cost of purchasing an additional unit of investment good and the benefit of having an extra unit of new capital, where the benefit includes the shadow value of the installed capital net of adjustment costs and the present value of the saved future adjustment costs. Equation (28) is the capital Euler equation, which equates the shadow price of capital to the present value of future marginal product of capital and the resale value of the un-depreciated capital, plus the value of capital as a collateral asset for borrowing. Equation (29) is the land Euler equation, which equates the price of the land to the present value of the future marginal product of land and the resale value, plus the value of land as a collateral asset for borrowing. Equation (30) is the bond Euler equation for the entrepreneur, which reveals that the borrowing constraint is binding (i.e., $\mu_{bt} > 0$) if and only if the interest rate is lower than the entrepreneur's intertemporal marginal rate of substitution.

I.2.2. *Stationary equilibrium.* We are interested in studying the fluctuations around the balanced growth path. For this purpose, we focus on a stationary equilibrium by appropriately transforming the growing variables. Specifically, we make the following transformations of the variables

$$\begin{gathered}
\tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t}, \quad \tilde{C}_{ht} \equiv \frac{C_{ht}}{\Gamma_t}, \quad \tilde{C}_{et} \equiv \frac{C_{et}}{\Gamma_t}, \quad \tilde{I}_t \equiv \frac{I_t}{Q_t \Gamma_t}, \quad \tilde{K}_t \equiv \frac{K_t}{Q_t \Gamma_t}, \\
\tilde{B}_t \equiv \frac{B_t}{\Gamma_t}, \quad \tilde{w}_t \equiv \frac{w_t}{\Gamma_t}, \quad \tilde{\mu}_{ht} \equiv \frac{\mu_{ht}\Gamma_t}{A_t}, \quad \tilde{\mu}_{et} \equiv \mu_{et}\Gamma_t, \quad \tilde{\mu}_{bt} \equiv \mu_{bt}\Gamma_t, \\
\tilde{q}_{lt} \equiv \frac{q_{lt}\bar{\lambda}_l^t}{\Gamma_t}, \quad \tilde{q}_{kt} \equiv q_{kt} Q_t, \quad \tilde{L}_{ht} \equiv \frac{L_{ht}}{\bar{\lambda}_l^t}, \quad \tilde{L}_{et} \equiv \frac{L_{et}}{\bar{\lambda}_l^t}
\end{gathered} \tag{31}$$

where $\Gamma_t \equiv [Z_t Q_t^{(1-\phi)\alpha} (\bar{\lambda}_l^t)^{\alpha\phi}]^{\frac{1}{1-(1-\phi)\alpha}}$.

Denote by $g_{\gamma t} \equiv \frac{\Gamma_t}{\Gamma_{t-1}}$ and $g_{qt} \equiv \frac{Q_t}{Q_{t-1}}$ the growth rates for the exogenous variables Γ_t and Q_t. Denote by g_γ the steady-state value of $g_{\gamma t}$ and $\lambda_k \equiv g_\gamma \bar{\lambda}_q$ the steady-state growth rate of capital stock. On the balanced growth path, investment grows at the same rate as does capital, so we have $\bar{\lambda}_I = \lambda_k$.

The stationary equilibrium is the solution to the following system of equations:

$$\tilde{\mu}_{ht} = \frac{1}{\tilde{C}_{ht} - \gamma_h \tilde{C}_{h,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_h}{\tilde{C}_{h,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_h \tilde{C}_{ht}}(1+\lambda_{a,t+1}), \tag{32}$$

$$\tilde{w}_t = \frac{\psi_t}{\tilde{\mu}_{ht}}, \tag{33}$$

$$\tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}}(1+\lambda_{a,t+1})\tilde{q}_{l,t+1} + \frac{\varphi_t}{\tilde{\mu}_{ht} L_{ht}}, \tag{34}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}} \frac{\Gamma_t}{\Gamma_{t+1}}(1+\lambda_{a,t+1}). \tag{35}$$

$$\tilde{\mu}_{et} = \frac{1}{\tilde{C}_{et} - \gamma_e \tilde{C}_{e,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_e}{\tilde{C}_{e,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_e \tilde{C}_{et}}, \tag{36}$$

$$(1-\alpha)\frac{\tilde{Y}_t}{N_t} = \left[1 + \phi_w \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}} R_t\right]\tilde{w}_t, \tag{37}$$

$$\tag{38}$$

$$\begin{aligned} 1 &= \tilde{q}_{kt}\left[1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2 - \Omega\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}}\right] \\ &\quad + \beta\Omega \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}\tilde{q}_{k,t+1}\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t} - \bar{\lambda}_I\right)\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t}\right)^2, \end{aligned} \tag{39}$$

$$\tilde{q}_{kt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha(1-\phi)\frac{\tilde{Y}_{t+1}}{\tilde{K}_t} + \tilde{q}_{k,t+1}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}(1-\delta)\right] + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t\phi_k \mathrm{E}_t \tilde{q}_{k,t+1}\frac{Q_t}{Q_{t+1}}, \tag{40}$$

$$\tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha\phi\frac{\tilde{Y}_{t+1}}{L_{et}} + \tilde{q}_{l,t+1}\right] + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t\phi_l \mathrm{E}_t \tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}, \tag{41}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{\Gamma_t}{\Gamma_{t+1}} + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}. \tag{42}$$

$$\tilde{Y}_t = \left(\frac{Z_t Q_t}{Z_{t-1}Q_{t-1}}\right)^{-\frac{(1-\phi)\alpha}{1-(1-\phi)\alpha}}(\bar{\lambda}_l)^{-\frac{\alpha\phi}{1-(1-\phi)\alpha}}[\tilde{L}_{e,t-1}^{\phi}\tilde{K}_{t-1}^{1-\phi}]^{\alpha}N_t^{1-\alpha}, \tag{43}$$

$$\tilde{K}_t = (1-\delta)\tilde{K}_{t-1}\frac{Q_{t-1}\Gamma_{t-1}}{Q_t\Gamma_t} + \left[1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2\right]\tilde{I}_t, \tag{44}$$

$$\tilde{Y}_t = \tilde{C}_{ht} + \tilde{C}_{et} + \tilde{I}_t, \tag{45}$$

$$\bar{L} = \tilde{L}_{ht} + \tilde{L}_{et}, \tag{46}$$

$$\tilde{Y}_t = \tilde{C}_{et} + \tilde{I}_t + \tilde{q}_{lt}(\tilde{L}_{et} - \tilde{L}_{e,t-1}) + \tilde{B}_{t-1}\frac{\Gamma_{t-1}}{\Gamma_t} + \tilde{w}_t N_t - \frac{\tilde{B}_t}{R_t}, \tag{47}$$

$$\tilde{B}_t = \theta_t \mathrm{E}_t \left[\phi_k \tilde{q}_{k,t+1} \tilde{K}_t \frac{Q_t}{Q_{t+1}} + \phi_l \tilde{q}_{l,t+1} \frac{\Gamma_{t+1}}{\Gamma_t} \tilde{L}_{et} \right] - \phi_w R_t \tilde{w}_t N_t. \tag{48}$$

We solve these 16 equations for 16 variables summarized in the vector

$$[\tilde{\mu}_{ht}, \tilde{w}_t, \tilde{q}_{lt}, R_t, \tilde{\mu}_{et}, N_t, \tilde{I}_t, \tilde{Y}_t, \tilde{C}_{ht}, \tilde{C}_{et}, \tilde{q}_{kt}, L_{et}, L_{ht}, \tilde{K}_t, \tilde{B}_t, \tilde{\mu}_{bt}]'.$$

I.2.3. *Steady state.* To solve the model's equilibrium dynamics, we log-linearize the stationary equilibrium conditions summarized in (32)-(48) around a deterministic steady state. The set of steady-state values required for solving the model include the shadow value of the loanable funds $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, the labor income share $\frac{\tilde{w}N}{\tilde{Y}}$, the ratio of commercial real estate to aggregate output $\frac{\tilde{q}_l \tilde{L}_e}{\tilde{Y}}$, the ratio of residential land to commercial real estate $\frac{\tilde{L}_h}{\tilde{L}_e}$, the ratio of loanable funds to output $\frac{\tilde{B}}{\tilde{Y}}$, the capital-output ratio $\frac{\tilde{K}}{\tilde{Y}}$, and the "big ratios" $\frac{\tilde{C}_h}{\tilde{Y}}$, $\frac{\tilde{C}_e}{\tilde{Y}}$, and $\frac{\tilde{I}}{\tilde{Y}}$.

To get the steady-state value for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, we use the stationary bond Euler equations (35) for the household and (42) to obtain

$$\frac{1}{R} = \frac{\beta(1+\bar{\lambda}_a)}{g_\gamma}, \quad \frac{\tilde{\mu}_b}{\tilde{\mu}_e} = \frac{\beta \bar{\lambda}_a}{g_\gamma}. \tag{49}$$

Since $\bar{\lambda}_a > 0$, we have $\tilde{\mu}_b > 0$ and the borrowing constraint is binding in the steady-state equilibrium.

Given $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, (37) implies that the labor share (denoted by s_n) is given by

$$s_n \equiv \frac{\tilde{w}N}{\tilde{Y}} = \frac{1-\alpha}{1+\frac{\phi_w \bar{\lambda}_a}{1+\bar{\lambda}_a}}. \tag{50}$$

To get the ratio of commercial real estate to output, we use the land Euler equation (41) for the entrepreneur, the definition of $\tilde{\mu}_e$ in (36), and the solution for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$ in (49). In particular, we have

$$\frac{\tilde{q}_l L_e}{\tilde{Y}} = \frac{\beta\alpha\phi}{1-\beta-\beta\bar{\lambda}_a\bar{\theta}\phi_l}. \tag{51}$$

To get the investment-output ratio, we first solve for the investment-capital ratio by using the law of motion for capital stock in (44) and then solve for the capital-output ratio using the capital Euler equation (40). Specifically, we have

$$\frac{\tilde{I}}{\tilde{K}} = 1 - \frac{1-\delta}{\lambda_k}, \tag{52}$$

$$\frac{\tilde{K}}{\tilde{Y}} = \left[1 - \frac{\beta}{\lambda_k}(\bar{\lambda}_a\bar{\theta}\phi_k + 1 - \delta)\right]^{-1} \beta\alpha(1-\phi), \tag{53}$$

where we have used the steady-state condition that $\tilde{q}_k = 1$, as implied by the investment Euler equation (39). The investment-output ratio is then given by

$$\frac{\tilde{I}}{\tilde{Y}} = \frac{\tilde{I}}{\tilde{K}}\frac{\tilde{K}}{\tilde{Y}} = \frac{\beta\alpha(1-\phi)[\lambda_k - (1-\delta)]}{\lambda_k - \beta(\bar{\lambda}_a\bar{\theta}\phi_k + 1 - \delta)}. \tag{54}$$

Given the solution for the ratios $\frac{\tilde{q}_l L_e}{\tilde{Y}}$ and $\frac{\tilde{K}}{\tilde{Y}}$ in (51) and (53), the binding borrowing constraint (48) implies that

$$\frac{\tilde{B}}{\tilde{Y}} = \bar{\theta}g_\gamma\phi_l\frac{\tilde{q}_l L_e}{\tilde{Y}} + \frac{\bar{\theta}\phi_k}{\bar{\lambda}_q}\frac{\tilde{K}}{\tilde{Y}} - \phi_w R s_n. \tag{55}$$

The entrepreneur's flow of funds constraint (47) implies that

$$1 = \frac{\tilde{C}_e}{\tilde{Y}} + \frac{\tilde{I}}{\tilde{Y}} + \frac{\tilde{B}}{\tilde{Y}}\left(\frac{1}{g_\gamma} - \frac{1}{R}\right) + s_n. \tag{56}$$

The aggregate resource constraint (45) then implies that

$$\frac{\tilde{C}_h}{\tilde{Y}} = 1 - \frac{\tilde{C}_e}{\tilde{Y}} - \frac{\tilde{I}}{\tilde{Y}}. \tag{57}$$

To solve for $\frac{L_h}{L_e}$, we first use the household's land Euler equation (i.e., the housing demand equation) (34) and the definition for the marginal utility (32) to obtain

$$\frac{\tilde{q}_l L_h}{\tilde{C}_h} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)}{g_\gamma(1 - g_\gamma/R)(1 - \gamma_h/R)}, \tag{58}$$

where the steady-state loan rate is given by (49).

Taking the ratio between (58) and (51) results in the solution

$$\frac{L_h}{L_e} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)(1 - \beta - \beta\bar{\lambda}_a\bar{\theta}\phi_l)}{\beta\alpha\phi g_\gamma(1 - g_\gamma/R)(1 - \gamma_h/R)}\frac{\tilde{C}_h}{\tilde{Y}}. \tag{59}$$

Finally, we can solve for the steady-state hours by combining the labor supply equation (33) and the labor demand equation (37) to get

$$N = \frac{g_\gamma(1 - \gamma_h/R)}{g_\gamma - \gamma_h}\frac{s_n}{\bar{\psi}}\frac{\tilde{Y}}{\tilde{C}_h}. \tag{60}$$

I.2.4. *Log-linearized equilibrium system.* Upon obtaining the steady-state equilibrium, we log-linearize the equilibrium conditions (32) through (48) around the steady state. We define the constants $\Omega_h \equiv (g_\gamma - \beta(1+\bar{\lambda}_a)\gamma_h)(g_\gamma - \gamma_h)$ and $\Omega_e \equiv (g_\gamma - \beta\gamma_e)(g_\gamma - \gamma_e)$.

The log-linearized equilibrium conditions are given by

$$\begin{aligned}
\Omega_h\hat{\mu}_{ht} &= -[g_\gamma^2+\gamma_h^2\beta(1+\bar{\lambda}_a)]\hat{C}_{ht}+g_\gamma\gamma_h(\hat{C}_{h,t-1}-\hat{g}_{\gamma t}) \\
&\quad -\beta\bar{\lambda}_a\gamma_h(g_\gamma-\gamma_h)\mathrm{E}_t\hat{\lambda}_{a,t+1}+\beta(1+\bar{\lambda}_a)g_\gamma\gamma_h\mathrm{E}_t(\hat{C}_{h,t+1}+\hat{g}_{\gamma,t+1}), && (61)\\
\hat{w}_t+\hat{\mu}_{ht} &= \hat{\psi}_t, && (62)\\
\hat{q}_{lt}+\hat{\mu}_{ht} &= \beta(1+\bar{\lambda}_a)\mathrm{E}_t\left[\hat{\mu}_{h,t+1}+\hat{q}_{l,t+1}\right] \\
&\quad +[1-\beta(1+\bar{\lambda}_a)](\hat{\varphi}_t-\hat{L}_{ht})+\beta\bar{\lambda}_a\mathrm{E}_t\hat{\lambda}_{a,t+1}, && (63)\\
\hat{\mu}_{ht}-\hat{R}_t &= \mathrm{E}_t\left[\hat{\mu}_{h,t+1}+\frac{\bar{\lambda}_a}{1+\bar{\lambda}_a}\hat{\lambda}_{a,t+1}-\hat{g}_{\gamma,t+1}\right], && (64)\\
\Omega_e\hat{\mu}_{et} &= -(g_\gamma^2+\beta\gamma_e^2)\hat{C}_{e,t}+g_\gamma\gamma_e(\hat{C}_{e,t-1}-\hat{g}_{\gamma t})+\beta g_\gamma\gamma_e\mathrm{E}_t(\hat{C}_{e,t+1}+\hat{g}_{\gamma,t+1}), && (65)\\
\hat{Y}_t-\hat{N}_t &= \frac{\phi_w\bar{\lambda}_a}{1+\bar{\lambda}_a+\phi_w\bar{\lambda}_a}(\hat{R}_t+\hat{\mu}_{bt}-\hat{\mu}_{et})+\hat{w}_t, && (66)\\
\hat{q}_{kt} &= (1+\beta)\Omega\lambda_k^2\hat{I}_t-\Omega\lambda_k^2\hat{I}_{t-1}+\Omega\lambda_k^2(\hat{g}_{\gamma t}+\hat{g}_{qt}) \\
&\quad -\beta\Omega\lambda_k^2\mathrm{E}_t[\hat{I}_{t+1}+\hat{g}_{\gamma,t+1}+\hat{g}_{q,t+1}], && (67)\\
\hat{q}_{kt}+\hat{\mu}_{et} &= \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}\phi_k}{\bar{\lambda}_q}(\hat{\mu}_{bt}+\hat{\theta}_t)+\frac{\beta(1-\delta)}{\lambda_k}\mathrm{E}_t(\hat{q}_{k,t+1}-\hat{g}_{q,t+1}-\hat{g}_{\gamma,t+1})+\left(1-\frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}\phi_k}{\bar{\lambda}_q}\right)\mathrm{E}_t\hat{\mu}_{e,t+1} \\
&\quad +\frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}\phi_k}{\bar{\lambda}_q}\mathrm{E}_t(\hat{q}_{k,t+1}-\hat{g}_{q,t+1})+\beta\alpha(1-\phi)\frac{\tilde{Y}}{\tilde{K}}\mathrm{E}_t(\hat{Y}_{t+1}-\hat{K}_t), && (68)\\
\hat{q}_{lt}+\hat{\mu}_{et} &= \frac{\tilde{\mu}_b}{\tilde{\mu}_e}g_\gamma\bar{\theta}\phi_l(\hat{\theta}_t+\hat{\mu}_{bt})+\left(1-\frac{\tilde{\mu}_b}{\tilde{\mu}_e}g_\gamma\bar{\theta}\phi_l\right)\mathrm{E}_t\hat{\mu}_{e,t+1}+\frac{\tilde{\mu}_b}{\tilde{\mu}_e}g_\gamma\bar{\theta}\phi_l\mathrm{E}_t(\hat{q}_{l,t+1}+\hat{g}_{\gamma,t+1}) \\
&\quad +\beta\mathrm{E}_t\hat{q}_{l,t+1}+(1-\beta-\beta\bar{\lambda}_a\bar{\theta}\phi_l)\mathrm{E}_t[\hat{Y}_{t+1}-\hat{L}_{et}], && (69)\\
\hat{\mu}_{et}-\hat{R}_t &= \frac{1}{1+\bar{\lambda}_a}\left[\mathrm{E}_t(\hat{\mu}_{e,t+1}-\hat{g}_{\gamma,t+1})+\bar{\lambda}_a\hat{\mu}_{bt}\right], && (70)\\
\hat{Y}_t &= \alpha\phi\hat{L}_{e,t-1}+\alpha(1-\phi)\hat{K}_{t-1}+(1-\alpha)\hat{N}_t-\frac{(1-\phi)\alpha}{1-(1-\phi)\alpha}[\hat{g}_{zt}+\hat{g}_{qt}], && (71)\\
\hat{K}_t &= \frac{1-\delta}{\lambda_k}[\hat{K}_{t-1}-\hat{g}_{\gamma t}-\hat{g}_{qt}]+\left(1-\frac{1-\delta}{\lambda_k}\right)\hat{I}_t, && (72)\\
\hat{Y}_t &= \frac{\tilde{C}_h}{\tilde{Y}}\hat{C}_{ht}+\frac{C_e}{\tilde{Y}}\hat{C}_{e,t}+\frac{\tilde{I}}{\tilde{Y}}\hat{I}_t, && (73)\\
0 &= \frac{\tilde{L}_h}{\bar{L}}\hat{L}_{ht}+\frac{\tilde{L}_e}{\bar{L}}\hat{L}_{et}, && (74)\\
\hat{Y}_t &= \frac{\tilde{C}_e}{\tilde{Y}}\hat{C}_{e,t}+\frac{\tilde{I}}{\tilde{Y}}\hat{I}_t+\frac{\tilde{q}_l\tilde{L}_e}{\tilde{Y}}(\hat{L}_{et}-\hat{L}_{e,t-1})+s_n(\hat{w}_t+\hat{N}_t) \\
&\quad +\frac{1}{g_\gamma}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_{t-1}-\hat{g}_{\gamma t})-\frac{1}{R}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_t-\hat{R}_t), && (75)
\end{aligned}$$

$$\begin{aligned}\hat{B}_t &= \hat{\theta}_t + g_\gamma \bar{\theta}\phi_l \frac{\tilde{q}_l L_e}{\tilde{Y}}\frac{\tilde{Y}}{\tilde{B}}\mathrm{E}_t(\hat{q}_{l,t+1} + \hat{L}_{et} + \hat{g}_{\gamma,t+1}) \\ &+\bar{\theta}\frac{\phi_k}{\bar{\lambda}_q}\frac{\tilde{K}}{\tilde{Y}}\frac{\tilde{Y}}{\tilde{B}}\mathrm{E}_t(\hat{q}_{k,t+1} + \hat{K}_t - \hat{g}_{q,t+1}) - \phi_w s_n R\frac{\tilde{Y}}{\tilde{B}}(\hat{w}_t + \hat{N}_t + \hat{R}_t). \qquad (76)\end{aligned}$$

The terms $\hat{g}_{zt}$, $\hat{g}_{qt}$, and $\hat{g}_{\gamma t}$ are given by

$$\hat{g}_{zt} = \hat{\lambda}_{zt} + \hat{\nu}_{zt} - \hat{v}_{z,t-1}, \qquad (77)$$

$$\hat{g}_{qt} = \hat{\lambda}_{qt} + \hat{\nu}_{qt} - \hat{v}_{q,t-1}, \qquad (78)$$

$$\hat{g}_{\gamma t} = \frac{1}{(1-(1-\phi)\alpha)}\hat{g}_{zt} + \frac{(1-\phi)\alpha}{(1-(1-\phi)\alpha)}\hat{g}_{qt}. \qquad (79)$$

The technology shocks follow the processes

$$\hat{\lambda}_{zt} = \rho_z\hat{\lambda}_{z,t-1} + \hat{\varepsilon}_{zt}, \qquad (80)$$

$$\hat{\nu}_{zt} = \rho_{\nu_z}\hat{\nu}_{z,t-1} + \hat{\varepsilon}_{\nu_z t}, \qquad (81)$$

$$\hat{\lambda}_{qt} = \rho_q\hat{\lambda}_{q,t-1} + \hat{\varepsilon}_{qt}, \qquad (82)$$

$$\hat{\nu}_{qt} = \rho_{\nu_q}\hat{\nu}_{q,t-1} + \hat{\varepsilon}_{\nu_q t}. \qquad (83)$$

$$(84)$$

There preference shocks follow the processes

$$\hat{\lambda}_{at} = \rho_a\hat{\lambda}_{a,t-1} + \hat{\varepsilon}_{at}, \qquad (85)$$

$$\hat{\varphi}_t = \rho_\varphi\hat{\varphi}_{t-1} + \hat{\varepsilon}_{\varphi t}, \qquad (86)$$

$$\hat{\psi}_t = \rho_\psi\hat{\psi}_{t-1} + \hat{\varepsilon}_{\psi t}. \qquad (87)$$

The liquidity shock follows the process

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} + \hat{\varepsilon}_{\theta t}. \qquad (88)$$

We solve the 19 equations (61) through (79) for the 19 unknowns in the vector

$$x_t = [\hat{\mu}_{ht}, \hat{w}_t, \hat{q}_{lt}, \hat{R}_t, \hat{\mu}_{et}, \hat{\mu}_{bt}, \hat{N}_t, \hat{I}_t, \hat{Y}_t, \hat{C}_{ht}, \hat{C}_{et}, \hat{q}_{kt}, \hat{L}_{ht}, \hat{L}_{et}, \hat{K}_t, \hat{B}_t, \hat{g}_{\gamma t}, \hat{g}_{zt}, \hat{g}_{qt}]'.$$

The state variables consist of the predetermined variables and the exogenous forcing processes summarized in the vector

$$s_t = [\hat{C}_{h,t-1}, \hat{C}_{e,t-1}, \hat{I}_{t-1}, \hat{L}_{e,t-1}, \hat{K}_{t-1}, \hat{B}_{t-1}, \hat{\lambda}_{zt}, \hat{\nu}_t, \hat{\lambda}_{qt}, \hat{\mu}_t, \hat{\lambda}_{at}, \hat{\varphi}_t, \hat{\psi}_t, \hat{\theta}_t]'$$

We use Chris Sims's gensys algorithm to solve the model.

I.2.5. *Estimation results.* To estimate the model, we follow the literature by calibrating the value of ϕ_w at 0.75 (Christiano, Motto, and Rostagno, 2010). Since growth of land supply does not affect equilibrium dynamics, we set $\bar{\lambda}_l = 1$, corresponding to no growth in aggregate land supply, consistent with the evidence on land quantity discussed in Supplemental Appendix II.

As shown in Tables 1 and 2, the estimated parameters in the model with working capital (in the columns under the heading "WK") are similar to those in the benchmark model (in the column under "Bench"). The estimation results are even closer to those in the benchmark model when the value of ϕ_w is calibrated to be less than 0.75.

II. Other variations in the model and the data

This section presents a few other variations of the benchmark model and some alternative data for estimation.

II.1. **DSGE model estimated with CoreLogic data.** The land price series we use for the benchmark model is based on the FHFA home price index. In Supplemental Appendix II, we discuss advantages and disadvantages of using this price index series relative to using other land price indices. To examine whether our main findings are robust to different land price series, we fit our model to the data in which the FHFA land price series is replaced by the CoreLogic land price series.

Tables 1 and 2 show that the estimated parameters using CoreLogic data are similar to those using the FHFA data, except that the volatility of housing demand shocks is substantially higher. The CoreLogic data imply a more volatile land price series because CoreLogic has a broader coverage of home prices than does FHFA: it includes both conforming loans and jumbo loans and it covers distressed home sales including short sales and foreclosures.

With the CoreLogic land price data, housing demand shocks account for over 50% of investment fluctuations (see Table 4, the column under "CoreLogic"). Thus, our results are robust with this different measure of land prices.

II.2. **Regime switching models: VAR and DSGE.** Our land price series spans the sample from 1975 to 2010, covering several recession periods with changes in macroeconomic volatility (Stock and Watson, 2003; Sims and Zha, 2006; Taylor, 2007). It is therefore important to investigate how our results are affected when volatility changes are explicitly taken into account.

We first fit our bivariate BVAR with a regime switching process on the volatility of a shock to land prices, following the approach of Sims, Waggoner, and Zha (2008). We find that the best-fit model is a Markov-switching BVAR with two volatility regimes. Figure 1 shows that the high-volatility regime is associated with the periods in the late 1970s, the early 1980s, and the recent deep recession; and that the low-volatility regime corresponds to the Great Moderation period. In Figure 2 we display the joint dynamics of land prices and business investment for both volatility regimes. Comparing to the impulse responses estimated from the constant-parameter BVAR model (reported in Figure 2 of the paper), the qualitative patterns of the impulse responses do not change. The main difference lies in the magnitude of responses to the land price shock under the two volatility regimes.

To examine implications of regime shifts in shock volatility in our DSGE model, we generalize the benchmark model to allow for regime shifts in the volatility of a housing demand shock, with the following heteroskedastic process

$$\ln \varphi_t = (1 - \rho_\varphi) \ln \bar{\varphi} + \rho_\varphi \ln \varphi_{t-1} + \sigma_\varphi(s_t)\varepsilon_{\varphi t}, \tag{89}$$

where the shock volatility $\sigma_\varphi(s_t)$ varies with the regime s_t. We assume that the shock volatility switches between two regimes ($s_t = 1$ or $s_t = 2$), with the Markov transition probabilities summarized by the matrix $P = [p_{ij}]$, where $p_{ij} = Prob(s_{t+1} = i | s_t = j)$ for $i, j \in \{1, 2\}$, $p_{12} = 1 - p_{22}$, and $p_{21} = 1 - p_{11}$.

We estimate this regime-switching DSGE model using the approach described in Liu, Waggoner, and Zha (2010). In the estimation, we adopt the same prior distributions for the parameters and use the same data set as in our benchmark model. The posterior mode estimates of the structural parameters and the shock parameters are very similar to those in the benchmark model, as shown in Tables 1 and 2 (in the columns under the heading "RS").

The estimated volatility of a housing demand shock has two distinct regimes: a low-volatility regime (regime 1 with $\sigma_\varphi = 0.03$) and a high-volatility regime (regime 2 with $\sigma_\varphi = 0.08$). The posterior mode estimates of the Markov switching probabilities ($p_{11} = 0.9794$ and $p_{22} = 0.9662$) indicate that both regimes are highly persistent, although the low-volatility regime is more persistent than the high-volatility regime.

Figure 3 shows the probability of the high volatility regime throughout the sample periods. It indicates that the high volatility regime is associated with periods of large declines in land prices (covering the two recessions between 1978 and 1983 and the recent deep recession).

According to the estimated variance decompositions, a housing demand shock accounts for about 20% of investment fluctuations in the low-volatility regime and $55-65\%$ in the high-volatility regime (see the last two columns in Table 4). Since the high-volatility regime captures periods with both large recessions and large declines in the land price, a housing demand shock plays a more important role for explaining the dynamics in land prices and business investment during recessions. This finding is consistent with Claessens, Kose, and Terrones (2011), who find that a recession is typically deeper than other recessions if there is a sharp fall in housing prices.

In addition, we estimate our DSGE model by increasing the number of volatility regimes to 3 and compute the marginal data density for each of the three models: the benchmark DSGE model (2344.0), the DSGE with two volatility regimes (2354.1), and the DSGE model with three volatility regimes (2353.2). According to these marginal data density results, the data favor the DSGE with two volatility regimes. We did not estimate a DSGE model with possible shifts in coefficients partly because there is no consensus on which parameters should be allowed to switch regime and partly because the task of solving and estimating a DSGE model with coefficients switching regime continues to be daunting. (Farmer, Waggoner, and Zha (2009) and Farmer, Waggoner, and Zha (2011) discuss conceptual issues regarding a regime-switching rational expectations model.) Computing time for obtaining the accurate marginal data density for the DSGE model with two or three volatility regimes is two and a half weeks on a cluster of dual-core computers.

Although it is infeasible to estimate DSGE models with coefficients switching regimes, we explore estimation of various bivariate (land price and investment) BVAR models with both volatility regimes and coefficient regimes. We compute the marginal data density for each model. The BVAR model with 3 volatility regimes has the highest marginal data density (632.3). The marginal data density is 613.2 for the BVAR model with no regime switching, 623.6 for the BVAR model with 2 volatility regimes, and 632.0 for the BVAR model with 4 volatility regimes. When we allow the coefficients for the land price equation in the BVAR model to switch between two regimes, where the Markov process controlling coefficient changes is independent of the 3-regime volatility Markov process, the marginal data density (627.2) becomes considerably lower. These results confirm the finding by Sims and Zha (2006) that once volatility changes are allowed, it is difficult to find changes in coefficients favored by the data.

II.3. **No patience shocks.** An intertemporal preference (patience) shock λ_{at} has been used in the DSGE literature as one of important shocks in driving business cycles

(Smets and Wouters, 2007). In our estimated model, the patience shock accounts for a nontrivial fraction of investment fluctuations (about $15-20\%$). Therefore it is important to examine whether abstracting from this shock would change the model's quantitative implications in a significant way. We reestimate the model without patience shocks. The estimation results are shown in Tables 1 and 2 (under the heading "No patience"). The estimates are broadly similar to those in the benchmark model. Without patience shocks, we find that a housing demand shock remains to be the most important driving force for investment dynamics, accounting for about $30-40\%$ of investment fluctuations (see Table 4, the column under "No Patience").

II.4. **Latent IST shocks.** Justiniano, Primiceri, and Tambalotti (2011) argue that if the price of investment goods is not used in fitting the model, investment-specific shocks can be interpreted as "financial" shocks and may have a large impact on macroeconomic fluctuations. When we reestimate the model by treating the IST shocks as a latent variable (i.e., without fitting to the time-series data of the relative price of investment), we find that the estimation results are similar to the benchmark model and a housing demand shock still accounts for $23\%-46\%$ of investment fluctuations (see Table 4, the column under "Latent IST").

II.5. **How important is land as a collateral asset?** In the data, real estate represents a large fraction of firms' tangible assets and, as discussed in the introduction, changes in the real estate value have a significant impact on firms' investment spending. In our benchmark model, we assume that land is a collateral asset for firms. A positive housing demand shock raises the land price and thereby expands the firm's borrowing capacity, enabling the firm to finance expansions of investment and production.

How important is land as a collateral asset in our macroeconomic model? To answer this question, we study an alternative model specification with the general setup of a collateral constraint as

$$B_t \leq \theta_t \mathrm{E}_t \left[\omega_l \, q_{l,t+1} L_{et} + \omega_k q_{k,t+1} K_t, \right], \tag{90}$$

where ω_l is the weight put on land value and ω_k on capital value. The weight parameters ω_l and ω_k cannot be identified separately, but one can identify the relative weight ω_k/ω_l. This model nests our benchmark model as a special case when $\omega_k/\omega_l = 1$.

We estimate this alternative model using the same set of time series data. The estimation implies that the relative weight for capital value ω_k/ω_l is 1.2. As shown in Figure 4, the impulse responses to both a TFP shock and a housing demand shock are very close to those from the benchmark model (compare the dotted-dashed lines and the asterisk lines).

II.6. **Frisch labor supply elasticity.** In the benchmark model, we assume that labor is indivisible so that the utility of leisure is linear and aggregate labor supply elasticity is infinity (Hansen, 1985; Rogerson, 1988). We now consider the alternative specification of the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^{t}A_{t}\left\{\log(C_{ht}-\gamma_{h}C_{h,t-1})+\varphi_{t}\log L_{ht}-\psi_{t}\frac{N_{ht}^{1+\eta}}{1+\eta}\right\}, \tag{91}$$

where $\eta > 0$ is the inverse Frisch elasticity of labor supply.

With this utility function, all equilibrium conditions are identical except that the labor supply equation (21) is changed into

$$w_{t}=\frac{A_{t}}{\mu_{ht}}\psi_{t}N_{ht}^{\eta}. \tag{92}$$

This change affects the steady state labor as well as the log-linearized version of the labor supply decision. In particular, the steady-state solution for labor (60) becomes

$$N=\left\{\frac{(1-\alpha)g_{\gamma}(1-\gamma_{h}/R)}{\bar{\psi}(g_{\gamma}-\gamma_{h})}\frac{\tilde{Y}}{\tilde{C}_{h}}\right\}^{\frac{1}{1+\eta}}. \tag{93}$$

The linearized labor supply equation (62) is replaced by

$$\hat{w}_{t}+\hat{\mu}_{ht}=\hat{\psi}_{t}+\eta\hat{N}_{t}. \tag{94}$$

We reestimate the benchmark model with the disutility function of labor replaced by this more flexible form. The posterior mode estimate for η is about 2.1. The estimates for the other parameters are similar to those in the benchmark model. As shown in Figure 4, the impulse responses of the land price and business investment to a TFP shock and to a housing demand shock do not differ much from those in the benchmark model (compare the dashed lines and the asterisk lines).

II.7. **External capital producers.** In the benchmark model, capital adjustment is internal to the entrepreneurs, so that the price of capital reflect the shadow value of capital that is specific to the entrepreneurs and does not reflect the market value of capital. In this sense, capital is a less pledgable than land for external financing. We now consider an alternative specification such that capital adjustment is done by an external capital producer (Carlstrom and Fuerst, 1997; Bernanke, Gertler, and Gilchrist, 1999). Entrepreneurs still accumulate capital, but they need to purchase new capital at a market price. We assume that the capital producers are owned by the household sector, so that capital is as pledgable as land as a collateral for borrowing.

There is a continuum of identical capital producers. The representative capital producer purchases one unit of investment goods at the price $1/Q_t$ and transforms the goods into usable capital. The transformation from investment goods to capital incurs an adjustment cost. Capital goods are then sold to the entrepreneurs at the market price q_{kt}.

The capital producer chooses investment I_t to maximize the cum-dividend (including dividend) value

$$V_t = \sum_{j=0}^{\infty} \mathrm{E}_t \beta^j \frac{\mu_{h,t+j}}{\mu_{ht}} \{ q_{kt+j} \left[1 - \frac{\Omega}{2} \left(\frac{I_{t+j}}{I_{t+j-1}} - \bar{\lambda}_I \right)^2 \right] I_{t+j} - \frac{I_{t+j}}{Q_{t+j}} \}. \tag{95}$$

The first order condition is given by

$$\begin{aligned} \frac{1}{Q_t} &= q_{kt} \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 - \Omega \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right) \frac{I_t}{I_{t-1}} \right] \\ &\quad + \beta \Omega \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}} q_{k,t+1} \left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I \right) \left(\frac{I_{t+1}}{I_t} \right)^2, \end{aligned} \tag{96}$$

This equation replaces the investment Euler equation (27) in the benchmark model.

The representative entrepreneur purchases capital goods from the capital producers at the market price q_{kt}. The entrepreneur's problem is given by

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \left[\log(C_{et} - \gamma_e C_{e,t-1}) \right], \tag{97}$$

subject to the flow-of-funds constraint

$$C_{et} + q_{lt}(L_{et} - L_{e,t-1}) + B_{t-1} + q_{kt}(K_{t+1} - (1-\delta)K_t) = Z_t [L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}]^{\alpha} N_{et}^{1-\alpha} - w_t N_{et} + \frac{B_t}{R_t}, \tag{98}$$

and the borrowing constraint

$$B_t \leq \theta_t E_t [q_{l,t+1} L_{et} + q_{k,t+1} K_t]. \tag{99}$$

The flow-of-funds constraint (98) here differs from (11) in the benchmark model in that the entrepreneur does not pay any internal investment adjustment costs, but instead, the entrepreneur acquires capital at the market price.

This change does not affect the capital Euler equation (28) in the benchmark model, which is rewritten here for convenience of references:

$$q_{kt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} \left[\alpha(1-\phi) \frac{Y_{t+1}}{K_t} + q_{k,t+1}(1-\delta) \right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \mathrm{E}_t q_{k,t+1}. \tag{100}$$

In equilibrium, capital goods market clearing implies that gross investment equals new capital goods produced (net of adjustment costs) so that

$$K_{t+1} - (1-\delta)K_t = \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2\right] I_t. \tag{101}$$

This equation corresponds to the capital law of motion (10) in the benchmark model.

To summarize, if capital goods are produced by an external sector, then we need to change two equilibrium conditions in the benchmark model. First, the investment Euler equation should be replaced by (96). Second, the flow-of-funds constraint for the entrepreneur should be replaced by (98).

These changes do not affect the steady state conditions (since the steady-state adjustment cost is zero). They do affect the log-linearized equilibrium dynamics. In particular, the log-linearized investment Euler equation (67) is replaced by

$$\begin{aligned} \hat{q}_{kt} &= (1+\beta(1+\bar{\lambda}_a))\Omega\lambda_k^2 \hat{I}_t - \Omega\lambda_k^2 \hat{I}_{t-1} + \Omega\lambda_k^2(\hat{g}_{\gamma t} + \hat{g}_{qt}) \\ &\quad - \beta(1+\bar{\lambda}_a)\Omega\lambda_k^2 \mathrm{E}_t[\hat{I}_{t+1} + \hat{g}_{\gamma,t+1} + \hat{g}_{q,t+1}], \end{aligned} \tag{102}$$

and the linearized flow-of-funds equation for the entrepreneur (75) is replaced by

$$\begin{aligned} \alpha\hat{Y}_t &= \frac{\tilde{C}_e}{\tilde{Y}}\hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}}(\hat{q}_{kt} + \hat{I}_t) + \frac{\tilde{q}_l L_e}{\tilde{Y}}(\hat{L}_{et} - \hat{L}_{e,t-1}) \\ &\quad + \frac{1}{g_\gamma}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_{t-1} - \hat{g}_{\gamma t}) - \frac{1}{R}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_t - \hat{R}_t), \end{aligned} \tag{103}$$

The estimated results for this alternative model are similar to those for the benchmark. Figure 4 displays the impulse responses of the land price and business investment to a TFP shock and to a housing demand shock for both models. As one can see, the results do not differ much (compare the dotted lines and the asterisk lines).

III. Stock prices, land prices, and investment

In our model, there are two types of collateral assets: land and capital. We choose to fit our model to land prices but not to stock prices. We find that shocks to land prices can explain a substantial fraction of investment fluctuations. We choose not to fit the model to stock prices because our model, as most of the DSGE models in the literature, is not equipped with the necessary frictions and shocks to explain the joint dynamics between stock prices and macroeconomic variables.

To examine the joint dynamics between land prices, business investment, and stock prices in the U.S. data, we estimate a recursive Bayesian VAR with these three variables, where the land price is ordered first, investment second, and the stock price third. This identification implies that stocks prices respond to all variables instantly, consistent with the argument put forth in the literature (for example, Leeper, Sims, and Zha (1996) and Bloom (2009)).

Figure 5, below, displays the impulse responses estimated from the BVAR model. The first column reports the responses following a shock to the land price. The shock leads to persistent increases in the land price, investment spending, and the stock price. The second column shows the responses following a shock to the stock price. This stock-price shock leads to a large increase in the stock price as well as a persistent increase in investment spending. The land price, however, does not seem to respond to the stock price shock and, if anything, the point estimates show that the land price actually declines slightly. This BVAR evidence suggests that the positive comovements between land prices and investment spending are driven by land price shocks rather than stock price shocks.

Further, Figure 5 shows that, following a land price shock, the stock price rises but the magnitude of its increase is much smaller than that for the land price. This evidence supports our model's implication that, following a housing demand shock, the land price rises much faster than does the value of capital. Thus, the land price dynamics play a key role in the amplification mechanism in our model.

Stock price dynamics are likely to be driven by other economic shocks that differ from a housing demand shock. As we discuss in the conclusion section, a challenging task for future research is to build a model to explain the empirical facts revealed by the BVAR impulse responses. Our current model is not designed to meet this challenge, partly because our focus is on the link between land price dynamics and macroeconomic fluctuations, which, in our view, is of substantive interest by itself; and partly because such a task is beyond the scope of this paper. In a related but very different setup, however, Christiano, Motto, and Rostagno (2008) fit a DSGE model to stock prices along with other macroeconomic variables. A more ambitious project in future research should fit a DSGE model to both land prices and stock prices and we hope that our model is a step toward that direction.

Table 1. Posterior mode estimates of structural parameters

Parameter	Bench	Alt	Regime	WK	CoreLogic	No patience	Latent IST
γ_h	0.4976	0.5660	0.4922	0.4988	0.5323	0.5392	0.5819
γ_e	0.6584	0.8398	0.6592	0.6674	0.5444	0.7375	0.6335
Ω	0.1753	6.3513	0.1642	0.1603	0.2923	0.1997	0.1831
$100(g_\gamma - 1)$	0.4221	0.3736	0.4493	0.4072	0.4352	0.3200	0.4335
$100(\bar{\lambda}_q - 1)$	1.2126	1.2484	1.2206	1.2071	1.1889	1.2110	1.3750
β	0.9855	0.9706	0.9848	0.9860	0.9852	0.9884	0.9853
$\bar{\lambda}_a$	0.0089	0.0239	0.0099	0.0083	0.0094	0.0050	0.0093
$\bar{\varphi}$	0.0457	0.0495	0.0436	0.0468	0.0447	0.0535	0.0449
ϕ	0.0695		0.0694	0.0696	0.0695	0.0698	0.0694
δ	0.0368	0.0369	0.0364	0.0370	0.0369	0.0378	0.0351

Note: The columns of numbers are the posterior mode estimates in various models. “Bench” denotes the benchmark model; “Alt” denotes the alternative model in which land is not used as collateral; ”Regime” denotes the Markov regime switching model; “WK” denotes the model with working capital; “CoreLogic” denotes the benchmark model estimated using land prices constructed based on the CoreLogic home price index; “No patience” denotes the benchmark model with no patience shocks; and “Latent IST” denotes the benchmark model estimated by treating the investment-specific technology shocks as a latent variable.

Table 2. Posterior mode estimates of shock parameters

Parameter	Bench	Alt	RS	WK	CL	No patience	Latent IST
ρ_a	0.9055	0.0883	0.9034	0.9479	0.9040		0.9928
ρ_z	0.4263	0.8798	0.4256	0.4040	0.5333	0.6826	0.4671
ρ_{ν_z}	0.0095	0.8770	0.0106	0.0501	0.3804	0.8782	0.0000
ρ_q	0.5620	0.4172	0.5494	0.5671	0.6649	0.5719	0.5856
ρ_{ν_q}	0.2949	0.4471	0.2836	0.2838	0.3947	0.3246	0.2914
ρ_φ	0.9997	0.9690	0.9995	0.9998	0.9999	0.9998	0.9979
ρ_ψ	0.9829	0.9749	0.9816	0.9959	0.9810	0.9886	0.9997
ρ_θ	0.9804	0.9623	0.9793	0.9894	0.9884	0.9852	0.9663
σ_a	0.1013	3.8118	0.0887	0.0941	0.1463		0.0001
σ_z	0.0042	0.0012	0.0042	0.0043	0.0033	0.0033	0.0057
σ_{ν_z}	0.0037	0.0063	0.0037	0.0036	0.0044	0.0071	0.0037
σ_q	0.0042	0.0057	0.0042	0.0041	0.0041	0.0041	0.0062
σ_{ν_q}	0.0029	0.0001	0.0029	0.0030	0.0029	0.0029	0.0001
$\sigma_\varphi(1)$	0.0462	0.1985	0.0316	0.0485	0.1080	0.0436	0.0566
$\sigma_\varphi(2)$			0.0785				
σ_ψ	0.0073	0.0106	0.0073	0.0076	0.0081	0.0079	0.0087
σ_θ	0.0112	0.0171	0.0112	0.0114	0.0237	0.0116	0.0116
p_{11}			0.9792				
p_{22}			0.9664				

Note: The columns of numbers are the posterior mode estimates in various models. "Bench" denotes the benchmark model; "Alt" denotes the alternative model in which land is not used as collateral; "Regime" denotes the Markov regime switching model; "WK" denotes the model with working capital; "CoreLogic" denotes the benchmark model estimated using land prices constructed based on the CoreLogic home price index; "No patience" denotes the benchmark model with no patience shocks; and "Latent IST" denotes the benchmark model estimated by treating the investment-specific technology shocks as a latent variable.

Table 3. Contributions (in percent) to investment fluctuations from a collateral shocks in the alternative model

	Horizon				
	1Q	4Q	8Q	16Q	24Q
Mode	45.14	38.27	33.29	30.47	30.01
Interval	(42.31, 49.67)	(36.83, 42.97)	(31.32, 37.05)	(27.25, 36.51)	(26.77, 37.56)

Note: Each interval marks the bounds of the 68% posterior probability interval.

Table 4. Contributions (in percent) to investment fluctuations from a housing demand shock

Horizon	Bench	WK	No patience	Latent IST	CoreLogic	High vol	Low vol
1Q	35.46	37.26	34.10	41.10	55.74	60.49	19.19
4Q	41.19	40.17	39.31	46.35	58.68	66.31	23.39
8Q	38.71	38.18	37.27	39.02	57.90	63.96	21.59
16Q	33.70	34.99	31.74	28.48	54.60	58.85	18.16
24Q	30.67	33.51	28.66	23.48	52.18	55.46	16.19

Note: The column labeled by "Bench" reports the contributions in the benchmark model; the column "WK" reports those in the model with working capital; the column "No patience" reports the results in the benchmark model with no patience shocks; the column "Latent IST" reports those in the benchmark model estimated by treating the investment-specific technology shocks as a latent variable; the column "CoreLogic" reports the results from the benchmark model with the Core Logic data on the land price; and the columns "High vol" and "Low vol" report the contributions under the high and low volatility regimes from the regime-switching benchmark model.

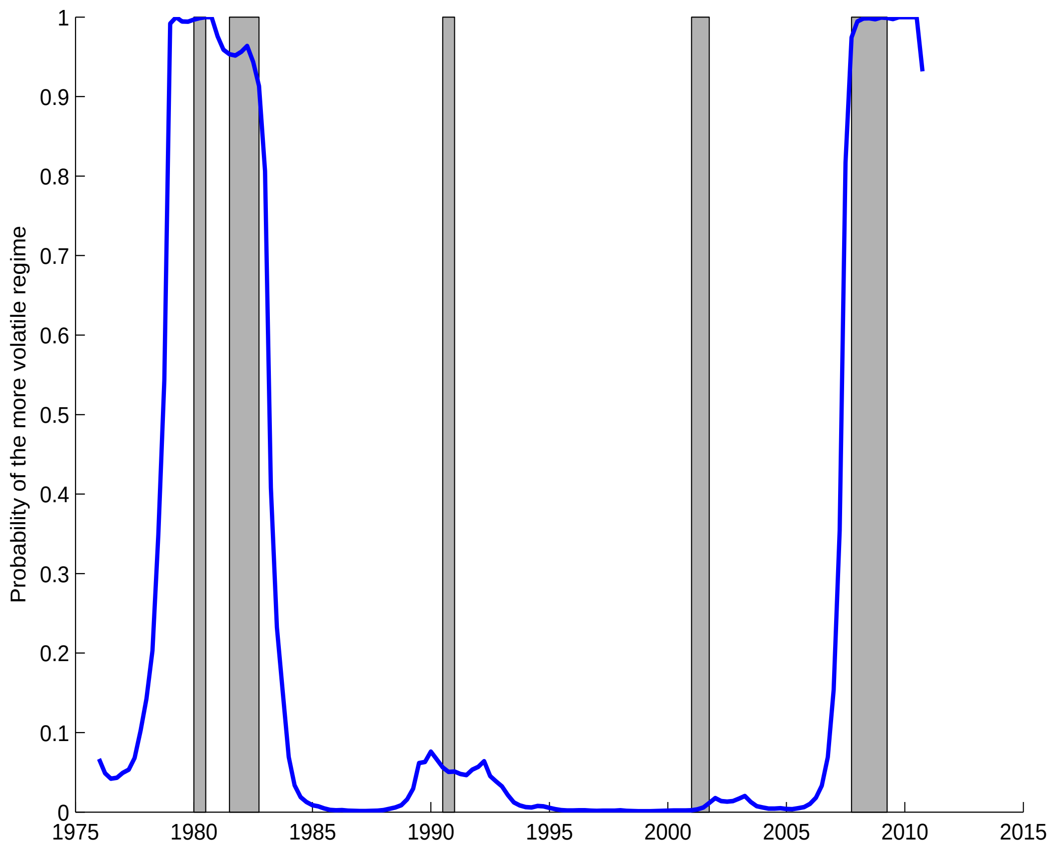

FIGURE 1. The posterior probability of the regime with a high volatility from the regime-switching BVAR model. The shaded area marks NBER recession dates.

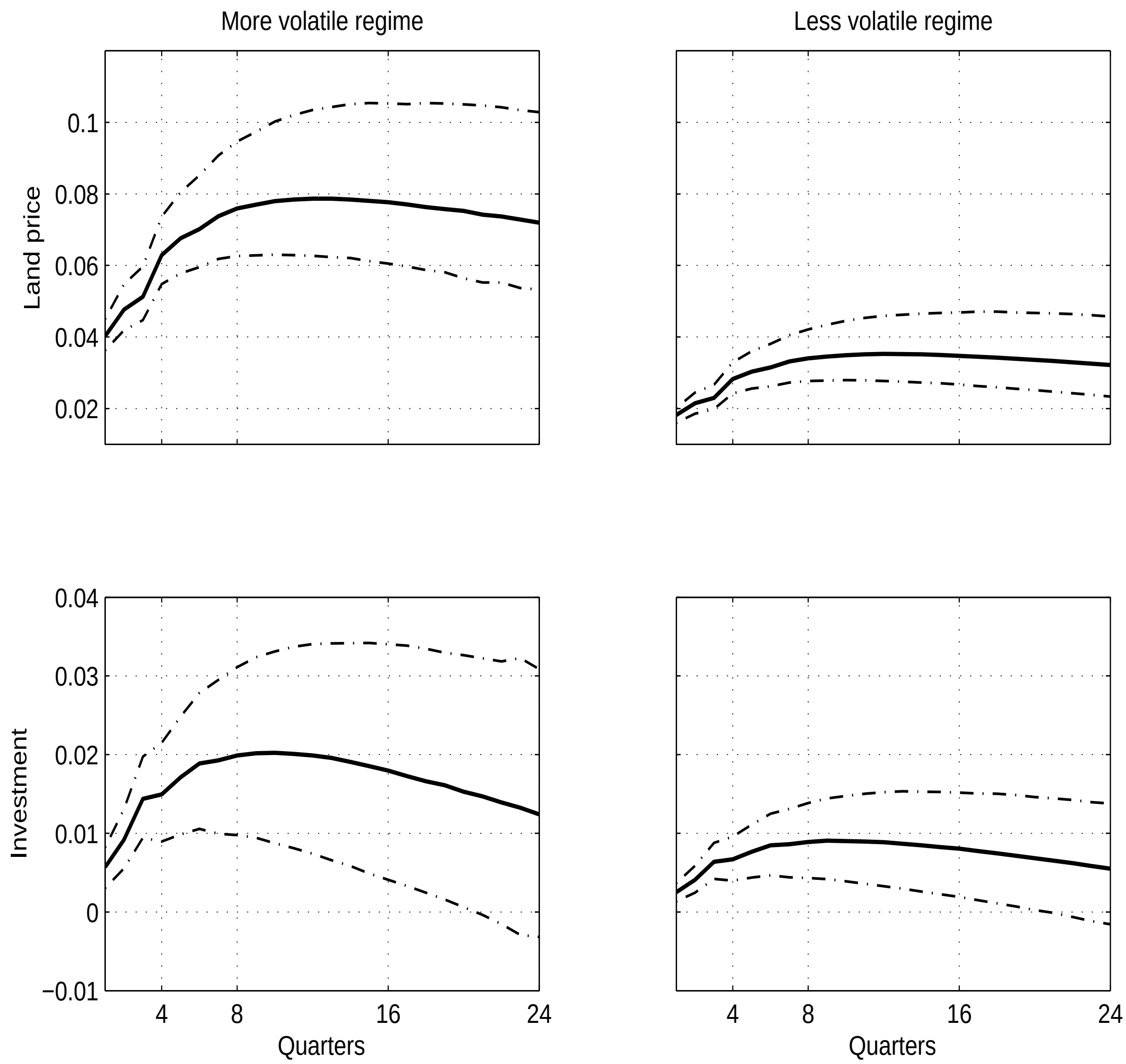

FIGURE 2. Impulse responses to a shock to the land price from a Markov-switching BVAR model.

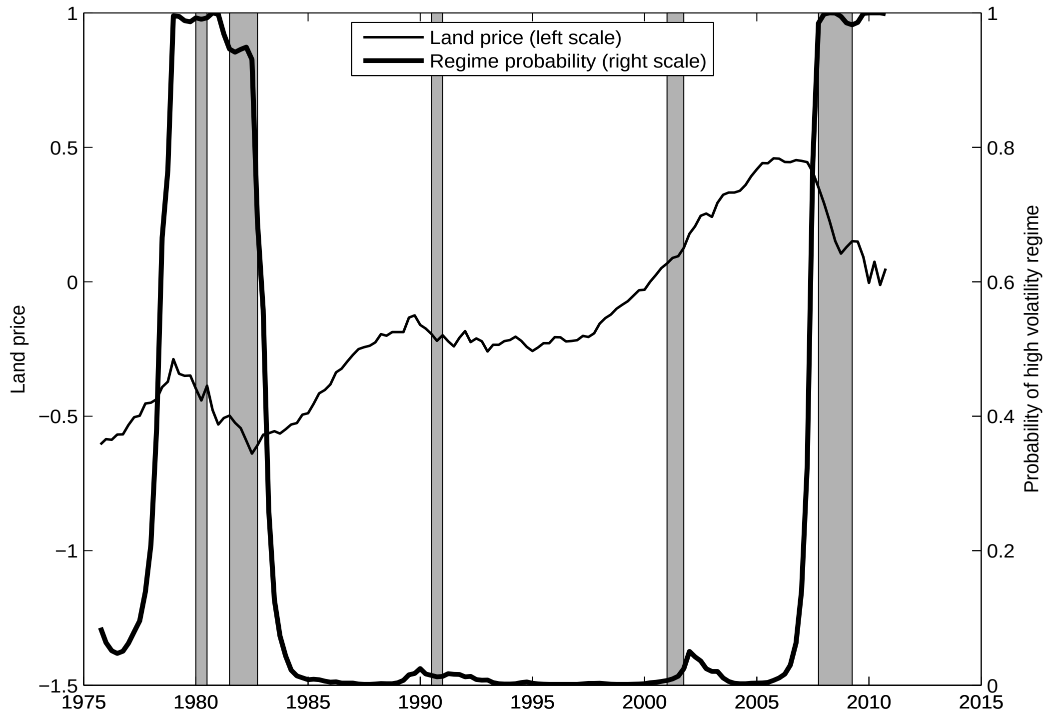

FIGURE 3. Log real land prices (left scale) and the posterior probability of the regime with larger volatility from the regime-switching model (right scale). The shaded area marks NBER recession dates.

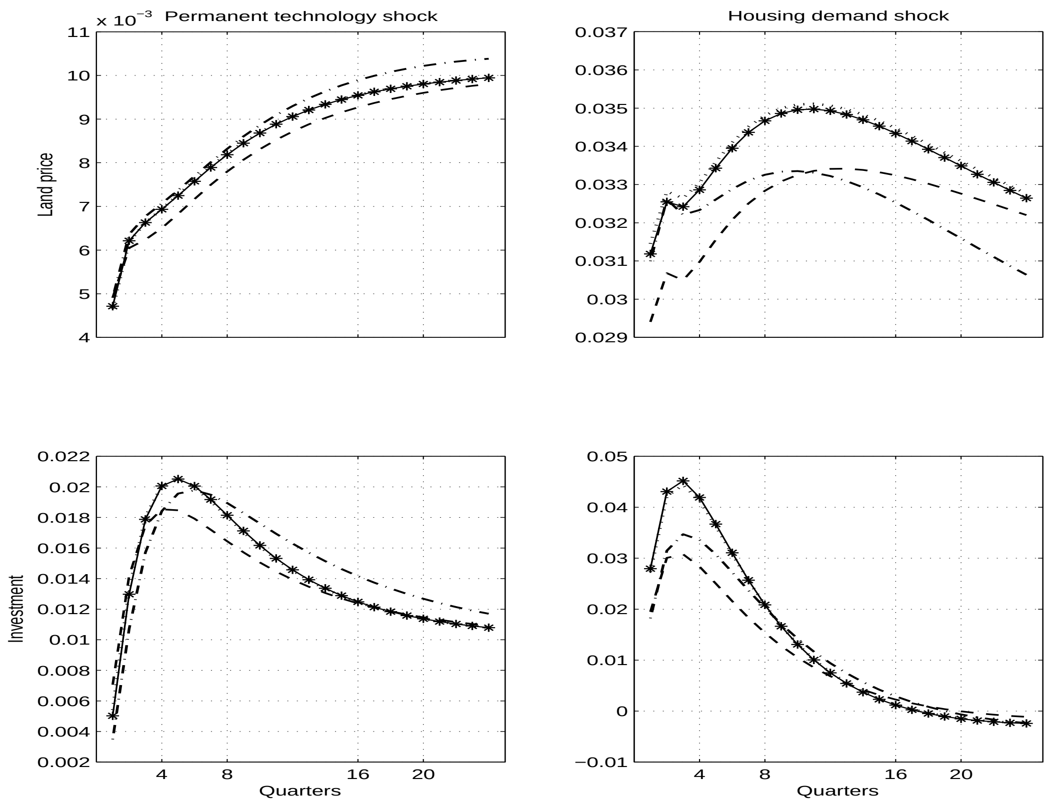

Figure 4. Impulse responses to a positive shock to neutral technology growth (left column) and to a positive shock to housing demand (right column). Lines marked by asterisks represent the responses for the benchmark model; dashed lines represent the model with a general form of the disutility function of labor; dotted-dashed lines represent the model with flexible relative weight on capital value in the credit constraint; dotted lines represent the model with external adjustments costs to capital. Note that the results are so close that some lines are on top of one another.

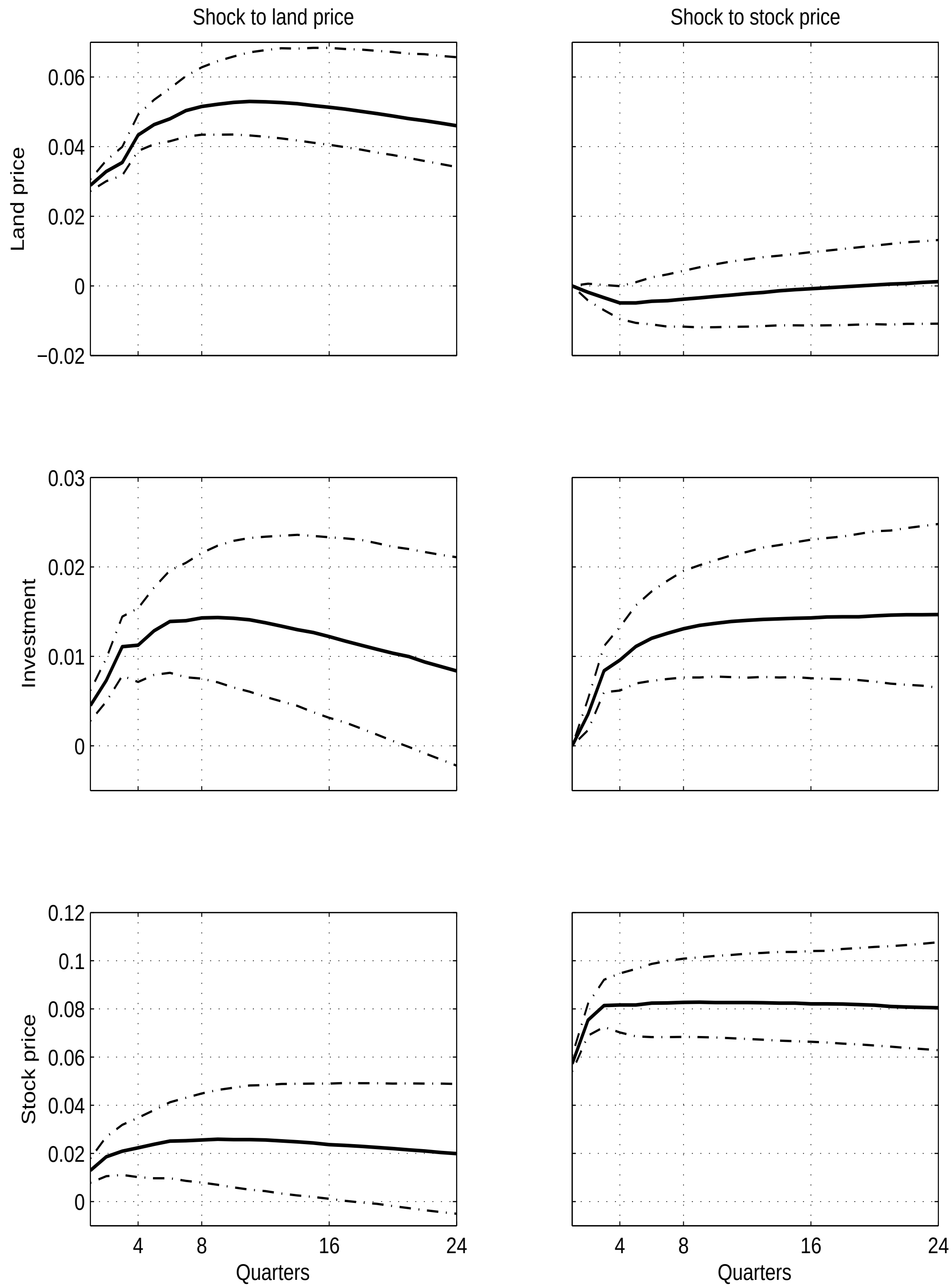

FIGURE 5. Impulse responses estimated from a BVAR model. Solid lines represent the estimated responses and dotted-dashed lines represent the 68% probability bands.

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