Endogenous Regime Switching Near the Zero Lower Bound

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August 2018

Suggested citation:

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August 27, 2018

Abstract

This paper develops a New Keynesian model in which the representative agent contemplates the possibility of an occasionally binding zero lower bound (ZLB) that is driven by switching between two local equilibria, labeled the "targeted" and "deflation" regimes, respectively. Sustained periods when the equilibrium real interest rate remains below its estimated long-run value can induce the agent to place a substantially higher weight on the deflation forecast rules, causing the deflation equilibrium to occasionally become fully realized. I solve for the time series of stochastic shocks and endogenous forecast rule weights that allow the model to exactly replicate the observed time paths of the U.S. output gap, quarterly inflation, and the federal funds rate since 1988. The data since the start of the ZLB episode in 2008.Q4 are best described as a mixture of the two local equilibria. In model simulations, raising the central bank's inflation target to 4% from 2% can reduce, but not eliminate, the endogenous switches to the deflation equilibrium.

Keywords: *Natural rate of interest, New Keynesian, Liquidity trap, Zero lower bound, Taylor rule, Deflation.*

JEL Classification: E31, E43, E52.

1 Introduction

The sample period from 1988 onwards is generally viewed as an example of consistent U.S. monetary policy aimed at keeping inflation low while promoting sustainable growth and full employment. Amazingly, the U.S. federal funds rate has been pinned close to zero for about one-fourth of the elapsed time since 1988. The U.S. economy is not alone in experiencing an extended period of zero or mildly negative nominal interest rates in recent decades.

Figure 1 plots three-month nominal Treasury bill yields in the United States, Japan, Switzerland, and the United Kingdom. Nominal interest rates in the United States encountered the zero lower bound during the 1930s and from 2008.Q4 though 2015.Q4. Nominal interest rates in Japan have remained near zero since 1998.Q3, except for the relatively brief period from 2006.Q4 to 2008.Q3. Nominal interest rates in Switzerland have been zero or slightly negative since 2008.Q4. Nominal interest rates in the United Kingdom have been approximately zero since 2009.Q1. Outside of these episodes, all four countries exhibit a strong positive correlation between nominal interest rates and inflation, consistent with the Fisher relationship.

Benhabib, Schmitt-Grohé and Uribe (2001a,b) show that imposing a zero lower bound (ZLB) on the nominal interest rate in a standard New Keynesian model gives rise to two long-run endpoints (steady states).[1] The basic idea is illustrated in Figure 2, which is adapted from Bullard (2010). The two intersections of the ZLB-augmented monetary policy rule (solid red line) with the Fisher relationship (dashed black line) define two long-run endpoints. I refer to these as the "targeted equilibrium" and "deflation equilibrium," respectively. The aim of this paper is to develop a quantitative New Keynesian model that can account for the pattern of inflation and interest rates observed in Figure 2 since 1988.

The model incorporates a shock to the agent's discount factor that gives rise to short-term movements in the equilibrium real interest rate. The long-run endpoint of the equilibrium real rate process is allowed to shift over time. This feature of the model captures the notion of a slow-moving natural rate of interest (r-star). R-star is an important benchmark for monetary policy because it determines the real interest rate that policymakers should aim for once shocks to the economy have dissipated and the central bank's macroeconomic targets have been achieved. The times series process for r-star in the model is calibrated to approximate

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[1] I use the terminology "long-run endpoints" rather than "steady states" because the model developed here allows for permanent shifts in the natural rate of interest which, in turn, can shift the long-run values of some macroeconomic variables.

the path of the U.S. natural rate series estimated by Laubach and Williams (2016).

As is well known, the New Keynesian deflation equilibrium is locally indeterminate. I therefore consider a minimum state variable (MSV) solution that rules out sunspot variables and extra lags of fundamental state variables. The decision rules associated with the deflation equilibrium induce more volatility in the output gap and inflation in response to shocks. Model variables in the deflation equilibrium have distributions with lower means and higher variances than those in the targeted equilibrium. But the significant overlap in the various distributions creates a dilemma for an agent who seeks to determine the likelihood that a string of recent data observations are drawn from one equilibrium or the other.

The representative agent in the model contemplates the possibility of an occasionally binding ZLB that is driven by switching between the two local equilibria. This view turns out to be true in the simulations, thus validating the agent's beliefs. The agent constructs forecasts using a form of model averaging, where the time-varying forecast weights are determined by recent performance, as measured by the root mean squared forecast errors for the output gap and inflation (the two variables that the agent must forecast). Sustained periods when the equilibrium real interest rate remains below its estimated long-run value can induce the agent to place a substantially higher weight on the deflation forecast rules, causing the deflation equilibrium to occasionally become fully realized. These episodes are accompanied by highly negative output gaps and a binding ZLB, reminiscent of the U.S. Great Recession. But even outside of recessions, the agent may continue to assign a nontrivial weight to the deflation equilibrium, causing the central bank to persistently undershoot its inflation target, similar to the U.S. economy since mid-2012.

I solve for the time series of stochastic shocks and endogenous forecast rule weights that allow the switching model to exactly replicate the observed time paths of the CBO output gap, quarterly PCE inflation, and the federal funds rate since 1988. The data since the start of the ZLB episode in 2008.Q4 are best described as a mixture of the two local equilibria. The model-implied weight on the targeted forecast rules starts to decline in 2008.Q4. At the end of the data sample in 2017.Q4, the weight on the targeted forecast rules is only 0.46, helping the switching model to account for the persistent undershooting of the Fed's inflation target since mid-2012. The path of expected inflation from the switching model starts to decline after 2008.Q4 and remains below the Fed's 2% inflation target through the end of the data sample. This pattern is very similar to the 1-year expected inflation rate derived from inflation swap contracts.

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The framework developed here is related to work by Aruoba and Schorfheide (2016) and Aruoba, Cuba-Borda, and Schorfheide (2018). These authors construct stochastic two-regime models in which the economy can switch back and forth between a targeted-inflation regime and a deflation regime, depending on the realization of an exogenous sunspot variable. They employ various model specifications to infer whether the interest rate and inflation observations in the data are more likely to have been generated by one regime or the other. Aruoba, Cuba-Borda, and Schorfheide (2018) conclude (p. 116) that: "the U.S. remained in the targeted-inflation regime during its ZLB episode, with the possible exception of the early part of 2009 where evidence is ambiguous."

An important premise underlying the results of Aruoba, Cuba-Borda, and Schorfheide (2018) is that the observed data must come from one regime or the other. In contrast, the switching model developed here generates data that is a mixture of the two regimes. This is due to the time-varying forecasts weights assigned by the representative agent in the model. The forecast weights, in turn, influence the model-generated data. So, the deflation regime can influence the model-generated data even if the deflation equilibrium is never fully realized. Another key difference is that the probability of transitioning between regimes is endogenous here and can therefore be influenced by a change in the monetary policy rule or other model parameters. The switching model shares some similarities with the work of Sargent (1999) in which the economy can endogenously transition between regimes of high versus low inflation, depending on policymakers' perceptions about the slope of the long-run Phillips curve. Here, the endogenous regime switching depends on the representative agent's perceptions about the best forecast weights.

Another related paper is one by Dordal-i-Carrera et al. (2016). These authors develop a New Keynesian model with volatile and persistent "risk shocks" (i.e., shocks that drive a wedge between the nominal policy rate and the short-term bond rate) to account for infrequent but long-lived ZLB episodes. A risk shock in their model is isomorphic to an equilibrium real rate shock here. Large adverse risk shocks are themselves infrequent and long-lived. As the binding ZLB episode becomes more frequent or more long-lived, the optimal inflation target increases. Unlike here, their analysis does not consider model solutions near the deflation equilibrium, but rather focuses on scenarios in which fundamental shocks are large enough to push the targeted equilibrium to a point where ZLB becomes binding.[2] In contrast, the model

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[2]This is also the methodology pursued by Reifschneider and Williams (2000), Schmitt-Grohé and Uribe (2010), Chung et al. (2012), Coibion, Gorodnichenko, and Wieland (2012), Dennis (2017), and Kiley and Roberts (2017).

developed here accounts for infrequent but long-lived ZLB episodes via endogenous switching between two local equilibria, i.e., the shock process itself is not the sole driving force for the infrequent and long-lived ZLB episodes.

As part of the quantitative analysis, I examine how raising the central bank's inflation target can influence the ZLB binding frequency and the volatility of macro variables in the switching model. I find that even with an inflation target of 4%, the ZLB binding frequency remains elevated at 10.5% and the average duration of a ZLB episode is 12.5 quarters. Once the deflation equilibrium is taken into account, raising the inflation target is a less effective solution for avoiding ZLB episodes. I also show that lower interest rate smoothing parameter in the Taylor-type rule serves to increase the ZLB binding frequency, but the episodes exhibit shorter duration on average.

It has been suggested by some that the deflation equilibrium should be ignored because it may not be learnable using standard algorithms.[3] Recently, however, Arifovic, Schmitt-Grohé, and Uribe (2018) demonstrate that both equilibria can be locally stable under a form of social learning. Armenter (2018) considers an extension of Benhabib, Schmitt-Grohé and Uribe (2001a,b) in which monetary policy is governed not by a Taylor-type rule, but rather by the optimal interest rate rule under discretion that minimizes the central bank's loss function. He shows that it may not be possible to achieve the targeted equilibrium if agents' initial inflation expectations are below the central bank's inflation target.

2   Model

The framework for the analysis is a three equation New Keynesian model, augmented by a zero lower bound on the short-term nominal interest rate. The log-linear version of the standard New Keynesian model is taken to represent a set of global equilibrium conditions, with the only nonlinearity coming from the ZLB. The setup is a reduced form version of a fully-specified nonlinear New Keynesian DSGE model, but has the advantage of delivering transparency of the model's dynamics.[4]

Private-sector behavior is governed by the following global equilibrium conditions:

$$y_t = E_t\, y_{t+1} - \alpha[i_t - E_t\, \pi_{t+1} - r_t] + \nu_t, \tag{1}$$

$$\pi_t = \beta E_t\, \pi_{t+1} + \kappa y_t + u_t, \tag{2}$$

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[3]See, for example, Evans, Guse, and Honkapohja (2008).

[4]Armenter (2018) adopts a similar approach in computing the optimal monetary policy under discretion in the presence of two steady states. Eggertsson and Sing (2016) show that the log-linear New Keynesian model behaves very similar to the true nonlinear model in the vicinity of the targeted equilibrium.

where equation (1) is the representative agents's consumption Euler equation and equation (2) is the Phillips curve that is derived from the representative firm's optimal pricing decision. The variable y_t is the output gap (the log deviation of real output from potential output), π_t is the quarterly inflation rate (log difference of the price level), i_t is the short-term nominal interest rate, r_t is the equilibrium real interest rate, i.e., the real interest rate that is consistent with efficient utilization of economic resources. Fluctuations in r_t can be interpreted as arising from changes in the agent's rate of time preference or changes in the expected growth rate of potential output.[5]

In solving the model, the expectations operator E_t will correspond to rational expectations in the vicinity of the model's long-run endpoints. None of the results in the paper are sensitive to the introduction of a discount factor applied to the term $E_t\, y_{t+1}$ in equation (1), along the lines of McKay, Nakamura, and Steinsson (2017). The variable ν_t represents a demand shock that is distinct from fluctuations in r_t. Along the lines of Bomfim (2001) and Lorenzoni (2009), the presence of ν_t in equation (1) can be motivated by the existence of measurement error or noisy signals about macroeconomic variables. Alternatively, the shock ν_t can be viewed as capturing fluctuations in the sensitivity of output to changes in i_t. The variable u_t in equation (2) represents a cost-push shock that can be motivated by a time-varying elasticity of substitution among goods or a time-varying tax rate on labor income. The shocks ν_t and u_t evolve according to following stationary laws of motion:

$$\nu_t = \rho_\nu \nu_{t-1} + \epsilon_{\nu,t}, \qquad \epsilon_{\nu,t} \sim N\left[0,\, \sigma_\nu^2 \left(1 - \rho_\nu^2\right)\right], \tag{3}$$

$$u_t = \rho_u u_{t-1} + \epsilon_{u,t}, \qquad \epsilon_{u,t} \sim N\left[0,\, \sigma_u^2 \left(1 - \rho_u^2\right)\right]. \tag{4}$$

The time series process for the equilibrium real interest rate is given by

$$r_t = \rho_r\, r_{t-1} + \left(1 - \rho_r\right) r_t^* + \varepsilon_t, \qquad \varepsilon_t \sim N\left(0,\, \sigma_\varepsilon^2\right), \tag{5}$$

$$r_t^* = r_{t-1}^* + \eta_t, \qquad\qquad\qquad \eta_t \sim N\left(0,\, \sigma_\eta^2\right). \tag{6}$$

Equations (5) and (6) summarize a "shifting endpoint" time series process since the long-run endpoint r_t^* can vary over time due to the permanent shock η_t. In any given period, r_t can deviate from r_t^* due to the temporary shock ε_t. The persistence of the "equilibrium real rate gap" $r_t - r_t^*$ is governed by the parameter ρ_r, where $|\rho_r| < 1$. Kozicki and Tinsely (2012) employ this type of process to describe U.S. inflation. When $\rho_r = 0$, we recover the

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[5]Specifically, we have $r_t \equiv -\log\left[\beta \exp\left(\zeta_t\right)\right] + \gamma E_t \Delta \bar{y}_{t+1}$, where ζ_t is a shock to the agent's time discount factor β, $\bar{y}_t$ is the logarithm of real potential output, and $\gamma = \alpha^{-1}$ is the coefficient of relative risk aversion. For the derivation, see Hamilton, et al. (2016) or Gust, Johannsen, and Lopez-Salido (2017).

random walk plus noise specification employed by Stock and Watson (2007) to describe U.S. inflation.[6] Given that r_t^* is the long-run endpoint of r_t, we can interpret r_t^* as the "natural rate of interest." This interpretation is consistent with the empirical strategies of Laubach and Williams (2016), Lubik and Matthes (2015), and Kiley (2015) which view the natural rate of interest as a longer-term economic concept.[7]

In the model, the agent's rational forecast for the equilibrium real rate gap at any horizon $h \geq 1$ is given by

$$E_t \left(r_{t+h} - r_{t+h}^* \right) = \left(\rho_r \right)^h \left(r_t - E_t r_t^* \right),$$
(7)

where $E_t r_t^*$ represents the current estimate of the natural rate computed using the Kalman filter. When $|\rho_r| < 1$ as assumed here, the gap between r_t and r_t^* is expected to shrink to zero as the forecast horizon h increases. The gap $r_t - E r_t^*$ captures a concept that has been emphasized by Fed policymakers in recent speeches, namely, a distinction between estimates of the "short-term natural of interest" and its longer-term counterpart (Yellen 2015, Dudley 2015, and Fischer 2016).

In Appendix A, I show that the Kalman filter expression for $E_t r_t^*$ is

$$E_t r_t^* = \lambda \left[\frac{r_t - \rho_r\, r_{t-1}}{1 - \rho_r} \right] + \left(1 - \lambda \right) E_{t-1} r_{t-1}^*$$
(8)

$$\lambda = \frac{-\left(1 - \rho_r \right)^2 \phi + \left(1 - \rho_r \right) \sqrt{\left(1 - \rho_r \right)^2 \phi^2 + 4\phi}}{2},$$
(9)

where λ is the Kalman gain parameter and $\phi \equiv \sigma_\eta^2 / \sigma_\varepsilon^2$. For the quantitative analysis, the values of ρ_r and λ are chosen so that the time path of $E_t r_t^*$ from equation (8) approximates the path of the U.S. natural rate series estimated by Laubach and Williams (2016, updated) for the sample period 1988.Q1 to 2017.Q4. Their estimation strategy assumes that the natural rate exhibits a unit root, consistent with equation (6). Hamilton, et al. (2016) present evidence that the ex-ante real rate of interest $i_t - E_t \pi_{t+1}$ in U.S. data is nonstationary, but they find that the gap between the ex-ante real rate and their estimate of the world long-run real rate appears to be stationary. Equations (5) and (8) imply that the gap $r_t - E_t r_t^*$ is stationary.

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[6] But unlike here, Stock and Watson (2007) allow for stochastic volatility in the permanent and temporary shocks.

[7] Some authors refer to r_t in equation (1) as the natural rate of interest. See, for example, Barsky, Justiniano, and Melosi (2014), Cúrdia (2015), and Del Negro, et al. (2017). In contrast, Cúrdia et al. (2015) refer to r_t in equation (1) as the efficient real interest rate.

The central bank's monetary policy rule is given by

$$i_t^* = \rho i_{t-1}^* + (1-\rho)\left[E_t r_t^* + \pi^* + g_\pi\left(\overline{\pi}_t - \pi^*\right) + g_y\left(y_t - y^*\right)\right], \tag{10}$$

$$\overline{\pi}_t = \omega\,\pi_t + (1-\omega)\,\overline{\pi}_{t-1}, \tag{11}$$

$$i_t = \max\left\{0,\ i_t^*\right\}, \tag{12}$$

where i_t^* is the desired nominal interest rate that responds to deviations of recent inflation $\overline{\pi}_t$ from the central bank's target rate π^* and to deviations of the output gap from its targeted long-run endpoint y^*. Recent inflation $\overline{\pi}_t$ is an exponentially-weighted moving average of past quarterly inflation rates so as to approximate the compound average inflation rate over the past 4 quarters—a typical central bank target variable.[8] Equation (12) is the ZLB that constrains the nominal policy interest rate i_t to be non-negative. The parameter ρ governs the degree of interest rate smoothing as i_t^* adjusts partially each period toward the value implied by the terms in square brackets. Similar to the policy rule employed by Dordal-i-Carrera et al. (2016), equation (10) keeps track of past negative values of i_t^*, thereby exhibiting a form of commitment to keep interest rates "lower for longer" whenever the ZLB becomes binding.[9]

The quantity $E_t r_t^* + \pi^*$ represents the targeted long-run endpoint of i_t^*. Including $E_t r_t^*$ in the policy rule implies that monetary policymakers continually update their estimate of the unobservable r_t^*. Support for this idea can be found in the Federal Open Market Committee's Summary of Economic Projections (SEP). Meeting participants provide their views on the projected paths of macroeconomic variables over the next three calendar years and in the longer run. Since the natural rate of interest is a longer-run concept, we can infer the median SEP projection for r_t^* by subtracting the median longer-run projection for inflation from the median longer-run projection for the nominal federal funds rate. The median SEP projection for r_t^* computed in this way has ratcheted down over time, as documented by Lansing (2016).[10]

2.1 Long-run endpoints

The Fisher relationship $i_t = r_t + E_t\,\pi_{t+1}$ is embedded in the non-stochastic version of equation (1).[11] Consequently, when $g_\pi > 1$, the model has two long-run endpoints (steady states)

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[8] Specifically, the value of ω is set to achieve $\overline{\pi}_t \simeq [\Pi_{j=0}^3(1 + \pi_{t-j})]^{0.25} - 1$

[9] Alstadheim and Henderson (2006) and Sugo and Ueda (2008) formulate interest rate rules that can preclude the deflation equilibrium.

[10] Gust, Johannsen, and Lopez-Salido (2017) show that a Taylor-type rule that includes a time-varying intercept that moves with perceived changes in the equilibrium real interest rate can achieve results that are similar to optimal discretionary policy.

[11] Cochrane (2018), Williamson (2018), and Garín, Lester, and Sims (2018) show that Fisherian effects can dominate the short-term comovement between the nominal interest rate and inflation in standard New Keyne-

as shown originally by Benhabib, Schmitt-Grohé, and Uribe (2001a,b). The novelty here is that the long-run endpoints can shift due to shifts in r_t^*. Straightforward computations using the model equations yield the following long-run endpoints that characterize the "targeted equilibrium" and the "deflation equilibrium," respectively.

Table 1. Long-run Endpoints

Targeted equilibrium	Deflation equilibrium
$\pi_t = \pi^*$	$\pi_t = -r_t^*$
$y_t = y^* = \pi^* \left(1 - \beta\right)/\kappa$	$y_t = -r_t^* \left(1 - \beta\right)/\kappa$
$i_t^* = r_t^* + \pi^*$	$i_t^* = \left(r_t^* + \pi^*\right)\left[1 - g_\pi - g_y \left(1 - \beta\right)/\kappa\right]$
$i_t = r_t^* + \pi^*$	$i_t = 0$

In the targeted equilibrium, long-run inflation is at the central bank's target rate π^* and the long-run output gap y^* is slightly positive for typical calibrations with $0.99 < \beta < 1$. The long-run desired nominal policy rate i_t^* conforms to the Fisher relationship and the ZLB is not binding such that $i_t = i_t^* > 0$, provided that $r_t^* > -\pi^*$. In the model simulations, I impose bounds on fluctuations in r_t^* that are based on the range of natural rate estimates observed in the data since 1988. In the deflation equilibrium, the long-run inflation rate, the long-run output gap, and the long-run desired nominal interest rate are all negative when $r_t^* > 0$.

2.2   Local linear forecast rules

Given the linearity of the model aside from the ZLB, it is straightforward to derive the agent's rational decision rules for y_t and π_t in the vicinity of the long-run endpoints associated with each of the two local equilibria. For the targeted equilibrium, the local decision rules are unique linear functions of the state variables r_t, $E_t r_t^*$, $\overline{\pi}_{t-1}$, i_{t-1}^*, ν_t, and u_t. For the deflation equilibrium, I solve for the minimum state variable (MSV) solution which abstracts from extraneous sunspot variables and extra lags of fundamental state variables.[12] For both equilibria, the current estimate of the natural rate $E_t r_t^*$ is a state variable, but for different reasons. For the targeted equilibrium, $E_t r_t^*$ is a state variable because its appears in the monetary policy rule (10). For the deflation equilibrium (in which the ZLB is presumed to be always binding), $E_t r_t^*$ is a state variable because it pins down the agent's current estimates of the long-run endpoints for π_t and y_t. Given the local linear decision rules, we can construct the agent's local linear forecast rules for y_{t+1} and π_{t+1} in each of the two equilibria.

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sian models. Uribe (2017) examines the strength of Fisherian effects in a structural VAR framework using data for the U.S. and Japan.

   [12]For background on MSV solutions, see McCallum (1999).

The decision rule coefficients applied to the gap $r_t - E_t r_t^*$ in the deflation equilibrium are larger in magnitude than those in the targeted equilibrium (see Appendices B and C). Consequently, the deflation equilibrium exhibits more volatility and undergoes a more severe recession in response to an adverse shock sequence that causes the gap $r_t - E_t r_t^*$ to become persistently negative. The higher volatility in the deflation equilibrium is due to the binding ZLB which prevents the central bank from taking action to mitigate the consequences of the adverse shock sequence.

The local linear forecast rules for the targeted equilibrium are derived under the assumption that $i_t^* > 0$ and hence do not take into account the possibility that a shock sequence could be large enough to cause the ZLB to become binding in the future. The error induced by the use of linear forecast rules will depend on the frequency and duration of ZLB episodes in the targeted equilibrium. Based on model simulations, the targeted equilibrium experiences a binding ZLB in only 3.98% of the periods, with an average duration of 6.6 quarters. The local linear forecast rules for the deflation equilibrium are derived under the assumption that $i_t^* \leq 0$ and hence do not take into account the possibility that a shock sequence could be large enough to cause the ZLB to become slack in the future. Based on model simulations, the deflation equilibrium experiences a binding ZLB in 77% of the periods, with an average duration of 34 quarters. The average duration of a slack ZLB episode in the deflation equilibrium is 10.0 quarters. It is important to recognize, however, that the agent in the switching model (described below) never encounters these particular statistics because they pertain to environments with no regime switching. In Section 4, I show that the agent's forecast errors in the switching model are close to white noise.

Aruoba, Cuba-Borda, and Schorfheide (2018) solve for piece-wise linear decision rules to account for the occasionally binding nature of the ZLB constraint within each of the two regimes of their model. They report (p. 104), that the probability of hitting the ZLB in the targeted-inflation regime is "virtually zero" given the pre-crisis distribution of shocks. In the deflation regime, the probability of hitting the ZLB is 89%. These statistics are similar to those obtained here using the linear forecast rules.

Focusing only on the targeted equilibrium, Richter and Throckmorton (2016) compare a linear model solution in which agents' forecasts do not account for the possibility of hitting the ZLB (but the ZLB is imposed in simulations, as done here) to a nonlinear model solution in which agents' forecasts do account for this possibility. They report that the posterior distributions and marginal likelihoods of the two models are similar. But the nonlinear model

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predicts higher output volatility and more-negative skewness in output and inflation during the ZLB episode.

2.3 Endogenous regime switching

I now consider an agent who contemplates the possibility of an occasionally binding ZLB that is driven by switching between the two local equilibria, which can be viewed as two separate regimes. This setup bears some resemblance to the "OccBin toolkit" solution method developed by Guerrieri and Iacoviello (2015). Despite taking into account the possibility of occasionally hitting the ZLB, the agent's use of linear forecast rules within each regime is fully rational only in a vicinity of the model's long-run endpoints.

The agent constructs forecasts using a form of model averaging—a technique that is often employed to improve forecast performance in situations where the true data generating process (or true regime) is unknown (Timmerman 2006). The agent in the switching model can be viewed as an econometrician trying to identify the best forecast rule for the environment. Along the lines of Bullard (2010), the agent is aware of the two local equilibria implied by the economic framework and is concerned about the possibility of getting stuck in a deflation trap. The forecast rules in the switching model are given by

$$\widehat{E}_t\, y_{t+1} = \mu_t E_t^{\text{targ}}\, y_{t+1} \; + \; (1 - \mu_t)\, E_t^{\text{defl}}\, y_{t+1}, \tag{13}$$

$$\widehat{E}_t\, \pi_{t+1} = \mu_t E_t^{\text{targ}}\, \pi_{t+1} \; + \; (1 - \mu_t)\, E_t^{\text{defl}}\, \pi_{t+1}, \tag{14}$$

where μ_t is the value that minimizes the root mean squared forecast error computed over a moving window of recent data. Specifically, μ_t is the value that minimizes:

$$
\begin{aligned}
RMSFE_{t-1} \;\; = \;\; \sum_{j=1}^{T_w} \; \Bigg\{ & \tfrac{1}{T_w} \left[y_{t-j} - \mu_t E_{t-j-1}^{\text{targ}}\, y_{t-j} - (1 - \mu_t)\, E_{t-j-1}^{\text{defl}}\, y_{t-j} \right]^2 \\
& + \tfrac{1}{T_w} \left[\pi_{t-j} - \mu_t E_{t-j-1}^{\text{targ}}\, \pi_{t-j} - (1 - \mu_t)\, E_{t-j-1}^{\text{defl}}\, \pi_{t-j} \right]^2 \Bigg\}^{0.5},
\end{aligned}
\tag{15}
$$

which shows that μ_t is computed using data dated $t-1$ or earlier. In the simulations, I impose the restriction $0 \leq \mu_t \leq 1$.

I show that qualitatively similar results are obtained if μ_t is determined by a logistic rule that is formulated along the lines recommended by Binning and Maih (2017) for solving models with occasionally-binding constraints. The logistic rule takes the form:

$$\mu_t = \frac{\exp\left(\psi\, i_{t-1}^*\right)}{1 \; + \; \exp\left(\psi\, i_{t-1}^*\right)}, \tag{16}$$

where $\psi > 0$ is a parameter. The logistic rule implies that μ_t will decline when the desired nominal policy rate approaches zero or crosses into negative territory. As ψ becomes larger, the resulting sequence for μ_t takes on values approaching either 1 or 0, with intermediate values less likely.

Given the representative agent's conditional forecasts from equations (13) and (14), the realizations of the endogenous macroeconomic variables y_t, π_t, i_t^* and i_t are determined simultaneously each period by solving the following nonlinear system of equations:

$$y_t = \widehat{E}_t y_{t+1} - \alpha \left[i_t - \widehat{E}_t \pi_{t+1} - r_t \right] + \nu_t, \tag{17}$$

$$\pi_t = \beta \, \widehat{E}_t \pi_{t+1} + \kappa y_t + u_t, \tag{18}$$

$$i_t^* = \rho i_{t-1}^* + (1 - \rho) \left[E_t r_t^* + \pi^* + g_\pi \left(\overline{\pi}_t - \pi^* \right) + g_y \left(y_t - y^* \right) \right], \tag{19}$$

$$i_t = 0.5 \, i_t^* + 0.5 \sqrt{(i_t^*)^2}, \tag{20}$$

where $\overline{\pi}_t = \omega \, \pi_t + (1 - \omega) \, \overline{\pi}_{t-1}$.

3   Parameter values

Table 2 shows the baseline parameter values used in the model simulations. The top group of parameters appear in the global equilibrium conditions (1) and (2). The middle group of parameters appear in the monetary policy rule (10). The bottom group of parameters pertain to the equilibrium real interest rate process and the forecast evaluation window for the switching model.

Table 2. Baseline Parameter Values

Parameter	Value	Description/Target
α	0.15	Interest rate coefficient in Euler equation.
β	0.995	Discount factor in Phillips curve.
κ	0.025	Output gap coefficient in Phillips curve.
σ_ν	0.004	Std. dev. of demand shock.
σ_u	0.010	Std. dev. of cost push shock.
ρ_ν	0.85	Persistence of demand shock.
ρ_u	0.33	Persistence of cost push shock.
π^*	0.02	Central bank inflation target.
ω	0.456	$\overline{\pi}_t \simeq$ 4-quarter inflation rate.
g_π	1.5	Policy rule response to inflation.
g_y	1.0	Policy rule response to output gap.
ρ	0.8	Interest rate smoothing parameter.
ρ_r	0.85	Persistence parameter for r_t.
λ	0.025	Kalman gain for $E_t r_t^*$.
σ_ε	0.0097	Std. dev. of temporary shock to r_t.
σ_η	0.0016	Std. dev. of permanent shock to r_t.
T_w	8	Window length in qtrs. for forecast evaluation.

The value $\alpha = 0.15$ for the interest rate sensitivity coefficient in equation (1) implies a coefficient of relative risk aversion of $1/\alpha \simeq 6.7$. This value is consistent with the small empirical sensitivity of consumption to changes in the interest rate, as shown originally by Campbell and Mankiw (1989). The Phillips curve parameters $\beta = 0.995$ and $\kappa = 0.025$ are identical to those employed by Evans et al. (2015) and are typical of values employed in the literature. The parameters that govern the demand shock in equation (3) and the cost push shock in equation (4) are computed from a quantitative exercise (described in section 4.3) in which I solve for the time series ν_t and u_t that allow the switching model to exactly replicate the observed time paths of the CBO output gap, quarterly PCE inflation, and the federal funds rate since 1988.

The inflation target of $\pi^* = 0.02$ is based on the Federal Open Market Committee's (FOMC) stated goal of 2% inflation, as measured by the 4-quarter change in the personal consumption expenditures (PCE) price index. I choose $\omega = 0.456$ to minimize the squared deviation between the 4-quarter PCE inflation rate and the exponentially-weighted moving average of quarterly PCE inflation computed using equation (11) for the period 1961.Q1 to 2017.Q4. When $\omega = 0.456$, the cumulative weight on the first four terms π_t through π_{t-3} in the moving average is 0.912. The monetary policy rule coefficients g_π, g_y and ρ are based on the Taylor (1999) rule, augmented to allow for a realistic amount of inertia in the desired nominal policy rate.

As is common in the literature, the equilibrium real interest rate r_t is considered observable by all agents. For calibration purposes, I use the r_t series estimated by Cúrdia (2015) which makes use of the New Keynesian model developed by Cúrdia, et al. (2015).[13] Figure 3 plots the r_t series from 1988.Q1 to 2017.Q4 together with the Laubach-Williams (2016) one-sided estimate of the (long-run) natural rate of interest.[14] When $\rho_r = 0.85$ and $\lambda = 0.025$, the Kalman filter estimate of the long-run endpoint of the r_t series constructed using equation (8) approximates the path of the Laubach-Williams one-sided estimate. Similar results are obtained if ρ_r and λ are chosen to approximate an alternative natural rate series estimated by Lubik and Matthes (2015). Given the considerable uncertainty surrounding estimates of r_t^*, any observed differences between the estimated series are not statistically significant.[15]

Given the values of ρ_r and λ, I solve for the value $\phi \equiv \sigma_\eta^2/\sigma_\varepsilon^2 = 0.028$ to satisfy the Kalman gain formula (9). Given ϕ, I solve for the value of σ_ε that allows the model-predicted standard deviation of Δr_t to match the corresponding value in the data for the period 1988.Q1 to 2017.Q4. Finally, given ϕ and σ_ε, we have $\sigma_\eta = \sigma_\varepsilon \sqrt{\phi}$.

The window length in quarters for computing the agent's forecast fitness measure from equation (15) is set to $T_w = 8$. Each period, the agent chooses the weight μ_t on the targeted forecast rules so as to minimize the root mean squared forecast errors over the past 2 years. In simulations, this choice produces a ZLB binding frequency of around 20%—reasonably close to the frequency observed in U.S. data since 1988. I also examine the sensitivity of the results to higher values of T_w. Higher values of T_w serve to reduce the ZLB binding frequency by reducing the likelihood of switches to the deflation equilibrium.

For the baseline simulation, I impose the bounds $-0.010 \leq r_t^* \leq 0.045$, which corresponds to the range of values for the model-implied $E_t r_t^*$ series plotted in Figure 3.[16] At the end of the data sample in 2017.Q4, the model-implied value of $E_t r_t^*$ remains mildly negative. Based on a life cycle model calibrated to the U.S. economy in 2015, Eggertsson, Mehrotra, and Robbins (2017) compute a steady-state natural rate of -1.5%. In a representative agent model, the long-run natural rate influences the mean real risk free rate of return. The mean risk free rate can be negative if the product of the coefficient of relative risk aversion and the variance of consumption growth are sufficiently high, implying a very strong precautionary saving

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[13] Updated data are from www.frbsf.org/economic-research/economists/vasco-curdia/.

[14] Updated data are from www.frbsf.org/economic-research/files/Laubach_Williams_updated_estimates.xlsx.

[15] According to Kiley (2015, p. 2), "the co-movement of output, inflation, unemployment, and real interest rates is too weak to yield precise estimates of r^*."

[16] Alternatively, one could model r_t^* as a "bounded random walk" along the lines described by Nicolau (2002). But this approach involves additional parameters and presumes that the agent has prior knowledge of the upper and lower bounds on r_t^*.

motive.[17] To allow for more negative values of the natural rate, I consider an alternative simulation that imposes the bounds $-0.02 \leq r_t^* \leq 0.02$.

4 Quantitative analysis

4.1 U.S. data around the ZLB episode

The top left panel of Figure 4 shows that the equilibrium real interest rate r_t has remained mostly below the model-implied value of $E_t r_t^*$ since the start of the Great Recession in 2007.Q4. This feature of the data implies persistently negative values for the gap $r_t - E_t r_t^*$, which is a state variable in the model. The bottom left panel shows that the nominal federal funds rate was approximately zero from 2008.Q4 through 2015.Q4. In the same panel, I plot the nominal federal funds predicted by a Taylor-type rule of the form (10) using the parameter values in Table 2 with $E_t r_t^*$ given by model-implied value, $\overline{\pi}_t$ given by the 4-quarter PCE inflation rate, and y_t given by the CBO output gap. The desired nominal funds rate predicted by the Taylor-type rule is negative starting in 2009.Q1 and remains negative through 2017.Q2.[18]

The top right panel of Figure 4 shows that the 4-quarter PCE inflation rate was briefly negative in 2009 and has remained mostly below the Fed's 2% inflation target since 2012.Q2. The bottom right panel shows that the Great Recession was very severe, pushing the CBO output gap down to -6.2% at the business cycle trough in 2009.Q2. This was the most severe economic contraction since 1947 as measured by the peak-to-trough decline in real GDP. The output gap finally turns positive in 2017.Q2, a full eight years after the Great Recession ended.

The various endpoints plotted in Figure 4 are computed using the expressions in Table 1, with r_t^* set equal to the model-implied value of $E_t r_t^*$. As $E_t r_t^*$ approaches zero or becomes negative, the "deflation" equilibrium is characterized by zero or low inflation, allowing this equilibrium to provide a better fit of recent U.S. inflation data. Aruoba, Cuba-Borda, and Schorfheide (2018) impose a relatively low value of $r^* = 0.86\%$, but they do not allow its value to vary over time.

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[17]In a representative agent model, $\log(R_{t+1}^f) = -\log(E_t M_{t+1})$, where R_{t+1}^f is the gross real risk free rate and M_{t+1} is the agent's stochastic discount factor. Assuming *iid* consumption growth and power utility, the mean risk free rate is given by $E[\log(R_{t+1}^f)] = -\log(\beta) + \gamma \overline{x} - \gamma^2 \sigma_x^2/2$, where β is the agent's time discount factor, γ is the coefficient of relative risk aversion, $\overline{x}$ is the mean growth rate of real per capita consumption and σ_x^2 is the corresponding variance. Assuming $\beta \simeq 1$ such that $\log(\beta) \simeq 0$, the condition $\gamma \sigma_x^2 > 2\overline{x}$ implies $E[\log(R_{t+1}^f)] < 0$. For details of the derivation, see Lansing and LeRoy (2014).

[18]Augmenting the Taylor-type rule to allow for a response to other variables (such as 4-quarter real GDP growth and an index of macroeconomic uncertainty) can produce a path for the desired nominal funds rate that turns positive somewhat earlier. See Lansing (2017).

4.2 Expected inflation in U.S. data

Figure 5 plots various measures of expected inflation in U.S. data. The top right panel shows 1-year and 5-year expected inflation rates derived from zero coupon inflation swap contracts that are traded in the over-the-counter market (Haubrich, Pennacchi, and Ritchken 2012). Expected inflation dropped sharply in 2008.Q4, coinciding with the start of the ZLB episode. In the top right panel, we see a similar pattern for 5-year and 10-year breakeven inflation rates derived from yields on Treasury Inflation Protected Securities (TIPS). The market-based measures have remained below the Fed's 2% inflation target for extended periods even after the Great Recession ended in 2009.Q2. The lower left panel in Figure 5 shows the median 1-year and 10-year expected inflation rates from the Survey of Professional Forecasters (SPF). The 1-year survey measure dropped sharply in 2008.Q4 and has recovered slowly, but to a level that remains below its pre-recession range. The 10-year survey measure does not exhibit a sharp drop in 2008.Q4, but has since trended downward to a level that is below its pre-recession range. The bottom right panel plots the Federal Reserve Bank of St. Louis' Price Pressures Measure (PPM). A set of common factors extracted from 104 separate data series are used to estimate the probability that the 4-quarter PCE inflation rate over the next year will exceed 2.5% (Jackson, Kliesen, and Owyang 2015). The PPM dropped sharply in 2008.Q4 and is currently hovering around a probability of 10%.[19]

Aruoba and Schorfheide (2016, p. 363) claim that "long-run (five-year-ahead) inflation expectations have been remarkably stable in the United States... despite falling policy rates." However, they do not consider the market-based measures of expected inflation shown in the top panels of Figure 5. Although not plotted in Figure 5, the Federal Reserve Bank of Atlanta's Business Inflation Expectation (BIE) survey shows that while most respondents understand that the Fed's inflation target is 2%, about two-fifths of respondents currently believe that the Fed is more likely to accept an inflation rate below target than to accept an inflation rate above target (Altig, Parker, and Meyer, 2017).

4.3 Replicating U.S. data since 1988 with the switching model

Given the U.S. data counterparts for the model variables i_t, r_t, $E_t r_t^*$, y_t, π_t, $\overline{\pi}_t \simeq \pi_{4,t}$, and i_t^*, we can use the calibrated switching model to "reverse engineer" the time series of the two

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[19]Expected inflation rates from swap contracts are from the Federal Reserve Bank of Cleveland. The TIPS breakeven inflation rates and the PPM are from the the Federal Reserve Bank of St. Louis' FRED data base. Expected inflation rates from the SPF are from the Federal Reserve Bank of Philadelphia.

persistent shocks ν_t and u_t that are needed to exactly replicate the data.[20] For this computation, the agent's subjective forecasts $\widehat{E}_t y_{t+1}$ and $\widehat{E}_t \pi_{t+1}$ that appear in the global equilibrium conditions (17) and (18) are constructed as the weighted averages shown in equations (13) and (14), but with U.S. data inserted for the state variables that appear in the two sets of local linear forecast rules. The variable i_t is the nominal federal funds rate, the gap $r_t - E_t r_t^*$ is the difference between the equilibrium real interest rate from Cúrdia (2015, updated) and the model-implied value of $E_t r_t^*$ from equation (8), the variable y_t is the CBO output gap, π_t is quarterly PCE inflation, and $\overline{\pi}_t$ is 4-quarter PCE inflation. During the ZLB episode in the data, i_t^* is given by the calibrated policy rule (10). But otherwise, $i_t^* = i_t$. The endogenous forecast weight μ_t is computed each period so as to minimize the $RMSFE$ from equation (15), where $T_w = 8$ quarters. The shock persistence parameters ρ_ν and ρ_u can influence the values of the coefficients in the local linear forecast rules. I start with initial guesses for ρ_ν and ρ_u and then repeat the exercise until convergence. The resulting standard deviations of ν_t and u_t are 0.004 and 0.010, respectively, with $\rho_\nu = 0.85$ and $\rho_u = 0.33$.

The results of the data replication exercise are plotted in Figure 6. The top left panel plots the model-implied shocks ν_t and u_t. Both shocks go from positive to negative at the start of the ZLB episode in 2008.Q4. These adverse shocks allow the model to exactly replicate the sharp drops in the CBO output gap and quarterly PCE inflation shown earlier in Figure 4.

The top right panel of Figure 6 compares the $RMSFE$ of the deflation forecast rules to the $RMSFE$ of the targeted forecast rules. The performance gap between the two sets of forecast rules initially narrows with the onset of the ZLB episode in 2008.Q4. This result leads to an initial sharp drop in the weight μ_t assigned to targeted forecast rules (bottom right panel). The performance gap narrows even further after 2012.Q2 when the Fed consistently undershoots its 2% inflation target, inducing a further decline in μ_t. At the end of the data sample in 2017.Q4, the value of μ_t is only 0.46, reflecting the relatively equal performance of the two sets of forecast rules over the prior 8 quarters. The fact that the model-implied value of $E_t r_t^*$ remains negative at the end of the data sample helps the deflation forecast rules to account for the observation of low positive inflation in the U.S. data.

The bottom left panel of Figure 6 compares the 1-year expected inflation rate from U.S. inflation swaps (shown earlier in Figure 5) to the path of $\widehat{E}_t \pi_{t+1}$ from the switching model. The correlation coefficient between the two series is 0.76. While the model-implied drop in expected inflation is somewhat more pronounced and more persistent than the swaps data,

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[20] Gelain, Lansing, and Natvik (2018) undertake a similar reverse-engineering excercise to identify the shocks needed to exactly replicate U.S. housing market data from 1993 onwards.

both series remain mostly below the Fed's 2% inflation target from 2008.Q4 through the end of the data sample. Due to the persistent nature of the shocks, the agent's subjective forecast $\widehat{E}_t \pi_{t+1}$ makes use of the identified shocks ν_t and u_t. These same shock values allow the switching model to exactly replicate the path of π_t in the data.

Of course, one could solve for different sequences of ν_t and u_t that would allow the targeted equilibrium with $\mu_t = 1$ for all t, or the deflation equilibrium with $\mu_t = 0$ for all t, to similarly replicate the U.S. data. But the $RMSFE$ minimization procedure employed by the representative agent in the model prefers interior solutions with $0 < \mu_t < 1$. Since μ_t is endogenous, the switching model provides us with an economic rationale for why measures of actual and expected U.S. inflation in the data have mostly not returned to their pre-recession levels.

Can the results of the data replication exercise be reconciled with those of Arouba, Cuba-Borda, and Schorfheide (2018), henceforth ACS? Possibly so. Recall that ACS conclude that "the U.S. remained in the targeted-inflation regime during its ZLB episode, with the possible exception of the early part of 2009 where evidence is ambiguous."[21] First, we should recognize that ACS compare the likelihood of the targeted-inflation regime to the likelihood of the deflation regime. According to their model, the economy must be in one regime or the other. In contrast, the data replication exercise tells us that the data since 2008.Q4 are best described as a mixture of the two regimes. But ACS do not allow for a mixed regime. Second, while ACS explicitly consider model uncertainty, a number of their model parameters are fixed during the estimation routine. The most notable of these is the natural rate of interest which is held constant at 0.86%. The uncertainty surrounding point estimates of the natural rate is extremely large (Kiley 2015). It could easily be the case that the true value of the steady state natural rate lies in negative territory, as maintained by Eggertsson, Mehrotra, and Robbins (2017). Indeed, the model-implied value of $E_t r_t^*$ in Figure 3 remains negative at the end of the data sample in 2017.Q4. When the natural rate is negative, the "deflation" equilibrium is characterized by positive inflation, allowing this equilibrium to provide a better fit of recent U.S. inflation data. The upshot of these points is that the conclusions of ACS may not be robust to plausible alternative environments.

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[21] More specifically, they acknowledge that across the six specifications of their model, "there is some uncertainty which vindicates Bullard (2010)'s concern of the possibility of a shift to the deflationary regime." (p. 111).

4.4 Switching model simulations

Numerical simulations can provide further insight into the behavior of the switching model. Figure 7 plots some key variables from a long simulation using the parameter values in Table 1. When the gap $r_t - E_t r_t^*$ is negative for a sustained interval (top panel), the resulting downward pressure on y_t and π_t serves to reduce the recent $RMSFE$ of the deflation forecast rules and increase the recent $RMSFE$ of the targeted forecast rules (middle panel). Around period 1725, the shift in relative forecast performance induces the agent to reduce the weight on the targeted forecast rules. The lower weight puts further downward pressure on y_t and π_t, causing the deflation equilibrium to eventually become fully realized around period 1750 (bottom panel). Then around period 1800, the gap $r_t - E_t r_t^*$ once again becomes positive, putting upward pressure on y_t and π_t. The $RMSFE$ of the deflation forecast rules starts to exceed the $RMSFE$ of the targeted forecast rules. The agent increases the weight on the targeted forecast rules which puts further upward pressure on y_t and π_t, eventually causing the targeted equilibrium to be restored.

Around periods 1610 and 1630, the switching model produces episodes where the ZLB is binding for an extended period, but $0.5 \lesssim \mu_t \lesssim 0.8$. In this case, the model-generated data are a mixture of the two local equilibria, similar to results obtained in the U.S. data replication exercise plotted earlier in Figure 6.

Qualitatively similar results are obtained if the agent employs the logistic rule (16) to compute the forecast weight each period. The bottom panel of Figure 7 shows the path of μ_t that is implied by the logistic rule with $\psi = 2000$, corresponding to the value employed by Binning and Maih (2017) in a similar modeling context. Interestingly, it is the representative agent's subjective belief that the deflation equilibrium is possible that allows it to become a reality. If the agent could somehow commit to employing the forecast rule weight $\mu_t = 1$ for all t, then the economy would always remain in the targeted equilibrium.

Figure 8 plots the distributions of macro variables in each of the three model versions. The macro variables in the deflation equilibrium have distributions with lower means but higher variances than those in the targeted equilibrium. But the significant overlap in the various distributions creates a dilemma for an agent who seeks to determine the likelihood that a string of recent data observations are generated by one equilibrium or the other. Variables in the switching model have means that are somewhat lower and variances that are somewhat higher than those in the targeted equilibrium. Consequently, the central bank in the switching model undershoots its inflation target and the volatilities of the output gap and inflation are

both higher relative to the targeted equilibrium.

Hills, Nakata, and Schmidt (2016) show that the risk of encountering the ZLB in the future can shift agents' expectations such that the central bank undershoots its inflation target in the present. Something similar is at work here. When the agent increases the weight on the deflation forecast rules, this can cause realized inflation to undershoot the central bank's target for a sustained interval, even when the ZLB is not binding. The switching model allows for low-frequency swings in the level of inflation that are driven solely by expectational feedback, not by changes in the monetary policy rule.[22]

As mentioned above, the U.S. output gap reached -6.2% at the trough of the Great Recession. The bottom right panel of Figure 8 shows that the likelihood of such an event in the targeted equilibrium is essentially zero. In contrast, a Great Recession-type episode is plausible in the switching model. Severe recessions can happen in the switching model due to the larger response coefficients on the gap $r_t - E_t r_t^*$ and the demand shock ν_t in the deflation equilibrium decision rule for y_t. Specifically, the response coefficient on $r_t - E_t r_t^*$ in the deflation equilibrium is 1.16 versus 0.58 in the targeted equilibrium. The response coefficient on ν_t in the deflation equilibrium is 7.73 versus 3.84 in the targeted equilibrium. The deflation equilibrium response coefficients have more influence on the switching model as the forecast weight μ_t declines, causing adverse realizations of $r_t - E_t r_t^*$ or ν_t to be transmitted more forcefully to the output gap.

Evans, Honkapohja, and Mitra (2016) argue that the deflation equilibrium does not provide a convincing explanation of the sluggish output recovery following the Great Recession because the steady state level of real activity in the deflation equilibrium is not much below the steady state level of real activity in the targeted equilibrium. However, their analysis does not take into account the notion that the gap $r_t - E_t r_t^*$ in U.S. data has remained significantly negative since the recession ended, as can be seen in the top left panel of Figure 4. A negative value for $r_t - E_t r_t^*$ puts stronger downward pressure on y_t in the deflation equilibrium, thus helping to explain the sluggish output recovery in U.S. data. Moreover, as we saw from the data replication exercise, putting some weight on the deflation equilibrium forecast rules can help to explain the pattern of U.S. data, even if the deflation equilibrium is never fully realized.

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[22]Lansing (2009) achieves a similar result in a model where the representative agent's forecast rule for quarterly inflation is based on a perceived law of motion that follows a Stock and Watson (2007) type time series process.

Table 3. Unconditional Moments: Data versus Model

	U.S. Data 1989.Q1-2017.Q4	Model Simulations		
		Targeted Equilibrium	Deflation Equilibrium	Switching Model
% periods $i_t = 0$	24.2%	3.98%	77.2%	19.4%
Mean ZLB duration	29 qtrs.	6.6 qtrs.	34.0 qtrs.	13.1 qtrs.
Max. ZLB duration	29 qtrs.	59 qtrs.	446 qtrs.	133 qtrs.
Mean y_t	-1.56%	0.40%	-0.41%	0.41%
Std. Dev.	1.68%	1.57%	3.48%	2.21%
Corr. Lag 1	0.94	0.78	0.84	0.82
Mean $\pi_{4,t}$	2.16%	1.99%	-1.79%	0.97%
Std. Dev.	1.10%	0.93%	1.95%	1.54%
Corr. Lag 1	0.89	0.85	0.96	0.94
Mean i_t^*	2.83%	3.78%	-2.71%	2.26%
Std. Dev.	3.37%	2.18%	3.54%	3.13%
Corr. Lag 1	0.99	0.99	0.98	0.99

Notes: The ZLB episode in U.S. data is from 2008.Q4 through 2015.Q4. Model results are computed from a 300,000 period simulation. $\pi_{4,t} \equiv [\Pi_{j=0}^{3}(1 + \pi_{t-j})]^{0.25} - 1$.

Table 3 provides a quantitative comparison between the U.S. data and the results of model simulations. For the most part, the statistics generated by the switching model compare favorably to those in U.S. data since 1988. For example, the switching model predicts a ZLB binding frequency of 19.4% versus 24.2% in the data. However, the mean 4-quarter inflation rate in the switching model is only 0.97% versus 2.16% in the data. This particular model prediction is more in line with data from Japan than the United States. But as shown later in Table 5, the mean 4-quarter inflation rate in the switching model can be increased by allowing r_t^* to dip further into negative territory during the simulations.

Using data from all advanced economies since 1950, Dordal-i-Carrera et al. (2016) estimate an average ZLB binding frequency of 7.5% and an average duration for ZLB episodes of 14 quarters. Excluding the high inflation period from 1968 to 1984 serves to raise the average ZLB binding frequency and the average ZLB duration to 10% and 18 quarters, respectively. For the period of consistent U.S. monetary policy since 1988, the single ZLB episode lasted 29 quarters.

Figure 9 plots the distribution of ZLB durations in each model version. Unlike the targeted equilibrium, the switching model can produce infrequent and long-lived ZLB episodes in response to small, normally distributed shocks. The average ZLB duration in the switching model is 13.1 quarters, with a maximum observed duration in the simulation of 133 quarters (Table 3). From Figure 9, we see that a 29 quarter ZLB episode is an extremely rare event in the targeted equilibrium but can occur with about 5% frequency in the switching model.

To account for infrequent and long-lived ZLB episodes in the targeted equilibrium, Dordal-i-Carreras, et al. (2016) develop a model with large, infrequent, and long-lived shocks.[23]

When $\omega = 0.456$, the exponentially-weighted moving average of quarterly inflation $\bar{\pi}_t$ computed from equation (12) provides a very good approximation of the 4-quarter inflation rate. Although not shown in Table 3, the mean, standard deviation, and first-order autocorrelation of $\bar{\pi}_t$ in the switching model are 0.98%, 1.55%, and 0.90, respectively. These values are very close to the corresponding statistics for $\pi_{4,t}$ of 0.97%, 1.54%, and 0.94.

The mean weight on the targeted forecast rules in the switching model is 0.71 with a standard deviation of 0.28. Larger values for the window length T_w that is used to compute the $RMSFE$ measure from equation (15) serve to reduce the frequency of regime switches and thereby raise the mean 4-quarter inflation rate. For example, when T_w is increased to 16 quarters, the mean value of μ_t is higher at 0.80 and the standard deviation is lower at 0.21. With $T_w = 16$, the ZLB binding frequency in the switching model drops to 12.5% from 19.4% and the average ZLB duration drops to 9.8 quarters from 13.1 quarters. The mean value of $\pi_{4,t}$ increases to 1.26% from 0.97%.

As noted earlier, the representative agent in the switching model considers the ZLB to be occasionally binding across regimes, but not within a given regime. Most of the time, the switching model fluctuates in a state that can be described as a mixture of the two regimes. Each period, the agent adjusts the forecast weight μ_t in a way that is perceived to improve forecast accuracy. In such an environment, the agent's failure to take into account a relatively rare within-regime event should not have much impact on overall forecasting performance. Table 4 summarizes the properties of the agent's forecast errors in each of the three model versions. The forecast error is given by $err^x_{t+1} = x_{t+1} - F_t x_{t+1}$ for $x_{t+1} \in \{y_{t+1}, \pi_{t+1}\}$, where $F_t x_{t+1}$ is the value predicted by the local linear forecast rule or, in the case of the switching model, the weighted-average forecast rule given by equation (13) or equation (14). The properties of the agent's forecast errors in the switching model are not much different from those in the targeted equilibrium. Notably, the forecast errors are close to white noise.

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[23]In a New Keynesian model with physical capital, Dennis (2017) shows that the introduction of capital adjustment costs can help to generate infrequent and long-lived ZLB episodes in the targeted equilibrium.

Table 4. Properties of Forecast Errors

| | Model Simulations | | |
Statistic	Targeted	Deflation	Switching
$Corr(err_{t+1}^y,\ err_t^y)$	0.003	-0.006	0.025
$Corr(err_{t+1}^\pi,\ err_t^\pi)$	0.003	0.002	0.069
$E\left(err_{t+1}^y\right)$	-0.003%	-0.065%	0.002%
$E\left(err_{t+1}^\pi\right)$	-0.002%	-0.004%	0.003%
$\sqrt{E[(err_{t+1}^y)^2]}$	0.988%	1.868%	1.255%
$\sqrt{E\left[\left(err_{t+1}^\pi\right)^2\right]}$	1.384%	1.439%	1.408%

Notes: Model results are computed from a 300,000 period simulation.

Recall that the long-run endpoint of π_t in the deflation equilibrium is $-r_t^*$. Allowing for more negative values of r_t^* in the simulation will therefore serve to increase the mean inflation rate in both the deflation equilibrium and the switching model. The wide confidence intervals around empirical estimates of the natural rate of interest would not rule out true values that are more negative. Table 5 compares the results of the original switching model simulation to an alternative simulation with $-0.02 \le r_t^* \le 0.02$. The lower bound of -0.02 allows r_t^* to dip further into negative territory. The upper bound of 0.02 reflects the view that the U.S. economy will not likely return to the high natural rate environment of the 1990s. The mean 4-quarter inflation rate in the alternative simulation increases to 1.38% from 0.97% in the original simulation. Moreover, the ZLB binding frequency increases to 29.4%, which is now higher than the U.S. data value of 24.2% shown in Table 3.

Table 5. Effect of Natural Rate Range in Switching Model

Statistic	$-0.010 \le r_t^* \le 0.045$	$-0.020 \le r_t^* \le 0.020$
% periods $i_t = 0$	19.4%	29.4%
Mean ZLB duration	13.1 qtrs.	13.0 qtrs.
Max. ZLB duration	168 qtrs.	154 qtrs.
Mean y_t	0.41%	0.29%
Std. Dev.	2.21%	2.32%
Corr. Lag 1	0.82	0.83
Mean $\pi_{4,t}$	0.97%	1.38%
Std. Dev.	1.54%	1.31%
Corr. Lag 1	0.94	0.92
Mean i_t^*	2.26%	1.01%
Std. Dev.	3.13%	2.76%
Corr. Lag 1	0.99	0.99

Notes: Model results are computed from a 300,000 period simulation.

4.5   Effect of raising the inflation target

In a press conference held on June 14, 2017, Fed Chair Janet Yellen stated that determining the appropriate level of the inflation target "is one of the most important questions facing monetary policy around the world in the future." Numerous authors make the case for a higher inflation target using frameworks that ignore the deflation equilibrium.[24] This methodology may understate the benefits of a higher inflation target because the analysis does not take into account the possibility that a higher target could help prevent switching to the volatile deflation equilibrium where recessions are more severe. Aruoba and Schorfheide (2016) examine the welfare implications of a 4% inflation target in a framework that does consider the possibility of switching to the deflation equilibrium via an exogenous sunspot shock. They conclude (p. 395) that "the case for a higher inflation target is not particularly strong." It's worth noting, however, that the probability of switching to the deflation equilibrium in the Aruoba-Schorfheide model is invariant to changes in the inflation target. In contrast, the framework developed here has the potential to reduce the probability of switching to the deflation equilibrium.

Table 6 shows the effects of raising the central bank's inflation target. As π^* increases, the ZLB binding frequency declines as does the mean duration of ZLB episodes. Higher values of π^* serve to reduce the volatility of y_t because the volatile deflation equilibrium is now avoided more often. But at the same time, higher values of π^* serve to *increase* the volatility of $\pi_{4,t}$. This is because higher values of π^* widen the spread between the two inflation distributions implied by the the two local equilibria. This causes the mean inflation rate to shift by a larger amount when a regime switch inevitably does occur.

From Table 6 we see that an increase in π^* can reduce, but not eliminate, the endogenous switches to the deflation equilibrium. Even with an inflation target of 4%, the ZLB binding frequency remains relatively high at 10.5% and the average duration of a ZLB episode is 12.5 quarters. The maximum observed duration in the simulation is 200 quarters or 50 years! Once the possibility of switching to the deflation equilibrium is taken into account, raising the inflation target proves to be a less effective solution for avoiding long-lived ZLB episodes.

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[24]See, for example, Blanchard, Dell'Ariccia, and Mauro (2010), Ball and Mazumder (2011), and Ball (2013).

Table 6. Effect of Raising the Inflation Target in Switching Model

Statistic	$\pi^* = 0.02$	$\pi^* = 0.03$	$\pi^* = 0.04$	$\pi^* = 0.05$
% periods $i_t = 0$	19.4%	14.6%	10.5%	6.9%
Mean ZLB duration	13.1 qtrs.	12.9 qtrs.	12.5 qtrs.	11.7 qtrs.
Max. ZLB duration	168 qtrs.	167 qtrs.	200 qtrs.	164 qtrs.
Std. Dev. y_t	2.21%	2.14%	2.06%	1.98%
Std. Dev. $\pi_{4,t}$	1.54%	1.62%	1.69%	1.71%
Std. Dev. i_t^*	3.13%	3.26%	3.26%	3.22%
Loss value, $\theta = 1$	2.88%	2.71%	2.80%	3.18%
Loss value, $\theta = 0.25$	2.15%	1.96%	2.11%	2.58%

Note: Model results computed from a 300,000 period simulation.

Coibion, Gorodnichenko, and Wieland (2012) calibrate their model to deliver a ZLB binding frequency equal to that observed in U.S. data going back to the year 1950. From the start of 1950 to 2017.Q4, the ZLB was binding in 29 out of 272 quarters, or 10.7% of the time. The average CPI inflation rate in U.S. data since 1950 is around 4%. Table 7 shows that the switching model with $\pi^* = 0.04$ delivers a ZLB binding frequency of 10.5%—very close to the U.S. value of 10.7% going back to 1950. Taking into account the micro-founded welfare costs of positive and variable inflation, Coibion, Gorodnichenko, and Wieland (2012) compute an optimal inflation rate for their model which is relatively low, less than 2% per year. Their analysis is extended by Dordal-i-Carrera et al. (2016) who modify the shock process in the model to match the now-higher ZLB binding frequency implied by additional years of data for the U.S. and other advanced economies. The optimal inflation rate for the modified model lies in the range of 2.5% to 4%.

Kiley and Roberts (2017) perform stochastic simulations using the DSGE model of Lindé, Smets, and Wouters (2016) which is estimated over the period 1965.Q1 to 2014.Q2. They consider constant values of the natural rate of interest as low as 1% and draws shocks from the estimated distributions of the model. When monetary policy follows a simple Taylor (1999) rule with no interest rate smoothing, they find that the ZLB binding frequency can be as high as 32.6% with an mean ZLB duration of 12 quarters. The very high ZLB binding frequency obtains even though the model solution considers only the targeted equilibrium. In contrast, the simulations here deliver a baseline ZLB binding frequency in the switching model of 19.4%, despite allowing for a natural rate of interest as low as -1.0% and further allowing for the possibility of switches to the deflation equilibrium. The much higher ZLB binding frequency obtained by Kiley and Roberts (2017) appears to be partly due to the shock distributions which are based on the more-volatile U.S. data sample going back to 1965. Here, in contrast,

the shock distributions are based the more-recent sample period of consistent monetary policy going back to 1988. Kiley and Roberts (2017) also do not allow for the possibility that the natural rate of interest may drift above 1% in their simulations.

In addition to the features noted above, Kiley and Roberts (2017) employ a Taylor rule with no interest rate smoothing. This feature of their model would also appear to contribute to a higher ZLB binding frequency. Table 7 shows the effect of the interest rate smoothing parameter ρ in the switching model. The baseline case with $\rho = 0.8$ is compared to the no smoothing case with $\rho = 0$. For all values of π^*, setting $\rho = 0$ results in a higher ZLB binding frequency, but the episodes exhibit a shorter duration on average. From a ZLB perspective, there appears to be no clear advantage to reducing the degree of interest rate smoothing in the monetary policy rule.

Table 7 Effect of Interest Rate Smoothing in Switching Model

Statistic	$\pi^* = 0.02$	$\pi^* = 0.03$	$\pi^* = 0.04$	$\pi^* = 0.05$
$\rho = 0.8$				
% periods $i_t = 0$	19.4%	14.6%	10.5%	6.9%
Mean ZLB duration	13.1 qtrs.	13.9 qtrs.	12.5 qtrs.	11.7 qtrs.
Max. ZLB duration	168 qtrs.	167 qtrs.	200 qtrs.	164 qtrs.
$\rho = 0$				
% periods $i_t = 0$	27.4%	23.3%	19.1%	15.1%
Mean ZLB duration	5.2 qtrs.	5.3 qtrs.	5.3 qtrs.	5.2 qtrs.
Max. ZLB duration	93 qtrs.	94 qtrs.	128 qtrs.	112 qtrs.

Note: Model results computed from a 300,000 period simulation.

Following Kiley and Roberts (2017), I use a simple loss function approach to quantify the various trade-offs that are involved in raising the inflation target. The trade-offs here include: (1) reducing the likelihood of endogenous switches to the volatile deflation equilibrium and thereby lowering the ZLB binding frequency, (2) inducing shifts in the volatilities of inflation and the output gap, and (3) introducing economic distortions that come from a higher average inflation. The loss function takes the form

$$Loss \;=\; E\left\{[\pi_{4,t} - 0.02]^2 \;+\; \theta\left[y_t - 0.02\left(1 - \beta\right)/\kappa\right]^2\right\}, \tag{21}$$

where 0.02 and $0.02\left(1 - \beta\right)/\kappa$ are the long-run endpoints in the targeted equilibrium when $\pi^* = 0.02$, as shown in Table 1. The presumption is that the central bank in the baseline calibration with $\pi^* = 0.02$ has chosen to target the "optimal" levels of $\pi_{4,t}$ and y_t. Hence, any shift away from the original target values when adopting $\pi^* > 0.02$ would introduce economic distortions that are taken into account by the loss function. Also following Kiley and Roberts

(2017), I consider two values for the weight θ on the second term that captures the loss from output gap deviations. The bottom rows of Table 7 show that the simple loss function approach would favor a modest increase in the central bank's inflation target. Specifically, the loss function is minimized at $\pi^* = 0.03$ when either $\theta = 1$ or $\theta = 0.25$. Notice, however, that the magnitude of the loss reduction in going from $\pi^* = 0.02$ to $\pi^* = 0.03$ is relatively minor. Moreover, this analysis does not consider any potential loss of central bank credibility that might arise from an upward shift in the inflation target. Overall, it would appear that the net gains from raising the inflation target in the switching model are quite small. This result is consistent with the conclusions of Aruoba and Schorfheide (2016).

5 Conclusion

Standard New Keynesian models subject to a ZLB exhibit two long-run endpoints (steady states) that are associated with two local equilibria. Most studies employing New Keynesian models focus on the targeted equilibrium and ignore the possibility of switching to the deflation equilibrium. But there appears to be no clear theoretical or empirical reason why the deflation equilibrium should be ruled out. Indeed, Bullard (2010) concludes that by "promising to remain at zero for a long time," central banks may inadvertently coordinate private-sector expectations so as to become stuck in the deflation equilibrium.

I examine a version of the New Keynesian model with a time-varying natural rate of interest and endogenous forecast rule switching based on past performance. The model can produce severe recessions when the equilibrium real interest rate remains persistently below its estimated long-run value, inducing the representative agent to place a significant weight on the forecast rules associated with the deflation equilibrium. Escape from the deflation equilibrium occurs endogenously when the equilibrium real rate eventually rises above its estimated long-run value. But even in normal times, a non-trivial weight on the deflation equilibrium forecast rules may cause the central bank to undershoot its inflation target and magnify the volatilities of macro variables relative to the targeted equilibrium.

I show that the switching model can exactly replicate the observed time paths of the U.S. output gap, quarterly PCE inflation, and the federal funds rate since 1988. The replication exercise tells us that the U.S. data since the start of the ZLB episode in 2008.Q4 are best described as a mixture of the targeted equilibrium and the deflation equilibrium. This result contrasts with that of Aruoba, Cuba-Borda, and Schorfheide (2018) who do not allow for a mixed regime but rather examine the likelihood that the U.S. data are best described as being

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drawn from one local equilibrium or the other.

A simple loss function approach favors a modest increase in the central bank's inflation target to around 3%. But even with an inflation target of 4%, the ZLB binding frequency remains relatively high at 10.5% and the average duration of a ZLB episode is 12.5 quarters. When the deflation equilibrium is taken into account, raising the inflation target is a less effective solution for avoiding long-lived ZLB episodes.

A Appendix: Kalman filter estimate of r-star

Straightforward computations using the laws of motion (5) and (6) yield the following expression for the unconditional variance of $\Delta r_t \equiv r_t - r_{t-1}$:

$$Var\left(\Delta r_t\right) = \sigma_\varepsilon^2 \left[\frac{(1 - \rho_r)\phi + 2}{1 + \rho_r}\right], \tag{A.1}$$

where $\phi \equiv \sigma_\eta^2/\sigma_\varepsilon^2$. Given ρ_r and ϕ, the value of σ_ε can be inferred from (A.1) using the observable value of $Var\left(\Delta r_t\right)$ from the data. Given ϕ and σ_ε, we have $\sigma_\eta = \sigma_\varepsilon \sqrt{\phi}$.

Solving equation (5) for r_t^* yields:

$$r_t^* = \underbrace{\frac{r_t - \rho_r r_{t-1}}{1 - \rho_r}}_{\text{Signal}} - \underbrace{\varepsilon_t}_{\text{Noise}}, \tag{A.2}$$

where the first term represents the signal and the second term represents the noise. Equation (6) shows that the Kalman filter estimate of r_t^*, denoted by $E_t r_t^*$, is a weighted average of the signal and the previous period's estimate $E_{t-1} r_{t-1}^*$, where the weight assigned to the signal is the Kalman gain parameter λ.

The one step ahead forecast error for r_{t+1} is given by

$$\begin{aligned}
err_{t+1} &= r_{t+1} - E_t r_{t+1}, \\
&= r_{t+1} - \left[\rho_r r_t + (1 - \rho_r) E_t r_t^*\right], \\
&= \varepsilon_{t+1} + (1 - \rho_r)\eta_{t+1} + (1 - \rho_r)\left(r_t^* - E_t r_t^*\right), \tag{A.3}
\end{aligned}$$

where the last term in (A.3) represents the estimation error for r_t^*. The optimal value of λ minimizes the mean squared forecast error, as given by

$$E\left(err_{t+1}^2\right) = \sigma_\varepsilon^2 \left[1 + (1 - \rho_r)^2\,\phi\right] + (1 - \rho_r)^2\,Var\left(r_t^* - E_t r_t^*\right). \tag{A.4}$$

where $Var\left(r_t^* - E_t r_t^*\right)$ is the unconditional variance of the estimation error.

The law of motion for the estimation error follows directly from equation (8) and can be written as

$$r_t^* - E_t r_t^* = \lambda \left[\frac{z_t - \rho_r\, z_{t-1}}{1 - \rho_r}\right] + (1 - \lambda)\left(r_{t-1}^* - E_{t-1} r_{t-1}^*\right) - \left[\frac{1 - \lambda - \rho_r}{1 - \rho_r}\right]\eta_t, \tag{A.5}$$

where $z_t \equiv r_t - r_t^*$ is the equilibrium real rate gap. The law of motion for z_t follows directly from equations (5) and (6) and can be written as

$$z_t = \rho_r\, z_{t-1} - \rho_r\,\eta_t + \varepsilon_t. \tag{A.6}$$

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Starting from equations (A.5) and (A.6), we can compute the following expression for the unconditional variance of the estimation error

$$Var\left(r_t^* - E_t r_t^*\right) = \sigma_\varepsilon^2 \left\{ \frac{\lambda\left(\rho_r^2\phi + 1\right) + \left(1 - \lambda - \rho_r\right)\left[\left(1 - \lambda\right)\left(1 - \rho_r\right)/\lambda + \rho_r\right]\phi}{\left(2 - \lambda\right)\left(1 - \rho_r\right)^2} \right\}, \qquad (A.7)$$

which can be substituted into equation (A.4) to obtain a complicated expression for $E\left(err_{t+1}^2\right)$ in terms of λ. From this expression, we can compute the gradient

$$\frac{\partial E\left(err_{t+1}^2\right)}{\partial \lambda} = \frac{2\left[\lambda^2 - \left(1 - \lambda\right)\left(1 - \rho_r\right)^2\phi\right]}{\left(2 - \lambda\right)^2 \lambda^2}. \qquad (A.8)$$

Setting the gradient equal to zero yields a quadratic equation in λ. The root that minimizes $E\left(err_{t+1}^2\right)$ is given by equation (9).

B Appendix: Targeted equilibrium

To solve for the local linear forecast rules associated with the targeted equilibrium, I assume that $i_t^* = i_t > 0$ for all t. This assumption is valid in a vicinity of the targeted equilibrium's long-run endpoints. Starting from equation (10) we have:

$$i_t^* = \rho i_{t-1}^* + \left(1 - \rho\right)\left[E_t r_t^* + \pi^* + g_\pi \omega\left(\pi_t - \pi^*\right) + g_\pi\left(1 - \omega\right)\left(\bar{\pi}_{t-1} - \pi^*\right) + g_y\left(y_t - y^*\right)\right] \quad (B.1)$$

where I have used equation (11) to eliminate $\bar{\pi}_t$.

Equation (B.1) together with the Euler equation (1) and the Phillips curve (2) form a linear system of three equations in the three unknown decision rules for y_t, π_t, and i_t^*. The state variables are: r_t, $E_t r_t^*$, $\bar{\pi}_{t-1}$, i_{t-1}^*, ν_t, and u_t. Standard techniques yield a set of linear decision rules of the form

$$\begin{bmatrix} y_t - \pi^*\left(1 - \beta\right)/\kappa \\ \pi_t - \pi^* \\ i_t^* - \left(E_t r_t^* + \pi^*\right) \end{bmatrix} = \mathbf{A} \begin{bmatrix} r_t - E_t r_t^* \\ \bar{\pi}_{t-1} - \pi^* \\ i_{t-1}^* - E_t r_t^* - \pi^* \\ \nu_t \\ u_t \end{bmatrix}, \qquad (B.2)$$

where $\mathbf{A}$ is a 3×5 matrix of decision rule coefficients. For the parameter values shown in Table 2, the matrix $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} 0.576 & -0.155 & -0.386 & 3.842 & -0.180 \\ 0.064 & -0.017 & -0.033 & 0.425 & 1.457 \\ 0.124 & 0.130 & 0.718 & 0.826 & 0.163 \end{bmatrix}. \qquad (B.3)$$

Iterating the linear decision rules in (B.2) ahead one period and then taking the conditional expectation of both sides yields the following set of linear forecast rules associated with the targeted equilibrium:

$$
\begin{aligned}
E_t y_{t+1} &= y^* + \mathbf{A}_{11}\rho_r \left(r_t - E_t r_t^*\right) + \mathbf{A}_{12}\left(\bar{\pi}_t - \pi^*\right) + \mathbf{A}_{13}\left(i_t^* - E_t r_t^* - \pi^*\right) + \\
&\quad \mathbf{A}_{14}\rho_\nu \nu_t + \mathbf{A}_{15}\rho_u u_t, \tag{B.4}
\end{aligned}
$$

$$
\begin{aligned}
E_t \pi_{t+1} &= \pi^* + \mathbf{A}_{21}\rho_r \left(r_t - E_t r_t^*\right) + \mathbf{A}_{22}\left(\bar{\pi}_t - \pi^*\right) + \mathbf{A}_{23}\left(i_t^* - E_t r_t^* - \pi^*\right) + \\
&\quad \mathbf{A}_{24}\rho_\nu \nu_t + \mathbf{A}_{25}\rho_u u_t, \tag{B.5}
\end{aligned}
$$

where $\mathbf{A}_{ij}$ represents the corresponding element of the matrix $\mathbf{A}$ and I have substituted in $E_t\left(r_{t+1} - E_{t+1}r_{t+1}^*\right) = \rho_r\left(r_t - E_t r_t^*\right)$. Notice that the forecast rules depend on the realization of π_t because $\bar{\pi}_t$ depends on π_t via equation (11). Also, the forecast rules depend on the realization of i_t^* due to the interest rate smoothing term in (B.1). Hence, the model solution allows for simultaneity between the forecasted and realized values of π_t and i_t^*. Neither the agent or the central bank are required to forecast i_{t+1}^*.

The local linear forecast rules (B.4) and (B.5) are derived under the assumption that the ZLB is never binding. However, in the stochastic simulation of the targeted equilibrium, I allow for an occasionally binding ZLB. When simulating the model, I substitute the local linear forecast rules given by (B.4) and (B.5) into the global equilibrium conditions (1) and (2). I allow for an occasionally binding ZLB by making the substitution $i_t = 0.5\,i_t^* + 0.5\sqrt{\left(i_t^*\right)^2}$ in the equilibrium condition (1). Together with the monetary policy rule (B.1), this procedure yields a system of three equations that are solved each period to obtain the three realizations y_t, π_t, and i_t^*.

C   Appendix: Deflation equilibrium

To solve for the local linear forecast rules associated with the deflation equilibrium, I assume $i_t^* \leq 0$ such that $i_t = 0$ for all t. This assumption is valid in a vicinity of the deflation equilibrium's long-run endpoints. Equation (B.1) applies unchanged to the deflation equilibrium, as does the Phillips curve (2). However, due to the binding ZLB, the Euler equation (1) now becomes

$$
y_t = E_t\,y_{t+1} + \alpha[E_t\,\pi_{t+1} + r_t] + \nu_t. \tag{C.1}
$$

Equation (C.1) together with equations (B.1) and (2) form a linear system of three equations in the three unknown decision rules for y_t, π_t, and i_t^*. The state variables are: r_t, $E_t r_t^*$,

$\overline{\pi}_{t-1}$, i^*_{t-1}, ν_t, and u_t. The minimum state variable (MSV) solution yields a set of linear decision rules of the form

$$
\begin{bmatrix} y_t - (-E_t r^*_t)(1-\beta)/\kappa \\ \pi_t - (-E_t r^*_t) \\ i^*_t - (E_t r^*_t + \pi^*)\left[1 - g_\pi - g_y(1-\beta)/\kappa\right] \end{bmatrix} = \mathbf{B} \begin{bmatrix} r_t - E_t r^*_t \\ \overline{\pi}_{t-1} - (-E_t r^*_t) \\ i^*_{t-1} - (E_t r^*_t + \pi^*)\left[1 - g_\pi - \frac{g_y(1-\beta)}{\kappa}\right] \\ \nu_t \\ u_t \end{bmatrix},
$$

(C.2)

where $\mathbf{B}$ is a 3×5 matrix of constant coefficients. The MSV solution implies $\mathbf{B}_{12} = \mathbf{B}_{22} = 0$ and $\mathbf{B}_{13} = \mathbf{B}_{23} = 0$. For the parameter values shown in Table 2, the matrix $\mathbf{B}$ is

$$
\mathbf{B} = \begin{bmatrix} 1.160 & 0 & 0 & 7.732 & 0.110 \\ 0.188 & 0 & 0 & 1.253 & 1.493 \\ 0.258 & 0.163 & 0.8 & 1.718 & 0.226 \end{bmatrix}.
$$

(C.3)

Comparing the first column of matrix $\mathbf{B}$ in (C.3) to the first column of matrix $\mathbf{A}$ in (B.3) shows that a shock to $r_t - E_t r^*_t$ will be transmitted more forcefully to macro variables in the deflation equilibrium than in targeted equilibrium. Specifically, we have

$$
\frac{\mathbf{B}_{11}}{\mathbf{A}_{11}} = 2.0, \qquad \frac{\mathbf{B}_{21}}{\mathbf{A}_{21}} = 2.9, \qquad \frac{\mathbf{B}_{31}}{\mathbf{A}_{31}} = 2.1.
$$

(C.4)

For the special case when $\rho = 0$ and $\omega = 1$, it is straightforward to derive the following analytical relationship between the decision rule coefficients for the two local equilibria:

$$
\frac{\mathbf{B}_{11}}{\mathbf{A}_{11}} = \frac{\mathbf{B}_{21}}{\mathbf{A}_{21}} = \frac{\mathbf{B}_{31}}{\mathbf{A}_{31}} = 1 + \frac{\alpha\left[\kappa g_\pi + (1 - \beta\rho_r) g_y\right]}{(1 - \beta\rho_r)(1 - \rho_r) - \alpha\kappa\rho_r} > 1.
$$

(C.5)

Iterating the linear decision rules in (C.2) ahead one period and then taking the conditional expectation of both sides yields the following set of local linear forecast rules for the deflation equilibrium:

$$
\begin{aligned}
E_t y_{t+1} &= -E_t r^*_t (1-\beta)/\kappa + \mathbf{B}_{11}\rho_r (r_t - E_t r^*_t) + \mathbf{B}_{14}\rho_\nu \nu_t + \mathbf{B}_{15}\rho_u u_t, & \text{(C.6)} \\
E_t \pi_{t+1} &= -E_t r^*_t + \mathbf{B}_{21}\rho_r (r_t - E_t r^*_t) + \mathbf{B}_{24}\rho_\nu \nu_t + \mathbf{B}_{25}\rho_u u_t, & \text{(C.7)}
\end{aligned}
$$

where the MSV solution implies $\mathbf{B}_{12} = \mathbf{B}_{22} = 0$ and $\mathbf{B}_{13} = \mathbf{B}_{23} = 0$ and I have substituted in $E_t\left(r_{t+1} - E_{t+1}r^*_{t+1}\right) = \rho_r (r_t - E_t r^*_t)$. Neither the agent or the central bank are required to forecast i^*_{t+1}.

The local linear forecast rules (C.6) and (C.7) are derived under the assumption that the ZLB is always binding. However, in the stochastic simulation of the deflation equilibrium,

I allow for an occasionally binding ZLB. When simulating the model, I substitute the local linear forecast rules given by (C.6) and (C.7) into the global equilibrium conditions (1) and (2). I allow for an occasionally binding ZLB by making the substitution $i_t = 0.5\, i_t^* + 0.5\sqrt{(i_t^*)^2}$ in the equilibrium condition (1). Together with the monetary policy rule (B.1), this procedure yields a system of three equations that are solved each period to obtain the three realizations y_t, π_t, and i_t^*.

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## Figure 1: Nominal Interest Rates and Inflation in Four Countries
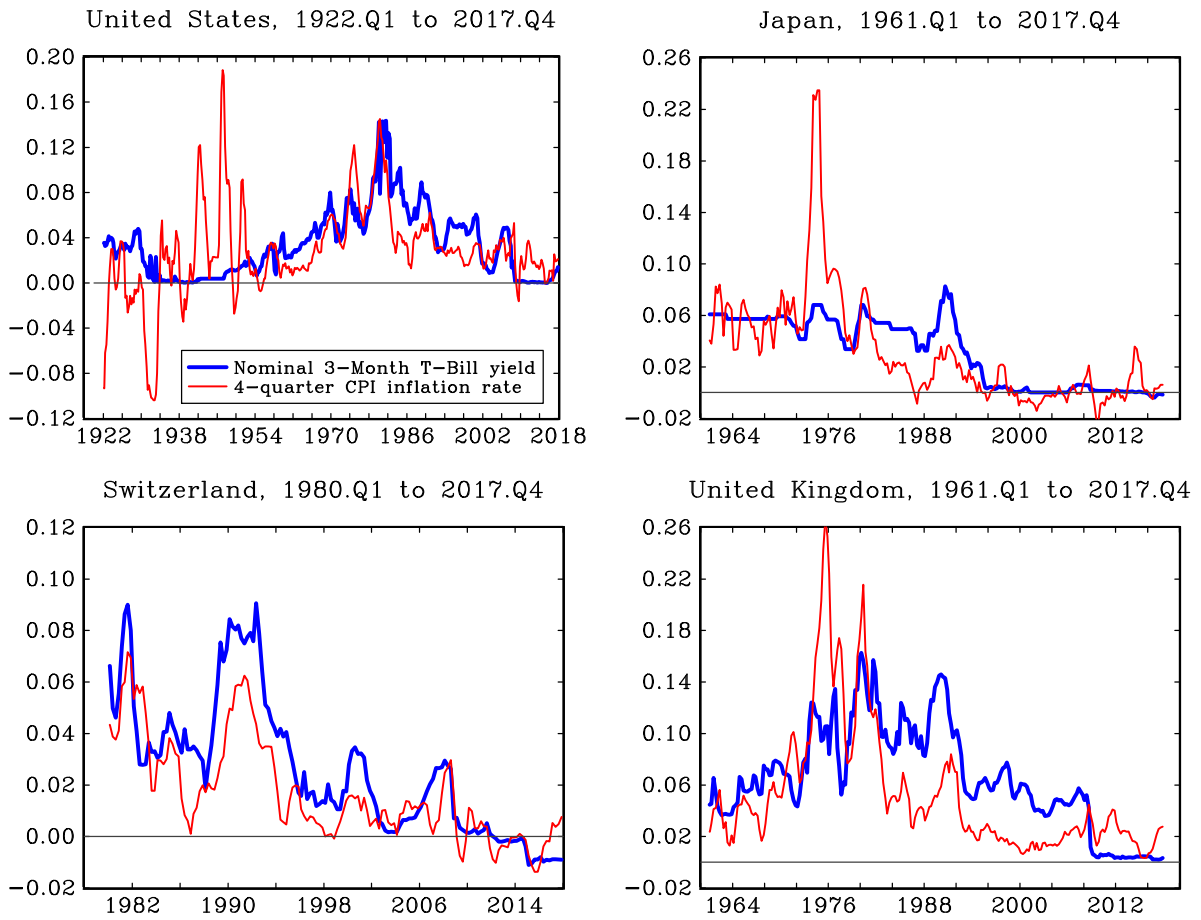
Notes: Nominal interest rates in the United States encountered the zero lower bound during the 1930s and from 2008.Q4 though 2015.Q4. Nominal interest rates in Japan have remained near zero since 1998.Q3 except for the relatively brief period from 2006.Q4 to 2008.Q3. Nominal interest rates in Switzerland have been zero or slightly negative since 2008.Q4. Nominal interest rates in the United Kingdom have been approximately zero since 2009.Q1. Outside of these episodes, all four countries exhibit a strong positive correlation between nominal interest rates and inflation, consistent with the Fisher relationship.

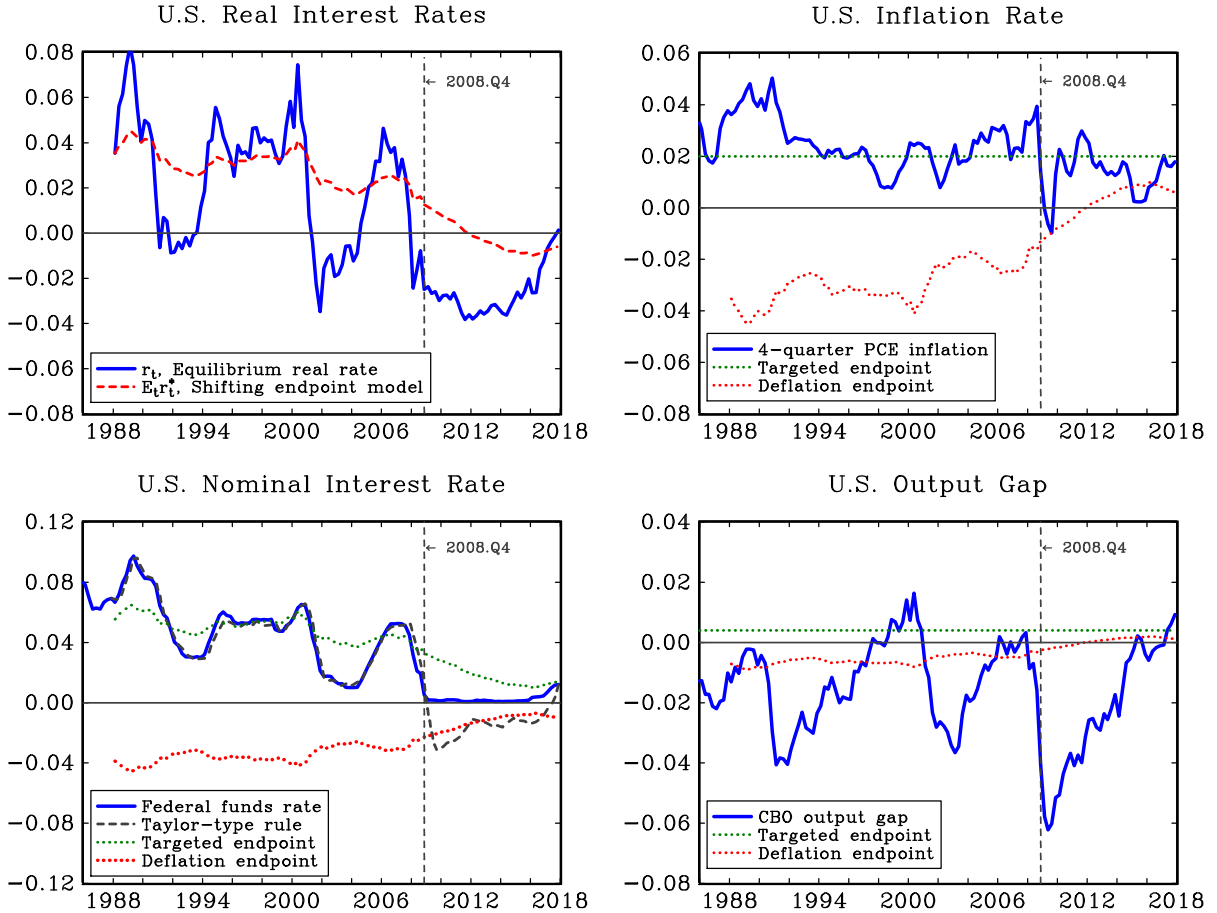Figure 2: U.S. Nominal Interest Rates and Inflation
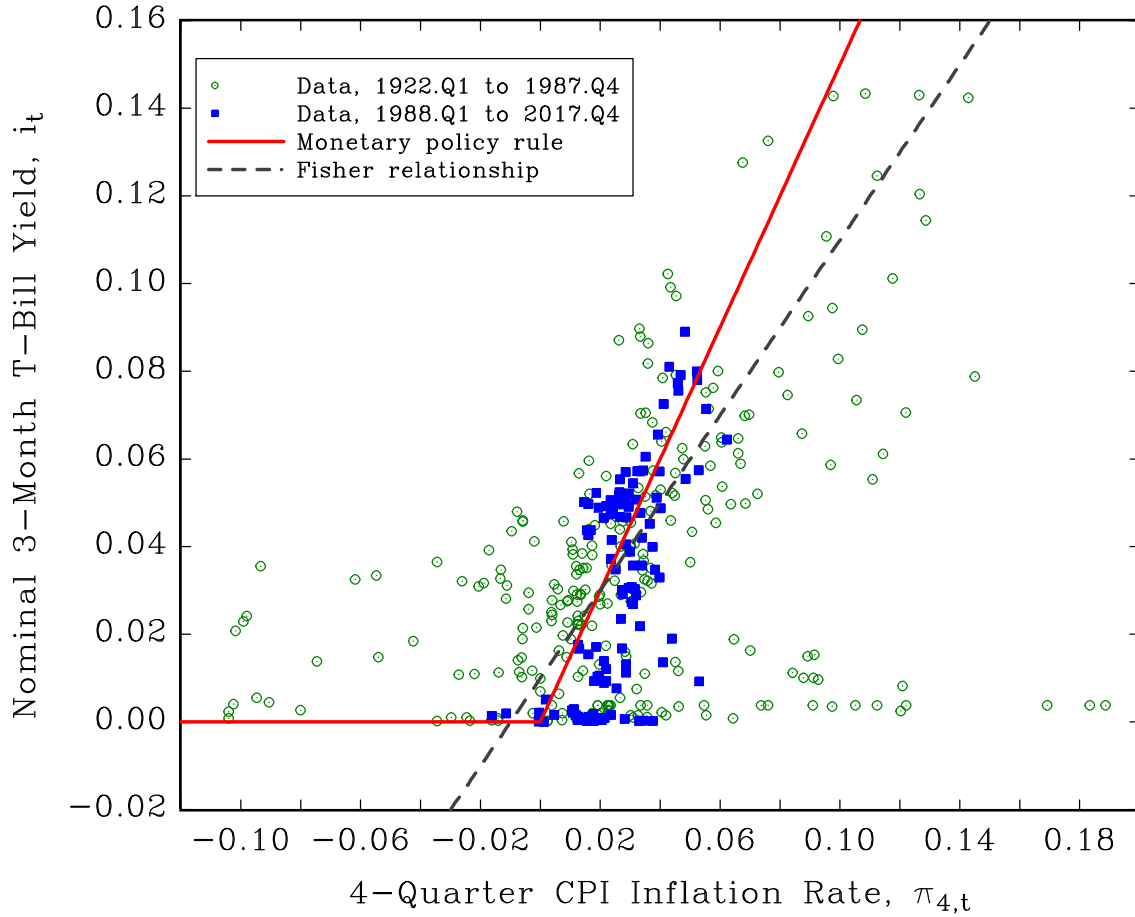
Notes: The two intersections of the ZLB-augmented monetary policy rule (solid red line) with the Fisher relationship (dashed black line) define two long-run endpoints, labeled the "targeted equilibrium" and "deflation equilibrium," respectively. The monetary policy rule is $i_t = r^* + \pi^* + g_\pi (\pi_{4,t} - \pi^*)$ with $r^* = 0.01$, $\pi^* = 0.02$ and $g_\pi = 1.5$. The Fisher relationship is $i_t = r^* + \pi_{4,t}$.

Figure 3: U.S. Real Interest Rates
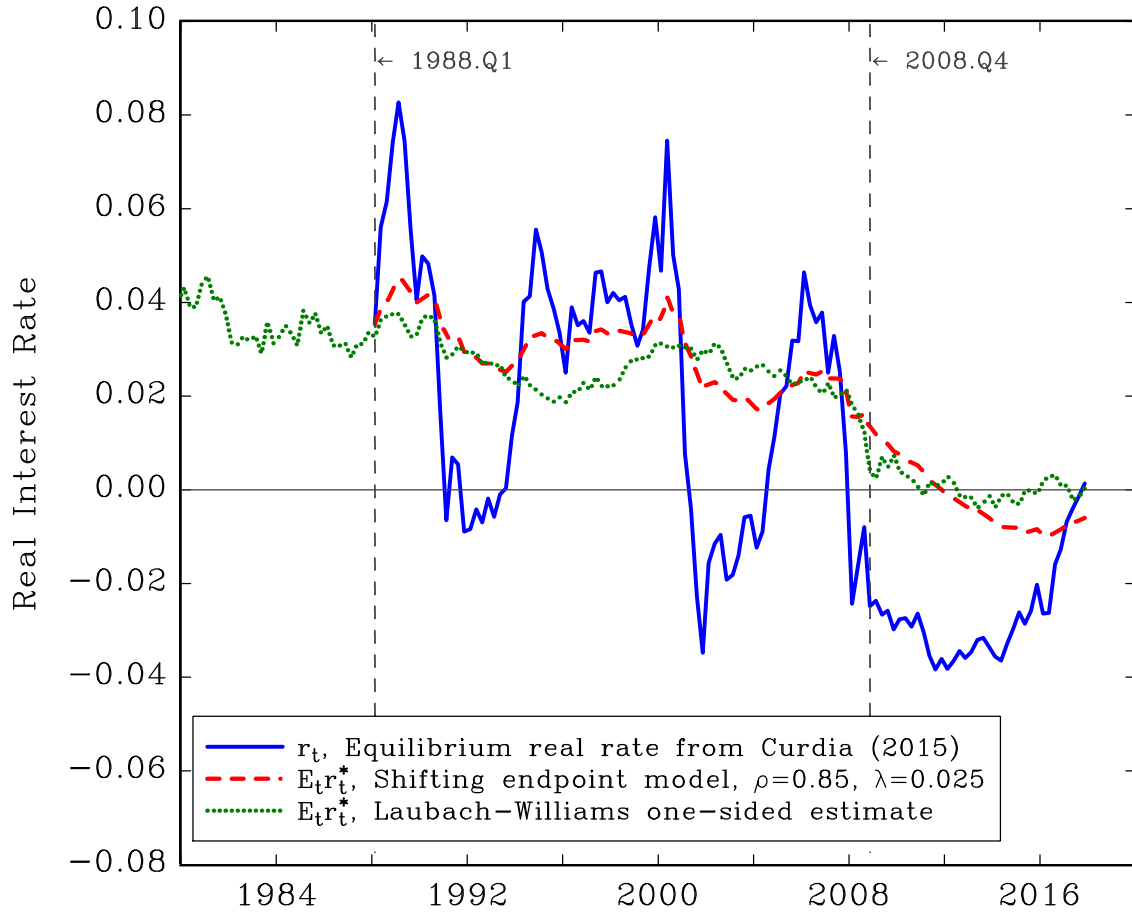
Notes: The equilibrium real interest rate (blue line) is from Cúrdia (2015, updated). The time series process for the Kalman filter estimate of the natural rate of interest in the model (dotted green line) is calibrated to approximate the one-sided estimate of the U.S. natural rate series (dashed red line) from Laubach and Williams (2016, updated).
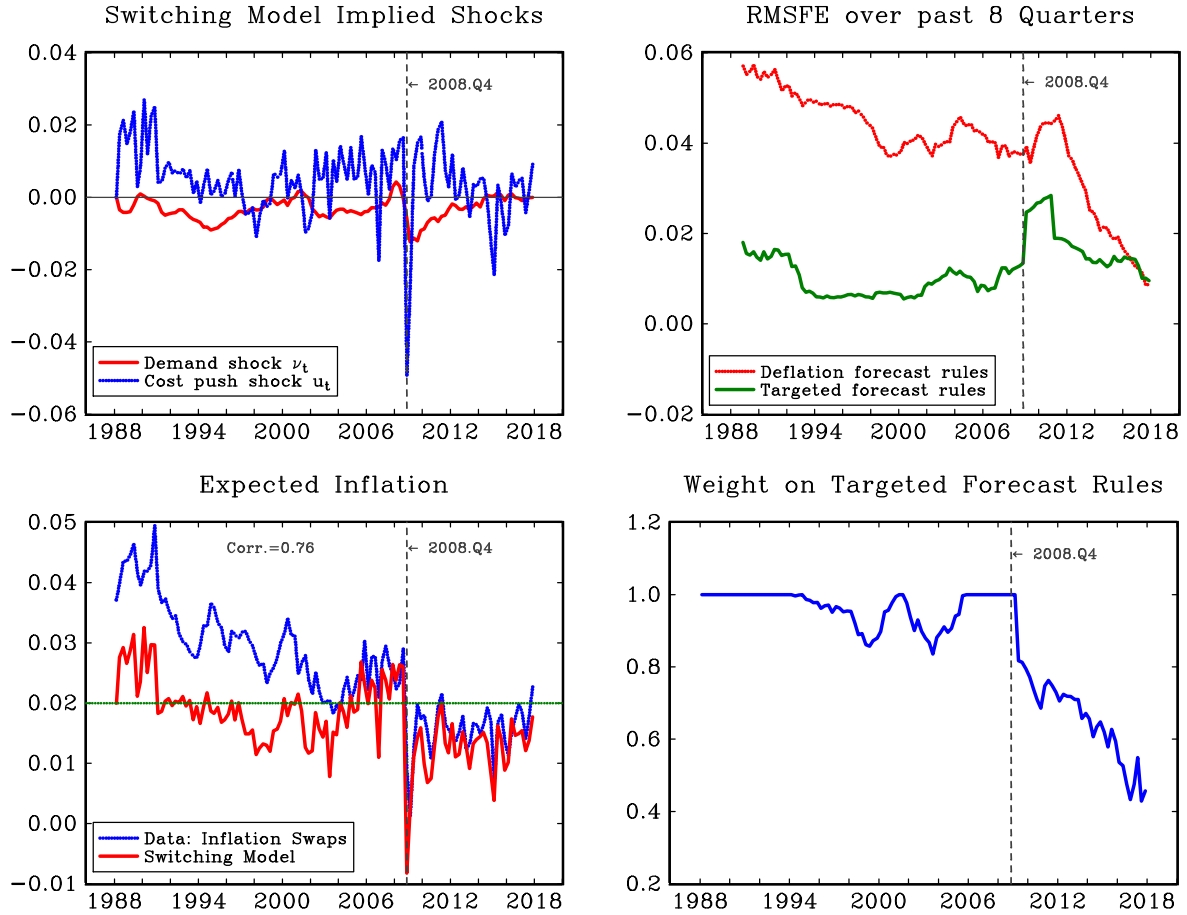
Figure 4: U.S. Data

U.S. Real Interest Rates

Legend:
- r_t, Equilibrium real rate
- $E_t r_t^*$, Shifting endpoint model

← 2008.Q4

U.S. Inflation Rate

Legend:
- 4-quarter PCE inflation
- Targeted endpoint
- Deflation endpoint

← 2008.Q4

U.S. Nominal Interest Rate

Legend:
- Federal funds rate
- Taylor-type rule
- Targeted endpoint
- Deflation endpoint

← 2008.Q4

U.S. Output Gap

Legend:
- CBO output gap
- Targeted endpoint
- Deflation endpoint

← 2008.Q4

Notes: The equilibrium real interest rate r_t has remained mostly below the model-implied value of $E_t r_t^*$ since the start of the Great Recession in 2007.Q4. The nominal federal funds rate was approximately zero from 2008.Q4 through 2015.Q4. A Taylor-type rule of the form (10) using the parameter values in Table 2, with $E_t r_t^*$ given by the model-implied value, $\overline{\pi}_t$ given by the 4-quarter PCE inflation rate, and y_t given by the CBO output gap predicts that the desired nominal funds rate was negative starting in 2009.Q1 and remains negative through 2017.Q2. The 4-quarter PCE inflation rate was briefly negative in 2009 and has remained mostly below the Fed's 2% inflation target since 2012.Q2. The Great Recession was very severe, pushing the CBO output gap down to -6.2% at the business cycle trough in 2009.Q2. The output gap finally turns positive in 2017.Q2, a full eight years after the Great Recession ended. The various endpoints plotted in the figure are computed using the expressions in Table 1, with r_t^* set equal to the model-implied value of $E_t r_t^*$.

## Figure 5: Measures of Expected U.S. Inflation
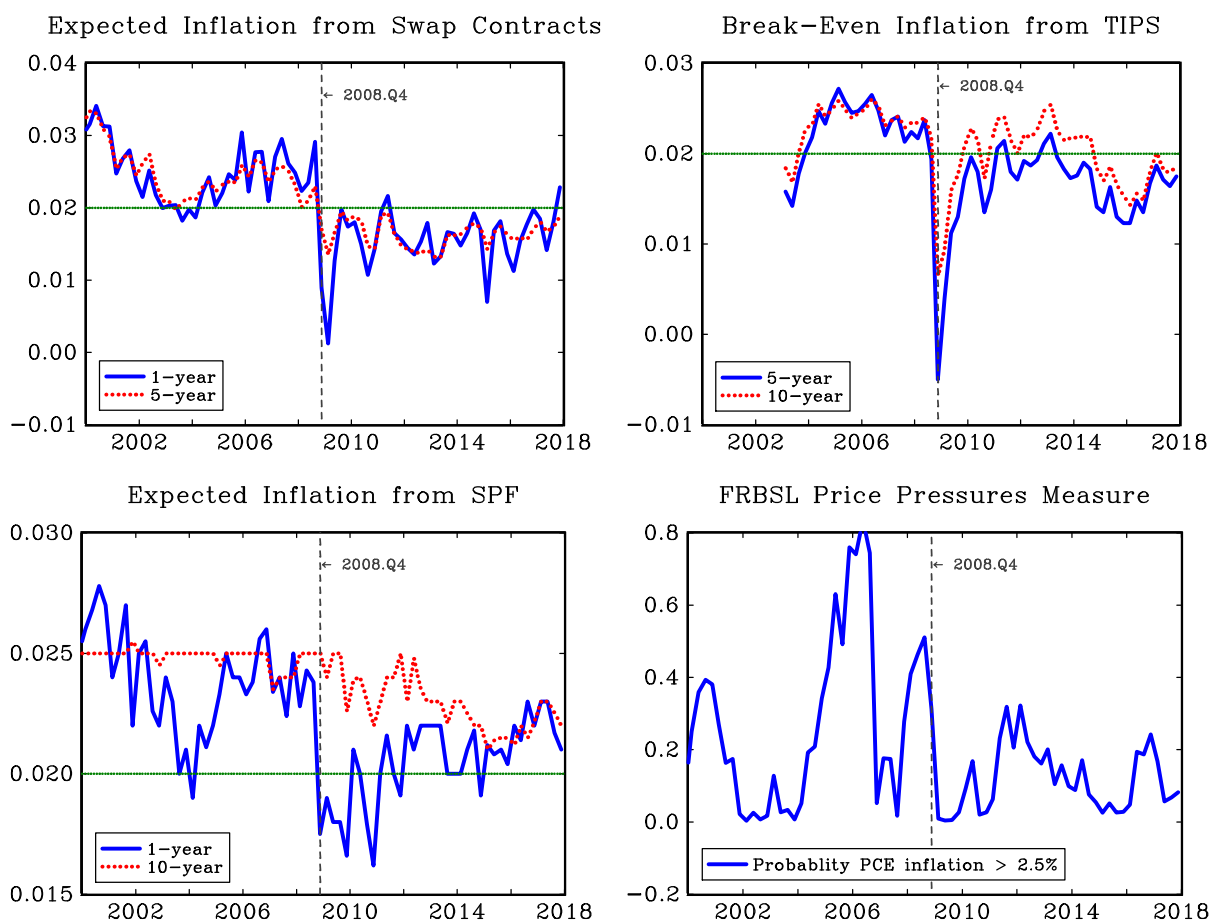
Notes: Market-based measures of expected inflation from zero coupon inflation swap contracts and Treasury Inflation Protected Securities (TIPS) dropped sharply in 2008.Q4, coinciding with the start of the ZLB episode. The market-based measures have remained below the Fed's 2% inflation target for extended periods even after the Great Recession ended in 2009.Q2. The median 1-year expected inflation rate from the Survey of Professional Forecasters (SPF) dropped sharply in 2008.Q4 and has recovered slowly to a level that is below its pre-recession range. The 10-year survey measure does not exhibit a sharp drop in 2008.Q4, but has since trended downward to a level that is below its pre-recession range. The Federal Reserve Bank of St. Louis' Price Pressures Measure (PPM) represents the probability that the 4-quarter PCE inflation rate over the next year will exceed 2.5%. The PPM dropped sharply in 2008.Q4 and is currently hovering around a probability of 10%.

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Figure 6: Replicating U.S. Data with the Switching Model

Switching Model Implied Shocks

RMSFE over past 8 Quarters

Expected Inflation

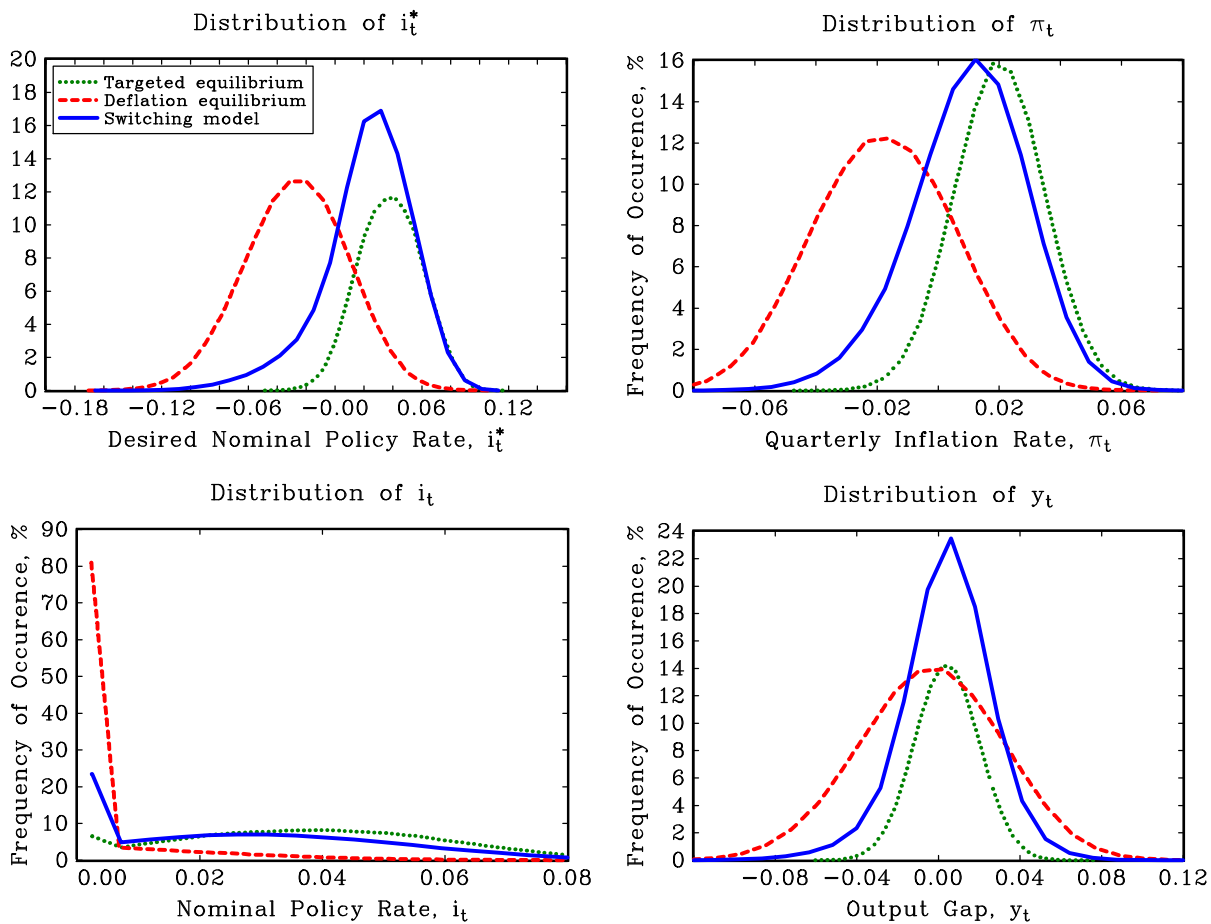**Weight on Targeted Forecast Rules**

Notes: Given the U.S. data counterparts for the model variables, I solve for the implied time series of stochastic shocks and endogenous forecast rule weights that allow the switching model to exactly replicate the observed U.S. time paths of the CBO output gap, PCE inflation, and the federal funds rate since 1988. The reverse-engineered shocks ν_t and u_t go from positive to negative at the start of the ZLB episode in 2008.Q4. The top right panel shows that the performance gap between the two sets of forecast rules initially narrows with the onset of the ZLB episode in 2008.Q4. The performance gap narrows even further after 2012.Q2 when the Fed consistently undershoots its 2% inflation target. At the end of the data sample in 2017.Q4, the weight μ_t assigned to the targeted forecast rules is only 0.46. The fact that the model-implied value of $E_t r_t^*$ remains negative at the end of the data sample helps the deflation forecast rules to account for the observation of low positive inflation in the data. The bottom left panel compares the 1-year expected inflation rate from U.S. inflation swaps to the path of $\widehat{E}_t \pi_{t+1}$ from the switching model. The correlation coefficient between the two series is 0.76.

Figure 7: Model Simulation: Endogenous Regime Switching
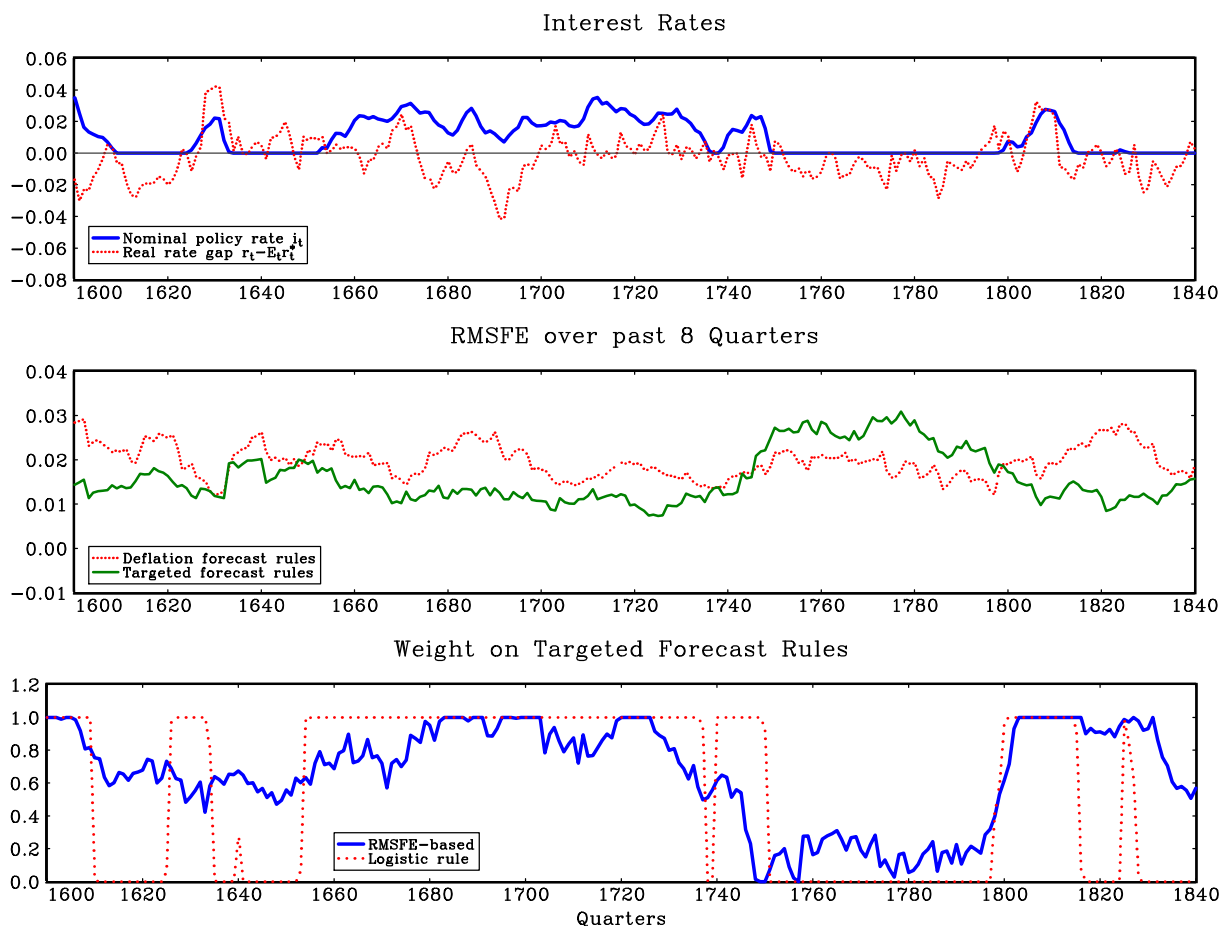
Notes: When the gap $r_t - E_t r_t^*$ is negative for a sustained period (top panel), the resulting downward pressure on y_t and π_t serves to reduce the recent $RMSFE$ of the deflation forecast rules and increase the recent $RMSFE$ of the targeted forecast rules (middle panel). Around period 1725, the shift in relative forecast performance reduces the weight on the targeted forecast rules. The lower weight puts further downward pressure on y_t and π_t, causing the deflation equilibrium to eventually become fully realized around period 1750 (bottom panel). Around period 1800, the gap $r_t - E_t r_t^*$ becomes positive, causing the targeted forecast rules to once again outperform the deflation forecast rules. The weight on the targeted forecast rules increases, causing the targeted equilibrium to be restored. Qualitatively similar results are obtained if the weight on the targeted forecast rules is determined by a logistic rule along the lines of Binning and Maih (2017).

Figure 8: Model Simulations: Distributions of Endogenous Variables

Notes: Model variables in the deflation equilibrium have distributions with lower means but higher variances than those in the targeted equilibrium. But the significant overlap in the various distributions creates a dilemma for an agent who seeks to determine the likelihood that a string of recent data observations are drawn from one equilibrium or the other. Variables in the switching model have means that are somewhat lower and variances that are somewhat higher than those in the targeted equilibrium. Consequently, the central bank in the switching model undershoots its inflation target and the volatilities of the output gap and inflation are both higher relative to the targeted equilibrium.

Figure 9: Model Simulations: Distribution of ZLB Durations
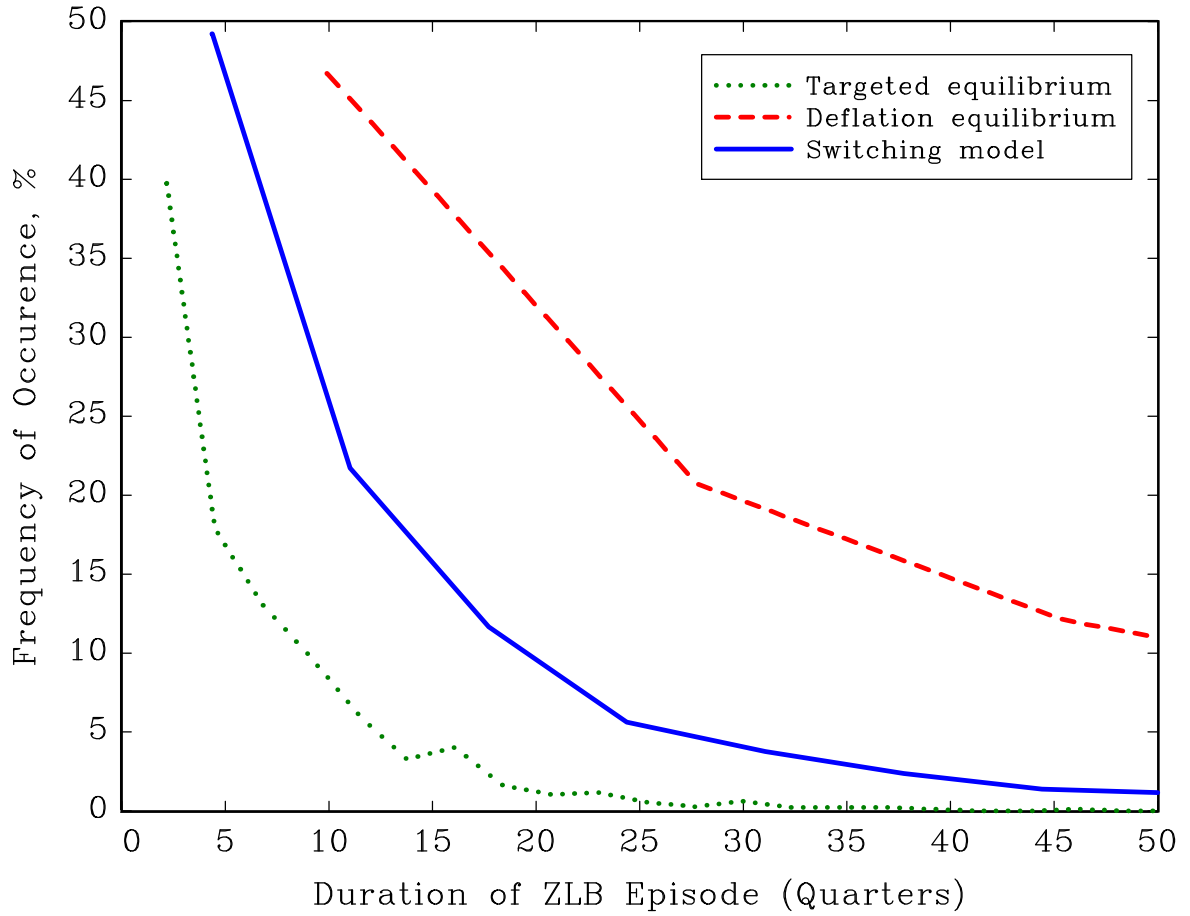
Notes: Unlike the targeted equilibrium, the switching model can produce infrequent but long-lived ZLB episodes in response to small, normally distributed shocks. A 29 quarter ZLB episode (the duration observed in U.S. data since 1988) is an extremely rare event in the targeted equilibrium but can occur with about 5% frequency in the switching model.