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Government spending multipliers and the zero lower bound

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A B S T R A C T

It is well-known that when the nominal interest rate hits the zero lower bound, the size of the fiscal multiplier can be large. The effectiveness of fiscal stimulus depends on the duration and the *expected* duration of the zero-lower-bound regime. Most studies fix this duration and therefore suffer from a bias. In this paper, we propose a way to estimate the government spending multiplier that allows this duration to be endogenously determined. Specifically, we incorporate into the Smets and Wouters (2007) model a monetary policy reaction function that follows a two-state Markov-switching process, and use data to pin down the transitional probability from one policy regime to the other. We then estimate the model and compute the fiscal multiplier using a data set that spans 1985:2–2015:3.

1. Introduction

The question of how big the government spending multiplier is has intrigued economists for decades. A large portion of the literature focuses on reduced form empirical estimates: from the earlier work of Evans (1969) and Barro (1981) to the more recent endeavor of Gordon and Krenn (2010) and Ramey (2011), various estimates have been produced using advanced econometrics techniques. The most cited range for the multiplier is between 0.8 and 1.5 (Ramey, 2011). A common difficulty that reduced form estimation encounters is the identification issue – truly exogenous spending shocks are rare, as most fiscal stimuli are responses to the state of the economy. More recently, macroeconomists have turned to dynamic stochastic general equilibrium (DSGE) models to seek more structured answers. Examples include Cogan et al. (2010), Christiano et al. (2011), Drautzburg and Uhlig (2011), and Woodford (2011), among others. In a standard new Keynesian model the multiplier is small, because there is a negative wealth effect on consumption when rational consumers anticipate higher future taxes. In order for the multiplier to increase, unconventional features are often considered, such as rule-of-thumb consumers and inelastic labor supply (Gali et al., 2007).

There is one scenario, however, that can produce a larger multiplier when modeled, and is also empirically relevant. It is when the nominal interest rate reaches the “zero lower bound.” As Eggertsson (2011), Christiano et al. (2011) and

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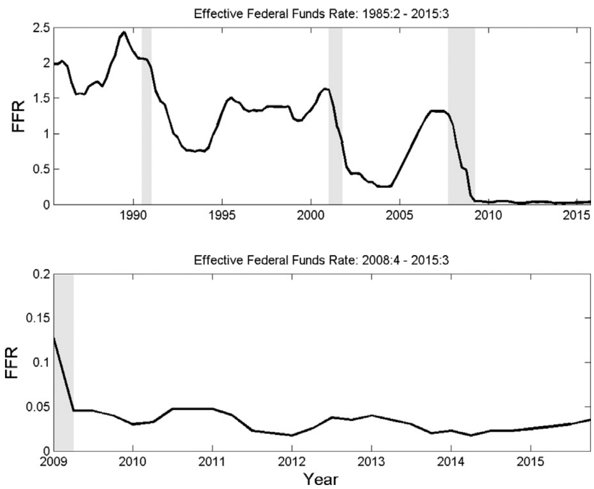

Fig. 1. The nominal interest rate.

Woodford (2011) show, the economy can fall into a deflationary cycle if consumption is weak and the desire for saving is high when the nominal interest rate hits the lower bound. In this case, higher government spending can raise output and expected inflation, which in turn lowers the real interest rate and boosts the economy further. Christiano et al. (2011)'s calibration is that if the zero lower bound is binding for 12 quarters when government spending increases, the multiplier can be as high as 2.3. Other authors, such as Eggertsson (2011) and Coenen et al. (2012), obtain similarly large estimates.

Thus, the duration and the *expected* duration of zero interest rate monetary policies can be crucial to the size of the multiplier. The duration matters because if it is not long enough to cover the period in which the fiscal policy is implemented, the size of the multiplier will be greatly reduced. Anticipated duration matters because economic agents are forward-looking (rational in DSGE models). Their response to the increased government spending naturally depends on how long they believe the zero lower bound policy will hold. In the aforementioned papers, the duration of the zero lower bound is treated rather "loosely." For example, Christiano et al. (2011) assume that the duration is 12 quarters, Drautzburg and Uhlig (2011) assume that it is 8 quarters, while Eggertsson (2011) experiment with 0, 4, and 8 quarters. This is problematic not just because the ex post duration of the policy in the U.S. is much longer, but also because by giving an exogenous duration, they have imposed an implicit assumption that agents all expect the duration to be a certain number of quarters. When taking the model to data, the estimates are surely biased by this arbitrary assumption.

Fig. 1 displays the time series for the nominal interest rate spanning the period from 1985:2 to 2015:3. The shaded areas correspond to the NBER-dated recessions. The top panel shows the federal funds rate for the entire sample period, while the lower panel focuses on the period in which the interest rate hits the lower bound. Near zero low interest rates prevailed from 2008 to 2015. Note that in the so-called "zero lower bound" regime, the nominal interest rate did not stay constant. It fluctuated around a near-zero but positive mean.

In this paper, we propose a way to estimate the government spending multiplier that allows firms and households to take the possibility of the economy reaching the zero lower bound regime into consideration. Specifically, we incorporate into the Smets and Wouters (2007) model, the workhorse model for structural estimation, a monetary policy reaction function that follows a two-state Markov-switching process. The reaction function has time variant parameters that depend on the state of the economy. In the normal state, the reaction function is a Taylor rule. In the other state, the nominal interest rate hits the zero lower bound. Therefore, the duration of the zero lower bound regime is endogenous, and the anticipated duration is no longer a fixed number. We take this model to the data and estimate the size of the government spending multiplier in the U.S. Our data set covers the most recent years of the U.S. economy, in which the monetary policy was essentially a zero lower bound regime. This is necessary because most of the earlier studies were done around 2009 and 2010, and the duration of the zero lower bound policy was invariably underestimated. It is a logical step forward to find out whether or not including newer data will change the results significantly. The fact that even economists equipped with the most advanced techniques can dramatically misjudge the duration of the zero interest rate regime accentuates the necessity to consider a model with endogenous durations.

This paper contributes to the literature of fiscal multipliers in two dimensions. First, this is the first paper that endogenizes the duration of the zero lower bound policy regime using a Markov-switching method. We create an economic environment in which agents are always aware that there is a probability that the policy regime may switch. By doing so, we can estimate the transitional probabilities from one regime to the other. In a rational expectations equilibrium, agents' expectations are consistent with the true probability distribution of random processes. Thus, we can use data to back out the *perceived* probability of regime switching. This is advantageous because one no longer has to arbitrarily impose a perceived duration.

Second, we make a contribution on the technical front. The possibility of a zero lower bound regime makes the monetary policy nonlinear, which often constitutes a challenge to researchers in computation and estimation. The Markov-switching method in this paper largely overcomes this challenge. Our approach is closely connected to the literature that models macroeconomic fluctuations using Markov-switching methods. For example, Liu et al. (2009, 2011) and Farmer et al. (2009, 2011) examine the sources of macroeconomic fluctuations by allowing for a Markov-switching setup that accounts for time variations in shock variances and the inflation target. They let the parameters in the monetary policy reaction function vary with regimes. One disparity between this paper and theirs is that rather than splitting the monetary policy into active (hawkish) and passive (dovish) regimes, this paper considers a Taylor rule regime and a zero lower bound regime. An innovation here is that the time-variant parameters are assigned values *a priori* in the zero lower bound regime (not for estimation) and are provided with prior distributions in the Taylor rule regime (for estimation). As a result, time-invariant parameters are estimated in tandem with time-variant ones.

The rest of the paper is organized as follows. Section 2 presents the model, data and estimation method. Section 3 discusses estimation results. Section 4 estimates the size of the multiplier. Section 5 is a discussion, and Section 6 concludes.

2. Model, data and estimation method

A workhorse DSGE model for empirical estimation is the Smets and Wouters (2007) model. Several papers in the literature have used it to estimate the government spending multiplier. We use a modified version of this model for our estimation.

As a brief recap, the Smets and Wouters (2007) model is an extended version of the basic New Keynesian model. It has five main features: staggered wages, intermediate goods prices indexed to lagged inflation, habit formation in consumption, investment adjustment costs, and variable utilization rate of capital. All these features contribute to the persistence of the dynamic system and fit with the data. The model we will estimate keeps all these features intact. The only difference lies in the monetary policy. In the original Smets and Wouters (2007) model, the monetary policy is characterized by a Taylor rule. Agents are not given any opportunity to anticipate a policy that keeps the interest rate at the zero lower bound for a prolonged period of time. We modify the interest rate policy rule as follows:

$$\hat{r}_t = \kappa_s + \rho_s \hat{r}_{t-1} + (1-\rho_s)\left(r_{\pi,s}\hat{\pi}_t + r_{y,s}\left(\hat{y}_t - \hat{y}_t^p\right)\right) + \varepsilon_{s,t},$$

where $\hat{r}_t$ denotes the nominal interest rate, $\hat{\pi}_t$ inflation, $\hat{y}_t$ output, and $\hat{y}_t^p$ the potential level of output.[1] The values of the policy parameters κ_s, ρ_s, $r_{\pi,\,s}$, $r_{y,\,s}$ and the policy shock $\varepsilon_{s,\,t}$ depend on which policy regime is in place. There are two states of the economy, each of which corresponds to a policy regime. In state 1 ($s=1$), the monetary policy is a Taylor rule: $\kappa_{s=1}=0$, $0 \le \rho_{s=1} \le 1$, $r_{\pi,s=1} \ge 1$, and $r_{y,s=1} \ge 0$. In state 2 ($s=2$), the monetary policy is one that keeps the nominal interest rate near the zero lower bound: $\kappa_{s=2} = -R$, $\rho_{s=2}=0$, $r_{\pi,s=2}=0$, and $r_{y,s=2}=0$[2]. Note that in this case, the *level* of the nominal interest rate is

$$i_t = \hat{r}_t + R = \varepsilon_{s=2,t},$$

where $\varepsilon_{s=2,t}$ is a white noise with a small positive mean and a small variance. In other words, the nominal interest rate is not exactly zero. It is a small number that fluctuates slightly above zero. In the zero lower bound regime, the monetary authority is assumed not to manipulate policy rates to affect the state of the economy.

The transition probabilities for the Markov-switching process are characterized by the following 2×2 matrix:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix},$$

where $p_{ij} = \text{Prob}(s_{t+1} = j | s_t = i)$ and elements of each row of $\mathbf{P}$ add up to one.

In solving the model, we closely follow the Newton method developed by Farmer et al. (2011), which was extended in Maih (2012). To construct the likelihood function, we apply the Sims et al. (2008) filter.[3] The model is then estimated by the standard Bayesian method using seven aggregate time series: real GDP, real consumption, real investment, real wages, hours worked, GDP deflator and the federal funds rate. Correspondingly, there are seven shocks in the dynamic system, which are technology, net worth, government spending, investment-specific technology, monetary policy, price markup and wage markup shocks.

The appendix offers a detailed description of the algorithm used to solve the model and to conduct the Bayesian estimation, but the following is a brief rundown of the procedures: First, the regime-dependent Smets and Wouters (2007) model is solved in its state-space form that relates endogenous variables to pre-determined variables. Second, we use the observation equations to introduce the seven aggregate time series data to the state space. Then, we construct the likelihood function using the Sims et al. (2008) filter. We derive the posterior density function by combining the likelihood function

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[1] To be more precise, $\hat{r}_t$ and $\hat{\pi}_t$ are deviations of the nominal interest rate and inflation from their respective steady states, and $\hat{y}_t$ and $\hat{y}_t^p$ are the *percentage* deviations of output and potential output from their respective steady states.

[2] R is the steady state value of the nominal interest rate.

[3] As in Maih (2012) and Alstadheim et al. (2013), we use a Matlab toolbox RISE (Rationality In Switching Environments) to solve the model. An introduction to this toolbox can be found at https://github.com/jmaih/RISE_toolbox and from which it can be downloaded.

Table 1
Model priors and posteriors.

Parameter	Prior distribution			Posterior distribution			
	Distribution	Low	High	Mode	Mean	Confidence interval 5%	Confidence interval 95%
φ	Normal	1.00	7.00	7.03	7.40	5.49	9.36
σ_c	Normal	0.75	2.25	0.99	0.97	0.79	1.15
h	Beta	0.30	0.70	0.61	0.62	0.51	0.71
ξ_w	Beta	0.30	0.70	0.89	0.87	0.82	0.92
$1/\sigma_l$	Beta	0.30	0.70	0.17	0.24	0.13	0.38
ξ_p	Beta	0.30	0.70	0.91	0.91	0.87	0.94
ι_w	Beta	0.30	0.70	0.49	0.50	0.31	0.69
ι_p	Beta	0.30	0.70	0.32	0.41	0.19	0.65
ψ	Beta	0.30	0.70	0.76	0.74	0.59	0.87
Φ	Normal	1.00	1.50	1.39	1.39	1.24	1.55
$r_{\pi,s_t=1}$	Gamma	0.50	5.00	3.07	3.31	2.42	4.34
$\rho_{s_t=1}$	Beta	0.30	0.70	0.89	0.89	0.86	0.92
$r_{y,s_t=1}$	Gamma	0.05	3.00	0.25	0.26	0.17	0.37
$\bar{\pi}$	Gamma	0.43	0.83	0.66	0.74	0.61	0.88
$100(\beta^{-1}-1)$	Gamma	0.05	0.80	0.28	0.70	0.13	1.50
$\bar{l}$	Normal	-4.00	4.00	1.67	1.08	-0.05	2.19
$\bar{\gamma}$	Normal	0.20	0.60	0.46	0.46	0.42	0.49
α	Normal	0.20	0.40	0.12	0.13	0.08	0.17
σ_a	Invgamma	0.01	2.00	0.44	0.44	0.39	0.50
σ_b	Invgamma	0.01	2.00	0.05	0.06	0.05	0.07
σ_g	Invgamma	0.01	2.00	0.41	0.42	0.37	0.46
σ_I	Invgamma	0.01	2.00	0.33	0.34	0.27	0.42
$\sigma_{r,s_t=1}$	Weibull	0.01	1.00	0.10	0.11	0.09	0.12
σ_p	Invgamma	0.01	2.00	0.10	0.11	0.09	0.13
σ_w	Invgamma	0.01	2.00	0.45	0.45	0.38	0.52
ρ_a	Beta	0.20	0.80	0.93	0.93	0.87	0.97
ρ_b	Beta	0.30	0.70	0.95	0.94	0.91	0.96
ρ_g	Beta	0.20	0.80	0.98	0.98	0.96	0.99
ρ_I	Beta	0.30	0.70	0.66	0.65	0.54	0.76
ρ_p	Beta	0.30	0.70	0.66	0.60	0.41	0.76
ρ_w	Beta	0.30	0.70	0.51	0.51	0.34	0.69
μ_p	Beta	0.30	0.70	0.54	0.49	0.34	0.66
μ_w	Beta	0.30	0.70	0.50	0.49	0.33	0.65
p_{12}	Beta	0.05	0.15	0.01	0.01	0.00	0.02
p_{21}	Beta	0.05	0.15	0.17	0.18	0.14	0.23

"Low" and "High" demarcate the bounds of the 90% probability interval for the prior distribution. All parameters are defined in the Appendix.

with prior distributions. Finally, we maximize the posterior density function and simulate the posterior distribution using the Metropolis–Hastings algorithm. All reported estimation results are based on five chains of 250,000 Monte Carlo Markov Chain (MCMC) simulations. For each chain, the first 125,000 draws are discarded as burn-in data.

3. Estimation result

Model parameters are partitioned into two sets: The first set includes ten calibrated parameters, some of which are related to the zero lower bound regime. Consistent with Smets and Wouters (2007), the depreciation rate on capital, the exogenous government spending-GDP ratio, the steady-state markup in the labor market, and the curvature parameters of the Kimball aggregators in the goods and labor markets are fixed at 0.025, 0.18, 1.5, 10 and 10, respectively. The second set includes the remaining parameters that are estimated, which we report in Table 1.

Note that for the purpose of accounting for very minor fluctuations in the nominal interest rate at the zero lower bound, we assume that the policy shock follows the same Markov transition process. In the Taylor rule regime, its standard deviation is estimated by the Bayesian method; in the zero lower bound regime, we calibrate it to be 0.02 – a value that we computed using the data.

It is well-known that, since the fourth quarter of 2008, the interest rate target of the Fed has stayed at the near zero lower bound. In the model, the switch of regimes is measured by the regime probability given to each regime. When the policy switches from the Taylor rule regime to the zero interest rate regime, the probability on the Taylor rule regime goes from a very high value to near 0 quickly. Our model setup allows these probabilities to be calculated endogenously using the data. An accurate estimate is an indication that our methodology is working properly.

Fig. 2 shows regime probabilities on the Taylor rule regime in the top panel and on the zero lower bound regime in the bottom panel.[4] The shaded areas correspond to the NBER-dated recessions. The sharp rise in the probability on the zero

[4] We follow the literature and use both the Hamilton (1994) filter and the Kim (1999) smoother in estimating the regime-switching DSGE model. During estimation, observations are introduced one by one from the sample and the Hamilton (1994) filter is used to determine to which regime the newly

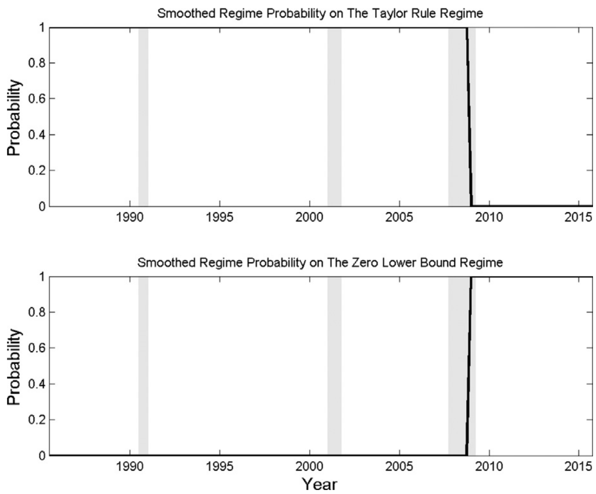

Fig. 2. Smoothed regime probabilities.

lower bound regime coincides very closely in time with the notable drop in the nominal interest rate in Fig. 1. From 2008:4 onwards, it stays at its upper limit of 1 and the policy rate fluctuates very negligibly near its lower bound.

4. Transition probabilities and the size of the multiplier

To compute the government spending multiplier, we calculate the impulse response functions of the estimated model on impact of a standard deviation of government spending shock. The relevant variables are output, consumption, investment, the inflation rate, the nominal interest rate, the real interest rate, and hours worked.

What is crucial to the size of the multiplier are the transition probabilities. As Table 1 indicates, p_{21} – the transition probability from the zero lower bound regime to the Taylor rule regime, is estimated to be about 17%. The other critical transition probability – the probability of transitioning to a zero lower bound regime when the economy is in the Taylor rule regime, p_{12}, is estimated to be 1%. This small number reflects the fact that before the last quarter of 2008, agents believed that there was almost no chance for the economy to reach the zero lower bound state. Given these estimates, the multipliers are computed and plotted in Fig. 3.

We note that the plotted multiplier is a *cumulative* multiplier: it is the area under the impulse response function of output divided by the area under the impulse response function of government spending. It shows the size of the fiscal multiplier at any given point in time. This follows the practice in the literature, such as in Ramey and Zubairy (2014). The appendix provides more mathematical details about this computation.

To measure the extent to which p_{21}, the perceived probability of transitioning from the zero lower bound regime to the Taylor rule regime, affects the magnitude of the multiplier, we also plot a counterfactual case in which it is much lower: $p_{21} = 5\%$. Fig. 4 indicates that the multiplier can be as large as more than 3. Evidently, a decrease in the value of p_{21} can substantially crowd in private consumption and investment, creating more hours and resulting in a larger multiplier. Plots of the responses of other aggregate variables also show an amplified response under the lower probability when compared with the case of a higher transition probability. Therefore, Fig. 4's message is that the size of the multiplier is directly affected by the transition probabilities. A low probability amplifies the multiplier, while a high probability does the opposite.

We emphasize that the transition matrix enters agents' information set when they make forecasts about future policy changes. Government stimulus tends to raise the inflation rate. If the nominal interest rate is at its lower bound, a rise in the inflation rate lowers the real interest rate, and consumers will have incentives to consume and invest. On the other hand, if the Taylor rule is operative, a rise in the inflation rate will eventually lead to a rise in the real interest rate due to the central bank's reaction. In this case consumers will have incentives to reduce spending and save. When agents make forecasts, they will consider the possibilities of both scenarios. The transition probabilities are the weights that agents give to each scenario when they form expectations. For example, if p_{21} is small, agents believe that it is more likely that the zero lower bound regime will continue in the future. This mechanism reinforces itself: if agents believe the real interest rate will be low, they will increase spending. But the higher spending boosts output which eventually leads to higher inflation rate and lowers the real interest rate. This is why the size of the multiplier is directly affected by the transition probabilities.

introduced data point belongs and to allocate it accordingly. The Kim (1999) smoother doesn't take part in the estimation process but is used to calculate regime probabilities using all the information in the sample, after the parameters are estimated.

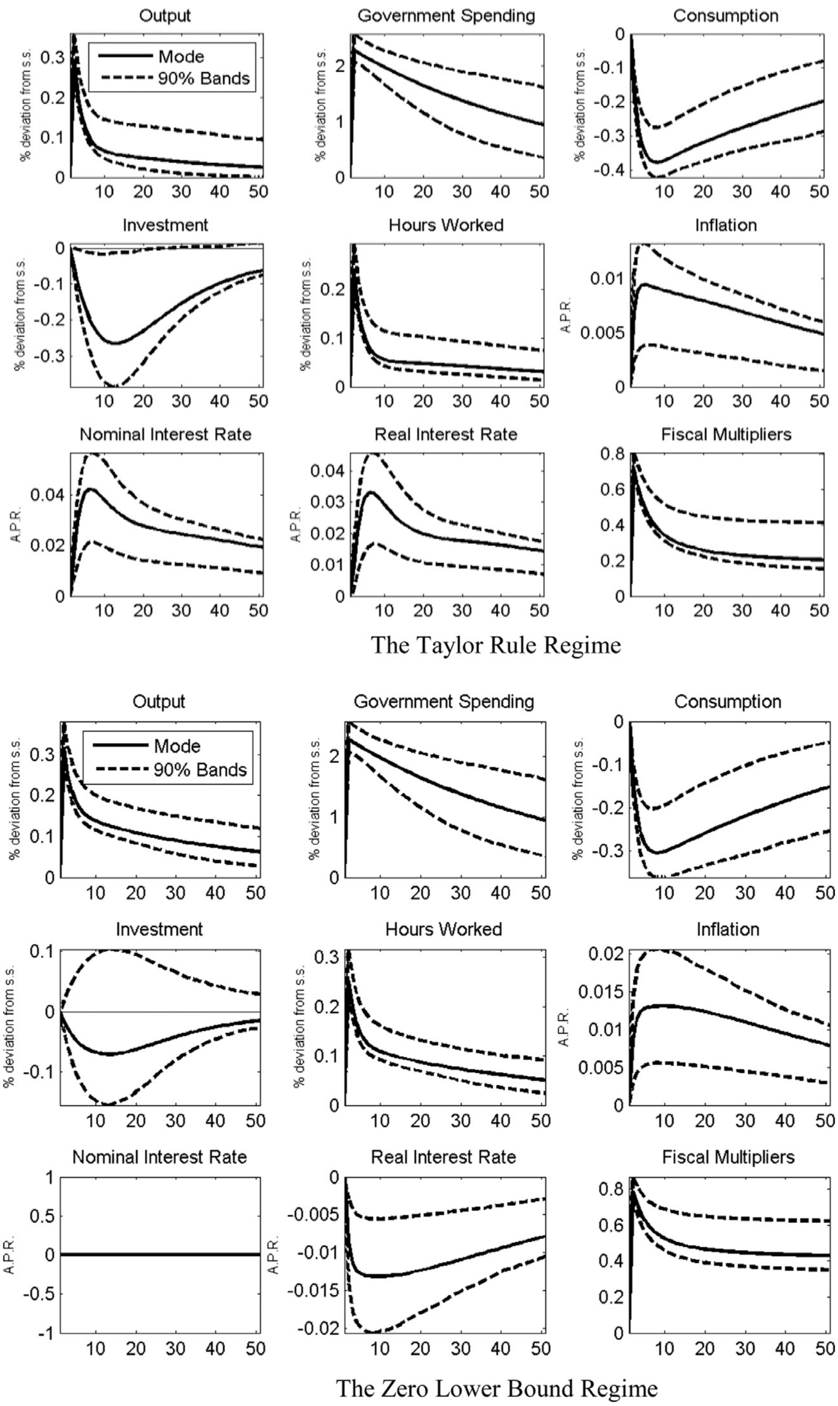

Fig. 3. Impulse responses to a standard deviation of government spending shock. All parameters are estimated from data. Solid lines indicate response functions and dashed lines indicate 5% and 95% posterior intervals. (1) The Taylor rule regime. (2) The zero lower bound regime.

One observation about Fig. 4 is that inflation in the Taylor rule regime is more volatile with the anticipation of a possible longer duration of the zero lower bound regime. This suggests that even if the economy is in the Taylor rule regime and agents believe that there is almost no possibility of it falling into the zero lower bound regime in the future (recall $p_{12} = 1\%$), the inflation cost of fiscal stimulus is large as long as agents believe that the zero lower bound regime will last very long once the economy inadvertently falls into it.

Likewise, we can also do a counterfactual experiment in which the perceived transitional probability from the Taylor rule regime to the zero lower bound regime is larger ($p_{12} = 13\%$). Fig 5 shows that other things equal, a positive government

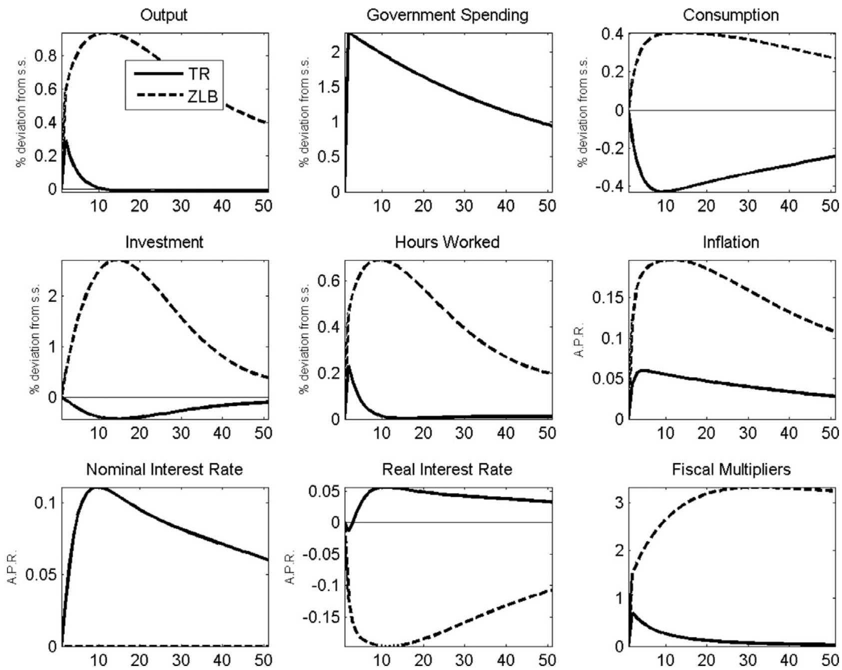

Fig. 4. Counterfactual impulse responses to a standard deviation of government spending shock. All parameters other than $p_{21} = 5\%$ are estimated from data. Solid lines indicate situations in the Taylor rule regime and dashed lines indicate situations in the zero lower bound regime.

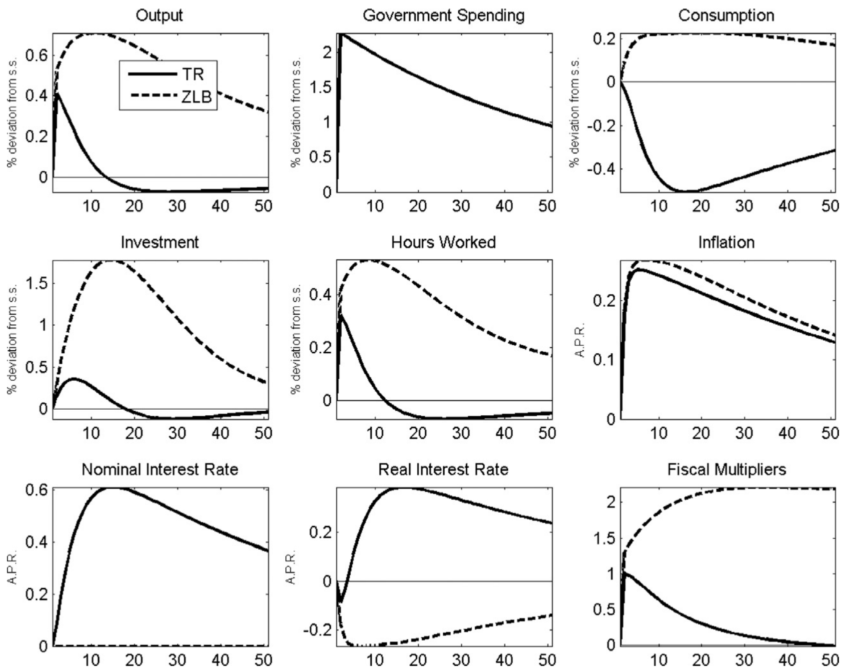

Fig. 5. Counterfactual impulse responses to a standard deviation of government spending shock. All parameters other than $p_{12} = 13\%$ are estimated from data. Solid lines indicate situations in the Taylor rule regime and dashed lines indicate situations in the zero lower bound regime.

spending shock triggers an inflation response that is higher than that in Fig 3. As expected, the multipliers are higher in both regimes.

We run numerical simulations to pin down the relationship between the probability of transitioning to the zero lower bound regime and the size of the fiscal multiplier. The two key numbers are p_{22}, the probability of staying in the zero lower bound regime while in the zero lower bound regime, and p_{12}, the probability of transitioning from the Taylor rule regime to the zero lower bound regime. We compute and compare impulse response functions while letting p_{22} vary from 0% to 95% of confidence bounds and letting p_{12} varying from 0% to 14% of confidence bounds, respectively. The result corroborates what we show in the figures: the higher the probability of the economy staying at the zero lower bound, the bigger the multiplier, and the higher the probability of the economy switching to the zero lower bound regime, the bigger the multiplier. If one converts the probability values into anticipated duration of the zero lower bound, then the result shows that the multiplier can be bigger than one only when agents expect a duration of staying in the zero lower bound regime for at least 4 years.

Finally, we run alternative simulations to test the robustness of our result. In our model setup, there are two possible states of the economy, each associated with a different policy regime; other parameters do not vary between states of the economy. In the robustness test, we relax this assumption and allow more parameters to change across regimes. The result is reported in Appendix A.5. As expected, the estimated parameter values are different, although all but one parameter (the investment adjustment cost parameter) have pretty stable values across different experiments. The main result is that peak response of the fiscal multiplier remains similar in magnitude to that in the benchmark setup, but the persistence of the responses has increased.

5. Discussion

To summarize, expected durations of regimes play an important role in the determination of the size of the multiplier. If the perceived probability of transitioning from the zero lower bound regime to the Taylor rule regime is low, then the fiscal multiplier can be large. If the perceived probability of transitioning from the Taylor rule regime to the zero lower bound regime is high, then the fiscal multiplier can be large too.

Agents use transition probabilities to form expectations. The model economy is a stationary dynamic system, and the transition probabilities are fixed parameters of the system. In other words, these probabilities are not conditional probabilities that vary when economic conditions change. The estimated p_{22} and p_{21} reflect agents' belief on average on the persistence of this regime during this period.[5] Ideally, however, one would prefer that agents' expectations vary as the economic situation changes on the ground. To proxy such a result, we estimate the model 28 times. Each time we extend the sample by one quarter to see how the probabilities vary as time moves away from the last quarter of 2008. We find that although there are variations, the estimated probabilities stay within a small range of values around the estimated posterior mode.

One could also condition the probabilities on output, inflation, and hours by using an indicator function such as that described in Davig and Leeper (2006) and Sims et al. (2008). However, three issues arise in this context. First, solving and estimating a medium-sized DSGE model with endogenous transition probabilities would be prohibitively complicated and computationally expensive. Second, it will be very difficult to effectively pin down threshold values when there is only one regime switch in the sample. Third, using an indicator function that only gives zeroes and ones cannot reconcile with the fact that in reality, regime persistence mostly falls in between these two extreme values.

This discussion is instructive nevertheless. It tells us that the actual size of the fiscal multiplier is likely larger than the estimated value obtained from the model in part of the sample period. This would happen whenever the actual expected duration of the zero lower bound regime is longer than what the the estimated transition probabilities imply.

Finally, we discuss how this paper is related to other papers that use deterministic simulations to estimate the effectiveness of government stimulus in the ZLB regime. The mechanism of deterministic simulations is to incorporate a non-linear switching process between the ZLB regime and the Taylor rule regime. The switch is triggered by a deterministic shock, and the period in which it is operative can be pre-specified. When it is operative, the nominal interest rate remains zero and after that the nominal interest rate follows the Taylor rule. One major disadvantage of this method is that the simulation periods for the ZLB regime, by construction, coincide exactly with its expected persistence. Note that it is the expected persistence that determines the magnitude of the multiplier in DSGE models. For instance, if one wants to reproduce the current situation in which the zero lower bound has bound for 28 quarters from the last quarter of 2008 to the third quarter of 2015, rational agents are forced to know this long duration in the very beginning of the simulation, i.e., at the last quarter of 2008. In our regime-switching model, the simulation periods for the ZLB regime can be very long even if the expected persistence is very short, since regime persistence is directly modeled and characterized by the estimated transition probabilities.

6. Conclusion

This paper addresses two important issues that prevent us from accurately estimating the magnitude of the government spending multiplier when the nominal interest rate is at the zero lower bound.

First, while the duration and expected duration of the zero lower bound monetary policy regime matter for the size of the multiplier, the literature does not have an effective and consistent approach that can help us pin down the duration. Second, the inherent non-linearity necessitated by the monetary policy reaction function in the zero lower bound poses practical difficulty in the estimation of DSGE models, primarily because the model is non-differentiable everywhere, and agents cannot take into account the possibility of the economy hitting the zero lower bound in the future when forming expectations.

[5] If agents did expect a longer persistence of the zero lower bound regime, such information would eventually be reflected by the estimated p_{22} and p_{21}.

This paper overcomes these two challenges by applying the Markov-switching algorithm that not only tackles the nonlinearity problem by splitting the monetary policy into the Taylor rule regime and the zero lower bound regime, but also reconciles well with the rational expectations view that agents take into account the transition probabilities between regimes when forming expectations. The expected duration of the zero lower bound can then be calculated from the estimated probabilities.

With new estimates of model parameters and the transition probabilities between regimes in hand, we compute impulse responses and the implied fiscal multiplier. The result shows a bigger multiplier in the zero lower bound regime than in normal times, although it is still less than one. Counterfactual exercises also show that the lower (higher) the probability of the economy switching to the Taylor rule (zero lower bound) regime, the bigger the multiplier.

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Appendix A

A1. Parameter definition

All parameters are defined as in Smets and Wouters (2007) when possible: ϕ stands for investment adjustment cost, σ_c inter-temporal elasticity of substitution, h habit persistence, ξ_w Calvo wage stickiness, σ_l elasticity of labor supply with respect to real wages, ξ_p Calvo price stickiness, ι_w labor market indexation, ι_p goods market indexation, ψ capital utilization, Φ fixed cost, $r_{\pi,s_t=1}$ policy weight on inflation for the Taylor rule regime, $\rho_{s_t=1}$ policy smoothing parameter for the Taylor rule regime, $r_{y,s_t=1}$ policy weight on long-run output gap for the Taylor rule regime, $\bar{\pi}$ quarterly steady-state inflation rate, β discount factor, $\bar{l}$ steady-state of labor, $\bar{r}$ quarterly trend growth rate to GDP, α labor share in production, p_{12} transition probability for switching from the Taylor rule regime to the zero lower bound regime, p_{21} transition probability for switching from the zero lower bound regime to the Taylor rule regime. σ_a & ρ_a, σ_b & ρ_b, σ_g & ρ_g, σ_I & ρ_I, $\sigma_{r,s_t=1}$, σ_p & ρ_p, and σ_w & ρ_w represent auto-correlations and standard deviations of technology, net worth, government spending, investment-specific technology, monetary policy for the Taylor rule regime, price markup and wage markup shocks, respectively. μ_p and μ_w are the MA coefficients for price and wage markup shocks, respectively.

A2. Model solution

Agents in our model take account of all possible future changes in regimes when forming expectations, which in turn affects their current decision-makings. The solution in each regime depends on solutions for all other regimes and our model is linear in variables only when conditional on regimes. Therefore, traditional solution methods such as gensys proposed by Sims (2002) for linear rational expectations model cannot be used here. Our model is solved by the RISE toolbox, the solution method of which is based on Maih (2012) and Farmer et al. (2011). Agents in our model are assumed to know the transition probabilities between regimes and use them to form expectations. When forming expectations, agents consider both the current regime and possible regime changes in the future. To any variable x, its expected value for the next period x_{t+1} is calculated by taking weighted average of all its expected values across regimes with the transition probabilities taken as weights. More specifically, the following expectations rule is used:

$$E_t(x_{t+1}|s_t=1) = p_{11}E_t x_{1,t+1} + p_{12}E_t x_{2,t+1}, \tag{A1}$$

where $E_t(x_{t+1}|s_t=1)$ is the time t's expectations of variable x_{t+1}, when the economy is in regime 1 at time t, $E_t x_{i,t+1}$ (for i=1 or 2) is the time t's expectations of variable x_{t+1} assuming the economy to be in regime i at time $t+1$, and p_{1i} is the transition probability of the economy going from regime 1 at time t to regime i at time $t+1$.

The 14 × 1 state vector $\mathbf{S}_t$ includes all predetermined and non-predetermined endogenous variables, which is defined as:

$$\mathbf{S}_t = [\hat{y}_t, \hat{c}_t, \hat{i}_t, \hat{q}_t, \hat{k}^s_t, \hat{k}_t, \hat{z}_t, \hat{r}^k_t, \hat{\mu}^p_t, \hat{\pi}_t, \hat{\mu}^w_t, \hat{w}_t, \hat{l}_t, \hat{r}_t]'.$$

The vector $\boldsymbol{\varepsilon}_t$ includes all structural shocks, which is defined as:

$$\varepsilon_t = \left[\hat{\varepsilon}^a_t, \hat{\varepsilon}^i_t, \hat{\varepsilon}^b_t, \hat{\varepsilon}^g_t, \hat{\varepsilon}^p_t, \hat{\varepsilon}^w_t, \hat{\varepsilon}^r_t\right]'.$$

The linearized dynamic system of our regime-switching DSGE model can be rewritten in a compact state-space form:

$$E_t\left\{\mathbf{C}^+_{s_{t+1}}\mathbf{S}_{t+1}(\cdot, s_t) + \mathbf{C}^0_{s_t}\mathbf{S}_t(s_t, s_{t-1}) + \mathbf{C}^-_{s_{t-1}}\mathbf{S}_{t-1}(s_{t-1}, s_{t-2}) + \mathbf{C}_{\varepsilon_t}\varepsilon_t\right\} = \mathbf{0}, \tag{A2}$$

where s_t follows a two-regime Markov chain, with matrix $\mathbf{P} = [p_{s_t,s_{t+1}}] = \left[\begin{smallmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{smallmatrix}\right]$ summarizing the transition probabilities between regimes. The rows of matrix $\mathbf{P}$ sum to one. $p_{s_t,s_{t+1}} = \text{Prob}(s_{t+1}|s_t)$ denotes the probability of the economy going from the current regime s_t to regime s_{t+1} in the next period. The dot in $\mathbf{S}_{t+1}(\cdot, s_t)$ assumes that agents in the current regime

s_t are uncertain about the regime that will prevail at time $t+1$ and have to use the expectations rule (A1) to forecast state variables. Regime-switching in volatilities is not assumed here, so $\mathbf{C}_{\varepsilon_t}$ remains constant as the economy moves across regimes.

The model solution results in the following regime-switching vector auto-regression:

$$\mathbf{S}_t = \boldsymbol{\Gamma}\left(s_t, \boldsymbol{\theta}^p, \mathbf{P}\right)\mathbf{S}_{t-1} + \mathbf{R}\left(s_t, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}\right)\varepsilon_t. \tag{A3}$$

The law of motion of the model economy (A3) depends on the structural parameters $\boldsymbol{\theta}^p$, the stochastic volatilities $\boldsymbol{\theta}^v$, the regime that prevails at time t s_t and the transition probabilities between regimes $\mathbf{P}$, meaning that agents' expectations on possible future regime changes matter for the dynamics that govern the model economy. Traditional stability concepts do not apply here. Following Svensson and Williams (2007) and Farmer et al. (2011), the stability of model solution is conducted by applying the concept of Mean Square Stability (MSS).

A3. Model estimation

The transition probability enters into the law of motion that governs the model economy (A3) and transforms our regime-switching DSGE model into a constant-parameter linear rational expectations model. If the transition probability is time-varying, our model is not linear in variables anymore, even conditional on regimes, which leads to model solution unfeasible by the currently available methods. Furthermore, we will have to deal with two more unknown variables (since the rows of the transition matrix sum to one the four transition probability variables reduce to two) and the number of unknown variables will be greater than that of model structural equations. There are no two more equations with economic meanings that can link the transition probability variables with state variables, so model parameters cannot be identified. The existing literature suggests that the regime could be time-varying but the transition probability cannot. We can use sample data to estimate the transition probabilities like any other model parameters.

In estimation, the measurement equations integrate the time t's state variables of each regime over the regime probabilities based on the time $t-1$'s information when connecting with the time t's observation. Mathematically, let $\mathbf{y}_t$ be a 7×1 vector including 7 observables:

$$\mathbf{y}_t = [dlGDP_t, dlCONS_t, dlINV_t, dlWAG_t, lHOURS_t, dlP_t, FEDFUNDS_t]',$$

let $\mathbf{a}$ be the corresponding steady-state vector:

$$\mathbf{a} = \left[\bar{\gamma}, \bar{\gamma}, \bar{\gamma}, \bar{\gamma}, \bar{l}, \bar{\pi}, \bar{r}\right]',$$

let $\mathbf{Y}_t = (\mathbf{y}_1, \ldots, \mathbf{y}_t)$ with $\mathbf{Y}_0$ taken as given, and let, $\mathbf{Q}_t = (s_0, \ldots s_t)$. The measurement equation can then be written in the following form:

$$\mathbf{y}_t = \mathbf{a} + \sum_{i=1}^{2} p\left(s_t = i | \mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}\right) \cdot \mathbf{H} \cdot \mathbf{S}_t, \tag{A4}$$

where matrix $\mathbf{H}$ maps the law of motion that governs the model economy (A3) into the seven observables.

The time t's regime probabilities based on the time $t-1$'s information $p(s_t = i|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P})$ are calculated by integrating the time $t-1$'s regime probabilities up to time $t-1$ information $p(s_{t-1} = i|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P})$ over the transition probabilities. More specifically, the following condition holds:

$$p\left(s_t = i | \mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}\right) = p_{1i} \cdot p\left(s_{t-1} = 1 | \mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}\right) + p_{2i} \cdot p\left(s_{t-1} = 2 | \mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}\right), \tag{A5}$$

where $p(s_{t-1} = i|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P})$ is recursively updated based on the Bayes' rule (we will be more specific on this later), and $p_{1i} = p(s_t = 1|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}, \mathbf{Q}_{t-1})$ and $p_{2i} = p(s_t = 2|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}, \mathbf{Q}_{t-1})$ are the transition probabilities that are assumed to be constant over time.

The probability of each regime at each point in time $p(s_t|\mathbf{Y}_t, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P})$ can be calculated and recursively updated by the Bayes' rule based on information about new observation $\mathbf{y}_t$, the calculated probability of each regime in the last period $p(s_t|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P})$ by (A5), and the model structure $f(\mathbf{y}_t|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}, s_t)$. The Bayes' rule is given as follows:

$$p\left(s_t = i | \mathbf{Y}_t, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}\right) = \frac{f\left(\mathbf{y}_t | \mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}, s_t = i\right) \cdot p\left(s_t = i | \mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}\right)}{\sum_{k=1}^{2} f\left(\mathbf{y}_t | \mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}, s_t = k\right) \cdot p\left(s_t = k | \mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}\right)}, \tag{A6}$$

where $f(\mathbf{y}_t|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^v, \mathbf{P}, s_t)$ is the time t's conditional likelihood function. This computation procedure is usually referred to as the Hamilton (1994) filter, and we can use it to plot the historical regime probabilities over the entire sample such as those in Fig 2.

The traditional Kalman filter is inappropriate here, since it cannot distinguish between regimes. To construct likelihood function, the non-standard Kalman filter that was proposed by Kim and Nelson (1999) is usually adopted, given the law of motion that governs the model economy (A3) and the measurement Eq. (A4). The filter in the RISE toolbox is based on Sims et al. (2008) which further improves the Kim and Nelson (1999) method. These two filtering procedures usually give essentially the same numerical results.

The likelihood function is constructed by integrating the conditional likelihood function at time t, $f(\mathbf{y}_t|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^\nu, \mathbf{P}, s_t)$, over the time t's regime probabilities based on the time $t-1$'s information $p(s_t = i|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^\nu, \mathbf{P})$.

$$L\left(\mathbf{Y}_T|\boldsymbol{\theta}^p, \boldsymbol{\theta}^\nu, \mathbf{P}\right) = \prod_{t=1}^{T}\left[\sum_{i=1}^{2} f\left(\mathbf{y}_t|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^\nu, \mathbf{P}, s_t = i\right)\cdot p\left(s_t = i|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^\nu, \mathbf{P}\right)\right]. \tag{A7}$$

We can use the Bayes' rule (A6) to recursively update the time $t-1$'s regime probabilities $p(s_{t-1}|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^\nu, \mathbf{P})$, use (A5) to calculate the time t's regime probabilities based on the time $t-1$'s information $p(s_t = i|\mathbf{Y}_{t-1}, \boldsymbol{\theta}^p, \boldsymbol{\theta}^\nu, \mathbf{P})$, and then recursively evaluate the above regime-dependent likelihood function (A7).

By combining the likelihood function (A7) with prior information, we can form posterior distribution. To maximize the posterior distribution and reach its global peak, we make use of both the derivative method and derivative-free stochastic grid search algorithm in the RISE toolbox. Until both methods converge to the same result, we stop searching and use the mode to initiate the Markov chain Monte Carlo (MCMC) procedure aiming to construct the whole posterior distribution.

A4. The multiplier

The multipliers in Fig 3 are cumulative multipliers that are calculated as follows. All variables in the linear model are expressed as percentage deviations from the steady state values. To compute cumulative multipliers some basic transformation is needed. In the model, we have $\hat{y}_t = (\ln Y_t - \ln Y)\times 100$ and $\hat{g}_t = \frac{G}{Y}(\ln G_t - \ln G)\times 100$, where $\hat{y}_t$ and $\hat{g}_t$ are the percentage deviations of output and government spending from their respective steady states. From these two equations, we can solve for their levels Y_t and G_t, which are $Y_t = e^{\frac{\hat{y}_t}{100}}Y$ and $G_t = e^{\frac{\hat{g}_t}{\frac{G}{Y}\cdot 100}}G$. The effect of the zero lower bound on output when there is no shock at all is $Y_{ZLB,t} = e^{\frac{\hat{y}_{ZLB,t}}{100}}Y$. So the difference between the path for Y_t and G_t under a government spending shock and under no shock at all is expressed as follows: $\Delta Y_t = Y_t - Y_{ZLB,t} = (e^{\frac{\hat{y}_t}{100}} - e^{\frac{\hat{y}_{ZLB,t}}{100}})Y$ and $\Delta G_t = G_t - G = (e^{\frac{\hat{g}_t}{\frac{G}{Y}\cdot 100}} - 1)G$. As in Ramey and Zubairy (2014), cumulative multipliers are constructed as $\frac{\sum_{i=1}^{M}\Delta Y_i}{\sum_{i=1}^{M}\Delta G_i} = \frac{\sum_{i=1}^{M}(e^{\frac{\hat{y}_i}{100}} - e^{\frac{\hat{y}_{ZLB,i}}{100}})}{\sum_{i=1}^{M}(e^{\frac{\hat{g}_i}{\frac{G}{Y}\cdot 100}} - 1)}\cdot\frac{Y}{G}$. We vary M from 2 to 51 quarters.

A5. Robustness test

The parameters of the model can be categorized into three groups. One group determines the volatility of structural shocks, one group the steady state of the model economy, and one group the price and wage rigidities. We need not change group 2, since there is a unique steady state. We did two separate experiments. In the first, we allow group 1 to vary across regimes, and in the second, we allow both group 1 and 3 to vary. The goal is to see if this affects the paper's main conclusion. Introducing more changing parameters makes the already non-Gaussian posterior distribution more non-Gaussian and imposes a fairly large burden on computation. Our optimization strategy is to use a derivative-free stochastic grid search algorithm to search for the global peak. Estimated results are shown in the following two tables:

The first table is computed based on the first experiment.

Prior distribution				Posterior distribution			
Parameter	Distribution	Low	High	Mode	Mean	Confidence interval 5%	Confidence interval 95%
$\varphi_{s_t=1}$	Normal	1.00	7.00	7.53	7.95	5.84	10.22
$\varphi_{s_t=2}$	Normal	1.00	7.00	3.76	4.03	2.53	6.03
σ_c	Normal	0.75	2.25	0.91	0.91	0.78	1.03
h	Beta	0.30	0.70	0.64	0.65	0.58	0.72
$\xi_{w,s_t=1}$	Beta	0.30	0.70	0.78	0.79	0.67	0.88
$\xi_{w,s_t=2}$	Beta	0.30	0.70	0.78	0.78	0.69	0.86
$1/\sigma_l$	Beta	0.30	0.70	0.31	0.36	0.20	0.51
$\xi_{p,s_t=1}$	Beta	0.30	0.70	0.84	0.84	0.75	0.89
$\xi_{p,s_t=2}$	Beta	0.30	0.70	0.88	0.87	0.80	0.93
$\iota_{w,s_t=1}$	Beta	0.30	0.70	0.49	0.49	0.32	0.67
$\iota_{w,s_t=2}$	Beta	0.30	0.70	0.52	0.52	0.34	0.72
$\iota_{p,s_t=1}$	Beta	0.30	0.70	0.30	0.37	0.21	0.53
$\iota_{p,s_t=2}$	Beta	0.30	0.70	0.49	0.52	0.30	0.76
$\psi_{s_t=1}$	Beta	0.30	0.70	0.66	0.65	0.53	0.77
$\psi_{s_t=2}$	Beta	0.30	0.70	0.72	0.69	0.56	0.83
Φ	Normal	1.00	1.50	1.41	1.42	1.27	1.58
$r_{\pi,s_t=1}$	Gamma	0.50	5.00	3.77	3.63	2.86	4.57

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Prior distribution				Posterior distribution			
Parameter	Distribution	Low	High	Mode	Mean	Confidence interval 5%	Confidence interval 95%
$\rho_{s_t=1}$	Beta	0.30	0.70	0.89	0.89	0.87	0.92
$r_{y,s_t=1}$	Gamma	0.05	3.00	0.22	0.21	0.11	0.33
$\bar{\pi}$	Gamma	0.43	0.83	0.71	0.76	0.66	0.85
$100(\beta^{-1}-1)$	Gamma	0.05	0.80	0.38	0.57	0.11	1.18
$\bar{l}$	Normal	-4.00	4.00	1.05	0.69	-0.15	1.59
$\bar{\gamma}$	Normal	0.20	0.60	0.46	0.45	0.39	0.49
α	Normal	0.20	0.40	0.11	0.11	0.08	0.15
σ_a	Invgamma	0.01	2.00	0.45	0.46	0.40	0.52
σ_b	Invgamma	0.01	2.00	0.06	0.06	0.05	0.08
σ_g	Invgamma	0.01	2.00	0.41	0.42	0.38	0.46
σ_I	Invgamma	0.01	2.00	0.34	0.35	0.28	0.42
$\sigma_{r,s_t=1}$	Weibull	0.01	1.00	0.10	0.11	0.09	0.12
σ_p	Invgamma	0.01	2.00	0.09	0.10	0.08	0.12
σ_w	Invgamma	0.01	2.00	0.41	0.44	0.37	0.51
ρ_a	Beta	0.20	0.80	0.93	0.94	0.89	0.97
ρ_b	Beta	0.30	0.70	0.92	0.92	0.88	0.94
ρ_g	Beta	0.20	0.80	0.98	0.97	0.95	0.99
ρ_I	Beta	0.30	0.70	0.64	0.64	0.55	0.75
ρ_p	Beta	0.30	0.70	0.70	0.73	0.57	0.86
ρ_w	Beta	0.30	0.70	0.72	0.62	0.39	0.85
μ_p	Beta	0.30	0.70	0.53	0.57	0.41	0.74
μ_w	Beta	0.30	0.70	0.63	0.55	0.37	0.76
p_{12}	Beta	0.05	0.15	0.01	0.02	0.00	0.02
p_{21}	Beta	0.05	0.15	0.18	0.18	0.14	0.23

The second table is computed based on the second experiment.

Prior distribution				Posterior distribution			
Parameter	Distribution	Low	High	Mode	Mean	Confidence interval 5%	Confidence interval 95%
$\varphi_{s_t=1}$	Normal	1.00	7.00	7.43	8.09	6.72	10.43
$\varphi_{s_t=2}$	Normal	1.00	7.00	2.89	3.87	2.18	5.03
σ_c	Normal	0.75	2.25	0.84	0.78	0.72	0.83
h	Beta	0.30	0.70	0.62	0.64	0.59	0.71
$\xi_{w,s_t=1}$	Beta	0.30	0.70	0.51	0.66	0.54	0.79
$\xi_{w,s_t=2}$	Beta	0.30	0.70	0.56	0.58	0.41	0.75
$1/\sigma_l$	Beta	0.30	0.70	0.46	0.37	0.27	0.54
$\xi_{p,s_t=1}$	Beta	0.30	0.70	0.77	0.81	0.75	0.86
$\xi_{p,s_t=2}$	Beta	0.30	0.70	0.81	0.82	0.70	0.89
$\iota_{w,s_t=1}$	Beta	0.30	0.70	0.45	0.46	0.29	0.64
$\iota_{w,s_t=2}$	Beta	0.30	0.70	0.54	0.61	0.46	0.78
$\iota_{p,s_t=1}$	Beta	0.30	0.70	0.33	0.38	0.22	0.51
$\iota_{p,s_t=2}$	Beta	0.30	0.70	0.51	0.49	0.39	0.59
$\psi_{s_t=1}$	Beta	0.30	0.70	0.66	0.64	0.56	0.72
$\psi_{s_t=2}$	Beta	0.30	0.70	0.72	0.69	0.61	0.77
Φ	Normal	1.00	1.50	1.49	1.46	1.29	1.64
$r_{\pi,s_t=1}$	Gamma	0.50	5.00	4.53	3.92	3.16	4.48
$\rho_{s_t=1}$	Beta	0.30	0.70	0.89	0.89	0.87	0.92
$r_{y,s_t=1}$	Gamma	0.05	3.00	0.10	0.14	0.06	0.20
$\bar{\pi}$	Gamma	0.43	0.83	0.76	0.82	0.72	0.91
$100(\beta^{-1}-1)$	Gamma	0.05	0.80	0.39	0.67	0.21	1.14
$\bar{l}$	Normal	-4.00	4.00	-0.12	0.12	-0.73	0.79
$\bar{\gamma}$	Normal	0.20	0.60	0.45	0.43	0.40	0.46
α	Normal	0.20	0.40	0.12	0.14	0.11	0.16
$\sigma_{a,s_t=1}$	Invgamma	0.01	2.00	0.39	0.39	0.33	0.46
$\sigma_{a,s_t=2}$	Invgamma	0.01	2.00	0.55	0.58	0.46	0.73
$\sigma_{b,s_t=1}$	Invgamma	0.01	2.00	0.05	0.06	0.05	0.07
$\sigma_{b,s_t=2}$	Invgamma	0.01	2.00	0.03	0.03	0.02	0.04
$\sigma_{g,s_t=1}$	Invgamma	0.01	2.00	0.40	0.41	0.39	0.45
$\sigma_{g,s_t=2}$	Invgamma	0.01	2.00	0.42	0.47	0.41	0.53
$\sigma_{I,s_t=1}$	Invgamma	0.01	2.00	0.32	0.36	0.31	0.42
$\sigma_{I,s_t=2}$	Invgamma	0.01	2.00	0.41	0.45	0.39	0.49

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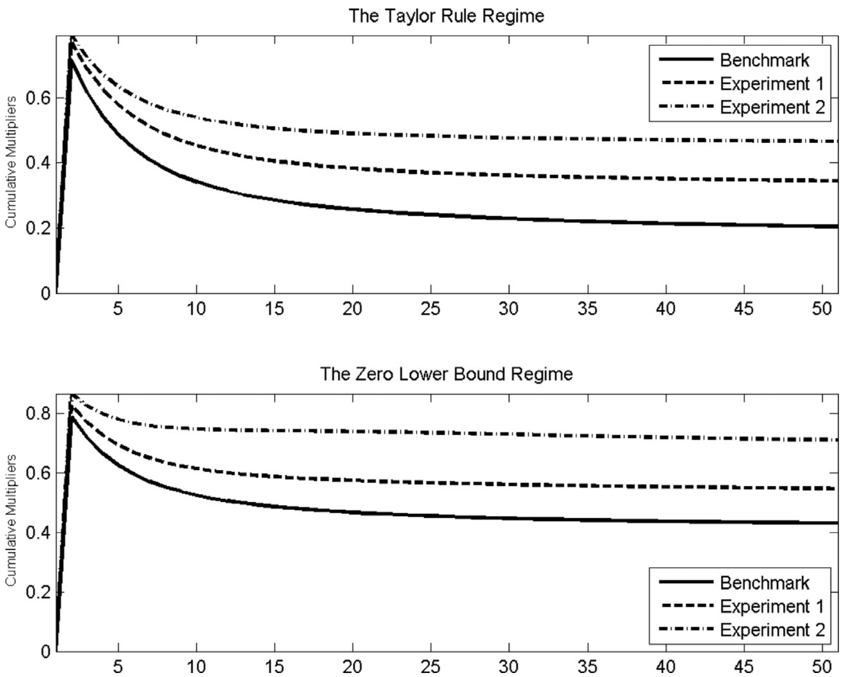
Prior distribution				Posterior distribution			
Parameter	Distribution	Low	High	Mode	Mean	Confidence interval 5%	Confidence interval 95%
$\sigma_{r,s_t=1}$	Weibull	0.01	1.00	0.13	0.12	0.11	0.13
$\sigma_{p,s_t=1}$	Invgamma	0.01	2.00	0.09	0.09	0.08	0.11
$\sigma_{p,s_t=2}$	Invgamma	0.01	2.00	0.12	0.13	0.11	0.15
$\sigma_{w,s_t=1}$	Invgamma	0.01	2.00	0.37	0.34	0.28	0.39
$\sigma_{w,s_t=2}$	Invgamma	0.01	2.00	0.60	0.73	0.62	0.85
ρ_a	Beta	0.20	0.80	0.92	0.92	0.89	0.96
ρ_b	Beta	0.30	0.70	0.95	0.94	0.93	0.96
ρ_g	Beta	0.20	0.80	0.98	0.97	0.96	0.99
ρ_I	Beta	0.30	0.70	0.62	0.57	0.46	0.64
ρ_p	Beta	0.30	0.70	0.83	0.77	0.67	0.90
ρ_w	Beta	0.30	0.70	0.80	0.85	0.72	0.95
μ_p	Beta	0.30	0.70	0.70	0.63	0.55	0.79
μ_w	Beta	0.30	0.70	0.70	0.63	0.56	0.69
p_{12}	Beta	0.05	0.15	0.02	0.01	0.00	0.02
p_{21}	Beta	0.05	0.15	0.19	0.17	0.14	0.21

The following figure shows the computed multiplier in both regimes. The peak response is similar to the benchmark case that we reported in the paper, but the persistence is higher in both experiments.



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