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Estimation of Operational Macromodels at the Zero Lower Bound*

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Abstract

We present and apply estimation techniques which can be used to estimate medium- and large-scale macromodels with forward-looking expectations at the zero lower bound (ZLB), and illustrate in detail the implications of a ZLB-episode in the observed sample in the Smets and Wouters (2007) and the Galí, Smets and Wouters (2011) models. We compare the merits of estimation methods in which the expected duration of the ZLB incident is modelled as endogenous and consistent with the policy rule forecast with Regime-Switching methods for which the **expected** ZLB duration is constant. Using the estimated models, we discuss the extent to which the ZLB impacts filtered shocks, impulse response funtions, and forecasts during the crisis. Moreover, we use the estimated models and shocks to assess the aggregate costs of the ZLB. Finally, we examine if the fit of the model is improved by allowing for breaks in policy rule coefficients, the influence of financial frictions, and the long-term equilibrium real rate since the start of the great recession.

JEL Classification: E52, E58

Keywords: Monetary policy, DSGE Models, Regime-Switching, Sigma Filter, Great Recession.

*The views, analysis, and conclusions in this paper are solely the responsibility of the authors and do not necessarily agree with the IMF, Norges Bank or the National Bank of Belgium, or those of any other person associated with these institutions.

Contents

1. Introduction

In this paper, we present and apply estimation techniques which can be used to estimate medium- and large-scale macro models with forward-looking expectations at the zero lower bound (ZLB). The recent Great Recession in United States and other advanced economies has had widespread implications for economic policy and economic performance, and triggered leading central banks as the Federal Reserve, European Central Bank, and the Bank of England to cut policy rates to zero or near zero. The crisis also led to historically low nominal interest rates and elevated unemployment levels in its aftermath.

The fact that the intensification of the crisis in the fall of 2008 became much deeper than central banks predicted and that the subsequent recovery was much slower, has raised many questions about the design of macroeconomic models at use in these institutions. One of these is the importance of accounting for the zero lower bound, and there is a rapidly expanding literature on assessing the empirical gains of explicit treatment of the ZLB episode. Some recent papers by Kulish, Morley and Robinson [77], Binning and Maih [16], Guerrieri and Iacoviello [69], Gust, Herbst, Lopez-Salido and Smith [70], and Richter and Throckmorton [88] argue that imposing and interest lower bound is key to understand the dynamics of quantities and prices during the recent great recession. Fratto and Uhlig [57], on the other hand, argues that the ZLB is largely irrelevant to understand the behaviour of the U.S. economy in the workhorse Smets and Wouters [92] model. The aim of this paper is to contribute to this literature.

To this end, we analyse the performance of benchmark macroeconomic models during the Great Recession. The specific models we use – variants of the well-known Smets and Wouters [92] model and the Galí, Smets and Wouters [64] model with unemployment – shares many features with the models currently used by central banks. To enhance the consistency between the policy rule based expectation of the ZLB duration and the market expectations, we include the 2-year Treasury yield as observables. Brave et al. [19] and Del Negro et al. [46] also use 2-year yields to discipline policy shocks during the crisis but do not consider the impact of a lower interest bound. Following the prescriptions of the beneficial effects of a "lower for longer" policy from Reifschneider and Williams [87] and Eggertsson and Woodford [49], we assume that the central bank in its policy rule smooth's over the shadow rate, i.e. the interest rate prescribed by the interest rate rule unconstrained by the lower bound, instead of the actual rate. Wu and Xia [95] argues that the use of the shadow rate captures the impact of unconventional monetary policy behaviour (forward guidance and long term asset purchases). Moreover, Wu and Zhang [96] claim that the Fed responding according to the shadow rate implies that no changes in propagation of demand and supply shocks (e.g. changes in fiscal and productivity) occurred during the recession and its aftermath. By assuming that the Fed smooths over the shadow rate in the monetary policy rule while being constrained by the ZLB, we can assess the validity of this claim.

More generally, by estimating and performing detailed posterior predictive analysis with and without imposing the interest lower bound on quarterly U.S. data 1965Q1-2016Q4, we illustrate the implications of a ZLB-episode in the observed sample. It is instructive to use both models as one of them is estimated using hours worked per capita as observable (SW model) whereas the other (GSW model) uses employment per capita and the unemployment rate as observables. Accordingly, the latter implies that the output gap is closed today whereas the former implies that the output gap remains sizably negative (since hours worked per capita is still notably below its post-war mean).

To impose the zero lower bound (ZLB henceforth) when estimating the model over the full sample, we use two basic alternative methods. First, we draw on Linde, Smets and Wouters ([82], LSW henceforth) and implement the ZLB as a binding constraint on the policy rule with an expected duration that is determined endogenously by the policy rule conditional on the state of the economy in each period. To account for future shock uncertainty, we use the Sigma point filter advocated by Binning and Maih [15] when imposing the ZLB in the estimations. The Sigma filter provides a numerically efficient method to approximate the time-varying and asymmetric forecast uncertainty by evaluating the forecast for a minimal number of sigma points at a one period ahead horizon.[1] Second, we use regime-switching methods, see e.g. Farmer, Waggoner and Zha [53], Davig and Leeper [38] and Maih [85], to impose the ZLB. At the ZLB regime this approach embodies a constant policy rule $R_t = 0$, and there is a constant probability each period that the economy will snap back to the normal state in which policy follows a policy rule which satisfies the Taylor principle.[2] Consequently, under this approach, the expected ZLB duration is exogenous to the state of the economy, which may appear as a very restrictive assumption. At the same time, the probability from switching from the ZLB to the normal regime is estimated *off* the

[1] Thus, our method used here differs in two important respects with that used by LSW. First, it takes into account the changing covariance structure of shocks at the ZLB. In LSW there was a treatment of the covariance matrix but it was simplistic: it assumed that the anticipated policy shocks used to impose the zero lower bound followed the same distribution as unanticipated policy shocks. A second difference is that we assume the central bank is smoothing over the shadow lagged interest rate in the policy rule: this makes the model less vulnerable to deflationary dynamics by assuming a "lower for longer policy" the zero lower bound.

[2] As shown by Davig and Leeper [38], the ZLB regime is still stationary provided that the probability is sufficiently high of snapping back to the regime where the Taylor principle is satisfied.

data. This feature will ensure a relatively well fitting model in our one ZLB episode sample. And from an empirical viewpoint, we believe it is interesting to contrast *endogenous* and *exogenous* ZLB duration approaches as the former implies that the impact of both demand and supply shocks will be an increasing function of the expected liquidity trap duration whereas the empirical model with exogenous duration will only feature a discrete shift in impulses (normal times vs ZLB).

Importantly, both models are log-linearized prior to subjecting them to the data, so the only non-linearity we introduce is the ZLB constraint. This facilitates for direct comparison of our estimation results to the voluminous literature on estimated linearized DSGE models, for instance the recent work by Fratto and Uhlig [57]. In addition, it allows us to evaluate "separately" the non-linear effects of the ZLB constraint in an otherwise standard linear macro-model

As noted earlier, our paper contributes to the growing literature aiming at investigating the influence of the ZLB on estimated structural macro models. Many of these papers argue convincing that the impact of non-linearities can be substantial, but often do to tease out the partial effect of the interest lower bound. Notably, Gust, Herbst, Lopez-Salido and Smith [70] show how to estimate a fully non-linear medium-scale DSGE model subject to the ZLB using Bayesian methods. While their paper represents a remarkable methodological achievement, their approach cannot readily be applied to large-scale models with many state variables and observables. Another important paper in this literature is Guerrieri and Iacoviello [69], who estimates a nonlinear DSGE model with housing allowing for both borrowing and collateral constraints. In their framework with two occasionally binding constraints, Guerrieri and Iacoviello [69] argues that a linearized model which neglects the fact that the collateral constraint is occasionally binding and only imposed nonlinearities stemming from the ZLB cannot capture the dynamics of the underlying non-linear model. We only deal with one nonlinear constraint, the non-negativity constraint on the nominal policy rate, so it is therefore not clear to what extent this finding applies to our approach. The paper by Richter and Throckmorton [88] cast some light on this, by comparing the estimation outcomes of their fully nonlinear model at the ZLB with estimation results for a variant which is linearized apart from the non-negativity constraint on the nominal interest rate. They argue that the nonlinear model performs better than the linearized ZLB model, but that the model which imposes the ZLB performs much better than a model which neglects the ZLB. But their analysis is confined to a small scale models with only 3-4 observable variables. Relative to these papers, our contribution is to examine if imposing the non-negativity constraint on the policy rate improves the log marginal data density and have economically significant effects relative to a standard linearized model in a framework with many observables and shocks. But clearly, an important limitation of our work is that we cannot assess the impact of estimating the fully non-linear models and fully allowing for future shock uncertainty. Even so, we believe that our work is first step in assessing the benefits of doing so: if the empirical gains from imposing the ZLB in an otherwise linearized model are small, the gains from a fully nonlinear approach are likely to be modest as well, as suggested by the work of Richter and Throckmorton [88].

Our key findings are as follows. First, unlike the findings in LSW [82], our best fitting models that take the ZLB explicitly into account improve considerably the marginal likelihood compared to models that ignore the ZLB. Even so, accounting for time-varying shock volatility and influence of financial frictions appears more important from a likelihood perspective as the improvement in posterior odds when considering these frictions are large compared to the gains when accounting for the ZLB. For instance, in our benchmark model the log marginal likelihood improves by 35 when accounting for the ZLB. In LSW, the gain in log marginal likelihood from accounting for time-varying shock uncertainty and financial frictions is over 100.

Second, our estimated models suggest that the impulse response function of various fundamental shocks changed considerably during the recent recession. This time-varying propagation mechanism affects the predictive densities and the covariance matrix of the forecast errors. For instance, an increase in the bond risk-premium shocks are much more contractionary when the ZLB binds than in normal times.

Third, the estimated endogenous duration of the ZLB based on the shadow policy rule moves the implied yield curve down in the direction of the financial market expectations. However, this goes with a cost as it further contribute to an overestimation of the speed of the economic recovery. This observation suggests that the model is missing a persistent decline in the natural real rate, perhaps resulting either from a decline in the fundamental real growth rate or from financial frictions that either deliver persistent negative effects from deleveraging on real demand and supply or an increased desire for safe and liquid assets that raises the spread between the risk free rate and the marginal return on capital. The results from the RS-approach provide a simple approach that confirms this last interpretation. Fourth, our models imply a high degree of nominal stickiness or real rigidities that moderate the response of prices and wages to low output gaps: their role is however highly dependent on model specification (output gap and policy rule) and measurement issues (labor supply and wage volatility).

The rest of the paper is structured as follows. Section 2 presents the prototype model – the estimated model of Smets and Wouters [91]. This model shares many features of models in use by central banks. It also discuss how the model can be tweaked to introduce unemployment following the approach of Gali, Smets and Wouters [64]. Next, we discuss the data, estimation methodology and estimation outcomes without imposing the ZLB. In Section 4, we briefly describe our ZLB estimation procedures and report ZLB estimation results. In Section 5 we perform the posterior predictive analysis, we compare the properties of the model estimated with and without the ZLB in a number of dimensions; filtered fundamental shocks, forecasts of key variables, impulse response functions to various shocks. In addition, we use our estimated ZLB models to quantify the impact of the ZLB on evolution of output during the crisis. Finally, section 6 summarizes our key findings and discussing some other key challenges for structural macro models used in policy analysis. Some appendices contains some technical details on the model, methods and data used in the analysis, as well as some additional results in the Gali, Smets and Wouters [64] model.

2. A Benchmark Macromodel

In this section, we present the benchmark model environment, which is the model of Smets and Wouters [92], SW07 henceforth. The SW07 model builds on the workhorse model by CEE, but allows for a richer set of stochastic shocks. In Section 3, we describe how we estimate it using aggregate times series for the United States.

2.1. Firms and Price Setting

Final Goods Production: The single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball [74], the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G_Y\left(\frac{Y_t(f)}{Y_t}\right) df = 1. \tag{2.1}$$

As in Dotsey and King [44], we assume that $G_Y(\cdot)$ is given by a strictly concave and increasing function:

$$G_Y\left(\frac{Y_t(f)}{Y_t}\right) = \frac{\phi_t^p}{1-(\phi_t^p-1)\epsilon_p}\left[\left(\frac{\phi_t^p+(1-\phi_t^p)\epsilon_p}{\phi_t^p}\right)\frac{Y_t(f)}{Y_t}+\frac{(\phi_t^p-1)\epsilon_p}{\phi_t^p}\right]^{\frac{1-(\phi_t^p-1)\epsilon_p}{\phi_t^p-(\phi_t^p-1)\epsilon_p}}+\left[1-\frac{\phi_t^p}{1-(\phi_t^p-1)\epsilon_p}\right], \tag{2.2}$$

where $\phi_t^p \geq 1$ denotes the gross markup of the intermediate firms. The parameter ϵ_p governs the degree of curvature of the intermediate firm's demand curve. When $\epsilon_p = 0$, the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When ϵ_p is positive the firms instead face a quasi-kinked demand curve, implying that a drop in the good's relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a large fall in demand. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementary in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost. Finally, notice that $G_Y(1) = 1$, implying constant returns to scale when all intermediate firms produce the same amount of the good.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index Y_t, taking the price $P_t(f)$ of each intermediate good $Y_t(f)$ as given. Moreover, final goods producers sell the final output good at a price P_t, and hence solve the following problem:

$$\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f) Y_t(f) \, df, \tag{2.3}$$

subject to the constraint in (2.1). The first order conditions (FOCs) for this problem can be written

$$\begin{aligned} \frac{Y_t(f)}{Y_t} &= \frac{\phi_t^p}{\phi_t^p - (\phi_t^p - 1)\epsilon_p} \left(\left[\frac{P_t(f)}{P_t} \frac{1}{\Lambda_t^p} \right]^{-\frac{\phi_t^p - (\phi_p - 1)\epsilon_p}{\phi_t^p - 1}} + \frac{(1-\phi_t^p)\epsilon_p}{\phi_t^p} \right) \\ P_t \Lambda_t^p &= \left[\int P_t(f)^{-\frac{1-(\phi_t^p - 1)\epsilon_p}{\phi_t^p - 1}} df \right]^{-\frac{\phi_t^p - 1}{1-(\phi_t^p - 1)\epsilon_p}} \\ \Lambda_t^p &= 1 + \frac{(1-\phi_t^p)\epsilon_p}{\phi_p} - \frac{(1-\phi_t^p)\epsilon_p}{\phi_t^p} \int \frac{P_t(f)}{P_t} df, \end{aligned} \tag{2.4}$$

where Λ_t^p denotes the Lagrange multiplier on the aggregator constraint in (2.1). Note that when $\epsilon_p = 0$, it follows from the last of these conditions that $\Lambda_t^p = 1$ in each period t, and the demand and pricing equations collapse to the usual Dixit-Stiglitz expressions, i.e.

$$\frac{Y_t(f)}{Y_t} = \left[\frac{P_t(f)}{P_t} \right]^{-\frac{\phi_t^p}{\phi_t^p - 1}}, P_t = \left[\int P_t(f)^{\frac{1}{1-\phi_t^p}} df \right]^{1-\phi_t^p}.$$

Intermediate Goods Production: A continuum of intermediate goods $Y_t(f)$ for $f \in [0, 1]$ is produced by monopolistic competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces the demand schedule in equation (2.4) from the final goods firms through the solution to the problem in (2.3), which varies inversely with its output price $P_t(f)$ and directly with aggregate demand Y_t.

Each intermediate goods producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = \varepsilon_t^a K_t(f)^\alpha \left[\gamma^t L_t(f)\right]^{1-\alpha} - \gamma^t \Phi,$$

where γ^t represents the labor-augmenting deterministic growth rate in the economy, Φ denotes the fixed cost (which is related to the gross markup ϕ_t^p so that profits are zero in the steady state), and ε_t^a is a total productivity factor which follows a Kydland-Prescott [76] style process:

$$\ln \varepsilon_t^a = \rho_a \ln \varepsilon_{t-1}^a + \eta_t^a, \eta_t^a \sim N(0, \sigma_a). \tag{2.5}$$

Firms face perfectly competitive factor markets for renting capital and hiring labor. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital R_{Kt} and the aggregate wage index W_t (defined below). Firms can without costs adjust either factor of production, thus, the standard static first-order conditions for cost minimization implies that all firms have identical marginal costs per unit of output.

The prices of the intermediate goods are determined by nominal contracts in Calvo [22] and Yun [97] staggered style nominal contracts. In each period, each firm f faces a constant probability, $1-\xi_p$, of being able to re-optimize the price $P_t(f)$ of the good. The probability that any firm receives a signal to re-optimize the price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, this is adjusted by a weighted combination of the lagged and steady-state rate of inflation, i.e., $P_t(f) = (1+\pi_{t-1})^{\iota_p} (1+\pi)^{1-\iota_p} P_{t-1}(f)$ where $0 \leq \iota_p \leq 1$ and π_{t-1} denotes net inflation in period $t-1$, and π the steady-state net inflation rate. A positive value of the indexation parameter ι_p introduces structural inertia into the inflation process. All told, this leads to the following optimization problem for the intermediate firms

$$\max_{\tilde{P}_t(f)} \mathrm{E}_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \frac{\Xi_{t+j} P_t}{\Xi_t P_{t+j}} \left[\tilde{P}_t(f) \left(\Pi_{s=1}^j (1+\pi_{t+s-1})^{\iota_p} (1+\pi)^{1-\iota_p}\right) - MC_{t+j}\right] Y_{t+j}(f),$$

where $\tilde{P}_t(f)$ is the newly set price and $\beta^j \frac{\Xi_{t+j} P_t}{\Xi_t P_{t+j}}$ the stochastic discount factor. Notice that given our assumptions, all firms that re-optimize their prices actually set the same price.

As noted previously, we assume that the gross price-markup is time-varying and given by $\phi_t^p = \phi^p \varepsilon_t^p$, for which the exogenous component ε_t^p is given by an exogenous ARMA(1,1) process:

$$\ln \varepsilon_t^p = \rho_p \ln \varepsilon_{t-1}^p + \eta_t^p - \vartheta_p \eta_{t-1}^p, \eta_t^p \sim N(0, \sigma_p). \tag{2.6}$$

2.2. Households and Wage Setting

Following Erceg, Henderson and Levin [51], we assume a continuum of monopolistic competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household's labor services $L_t(h)$, $h \in [0,1]$, as imperfect substitutes for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The aggregated labor index L_t has the Kimball [74] form:

$$L_t = \int_0^1 G_L\left(\frac{L_t(h)}{L_t}\right) dh = 1, \tag{2.7}$$

where the function $G_L(\cdot)$ has the same functional form as does (2.2), but is characterized by the corresponding parameters ϵ_w (governing convexity of labor demand by the aggregator) and a time-varying gross wage markup ϕ_t^w. The aggregator minimizes the cost of producing a given amount of the aggregate labor index L_t, taking each household's wage rate $W_t(h)$ as given, and then sells units of the labor index to the intermediate goods sector at unit cost W_t, which can naturally be interpreted as the aggregate wage rate. From the FOCs, the aggregator's demand for the labor hours of household h – or equivalently, the total demand for this household's labor by all goods-producing firms – is given by

$$\frac{L_t(h)}{L_t} = G_L'^{-1}\left[\frac{W_t(h)}{W_t}\int_0^1 G_L'\left(\frac{L_t(h)}{L_t}\right)\frac{L_t(h)}{L_t}dh\right], \tag{2.8}$$

where $G_L'(\cdot)$ denotes the derivative of the $G_L(\cdot)$ function in equation (2.7).

The utility function of a typical member of household h is

$$\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{1-\sigma_c}\left(C_{t+j}(h) - \varkappa C_{t+j-1}\right)\right]^{1-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_{t+j}(h)^{1+\sigma_l}\right), \tag{2.9}$$

where the discount factor β satisfies $0 < \beta < 1$. The period utility function depends on household h's current consumption $C_t(h)$, as well as lagged aggregate consumption per capita, to allow for external habit persistence (captured by the parameter $\varkappa$). The period utility function also depends inversely on hours worked $L_t(h)$.

Household h's budget constraint in period t states that expenditure on goods and net purchases of financial assets must equal to the disposable income:

$$\begin{aligned} &P_t C_t(h) + P_t I_t(h) + \frac{B_{t+1}(h)}{\varepsilon_t^b \tilde{R}_t} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h) \\ &= B_t(h) + W_t(h) L_t(h) + R_t^k Z_t(h) K_t^p(h) - a(Z_t(h)) K_t^p(h) + \Gamma_t(h) - T_t(h). \end{aligned} \tag{2.10}$$

Thus, the household purchases part of the final output good (at a price of P_t), which is chosen to be consumed $C_t(h)$ or invest $I_t(h)$ in physical capital. Following Christiano, Eichenbaum, and Evans [28], investment augments the household's (end-of-period) physical capital stock $K^p_{t+1}(h)$ according to

$$K^p_{t+1}(h) = (1-\delta)K^p_t(h) + \varepsilon^i_t \left[1 - S\left(\frac{I_t(h)}{I_{t-1}(h)}\right)\right] I_t(h). \tag{2.11}$$

The extent to which investment by each household turns into physical capital is assumed to depend on an exogenous shock ε^i_t and how rapidly the household changes its rate of investment according to the function $S\left(\frac{I_t(h)}{I_{t-1}(h)}\right)$, which we assume satisfies $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) = \varphi$ where γ is the steady state gross growth rate of the economy. The stationary investment-specific shock ε^i_t follows the process:

$$\ln \varepsilon^i_t = \rho_i \ln \varepsilon^i_{t-1} + \eta^i_t, \eta^i_t \sim N(0, \sigma_i).$$

In addition to accumulating physical capital, households may augment their financial assets through increasing their overall nominal bond holdings (B_{t+1}), from which they earn an interest rate of $\tilde{R}_t$. This return is assumed to be a convex function of the interest rate set by the central bank (R_t) and the return on short-term goverment yields (R^G_t), so

$$\tilde{R}_t = R_t^{1-\kappa} \left(R^G_t\right)^{\kappa}, \tag{2.12}$$

where $0 \leq \kappa \leq 1$. By estimating κ, we allow the data to inform us about the relative importance of the different returns for households consumption and investment decisions. Finally, the return on the bond portfolio is also subject to a risk-shock, ε^b_t, which follows an ARMA(1,1) process:

$$\ln \varepsilon^b_t = \rho_b \ln \varepsilon^b_{t-1} + \eta^b_t - \vartheta_b \eta^b_{t-1}, \eta^b_t \sim N(0, \sigma_b). \tag{2.13}$$

Fisher [56] shows that this shock can be given a structural interpretation.

We assume that agents can engage in friction-less trading of a complete set of contingent claims to diversify away idiosyncratic risk. The term $\int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of these state-contingent domestic bonds, with $\xi_{t,t+1}$ denoting the state-dependent price, and $B_{D,t+1}(h)$ the quantity of such claims purchased at time t.

On the income side, each member of household h earns labor income $W_t(h) L_t(h)$, capital rental income of $R^k_t Z_t(h) K^p_t(h)$, and pays a utilization cost of the physical capital equal to $a(Z_t(h)) K^p_t(h)$ where $Z_t(h)$ is the capital utilization rate. The capital services provided by household h, $K_t(h)$ thereby equals $Z_t(h) K^p_t(h)$. The capital utilization adjustment function $a(Z_t(h))$ is assumed to satisfy $a(1) = 0$, $a'(1) = r^k$, and $a''(1) = \psi/(1-\psi) > 0$, where $\psi \in [0,1)$ and a higher value of ψ

implies a higher cost of changing the utilization rate. Finally, each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (regarded as taxes net of any transfers).

In every period t, each member of household h maximizes the utility function in (2.9) with respect to consumption, investment, (end-of-period) physical capital stock, capital utilization rate, bond holdings, and holdings of contingent claims, subject to the labor demand function (2.8), budget constraint (2.10), and transition equation for capital (2.11).

Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described previously. Thus, the probability that a household receives a signal to re-optimize its wage contract in a given period is denoted by $1-\xi_w$. In addition, SW07 specify the following dynamic indexation scheme for the adjustment of wages for those households that do not get a signal to re-optimize: $W_t(h) = \gamma(1+\pi_{t-1})^{\iota_w}(1+\pi)^{1-\iota_w} W_{t-1}(h)$. All told, this leads to the following optimization problem for the households

$$\max_{\tilde{W}_t(h)} \mathrm{E}_t \sum_{j=0}^{\infty} (\beta\xi_w)^j \frac{\Xi_{t+j}P_t}{\Xi_t P_{t+j}} \left[\tilde{W}_t(h)\left(\Pi_{s=1}^{j}\gamma(1+\pi_{t+s-1})^{\iota_w}(1+\pi)^{1-\iota_w}\right) - W_{t+j}\right] L_{t+j}(h),$$

where $\tilde{W}_t(h)$ is the newly set wage and $L_{t+j}(h)$ is determined by equation (2.7). Notice that with our assumptions all households that re-optimize their wages will actually set the same wage.

Following the same approach as with the intermediate-goods firms, we introduce a shock ε_t^w to the time-varying gross markup, $\phi_t^w = \phi^w \varepsilon_t^w$, where ε_t^w is assumed being given by an exogenous ARMA(1,1) process:

$$\ln\varepsilon_t^w = \rho_w \ln\varepsilon_{t-1}^w + \eta_t^w - \vartheta_w \eta_{t-1}^w, \eta_t^w \sim N(0,\sigma_w). \tag{2.14}$$

2.3. Market Clearing Conditions and Monetary Policy

Government purchases G_t are exogenous, and the process for government spending relative to trend output in natural logs, i.e. $g_t = G_t/\left(\gamma^t Y\right)$, is given by the following exogenous AR(1) process:

$$\ln g_t = \left(1-\rho_g\right)\ln g + \rho_g\left(\ln g_{t-1} - \rho_{ga}\ln\varepsilon_{t-1}^a\right) + \varepsilon_t^g, \varepsilon_t^g \sim N(0,\sigma_g). \tag{2.15}$$

Government purchases neither have any effects on the marginal utility of private consumption, nor do they serve as an input into goods production. The consolidated government sector budget constraint is

$$\frac{B_{t+1}}{\tilde{R}_t} = G_t - T_t + B_t,$$

where T_t are lump-sum taxes. By comparing the debt terms in the household budget constraint in equation (2.10) with the equation above, one can see that receipts from the risk shock are subject to iceberg costs, and hence do not add any income to the government.[3] We acknowledge that this is an extremely simplistic modeling of the fiscal behavior of the government relative to typical policy models, and there might be important feedback effects between fiscal and monetary policies that our model does not allow for.[4] As discussed by Benigno and Nisticó [12] and Del Negro and Sims [43], the fiscal links between governments and central banks may be especially important today when central banks have employed unconventional tools in monetary policy. Nevertheless, we maintain our simplistic modeling of fiscal policy throughout the paper, as it allows us to examine the partial implications of amending the benchmark model with the zero lower bound (ZLB, henceforth) constraint more directly.

The conduct of monetary policy is assumed to be approximated by a Taylor-type policy rule (here stated in non-linearized form)

$$R_t \;= \max\left\{1, \left[R \; \left(\frac{\Pi_t}{\Pi}\right)^{r_\pi} \; \left(\frac{Y_t}{Y_t^{pot}}\right)^{r_y} \left(\frac{Y_t}{Y_t^{pot}} / \frac{Y_{t-1}}{Y_{t-1}^{pot}}\right)^{r_{\Delta y}} \left(\frac{R_t^G}{R_t}\right)^{r_s} \left(\eta_t^b\right)^{r_b}\right]^{(1-\rho_R)} R_{t-1}^{\rho_R} \varepsilon_t^r \right\}, \tag{2.16}$$

where Π_t denotes the is gross inflation rate, Y_t^{pot} is the level of output that would prevail if prices and wages were flexible, and variables without subscripts denote steady state values. The policy shock ε_t^r is supposed to follow an AR(1) process in natural logs:

$$\ln \varepsilon_t^r = \rho_r \ln \varepsilon_{t-1}^r + \eta_t^r, \eta_t^r \sim N\left(0, \sigma_r\right). \tag{2.17}$$

Relative to the basic SW07 model, the policy rule in equation (2.16) allows for the possibility that policymakers responds to R_t^G/R_t, i.e. the spread between the yield on government bonds and the policy rate set by the central bank. Notice that the sign for r_s is not evident. To the extent that a positive spread R_t^G/R_t reflects elevated future output gaps and inflation not captured by current outcomes for these variables, the sign of r_s should be positive. On the other hand, if we think about the wedge between the returns as reflecting some sort of risk premium, one should rather expect a negative r_s: the central bank will normally try to offset adverse influences from a rising risk premium by cutting its policy rate. To allow for an influence of term-premium shocks, we

[3] But even if they did, it would not matter as the government is assumed to balance its expenditures each period through lump-sum taxes, $T_t = G_t + B_t - B_{t+1}/\tilde{R}_t$, so that government debt $B_t = 0$ in equilibrium. Furthermore, as Ricardian equivalence (see Barro, [11]) holds in the model, it does not matter for equilibrium allocations whether the government balances its debt or not in each period.

[4] See e.g., Leeper and Leith [79], and Leeper, Traum and Walker [80].

assume that the short-term government yield equals the policy rate plus a term-premium, i.e.

$$R_t^G = R_t \varepsilon_t^{tp}, \tag{2.18}$$

where the term premium is given by

$$\ln \varepsilon_t^{tp} = \rho_{tp} \ln \varepsilon_{t-1}^{tp} + \eta_t^{tp}, \eta_t^{tp} \sim N\left(0, \sigma_{tp}\right). \tag{2.19}$$

To facilitate measurement of the term-premium shocks, we include a 2-year government yield in the estimation of the model as described in further detail in Section 3 below. In addition, following the recent evidence in Caldara and Herbst [21] (who argues that the Fed reacts strongly to credit spreads), the policy rule specification also allows for the possibility that the central bank responds directly to the risk premium innovation η_t^b (see eq. 2.10) through the coefficient r_b.

Finally, total output of the final goods sector is used as follows:

$$Y_t = C_t + I_t + G_t + a\left(Z_t\right) \bar{K}_t,$$

where $a\left(Z_t\right) \bar{K}_t$ is the capital utilization adjustment cost.

2.4. Extension to an Environment with Unemployment

The Smets and Wouters models ([91], [92]) did not feature unemployment, which rose sharply during the recent crisis. To cross-check the robustness of our empirical findings, we will also report some results for a variant of the model which offers a simplistic framework to match variations in the unemployment rate. The specific environment we use for this purpose is the Galí, Smets and Wouters [64] model, GSW henceforth. Following Gali (2011a,b), Galí, Smets and Wouters reformulates the SW-model to allow for involuntary unemployment, while preserving the convenience of the representative household paradigm. Unemployment in the model results from market power in labor markets, reflected in positive wage markups. Variations in unemployment over time are associated with changes in wage markups, either exogenous or resulting from nominal wage rigidities.[5]

[5] The introduction of unemployment allows GSW to overcome an identification problem pointed out by Chari, Kehoe and McGrattan (2008) as an illustration of the immaturity of New Keynesian models for policy analysis. Their observation is motivated by the SW finding that wage markup shocks account for almost 50 percent of the variations in real GDP at horizons of more than 10 years. However, without an explicit measure of unemployment (or, alternatively, labor supply), these wage markup shocks cannot be distinguished from preference shocks that shift the marginal disutility of labour. The policy implications of these two sources of fluctuations are, however, very different.

To save space, we will not provide an indepth exposition of the theoretical model here, instead we refer the interested reader to GSW. The only difference relative to the GSW model is that we allow for the possibility that the 2-year government yield affects the effective interest rate for households and firms via a positive κ in eq. (2.12) and through r_s in the policy rule (2.16).

3. Estimation on Data Without Imposing the ZLB

We now proceed to discuss how the model is estimated without imposing the ZLB. Subsequently, we will estimate the model when we impose the ZLB.

3.1. Solving the Model

Before estimating the models, we log-linearize all the equations. The log-linearized representation is provided in Appendix A. To solve the system of log-linearized equations, we use the code packages Dynare and RISE which provides an efficient and reliable implementation of the method proposed by Blanchard and Kahn [17].

3.2. Data

We use eight key macro-economic quarterly US time series as observable variables when estimating the SW model: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log difference of the GDP deflator, the federal funds rate, and a two-year Government yield. These observables are identical to those used by Smets and Wouters ([91], [92]), with the exception that we follow Brave et al. [19] and Del Negro et al. [46] by adding the two-year government yield as observable. The key rationale for doing so is to discipline the policy shocks during the crisis (i.e. mitigate possible tensions between the model projected path and the anticipated market path), while the term premium shocks allow for some deviations. Another interesting feature of this modeling is that it provides a role for policy to affect macroeconomic outcomes by lowering the term premium (ε_t^{tp}, see eq. 2.19). Still, we acknowledge that our approach is ad hoc in the way we introduce a role for long-term rates and term-premium shocks into the model. Therefore, we also report results for the original SW model, i.e. a variant of the model which omits the 2-year yield as observable and sets $\kappa = 0$, implying that the term-premium shocks are irrelevant for the dynamics in the model.

A full description of the data used is given in Appendix B. In this appendix, we also discuss the data used to estimate the GSW model. The solid blue line in Figure 3.1 shows the data for the full sample, which spans 1965Q1–2016Q4.[6] From the figure, we see the extraordinary large fall in private consumption, which exceeded the fall during the recession in the early 1980s. The strains in the labor market are also evident, with hours worked per capita falling to a post-war bottom low in early 2010. Finally, we see that the Federal reserve cut the federal funds rate to near zero in 2009Q1 (the FFR is measured as an average of daily observations in each quarter). Evidently, a federal funds rate near zero was perceived as an effective lower bound by the FOMC committee, and they kept it as this level during the crisis and adopted alternative tools to make monetary policy more accommodating (see e.g. Bernanke [14]). Meanwhile, inflation fell to record lows and into deflationary territory by late 2009. Since then, inflation has rebounded close to the new target of 2 percent announced by the Federal Reserve in January 2012.

The measurement equation, relating the variables in the model to the various variables we match in the data, is given by:

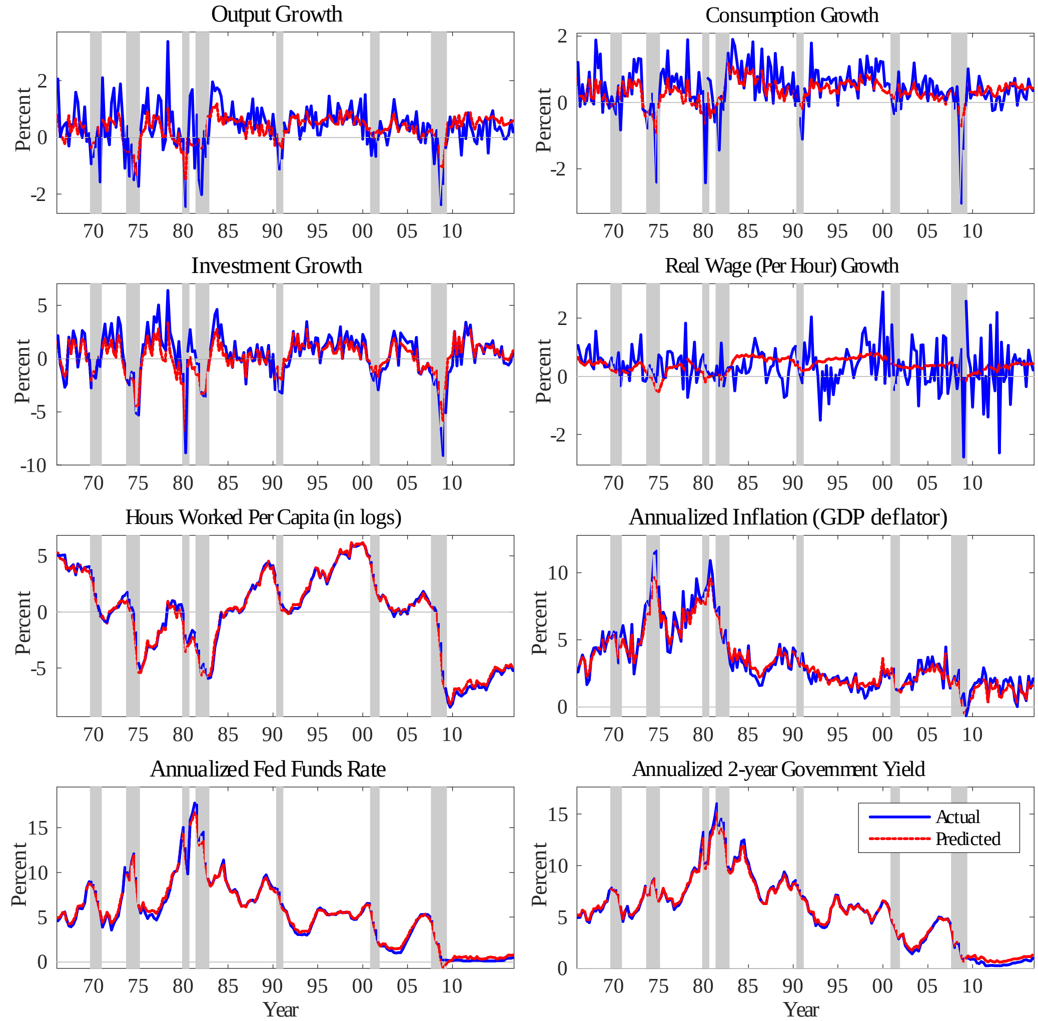
$$Y_t^{obs} = \begin{bmatrix} \Delta \ln GDP_t \\ \Delta \ln CONS_t \\ \Delta \ln INVE_t \\ \Delta \ln W_t^{real} \\ \ln HOURS_t \\ 4\Delta \ln PGDP_t \\ FFR_t \\ RG_t^{2y} \end{bmatrix} = \begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln I_t - \ln I_{t-1} \\ \ln (W/P)_t - \ln (W/P)_{t-1} \\ \ln L_t \\ 4 \ln \Pi_t \\ 4 \ln R_t \\ (4/8)\Sigma_{j=0}^{7} \ln R_{t+j|t}^{G} \end{bmatrix} \approx \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ 4\bar{\pi} \\ 4\bar{r} \\ 4\left(\bar{r}^G - \bar{r}\right) \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{\imath}_t - \hat{\imath}_{t-1} \\ \hat{w}_t^{real} - \hat{w}_{t-1}^{real} \\ l_t \\ 4\pi_t \\ 4\hat{R}_t \\ (4/8)\Sigma_{j=0}^{7} \ln \hat{R}_{t+j|t}^{G} \end{bmatrix} \tag{3.1}$$

where ln and $\Delta \ln$ stand for log and log-difference respectively, $\bar{\gamma} = 100\,(\gamma - 1)$ is the common quarterly trend growth rate to real GDP, consumption, investment and wages, $\bar{\pi} = 100\pi$ is the quarterly steady-state inflation rate and $r = 100\left(\beta^{-1}\gamma^{\sigma_c}\,(1+\pi) - 1\right)$ is the steady-state nominal interest rate. Notice, however, that inflation, the federal funds rate and the two year government yield are expressed in annualized rates. Given the estimates of the trend growth rate and the steady-state inflation rate, the latter will be determined by the estimated discount rate. Finally, $\bar{l}$ is steady-state hours-worked, which is normalized to be equal to zero.

Structural models impose important restrictions on the dynamic cross-correlation between the variables but also on the long run ratios between the macro-aggregates. Our transformations in (3.1) impose a common deterministic growth component for all quantities and the real wage, whereas hours worked per capita, the real interest rate and the inflation rate are assumed to have a constant

[6] The figure also includes a red-dashed line, whose interpretation will be discussed in further detail within Section 3.

Figure 3.1: Actual and Predicted Data in Benchmark Model.



mean. These assumptions are not necessarily in line with the properties of the data and may have important implications for the estimation results. Some prominent papers in the literature assume real quantities to follow a stochastic trend, see e.g. Altig et al. [9]. Fisher [45] argues that there is a stochastic trend in the relative price of investment and examines to what extent shocks that can explain this trend matter for business cycles. There is also an ongoing debate on whether hours worked per capita should be treated as stationary or not, see e.g. Christiano, Eichenbaum and Vigfusson [31], Galí and Rabanal [63], and Boppart and Krusell [18]. Within the context of DSGE modeling for policy analysis, it is probably fair to say that less attention and resources have been spent to mitigate possible gaps in the low frequency properties of models and data, presumably partly because the jury is still out on the deficiencies of the benchmark specification, but also partly because the focus is on the near-term behavior of the models (i.e. monetary transmission mechanism, near-term forecasting performance, and historical decomposition) and these shortcomings do not seriously impair the model's behavior in this dimension.

3.3. Estimation Methodology

Following SW07, Bayesian techniques are adopted to estimate the parameters using the seven U.S. macroeconomic variables in equation (3.1) during the period 1965Q1–2016Q4. Bayesian inference starts out from a prior distribution that describes the available information prior to observing the data used in the estimation. The observed data is subsequently used to update the prior, via Bayes' theorem, to the posterior distribution of the model's parameters which can be summarized in the usual measures of location (e.g. mode or mean) and spread (e.g. standard deviation and probability intervals).[7]

Some of the parameters in the model are kept fixed throughout the estimation procedure (i.e., having infinitely strict priors). We choose to calibrate the parameters we think are weakly identified by the variables included in $\tilde{Y}_t$ in equation (3.1). In Table 3.1 we report the parameters we have chosen to calibrate. These parameters are calibrated to the same values as in SW07.

Table 3.1: Calibrated parameters.

Parameter	Description	Calibrated Value
δ	Depreciation rate	0.025
ϕ_w	Gross wage markup	1.50
g_y	Government G/Y ss-ratio	0.18
ϵ_p	Kimball Elast. GM	10
ϵ_w	Kimball Elast. LM	10

Note: The calibrated parameters are adapted from SW07.

The remaining 43 parameters, which mostly pertain to the nominal and real frictions in the model as well as the exogenous shock processes, are estimated. The first three columns in Table 2.2 shows the assumptions for the prior distribution of the estimated parameters. The location of the prior distribution is identical to that of SW07. We use the beta distribution for all parameters bounded between 0 and 1. For parameters assumed to be positive, we use the inverse gamma distribution, and for the unbounded parameters, we use the normal distribution. The exact location and uncertainty of the prior can be seen in Table 2.2, but for a more comprehensive discussion of our choices regarding the prior distributions we refer the reader to SW07.

[7] We refer the reader to Smets and Wouters [91] for a more detailed description of the estimation procedure.

Given the calibrated parameters in Table 3.1, we obtain the joint posterior distribution mode for the estimated parameters in Table 3.2 in two steps. First, the posterior mode and an approximate covariance matrix, based on the inverse Hessian matrix evaluated at the mode, is obtained by numerical optimization on the log posterior density. Second, the posterior distribution is subsequently explored by generating draws using the Metropolis-Hastings algorithm. The proposal distribution is taken to be the multivariate normal density centered at the previous draw with a covariance matrix proportional to the inverse Hessian at the posterior mode; see Schorfheide [89] and Smets and Wouters [91] for further details. The results in 3.2 shows the posterior mode of all the parameters along with the approximate posterior standard deviation obtained from the inverse Hessian at the posterior mode. Finally, the last column reports the posterior mode in the SW07 paper.

3.4. Posterior Distributions of the Estimated Parameters

To learn how the 2-year government yield and the term-premium shock affects the estimation outcome, Table 3.2 reports results of the model without this observable and shock, referred to as "7-observables". The estimation results with all observables is referred to as the "8-observables" model.

Table 3.2: Prior and Posterior Distributions for Benchmark Model Without the ZLB.

Parameter		Prior distribution			Posterior distribution				SW07 results
					7 observables		8 observables		Posterior
		type	mean	std.dev. /df	mode	std.dev. Hess.	mode	std.dev. Hess.	mode
Calvo prob. wages	ξ_w	beta	0.50	0.10	0.82	0.041	0.83	0.037	0.73
Calvo prob. prices	ξ_p	beta	0.50	0.10	0.80	0.050	0.80	0.045	0.65
Indexation wages	ι_w	beta	0.50	0.15	0.64	0.132	0.54	0.133	0.59
Indexation prices	ι_p	beta	0.50	0.15	0.26	0.089	0.24	0.083	0.22
Gross price markup	ϕ_p	normal	1.25	0.12	1.51	0.076	1.54	0.10	1.61
Capital production share	α	normal	0.30	0.05	0.17	0.017	0.16	0.017	0.19
Capital utilization cost	ψ	beta	0.50	0.15	0.74	0.096	0.77	0.089	0.54
Investment adj. cost	φ	normal	4.00	1.50	4.61	0.918	4.38	0.879	5.48
Habit formation	$\varkappa$	beta	0.70	0.10	0.66	0.056	0.52	0.065	0.71
Inv subs. elast. of cons.	σ_c	normal	1.50	0.37	1.29	0.161	1.35	0.156	1.59
Labor supply elast.	σ_l	normal	2.00	0.75	1.85	0.604	2.16	1.241	1.92
Log hours worked in S.S.	$\bar{l}$	normal	0.00	2.00	0.92	1.453	-0.42	2.535	-0.10
Discount factor	$100(\beta^{-1}-1)$	gamma	0.25	0.10	0.13	0.052	0.12	0.050	0.16
Quarterly Growth in S.S.	$\bar{\gamma}$	normal	0.40	0.10	0.39	0.037	0.39	0.017	0.43
Stationary tech. shock	ρ_a	beta	0.50	0.20	0.98	0.013	0.98	0.010	0.95
Risk premium shock	ρ_b	beta	0.50	0.20	0.87	0.038	0.92	0.020	0.18
Invest. spec. tech. shock	ρ_i	beta	0.50	0.20	0.77	0.070	0.74	0.052	0.71
Gov't cons. shock	ρ_g	beta	0.50	0.20	0.98	0.008	0.97	0.007	0.97
Price markup shock	ρ_p	beta	0.50	0.20	0.91	0.047	0.92	0.036	0.90
Wage markup shock	ρ_w	beta	0.50	0.20	0.99	0.010	0.99	0.005	0.97
Response of g_t to ε_t^a	ρ_{ga}	beta	0.50	0.20	0.50	0.073	0.49	0.075	0.52
Stationary tech. shock	σ_a	invgamma	0.10	2.00	0.46	0.026	0.45	0.026	0.45
Risk premium shock	σ_b	invgamma	0.10	2.00	1.15	0.142	0.58	0.090	2.22
MA(1) risk premium shock	ϑ_b	beta	0.50	0.20	0.67	0.095	0.69	0.094	
Invest. spec. tech. shock	σ_i	invgamma	0.10	2.00	0.35	0.037	0.35	0.035	0.45
Gov't cons. shock	σ_g	invgamma	0.10	2.00	0.47	0.024	0.48	0.024	0.52
Price markup shock	σ_p	invgamma	0.10	2.00	0.13	0.012	0.14	0.012	0.14
MA(1) price markup shock	ϑ_p	beta	0.50	0.20	0.82	0.071	0.83	0.055	0.74
Wage markup shock	σ_w	invgamma	0.10	2.00	0.37	0.021	0.37	0.021	0.24
MA(1) wage markup shock	ϑ_w	beta	0.50	0.20	0.97	0.012	0.98	0.006	0.88
Quarterly infl. rate. in S.S.	$\bar{\pi}$	gamma	0.62	0.10	0.77	0.111	0.74	0.130	0.81
Inflation response	r_π	normal	1.50	0.25	1.83	0.168	1.71	0.200	2.03
Output gap response	r_y	normal	0.12	0.05	0.08	0.023	0.04	0.024	0.08
Diff. output gap response	$r_{\Delta y}$	normal	0.12	0.05	0.23	0.025	0.19	0.036	0.22
Mon. pol. shock std	σ_r	invgamma	0.10	2.00	0.22	0.012	0.17	0.022	0.24
Mon. pol. shock pers.	ρ_r	beta	0.50	0.20	0.14	0.068	0.10	0.057	0.12
Interest rate smoothing	ρ_R	beta	0.75	0.10	0.84	0.022	0.85	0.025	0.81
Term premium response	r_s	normal	0.00	0.50			0.09	0.072	
Risk premium response	r_b	normal	-0.00	0.20			-0.21	0.065	
Gov't yield weight.	κ	beta	0.50	0.20			0.36	0.213	
Quarterly term spread in S.S.	$\bar{r}^G$-$\bar{r}$	gamma	0.20	0.10			0.13	0.059	
Term premium pers.	ρ_{tp}	beta	0.50	0.20			0.86	0.040	
Term premium shock	σ_{tp}	invgamma	0.10	2.00			0.19	0.019	
Log marginal likelihood					Laplace	-1086.22	Laplace	-1200.10	

Note: Data for 1965Q1–1965Q4 are used as pre-sample to form a prior for 1966Q1, and the log-likelihood is evaluated for the period 1966Q1–2016Q4. A posterior sample of 250,000 post burn-in draws was generated in the Metropolis-Hastings chain. Convergence was checked using standard diagnostics such as CUSUM plots and the potential scale reduction factor on parallel simulation sequences. The MCMC marginal likelihood was numerically computed from the posterior draws using the modified harmonic estimator of Geweke [66].

There two important features to notice with regards to the posterior parameters in Table 3.2. First, the policy- and deep-parameters are generally very similar to those estimated by SW07, reflecting a largely overlapping estimation sample (SW07 used data for the 1965Q1–2004Q4 period to estimate the model). The only noticeable difference relative to SW07 is that the estimated degree of wage and price stickiness is somewhat more pronounced (posterior mode for ξ_w is about 0.83

instead of 0.73 in SW07, and the mode for ξ_p has increased from 0.65 (SW07) to about 0.80). The tendency of an increased degree of price and wage stickiness in the extended sample is supported by Del Negro, Giannoni and Schorfheide [40], who argue that a New Keynesian model similar to ours augmented with financial frictions points towards a high degree of price and wage stickiness to fit the behavior of inflation during the Great Recession. Second, in terms of stochastic shock processes, the profile of the risk premium shock changed consirably with the longer sample including the financial crisis. While the risk premium shock had a high volatility and low persistence in the original SW07 model, the process becomes much more persistent in the updated sample. Third, the inclusion of the 2-year yield as observable does not materially change the estimation results for the other parameters. Even so, the posterior mode κ is fairly high (0.34), but the uncertainty about the mode is substantial. There is also little evidence of a vigorous response to the term-spread in the policy rule (r_s is low), but there is clear evidence that the Fed responds strongly to the risk-premium (r_b is -0.21). The inclusion of the financial variables in the policy rule and the inclusion of the 2-year yield as observable leads to a higher degree of interest smoothing and lower responses to inflation, the output gap, and the change in the output gap. Moreover, including the extra information in the policy rule reduces the magnitude of the policy shock.

4. Estimation of Benchmark Model When Imposing the ZLB

We now extend the analysis in Section 3 by the influence of the zero lower bound on policy rates. Basically, two different methods are used to estimate the model subject to the ZLB. First, we build on the simple method in Lindé, Smets and Wouters [82], but to take shock uncertainty into account we use the Sigma filter advocated by Binning and Maih [15]. Under this approach, the duration of a ZLB episode is determined endogenously by the state of the economy. Our second method, instead, examines the merits of a Regime-Switching approach (see e.g. Farmer, Waggoner and Zha [53], Davig and Leeper [38] and Maih [85]) to impose the ZLB. In this latter approach, the incidence and duration of the ZLB is exogenously given. Presenting results for both methods will allow us to assess the empirical merits of modelling the incidence and duration of the ZLB as an endogenous outcome, or if a regime-switching approach which approximates the ZLB as an exogenous incident with a fixed **expected** duration is sufficient from an empirical perspective. Below, we first present our method to impose the ZLB through endogenous methods, and then turn to discuss the results. Regime switching methods are already well-described elsewhere in the literature, and the reader interested in more details about those methods is hence referred elsewhere (see e.g. the references above).

4.1. ZLB Estimation Methodologies

When estimating the model subject to the ZLB constraint, we make use of the same linearized model equations (stated in Appendix Appendix A), except that we impose the non-negativity constraint on the federal funds rate. To do this, we adopt the following policy rule for the federal funds rate when the ZLB binds:

$$\begin{aligned}\widehat{R}_t^* \;&=\; \rho_R \widehat{R}_{t-1}^* \;+\; (1-\rho_R)\left[r_\pi \widehat{\pi}_t \;+\; r_y(\widehat{ygap}_t) + r_{\Delta y}\Delta(\widehat{ygap}_t) + r_s \hat{\varepsilon}_t^{tp} + r_b \widehat{\eta}_t^b\right] \;, \\ \widehat{R}_t &= \max\left(-\bar{r}, \widehat{R}_t^*\right). \end{aligned} \tag{4.1}$$

The policy rule in (4.1) assumes that the central bank will keep it actual interest rate $\widehat{R}_t$, if constrained by the ZLB, at its lower bound ($-\bar{r}$) as long as the shadow rate, $\widehat{R}_t^*$, is below the lower bound. Note that $\widehat{R}_t$ in the policy rule (4.1) is measured as percentage point deviation of the federal funds rate from its quarterly steady state level ($\bar{r}$), so restricting $\widehat{R}_t$ not to fall below $-\bar{r}$ is equivalent to imposing the ZLB on the nominal policy rate.[8] In its setting of the shadow-rate at the ZLB, we assume that the Fed is smoothing over the lagged shadow-rate $\widehat{R}_{t-1}^*$, as opposed to the actual lagged rate $\widehat{R}_{t-1}$. This is line with Reifschneider and Willams [87] and Eggertsson and Woodford [49] "Lower for longer policy", but differs from LSW [82] who assumed smoothing over the actual policy rate.[9].

Our first method to impose the policy rule (4.1) in estimation draws on the work by Hebden, Lindé and Svensson [72] and Maih [84]. This method is convenient because it is quick even when the model contains many state variables, and we provide further details about the algorithm in Appendix D.[10] In a nutshell, the algorithm imposes the non-linear policy rule in equation (4.1) through current and anticipated shocks (add-factors) to the policy rule. More specifically, if the projection of $\widehat{R}_{t+h}$ in (4.1) given the filtered state in period t in any of the periods $h = 0, 1, ..., T$ for some sufficiently large non-negative integer T is below $-\bar{r}$, the algorithm adds a sequence of anticipated policy shocks $\widehat{\varepsilon}_{t+h|t}^r$ such that $\widehat{R}_{t+h|t} = \widehat{R}_{t+h|t}^* + \widehat{\varepsilon}_{t+h|t}^r \geq 0$ for all $h = \tau_1, \tau_1 + 1, ..., \tau_2$. If

[8] See (3.1) for the definition of $\bar{r}$. If writing the policy rule in levels, the first part of (4.1) would be replaced by (2.16) (omitting the policy shock), and the ZLB part would be $R_t = \max(1, R_t^*)$.

[9] To implement the change in smoothing concept between normal (eq. 2.16) and constrained (eq. 4.1) periods in the policy rule, we use a regime switch where the switch happens immediately and unexpectedly whenever the ZLB constraint binds. This complication is necessary because estimating the model with smoothing over the shadow rate in normal times as well would result in a highly persistent monetary policy shock $\widehat{\varepsilon}_t^r$ (with a persistence of the shock similar to ρ_R in magnitude). With such a persistent policy shock, the "lower for longer" advantage of the shadow rate would disappear.

[10] Iacoviello and Guerrieri [71] shows how this method can be applied to solve DSGE models with other types of asymmetry constraints.

the added policy shocks put enough downward pressure on the economic activity and inflation, the duration of the ZLB spell will be extended both backwards (τ_1 shrinks) and forwards (τ_2 increases) in time. Moreover, as we think about the ZLB as a constraint on monetary policy, we require all current and anticipated policy shocks to be *positive* whenever $\widehat{R}_t^* < -\bar{r}$. Imposing that all policy shocks are strictly positive whenever the ZLB binds, amounts to think about these shocks as Lagrangian multipliers on the non-negativity constraint on the interest rate, and implies that we should not necessarily be bothered by the fact that these shocks may not be normally distributed even when the ZLB binds for several consecutive periods $t, t+1, ..., t+T$ with long expected spells each period (h large).

Importantly, this method implies that both the incidence and the duration of the ZLB is *endogenously* determined by the model subject to the criterion to maximize the log marginal likelihood. The use of the two-year yield as observable along with the policy rate also disciplins the analysis in terms of expected ZLB durations. In this context, it is important to understand that the non-negativity requirement on the current and anticipated policy shocks for each possible state and draw from the posterior, forces the posterior itself to move into a part of the parameter space where the model can account for long ZLB spells which are contractionary to the economy. Without this requirement, DSGE models with endogenous lagged state variables may experience sign switches for the policy shocks, so that the ZLB has a stimulative rather than contractionary impact on the economy even for fairly short ZLB spells as documented by Carlstrom, Fuerst and Paustian [25].[11] As discussed in further detail in Hebden et al., the non-negativity assumption for all states and draws from the posterior also mitigates the possibility of multiple equilibria (indeterminacy). Notice that when the ZLB is not a binding constraint, we assume the contemporaneous policy shock $\widehat{\varepsilon}_t^r$ in equation (4.1) can be either negative or positive; in this case we do not use any anticipated policy shocks as monetary policy is unconstrained. For the term premium shock ε_t^{tp} in eq. (2.19), we never impose any sign restrictions.

However, a potentially serious shortcoming of this method as implemented in e.g. LSW [82] is that it relies on perfect foresight and hence does not explicitly account for the role of future shock uncertainty as in the work of e.g. Adam and Billi [1] and Gust, Herbst, Lopez-Salido and Smith [70].[12] To mitigate this shortcoming, we use the Sigma filter advocated by Binning and Maih [15]

[11] This can be beneficial if we think that policy makers choose to let the policy rate remain at the ZLB although the policy rule dictated that the interest rate should be raised ($\hat{R}_t^*$ is above $-\bar{r}$). In the case of the U.S., this possibility might be relevant in the aftermath of the crisis and we therefore subsequently use an alternative method which allows for this.

[12] Even so, we implicitly allow for parameter and shock uncertainty by requiring that the filtered current and

when imposing the ZLB in the estimation. The Sigma filter provides a numerically efficient method to approximate the time-varying and asymmetric forecast uncertainty by evaluating the forecast for a minimal number (n_σ) of sigma points at a one period ahead horizon. The forecasting step for the state vector (s_t) in the filter becomes:

$$s_{t+1|t} = \sum_{i=0}^{n_\sigma} w_i \chi^i_{t+1|t} = \sum_{i=0}^{n_\sigma} w_i (T s_{t|t} + \sum_{h=1}^{ZLB_{Dur}} R^h \widehat{\varepsilon}^r_{t+h|t} \pm R\sqrt{n_\varepsilon + \kappa}\sigma^i), \tag{4.2}$$

The final forecast $s_{t+1|t}$ is a weighted average of the forecasts ($\chi^i_{t+1|t}$) evaluated at the individual sigma points, where the number of sigma points n_σ is equal to (1+2n_ε) (with n_ε the number of stochastic shocks), and the sigma point weights (w_i) are equal to $1/(2(n_\varepsilon + \kappa))$ except for $w_0 = \kappa/(n_\varepsilon + \kappa)$ so that $\Sigma w_i = 1$. We use a large $\kappa = n_\varepsilon$ so that the probability of a future ZLB-constraint is detected early even with a one period stochastic horizon. T and R are the standard state transition matrixes corresponding with the rational expectation solution in a first order perturbation of the model. σ^i represents the diagonal matrix of standard errors of the fundamental shocks in which only shock i is activated. Finally R^h corresponds with the impact effects of an anticipated monetary shock h-periods ahead and the anticipated shocks ($\widehat{\varepsilon}^r_{t+h|t}$) are calculated for all future periods during which the lower bound is expected to be binding in the projection (ZLB_{Dur}).

The corresponding covariance matrix for the one step ahead forecast error is given by:

$$P_{t+1|t} = TP_{t|t}T' + \sum_{i=1}^{n_\sigma} w_i (s_{t+1|t} - \chi^i_{t+1|t})(s_{t+1|t} - \chi^i_{t+1|t})'. \tag{4.3}$$

With this notation, it is easy to see how the covariance matrix of the forecast errors will adjust whenever the propagation of shocks is affected by the ZLB constraint. Note also that the sigma filter converges to the Kalman filter when the ZLB is not binding and the sigma points produce symmetric forecasts around the zero-shock forecast $\chi^0_{t+1|t}$. To assess the impact of using the Sigma filter in estimation, Table 4.1 also includes results when we follow LSW [82] and use the standard Kalman filter to evaluate the likelihood.

anticipated policy shocks in each time point are positive for all parameter and shock draws from the posterior whenever the ZLB binds. More specifically, when we evaluate the likelihood function and find that $\mathrm{E}_t \widehat{R}_{t+h} < 0$ in the modal outlook for some period t and horizon h conditional on the parameter draw and associated filtered state, we draw a large number of sequences of fundamental shocks for $h = 0, 1, ..., 12$ and verify that the policy rule (4.1) can be implemented for all possible shock realizations through positive shocks only. For those parameter draws this is not feasible, we add a smooth penalty to the likelihood which is set large enough to ensure that the posterior will satisfy the constraint. For example, it turns out that the model in 2008Q4 implies that the ZLB would be a binding constraint in 2009Q1 through 2009Q3 in the modal outlook. For this period we generated 1,000 shock realizations for 2009Q1,2009Q2,...,2011Q4 and verified that we could implement the policy rule (4.1) for all forecast simulations of the model through non-negative current and anticipated policy shocks.

Our second approach to impose the ZLB in the estimations rely on regime-switching methods. Under this approach, the conduct of monetary policy alternates between a "Normal" regime when monetary policy follows an unconstrained Taylor rule

$$R_t - R \; = \rho_R \left(R_{t-1} - R\right) \; + \; (1-\rho_R) \left[r_\pi \widehat{\pi}_t \; + \; r_y(\widehat{ygap}_t) + r_{\Delta y} \Delta(\widehat{ygap}_t) + r_s \hat{\varepsilon}_t^{tp} + r_b \widehat{\eta}_t^b \right] + \hat{\varepsilon}_t^r \; , \quad (4.4)$$

and a "ZLB" regime when

$$R_t \; = \hat{\varepsilon}_t^r. \quad (4.5)$$

The switch from the Normal to the ZLB regime is modelled as an "exogenous" event, but occurs stochastically with probability p_{12} in each period, whereas the probability that the economy switch back from the ZLB to the Normal regime is given by p_{21}. As the switches from one regime to another, the expected duration of the ZLB regime is given by $1/p_{21}$. When the economy is in the ZLB regime, monetary policy is passive. Even so, as discussed in further detail in Davig and Leeper [38], the equilibrium is determinate and unique provided that p_{21} is sufficiently high relative to p_{12}. Finally, notice that the policy rule in the ZLB regime (eq. 4.5) is associated with a shift in the intercept, i.e. $R = 0$. In the results reported below, we assume that the risk shock ε_t^b permanently shifts up and that the central bank – in order to offset this upward shift – adjust the intercept in the policy rule downward by the same amount and hence set $R = 0$.[13] Notice that in the ZLB regime we still allow for a shock to the policy – albeit with a reduced variance – to account for the fact that the federal funds rate in the data is not exactly constant and zero during the ZLB episode (see Figure 3.1).

4.2. Estimation Results

The posterior mode and standard deviation when the model is estimated subject to the ZLB are shown in Table 4.1. The left two columns report results when the incidence and duration of the ZLB is endogenously determined; results for the LSW [82] approach ("ZLB – LSW approach") is reported first followed by our method which approximates future shock uncertainty through the Sigma filter ("ZLB – Sigma filter"). Last, the table reports results for the regime-switching

[13] Since standard measures of risk-spreads receded after the most intensive phase of the financial crisis, our assumption of a permanent increase in the risk-premium in the ZLB regime may be not be supported by the data had we included the risk-spread as an observable in estimation. However, we have estimated a variant of the model where we allow for the possibility that multiple breaks in the discount factor, inflation rate, trend growth rate and the risk-premium shock all contribute to the zero intercept in the policy rule in the ZLB regime. The results are similar to ones reported below. So by a Occam's razor argument, we proceeded with the most simple specification. Note that the permanent drop in the policy rate might be consistent with the evidence on the lower r*, in particular, our version is consistent with the interpretation of Del Negro et al. [47], who interpret the decline in the natural rate as a result of an increased desire for safety and liquidity.

approach. Notice that the only difference between these results and those reported in Table 3.2 is the modelling of the ZLB. All other aspects of the analysis (sample, number of observables and shocks etc.) are identical.

By comparing the posterior mode between the alternative approches to impose the ZLB, we see that the differences are generally quite small, especially between the first two variants with endogenous duration (LSW-approach and Sigma filter). However, the regime-switching results are somewhat different. First, this method generates a higher degree of real rigidities ($\varkappa$ and φ), which enhances endogenous propagation of shocks. Moreover, the Frisch elasticity of labour supply ($1/\sigma_l$) is larger with this method (about 1/2 instead of 1/3 with the other two methods). Turning to the financial side of the economy, we see that the policy rule coefficients to real economic activity r_y and $r_{\Delta y}$ are larger, whereas the reaction to the risk-premium spread is notably lower -0.01 instead of -0.19) with the other two methods. The policy rule coefficient for the term-premium, r_s, is, however, notably larger (0.34 instead of 0.04), but since κ is substantially smaller (0.14 instead of about 0.45) the feedback to the borrowing rate is about the same. The smaller reaction to the bond risk-premium will imply that the adverse effects of such shocks will be notably more elevated in the regime-switching model compared to the variants with endogenous ZLB duration. Another interesting finding is the probabilites w.r.t. to the Normal and ZLB regimes. The posterior mode for switching from the Normal to the ZLB regime (p_{12}) equals 0.01, whereas the exit probability from the ZLB regime to the Normal regime (p_{21}) equals 0.15 in each period. Hence, the expected ZLB duration according to the regime-switching approach equals 6.67 quarters.

[Jesper: Should we show a figure of the estimated regime probabilities?]

If we compare our results in Table 4.1 with the 8-variables estimation results without imposing the ZLB, we note that the ZLB has little impact on the estimated parameters. Essentially, the no ZLB results are very similar to estimation outcomes with ZLB endogenous duration, but there are some differences w.r.t. the regime-switching approach (which were true for the ZLB estimation results as well). The ZLB estimation results confirm the notably higher degree of nominal stickiness, consistent with the findings by Del Negro, Giannoni and Schorfheide [40] and LSW [82].[14]

[14] As different models make alternative assumptions about strategic complements in price and wage setting, we have the reduced form coefficient for the wage and price markups in mind when comparing the degree of price and wage stickiness. For our posterior mode in the ZLB model this coefficient equals 0.009 at the posterior mode for the New Keynesian Phillips curve which is in between the estimate of Del Negro et al. [46] (0.016) and Brave et al. [19] (0.002).

Table 4.1: Posterior Distributions in Benchmark Model imposing the ZLB.

Parameter		ZLB-LSW method Posterior mode	ZLB-LSW method Posterior std.dev. Hess.	ZLB-Sigma filter Posterior mode	ZLB-Sigma filter Posterior std.dev. Hess.	ZLB - Regime-Switching Posterior mode	ZLB - Regime-Switching Posterior std.dev. Hess.
Calvo prob. wages	ξ_w	0.86	0.040	0.86	0.040	0.82	0.023
Calvo prob. prices	ξ_p	0.78	0.043	0.77	0.043	0.87	0.023
Indexation wages	ι_w	0.56	0.128	0.55	0.128	0.47	0.119
Indexation prices	ι_p	0.26	0.086	0.25	0.086	0.20	0.077
Gross price markup	ϕ_p	1.54	0.098	1.60	0.098	1.65	0.112
Capital production share	α	0.17	0.016	0.17	0.016	0.17	0.019
Capital utilization cost	ψ	0.78	0.086	0.76	0.086	0.71	0.017
Investment adj. cost	φ	4.15	0.868	4.00	0.868	5.63	0.695
Habit formation	$\varkappa$	0.49	0.062	0.48	0.062	0.69	0.002
Inv subs. elast. of cons.	σ_c	1.36	0.142	1.44	0.142	1.11	0.002
Labor supply elast.	σ_l	2.89	1.199	3.21	1.199	2.10	0.095
Log hours worked in S.S.	$\bar{l}$	-1.95	2.318	-0.04	2.318	-0.63	0.004
Discount factor	$100(\beta^{-1}-1)$	0.11	0.047	0.13	0.047	0.12	0.010
Quarterly Growth in S.S.	$\bar{\gamma}$	0.39	0.016	0.40	0.016	0.43	0.005
Stationary tech. shock	ρ_a	0.97	0.008	0.97	0.008	0.96	0.011
Risk premium shock	ρ_b	0.92	0.005	0.91	0.005	0.83	0.022
Invest. spec. tech. shock	ρ_i	0.76	0.046	0.76	0.046	0.75	0.046
Gov't cons. shock	ρ_g	0.98	0.007	0.97	0.007	0.97	0.007
Price markup shock	ρ_p	0.93	0.029	0.92	0.029	0.81	0.047
Wage markup shock	ρ_w	0.99	0.005	0.99	0.005	0.98	0.008
Response of g_t to ε_t^a	ρ_{ga}	0.49	0.074	0.47	0.074	0.49	0.080
Stationary tech. shock	σ_a	0.45	0.026	0.44	0.026	0.44	0.030
Risk premium shock	σ_b	0.65	0.093	0.64	0.093	1.03	0.112
MA(1) risk premium shock	ϑ_b	0.71	0.057	0.65	0.057	0.59	0.064
Invest. spec. tech. shock	σ_i	0.36	0.032	0.37	0.032	0.34	0.038
Gov't cons. shock	σ_g	0.48	0.024	0.48	0.024	0.48	0.010
Price markup shock	σ_p	0.14	0.012	0.14	0.012	0.14	0.013
MA(1) price markup shock	ϑ_p	0.84	0.047	0.84	0.047	0.72	0.072
Wage markup shock	σ_w	0.37	0.020	0.37	0.020	0.36	0.023
MA(1) wage markup shock	ϑ_w	0.98	0.006	0.98	0.006	0.97	0.012
Quarterly infl. rate. in S.S.	$\bar{\pi}$	0.77	0.101	0.84	0.101	0.76	0.010
Inflation response	r_π	1.61	0.147	1.94	0.147	1.98	0.201
Output gap response	r_y	0.02	0.014	0.06	0.014	0.11	0.025
Diff. output gap response	$r_{\Delta y}$	0.17	0.024	0.19	0.024	0.26	0.041
Mon. pol. shock std	σ_r	0.17	0.021	0.18	0.021	0.23	0.016
Mon. pol. shock pers.	ρ_r	0.07	0.045	0.03	0.045	0.20	0.053
Interest rate smoothing	ρ_R	0.87	0.015	0.90	0.015	0.78	0.024
Term spread response	r_s	0.03	0.043	0.01	0.043	0.34	0.003
Risk spread response	r_b	–0.18	0.051	–0.19	0.051	–0.01	0.022
Gov't yield weight.	κ	0.38	0.185	0.53	0.185	0.14	0.194
Average term spread	$\bar{r}^G-\bar{r}$	0.13	0.056	0.08	0.056	0.21	0.005
Term premium pers.	ρ_{tp}	0.85	0.036	0.82	0.036	0.87	0.017
Term premium shock	σ_{tp}	0.19	0.018	0.20	0.018	0.15	0.011
Trans. Prob. – R1 to R2	p_{12}					0.01	0.003
Trans. Prob. – R2 to R1	p_{21}					0.15	0.034
Log marginal likelihood		Laplace	-1196.58	Laplace	-1166.38	Laplace	-1176.76

Note: See notes to Table 2.2. The "ZLB–No Shock Uncer." allows the duration of the ZLB to be endogenous as described in the main text but does not take future shock uncertainty into account, whereas the "ZLB–Sigma filter" results incorporates the time-varying and asymmetric forecast uncertainty in the covariance matrix. Finally, the "RS-approach" assumes that the central bank follows the Taylor rule (4.1) in Regime 1 and while $R_t = 0$ in Regime 2. All priors are adopted from Table 2.2. For the transition probabilities p_{12} and p_{21} in the RS-approach, we use a beta distribution with means 0.10 and 0.30 and standard deviations 0.05 and 0.10 respectively.

In terms of marginal likelihood, we see by comparing the results in Table 4.1 with the log marginal likelihood (LML, henceforth) in Table 3.2 (-1200.10), that all variants in which we impose the ZLB is associated with an improvement in LML. Given that LSW [82] found that imposing the ZLB was associated with a deterioration of LML, the improvement in the table may seem surprising at first glance. The reason why we obtain an improvement here is that we assume the central bank is smoothing over the shadow rate in 4.1. Had we instead followed LSW and assumed that the CB smoothed over the actual interest rate, LML equals -1209.19 which represents a noticeable deterioration in posterior odds.[15] When we impose the ZLB with the Sigma filter or through a regime-switching approach, the improvement relative to the LSW-approach is substantial. Moreover, the results suggest that our Sigma filter approach with endogenous ZLB duration is preferred by the data visavi the regime-switching approach. Even so, it has to be recognized that the endogenous ZLB regime has an advantage over the regime-switching model in that we assume a lower for longer interest rate policy (smoothing over the shadow rate) can stabilize the economy. In the RS-approach, we do not allow for this possibility. If we instead in the regime-switching framework allows for an active fiscal policy in the ZLB regime, so that government spending in eq. (2.15) reacts to the output gap, we find that LML improves to -1168.29. This is very close to the LML obtained for the endogenous ZLB model with the Sigma filter. Hence, we conclude that the posterior odds are similar for both approaches.

To sum up, we can safely conclude that imposing the ZLB in estimation does not seem to entail any considerable changes in model parameters, apart from the fact that the estimated degree of stickiness in price and wage setting is enhanced when including data following the great moderation period. By and large, the same findings hold up for our variant of the Gali, Smets and Wouters [64] model, with the possible exception that the degree of wage and price stickiness do not increase when extending the sample in this model (see Appendix C for further details). However, even if the parameters are not much affected by imposing the ZLB, it is concievable that the ZLB impacts the models' properties in other dimesions. We explore this next.

5. Assessing the Empirical Impact of the ZLB

In this section, we do posterior predictive analysis aimed at quantifying the impact of imposing the ZLB. We will compare properties of the models estimated without the ZLB to the best-fitting model with endogenous ZLB duration (i.e. the empirical ZLB model with the Sigma filter). We do four experiments. First, we compare filtered shocks in the no-ZLB with those obtained in the

[15] This deterioration of the LML was induced mainly by requiring that all current and anticipated policy shocks should be positive for the predictive density of the model. This constraint on the shocks was very important in the model without shadow rate smoothing and resulted in higher estimates of the nominal stickiness in prices and wages. With smoothing over the shadow rate, the "lower for longer" argument makes this constraint less binding even for models with more moderate degree of stickiness.

ZLB model. Next, we study the extent to which the ZLB alters the forecasts at different points during the ZLB episode. Third, we study impulse responses for some key shocks in normal times and in the ZLB model during the crisis. Fourth and finally, we quantify the costs of the ZLB in our models and compare our numbers with existing findings in the literature.

5.1. Historical Shock Estimates

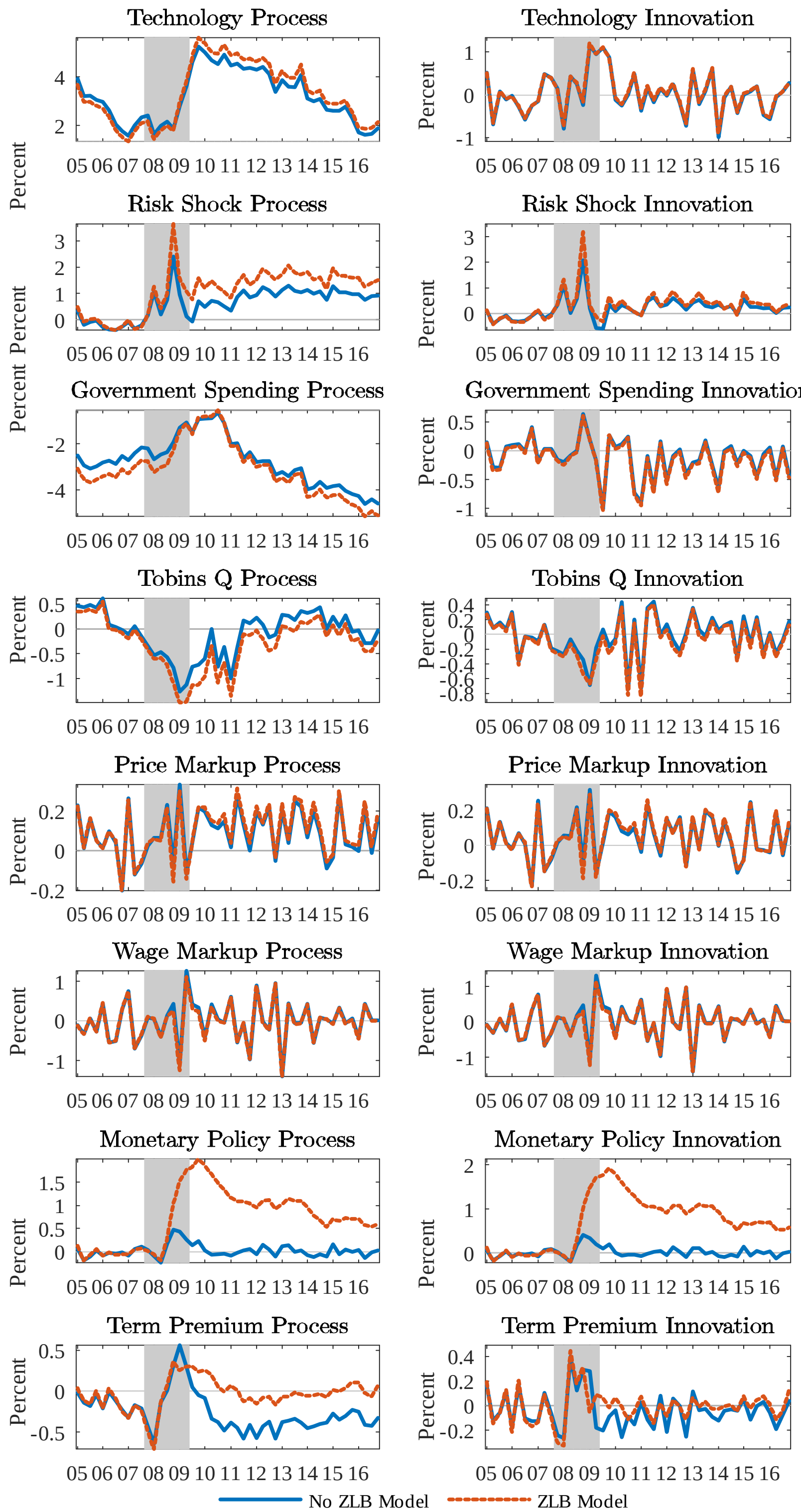
In Figure 5.1, we report the filtered shocks (i.e. the $\hat{\varepsilon}_t$'s, left column) and the innovations to these shocks (i.e. the η_t's, right column) for the benchmark model with and without imposing the ZLB for the period $2005Q1 - 2016Q4$.[16] The ZLB results pertain to the variant estimated with the Sigma-filter (see middle column in Table 4.1).

As can be seen from the figure, there are generally very small differences for the non-monetary policy shocks. So the ZLB does not change much the filtering of those shocks. For the two shocks influenced by the central bank, the monetary policy shock $\hat{\varepsilon}_t^r$ and the term-premium shock $\hat{\varepsilon}_t^{tp}$, the differences are larger. In particular, the filtered monetary policy shocks becomes and remain strongly positive in the ZLB model following the intensification of the recession in 2008Q3. In the no ZLB model, the monetary policy shocks are positive – but smaller – during the accute phase of the crisis but then disippates after the end of the recession and hovers around nil from 2010 and onwards.

Before we turn to the differences between filtered monetary policy and term-premium shocks in the no-ZLB and ZLB models in more detail, it is useful to discuss which fundamental shocks the ZLB and no ZLB model singles out as the drivers behind the intensificiation of the crisis in the fall of 2008. Closer inspection of Figure 5.1 documents that the key innovations happened to technology, investment specific technology (the Tobin's Q-shock), and the risk-premium shock during the most intense phase of the recession. More specifically, the model filters out a very large positive shock to technology (about 1.5 percent as shown in the upper left panel, which corresponds to a 3.4 standard error shock) in 2009Q1. In 2008Q4 and 2009Q1, the model also filters out two negative investment specific technology shocks (about -1 and -1.5 percent – or 2.0 and 3.7 standard errors – respectively). The model moreover filters out a large positive risk shocks in 2008Q3–Q4, and in 2009Q1 (0.5, 1.5, and 0.5 percent respectively, equivalent to 1.9, 6.0 and 2.8 standard errors).[**Jesper: need to check these numbers!**] These filtered shocks account for

[16] The shocks and innovations reported in the figure are the updated shocks and innovations conditional on information in period t, i.e. $\varepsilon_{t|t}$ and $\eta_{t|t}$.

Figure 5.1: Assesing the Impact of ZLB on Filtered Shocks in Benchmark Model.



the bulk of the sharp decline in output, consumption and investment during the acute phase of the crisis at the end of 2008 and the beginning of 2009. Our finding of a large positive technology shock in the first quarter of 2009 may at first glance be puzzling, but is driven by the fact that labor productivity rose sharply during the most acute phase of the recession. The model replicates this feature of the data by filtering out a sequence of positive technology shocks.[17] These technology shocks will stimulate for output, consumption and investment. The model thus needs some really adverse shocks that depresses these quantities even more and causes hours worked per capita to fall, and this is where the positive risk premium and investment specific technology shocks come into play. These shocks cause consumption (risk premium) and investment (investment specific) – and thereby GDP – to fall. Lower consumption and investment also causes firms to hire less labor, resulting in hours worked per capita to fall.

Two additional shocks that helps account for the collapse in activity at the end of 2008 is the monetary policy and term-premium shocks shown in the bottom panels (expressed at a quarterly rate). These shock becomes quite positive in 2008Q4 and 2009Q1 in the no-ZLB model; in annualized terms the monetary policy shocks equals roughly 150 (1.6 standard errors) and 250 (2.8 standard errors) basis points in each of these quarters respectively. Moreover, the annulized term premium in the no-ZLB model increases 2 percent between 2008Q3 (when it is roughly nil) and 2009Q1. As the actual observations for the annualized federal funds rate is about 50 and 20 basis points, these sizable shocks suggests that the zero lower bound is likely to have been a binding constraint, at least in these quarters. This finding is somewhat different from those of Del Negro and Schorfheide [41] and Del Negro, Giannoni, and Schorfheide [40], who argued that the zero lower bound was not a binding constraint in their estimated models.

In the ZLB variant of the model, we see from Figure 5.1 that the filtered monetary policy shock (and innovation) rises even more in 2008Q4 – i.e. the period the lower bound becomes binding for the federal funds rate – and then remains elevated relative to the no ZLB model for the remainder of the sample period. The large positive monetary policy shocks starting in 2008Q4 reflect that our filtered estimate of the shadow rate $\hat{R}_t^*$ falls and remains well below zero for a protracted

[17] Our finding of a very persistent rise in the exogenous component of total factor productivity (TFP) during the crisis is seemingly at odds with Christiano, Eichenbaum and Trabandt [29], who reports that TFP fell during the recession. Gust et al. [70] also report negative innovations to technology in 2008 (see Figure 6 in their paper). While a closer examination behind the differences would take us too far, we note that our filtered innovations to technology are highly correlated with the two TFP measures computed by Fernald [55]. When we compute the correlations between our technology innovations η_t^a, shown in the right column in Figure 5.1, and the period-by-period change in the raw and utilization-corrected measure of TFP by Fernald, we learn that the correlation between our innovations and his raw measure is almost 0.5 and as high as 0.6 for his utilization adjusted series. As we are studying first differences and innovations, this correlation must be considered quite high and lends support for our basic result that weak TFP growth was not a key contributing factor to the crisis; see LSW [82] for more details.

time period. Because the actual federal funds rate did not fall below nil, the policy rule the fed is assumed to follow while constrained by the lower bound – see the policy rule (4.1) – requires positive monetary policy shocks so that $\hat{R}_t^* + \varepsilon_t^r = -\bar{r}$ and thus that the actual policy does not fall below zero. Note that "nature" of the policy shock changes when the zlb starts to bind on impact: the policy shock is no longer an independent shock orthogonal to other fundamental shocks, instead, the policy shock becomes endogenous and dependent on the other fundamental shocks that affect the gap between the shadow policy rate ($\hat{R}_t^*$) and the constraint ($-\bar{r}$).

The bottom left panel in the figure also shows that the ZLB drives up the filtered estimate of the term-premium shock persistently. This reflects that the ZLB model, in which the central bank smooths over the notional and not the actual policy interest rate is associated with a lower policy rate path relative to the no-ZLB model which generates a projection with a notably quicker normalization of policy rates (we will discuss the interest forecasts from the alternative models next). Since the two-year yield we use as observable when estimating the model is the sum of the expected policy rate path and the term-premium shock, we need an elevated term-premium in the ZLB model to account for the difference. The elevated term-premium puts some persistent downward pressure on the economic activity and inflation according to our estimated model, albeit the "lower for longer" policy (see Eggertsson and Woodford [49] and Reifschneider and Williams [87]) embedded into the policy rule (4.1) offsets some of this pressure.

Accounting endogenously for the ZLB constraint also affects the estimates of the risk premium and Tobin's Q shocks necessary for explaining the drop in consumption and investment. To the extent that these shocks were among the main drivers of the recession, they are also responsable for a large fraction of the induced ZLB constraint and the corresponding policy shocks. Actually, it turns out that the magnitude of these financial shocks is even reinforced in the model with ZLB compared to the model without ZLB, as they take over the impact of the independent policy shocks. As these shocks enter the model in the intertemporal first order conditions alongside the policy rate, their identification is most sensitive to the assumptions made about the ZLB and the anticipated policy rates. The presence of this type of shocks that act as close substitute for the policy-shocks also explain why the identification of other fundamental shocks is much less affected by the ZLB-assumption.

To preserve space, we report the results of the same exercise in our variant of the GSW model with unemployment in Appendix C. The results in this model are totally in line with those discussed here.

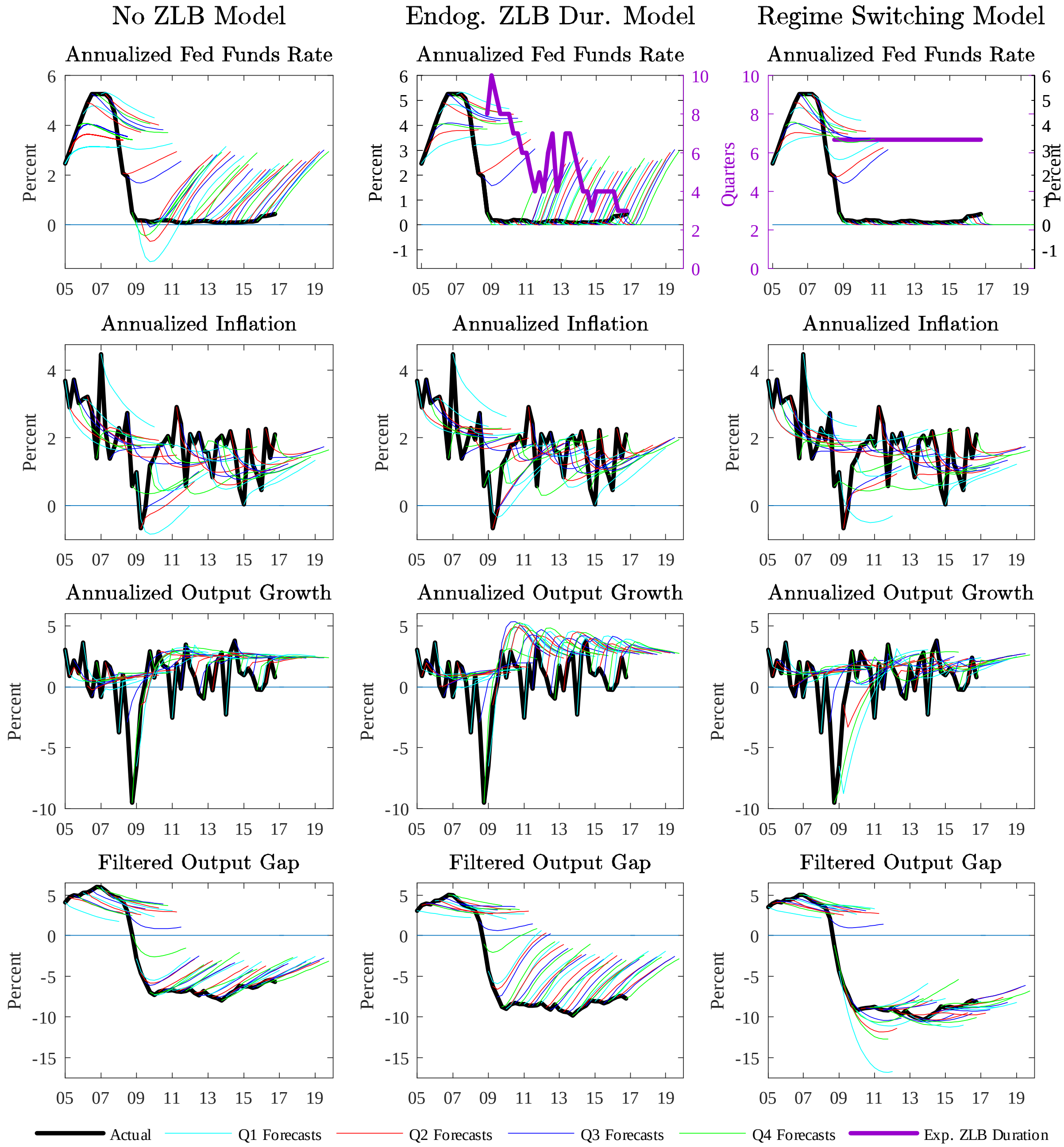
5.2. Influence on Forecasts

We now to turn to analyze the impact on the projections from the model. We will do this in two ways. First, we will show the impact on the rolling point forecasts. For all quarters between 2005Q1-2016Q4, we compute a three year ahead forecasts for the nominal policy rate, inflation, output growth, and the filtered output gap (treating $y_{t|T}^{gap}$ as actual data). As this exercise is aimed at being illustrative of the influence of alternative ways of impose the ZLB rather than focused on RMSEs on different projection horizons, we do not use realtime data. Studying the rolling forecasts during the crisis when the expected duration of the ZLB varies will provide us with useful diagnostics to understand the influence of the ZLB. Next, we complement this analysis by studying the complete prediction intervals for the two recessionary quarters 2008$Q4$ and 2009$Q1$.

In Figure 5.2 we report the results of the first exercise. The left column report results for the no-ZLB model, the middle column results for the ZLB-sigma filter model with endogenous duration, and the right column results for the regime-switching model with exogenous ZLB duration. The solid black line shows actual outcomes, the thin lines the 12-quarter rolling forecasts (with different colors for each quarter). Finally, the purple line in the upper middle and right panels for the ZLB models shows the expected duration of the liquidity trap in a given quarter, with the number of ZLB-quarters given on the right (model with endogenous duration) and left (regime-switching model) axes. As seen from the figure, the no-ZLB model needs positive monetary policy shock to respect the interest lower bound. But absent uncertainty about future shocks, the ZLB was not a binding constraint for a long time period; according to the historical rule the lower bound was only binding as of 2008Q4 and in 2009. After this period, the rule with constant parameters and steady real interest rate prescribes interest rate hikes as inflation and output growth rebounded, and the output gap was predicted to improve. Regarding the filtered output gap, it is interesting to notice that the model consistent output gap dropped roughly 10 percent during the crisis according to all three variants of the model. This fall is comparable in magnitude to the overall fall in actual output, which implies that potential gdp remained about unchanged during the crisis, due to the offsetting impact of higher productivity (which drive potential output up) and negative investment-specific and bond risk-premium shocks (which drive potential output down). Contractionary monetary and term-premium shocks then account for the large drop in actual output and the output gap.

Perhaps surprisingly, the rolling projections for the model in which we impose the ZLB are even more positive, for inflation, output growth and the output gap than in the no ZLB model. How is

Figure 5.2: Actual Outcomes and Rolling 12-Quarter Ahead Forecasts With and Without the ZLB in Benchmark Model.



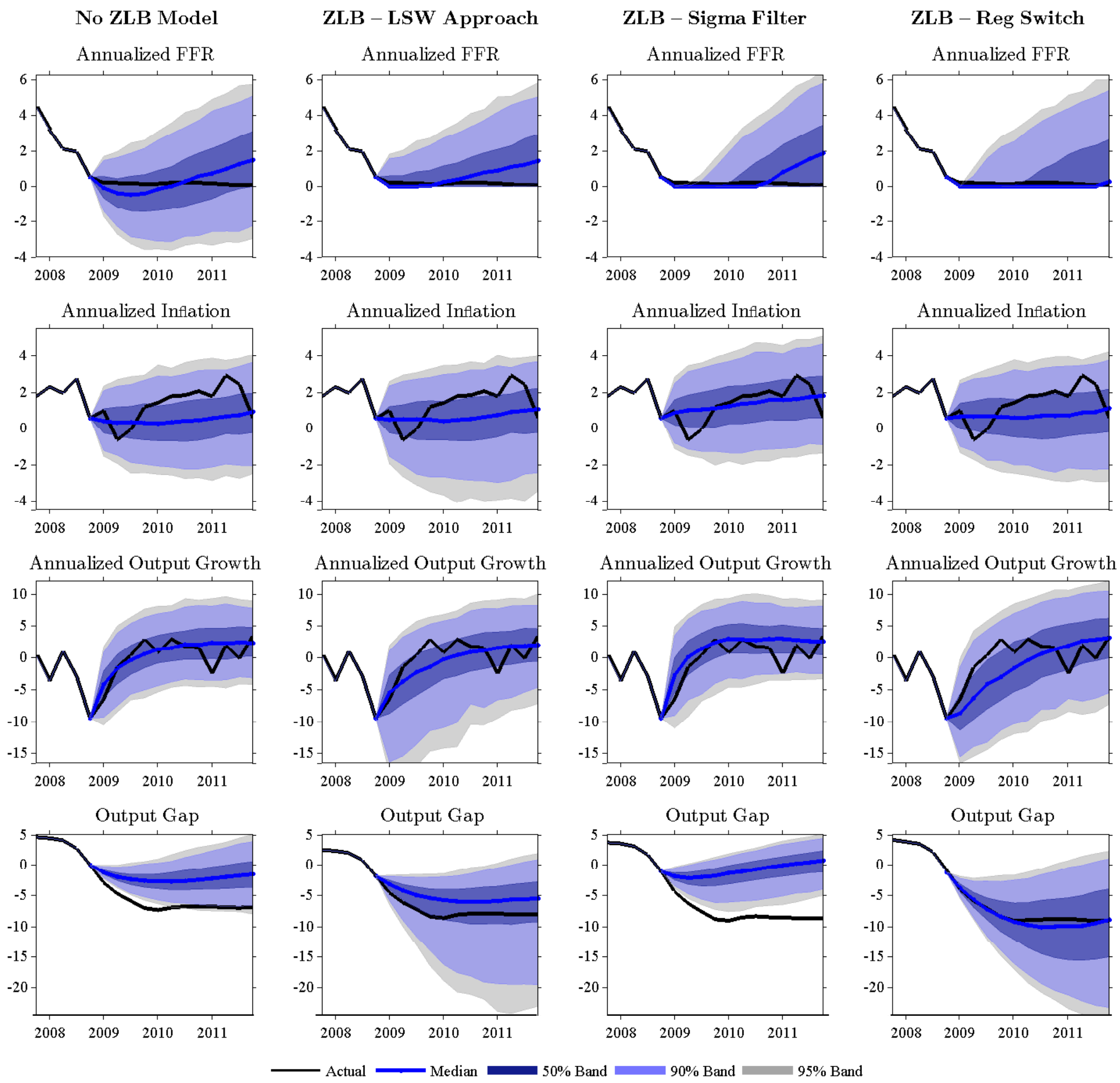
this possible? The explanation is our assumption that the central bank pursues a "lower for longer" policy at the ZLB (see 4.1) and smooths over the shadow rate (which is strongly negative) instead of the actual interest rate (which is around nil). The "lower for longer" dimension of the estimated policy rule implies that the expected ZLB duration is as long as 10 quarter in the beginning of 2009, well exceeding the number of quarters the no-ZLB model predicts the federal funds rate to be below zero. Essentially, this feature prolongs the ZLB episode but generates much more optimistic projections.

Finally, we have the regime-switching model in which the expected duration is 6.67 quarters in the ZLB regime, as seen from the left-hand axis in the upper right figure. Although the expected duration is only 6.67 quarters, we here assume that the regime is unaffected which leads to an unchanged federal funds rate projection in the ZLB regime. Because monetary policy is unresponsive in the ZLB regime and cannot act to stabilize the output gap and inflation in this regime, the projected paths are less benign compared to an approach where we explicitly allowed for the possibility that the conduct of monetary policy could switch back to the Normal regime each period. (We will allow for this possibility when we study predictive densities next.) Even so, it is important to recognize that the decision rules in the ZLB regime incorporates the expectations that this particular regime will not be lasting as long as in our simulations, and this implicitly provides some offset to this effect. Now, given that the central bank is prevented from stimulating the economy, the projections are much more pessimistic in the ZLB RS-model compared to the endogenous ZLB model. The RS-model suggests that a straight out deflationary scenario with a much larger drop in economic activity was possible in 2008Q4. In the absense of fiscal stimulus or other policy actions, it also features a notably more pessimistic view about the recovery in the aftermath of the crisis.

Figure 5.3 shows the forecast distribution (given the state in 2008Q4) in four variants of the benchmark model. The left column gives the results when the ZLB is counterfactually neglected, whereas the two columns to the right shows the results when the ZLB is imposed using the Sigma filter and Regime-Switching methods (the two estimated models in Table 4.1). As a reference point, the second-left column shows the same variant as in LSW, i.e. the ZLB is imposed without the Sigma filter and the central bank is assumed to smooth over the actual interest rate instead of the shadow interest rate.[18] Comparing this variant with the ZLB–Sigma filter model provides us with an assessment of the role of lagged interest rate concept (shadow versus actual interest rate).

[18] Thus, t**his variant differs from the model estimated in Table because this variant assumes the CB smooths over shadow rate (see the policy rule 4.1)**

Figure 5.3: Assesing the Impact of ZLB on Predictive Densisites 2008Q4 in Benchmark Model.



As expected, we see that the forecast distribution (black line is actual outcome and the blue line with dots is the median projection which is shown together with 50, 90 and 95 percent bands) in the variant of the model which counterfactually neglects the ZLB features symmetric uncertainty bands around the modal outlook, and the prediction densities covers well the actual outcomes for the fed funds rate, inflation and output growth. However, the filtered outcome for the output gap is well below the prediction densities from the model. So the no ZLB model is overly optimistic about the rebound from the recession. The variant of the ZLB model which imposes the ZLB when smoothing of the actual interest rate, the second column in the figure, have much less optimistic view about the recovery, and the uncertainty bands cover the filtered outcome for the output gap. Perhaps surprisingly, the modal outlook given the state in 2008Q4 for this variant ("ZLB – LSW approach") differs very little to the modal outlook in the "No ZLB model" which completely neglects the ZLB . Obviously, a key difference is that the median path of the federal funds rate is constrained by the lower bound in 2009, but below nil in the unconstrained version of the model. Still, the quantitative difference for the median projection for inflation and output growth is small. The most noticeable difference between the No ZLB model and this ZLB variant of the model for output growth and inflation are the uncertainty bands: they are wider and downward scewed in the model that imposes the ZLB constraint (the second to left column of Figure 5.3) compared to the No ZLB model that neglects the presence of the ZLB constraint.

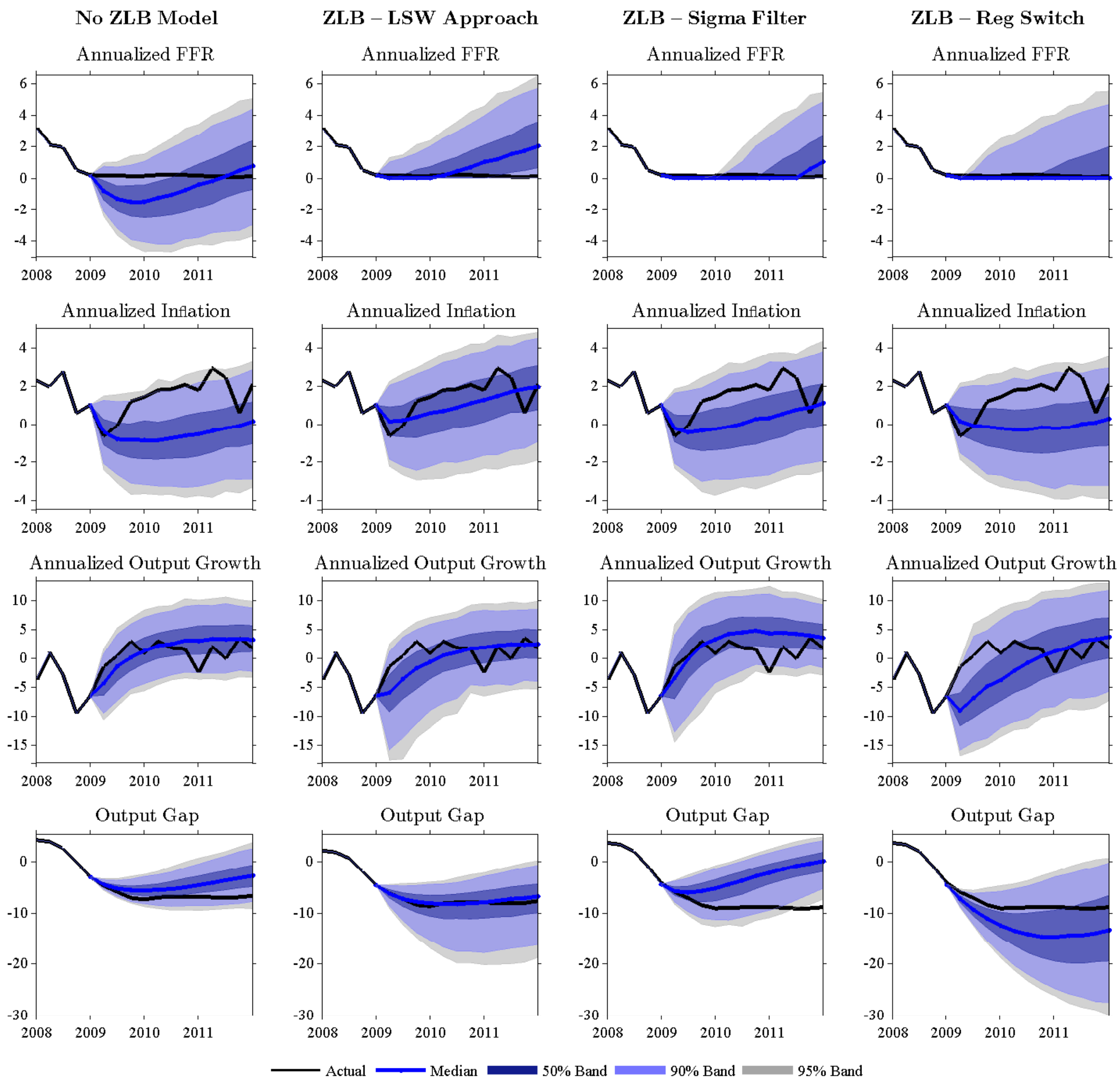
Turning to the model estimated with the Sigma filter and smoothing over the shadow rate, however, we see that the predictive densities are similar to those in the model in which we do not impose the ZLB. All the downside risk is gone, because the commitment to a lower-for-longer policy provides more stimulus for a prolonged period and thereby avoids the worst outcomes. The lower for longer policy is evident in the predictive densities for the annualized federal funds rate, which for the prediction contingent on the state in 2008Q4 imply zero probability before the end of 2009, in contrast to the case when smoothing of the actual interest rate. Another interesting difference is that the predictive density for the output gap is again notably higher than the actual outcome when smoothing over the notional rate. As the predictive density for output growth for this model variant is well in line with the actual outcome, this result may seem surprising at first glance. The explanation is the combination of very positive risk premium realisations and positive technology shocks. The risk shocks cause negative growth rates in actual output but not in potential output (which we assume is only affected by efficient shocks, so the output gap falls. At the same time, the positive technology shocks generate strong gains in potential growth but with slow price and wage adjustment the effects on actual growth are moderate. Hence, also this type of shock reduces the output gap. As a result, both shocks which our estimation procedure identify as key to explain the crisis pushes the output gap strongly negative, while partly offsetting each other for actual output growth.

The Regime-switching model, on the other hand, produces much less benign outlook prospects even when we, in contrast to Figure 5.2, allow for the possibility that the regime switches back to the Normal regime when we simulate the predictive densities. Because of the asymmetry created by the ZLB, and estimated relatively high probability of remaining in the ZLB regime, the lower percentiles of the forecast densities for the output gap are notably downward scewed and the modal outlook for the output gap is even lower than in the ZLB–LSW approach although the policy rate in the modal outlook remains at the lower bound for a notably longer period in the ZLB–RS model. This underscores the importance of providing policy accommodation at the ZLB.

We now turn the to the predictive densities conditional on the state in 2009Q1. These are reported in Figure 5.4. From the figure, we see that the overall message is similar to the previous densities conditional on 2008Q4. However, there are some important differences. First, all models imply a notably grimmer outlook, with straightout deflationary pressure in the No ZLB model. In the models with endogenous ZLB duration, the predicted output gap is well in line with the proceeding outcome in the ZLB – LSW model and in the model where the CB smooths over the shadow rate (ZLB – Sigma filter) the uncertainty bands now cover the filtered output gap series with the exception of the last part of the projection horizon. The "lower for longer" policy rule in this variant of the model now calls for a prolonged ZLB incident, lasting well into 2011 in the modal outlook. For the ZLB – Regime-Switching model, we do not see lift-off in the model outcome during the forecast horizon in the modal projection, and the predicted output gap is at low as negative 15 percent in 2011.

Overall this suggests that taking the ZLB into account in the estimation stage may be of key importance in forecasting. But assessing its economic consequences depends on the specifics of how the ZLB is modelled and the conduct of monetary policy. In our preferred model with endogenous ZLB duration, we find empirical support that a lower for longer approach by the Fed substantially mitigated the risk of adverse outcomes of the recession. But perhaps ironically, our empirical findings suggest that this model, from a forecasting perspective, is reasonably well approximated by a no ZLB model. Even so, drawing the conclusion, like Fratto and Uhlig [57], that the ZLB therefore is irrelevant can be missleading. It may be a reasonable approximation

Figure 5.4: Assesing the Impact of ZLB on Predictive Densisites 2009Q1 in Benchmark Model.



for some purposes, but clearly not in policy deliberations in a long-lived liquidity trap. For the Regime-switching model, the results clearly shows the importance of providing stimulus through fiscal policy or unconventional monetary policy actions when the ZLB binds.

5.3. Influence on Impulse Responses

In this section, we study the effects of a selected set shocks in during the crisis. Figure 5.5 shows the effects of positive technology and government spending shocks whereas Figure 5.6 shows the effects of positive wage markup and risk-premium shocks. The model we use to generate the impulses is the ZLB-model estimated with the Sigma filter (i.e. the middle column results in Table 4.1. We use the estimated model to compute impulse response functions given the state in each quarter 2008Q1-2016Q3. Since the ZLB is not expected to bind 2008Q1-2008Q3, the first three lines in the figures are identical and simply shows the impulses to one-standard deviation innovations for these shocks when monetary policy is constrained. However, from 20088Q4 and onwards. the ZLB is often expected to bind for multiple periods ahead, and the impulses reported in the figures then shows the partial impact of such a shock given the expected ZLB duration prevailing at each point in time.[19]

Turning to the results for the technology shock and government spending shock in Figure 5.5, we see that the effects of both shocks becomes elevated when the ZLB starts to bind in 2008Q4. For the technology shocks, we find that they have notably smaller positive effects on the economy in the near term, due to the fact that the deflationary pressure they imply are not met with nominal interest rate cuts by the central bank. However, in contrast to Eggertsson [50], they still have expansionary effects on the economy. The effects of a positive one std shock to government (about 0.5% of baseline GDP) shocks also becomes somewhat elevated when the ZLB starts to bind, but the rise in multiplier is modest. The modest rise reflects several issues pertaining to the estimated model. First, the government spending process is highly persistent (ρ_g varies between 0.97 and 0.98). This implies that a large part of the spending hike will occur when the ZLB is no longer binding, and this dampens the multiplier at the ZLB considerably by limiting its effects on both the actual and potential real interest rate (see e.g. Erceg and Lindé [52] for further discussion). Furthermore, because prices and wages are extremely sticky, the rise in inflation is modest and this dampens the decline in the actual rates. As a result, a highly persistent spending hike will only

[19] Specifically, to generate the impulses we first construct an unconditional forecast given the filtered state in each period 2008Q1-2016Q3, as in Figure 5.2. Next, we add a one standard error innovation for each of the shocks to the filtered state, and recompute a conditional forecast. The impulses are then computed as the difference between the conditional and unconditional projections.

Figure 5.5: Impulse Responses to Technology and Government Spending Shocks in Benchmark Model.

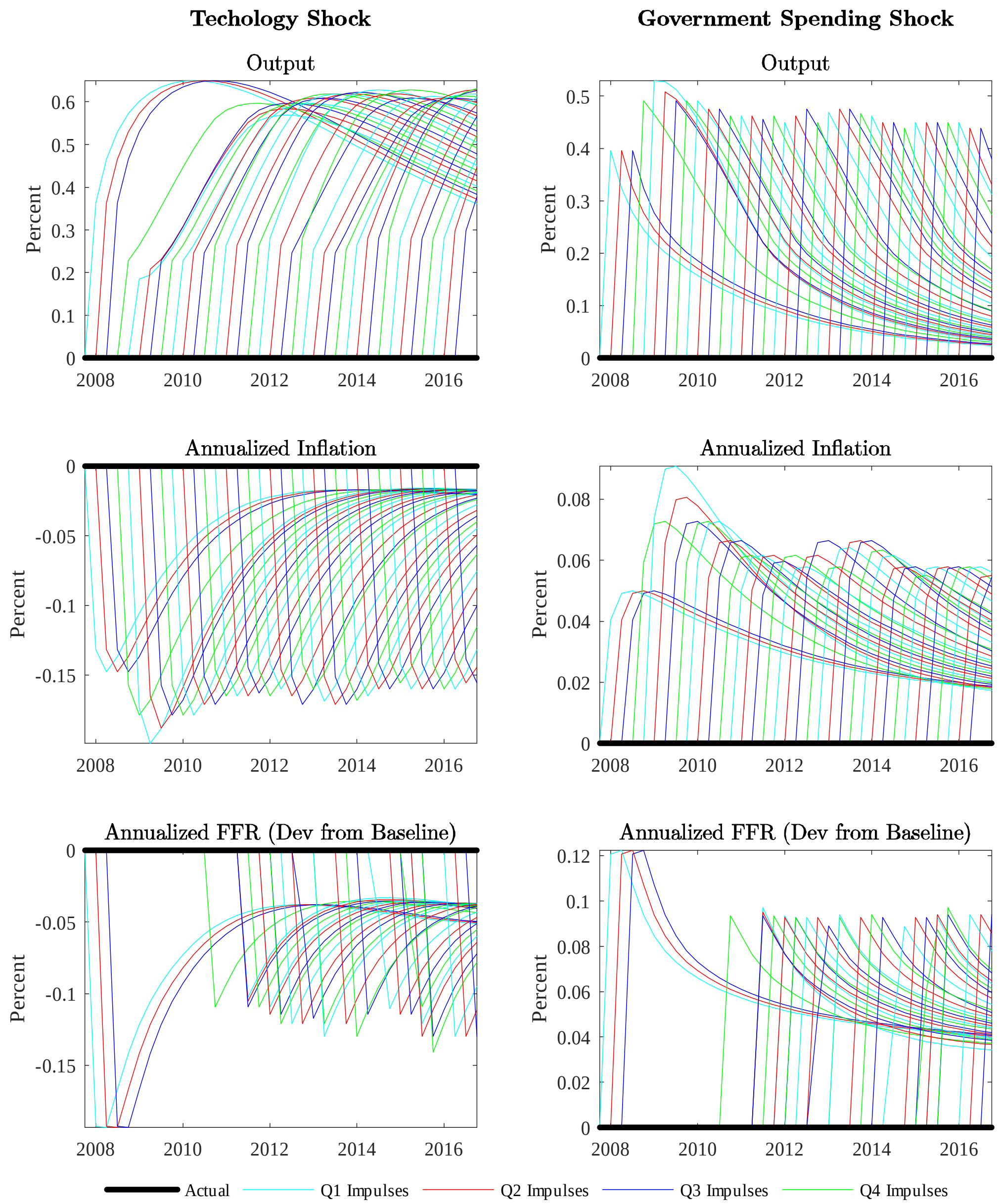
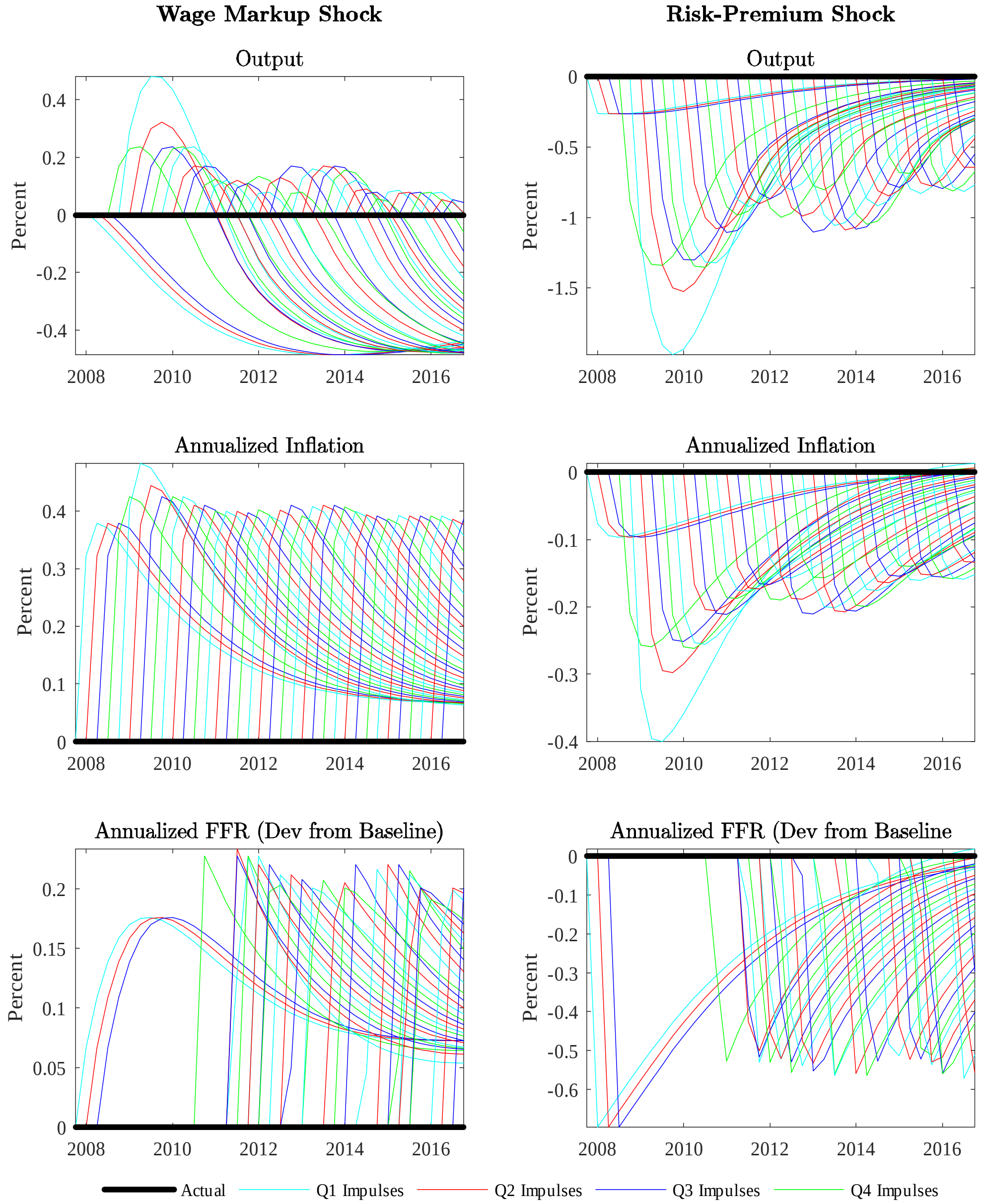


Figure 5.6: Impulse Responses to Wage Markup and Risk-Premium Shocks in Benchmark Model.



moderately lower the real interest rate gap and the spending multiplier hence remains muted even in 2009 although the expected ZLB duration is fairly long.

Moving on the wage markup shocks and risk-premium shocks in Figure 5.6, we have more interesting findings.[**Remains to be written.**]

5.4. The Macroeconomic Costs of the ZLB

We now turn to discussing the macroeconomic costs of the ZLB. To compute the macroeconomic costs of the interest rate lower bound, we condition on the state in 2008Q3 and make a counterfactual simulation of the economy in which all current and anticipated policy shocks ($\widehat{\varepsilon}^{r}_{t+h|t}$ for $t = 2008Q4, ..., 2016Q4$ and $h = 0, ..., H$) are set to nil and the switch from the normal times policy rule (smoothing over the actual lagged policy rate, see eq. 2.16) to the policy rule smoothing over the shadow rate (eq. 4.1) never occurs. For the Regime-switching model, we assume that the shift to the ZLB regime never happens, and use the smoothed estimates of the innovations as estimated by the active regime for each period (i.e. the best estimates of innovations) to compute the counterfactual simulation. We then compute the difference between the output paths generated in these simulations with the actual path for (log output).[20] Figure 5.7 shows the results of this exercise for three variants of the model. The No ZLB model is the 8-variables model in Table 3.2, the "Endogenous ZLB Duration model" is the ZLB-Sigma filter model in Table 4.1 where the Fed smooths over the shadow rate, and the regime-switching model is the ZLB–RS model in Table 4.1.

As can be seen from figure...[**Remains to be written.**]

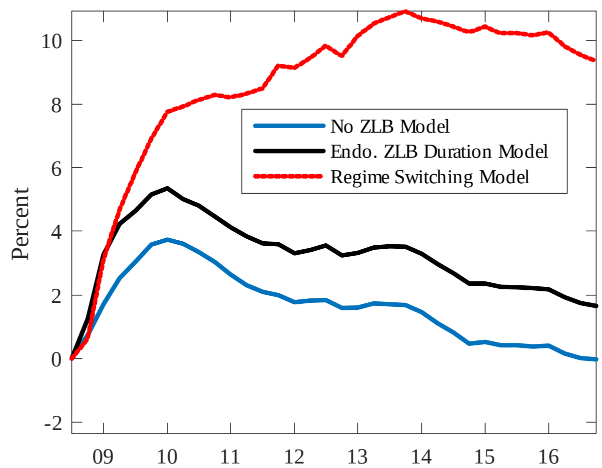
6. Concluding Remarks

Briefly summarize key findings. [**Remains to be done.**]

While our paper have taken some steps in assessing the role of the ZLB in workhorse macro-models, there are several important extensions warranted. An obvious extension would be to assess how our results hold up in a nonlinear framework. However, before such an exercise can be done in model environments with many shocks, endogenous state variables and observables used in the estimation, we need better numerical and estimation techniques to handle nonlinear models.

[20] For the endogenous ZLB duration model we use updated estimates of the innovations ($\eta_{t|t}$) instead smoothed innovations ($\eta_{t|T}$) as our algorithm used to estimate this variant of the model does not allow us to compute smoothed estimates. The updated innovations does not reproduce the actual data exactly, but the differences are small (the largest deviation for GDP is XX percent 2008Q4-2016Q4). To account for this discrepancy, we treat the simulated data series imposing the ZLB as the actual data when quantifying the cost of the ZLB for this model.

Figure 5.7: Output Costs of the Interest Rate Lower Bound in Alternative Variants of the Benchmark Model.



It would also be interesting to use our framework to examine if the steady state natural real rate has fallen (e.g. due to lower trend growth) and this has caused the (gross) steady state nominal interest rate R in equation (2.16) to fall; ceteris paribus this would call for an extended ZLB duration. Another interesting hypothesis to examine if the Federal Reserve decided to respond more vigorously to the negative output gap (i.e. r_y in equation (4.1) increased) from the outset of the Great Recession and thereafter.

Another extension is to integrate a better modelling of the financial sector in the models. Linde, Smets and Wouters [82] documented that some of the smoothed fundamental shocks are non-Gaussian, and strongly related to observable financial variables such as the Baa-Aaa and term spread, suggesting the importance of including financial shocks and frictions to account for large recessions. While we have in this paper implicitly allowed for financial frictions in the form of risk-premium, term-premium and Tobin's Q-shocks, it is important to examine the benefits of a more explicit structural approach to modelling the financial sector. This may involve a regime-switching approach as LSW documented that the impact of financial frictions appear to be highly state-dependent.

Another shortcoming of the model is its inability to explain the sharp sustained drop in output without generating deflation. In the data, output fell persistently with 10 percent while core inflation and inflation expectations remained well above deflationary territory. Even when accounting for the ZLB, the models we have analysed cannot account for this feature without positive markup shocks because the ZLB will exacerbate the effects on both output and inflation of fundamental demand and supply shocks. This suggest that important state-dependent asymmetries in the elasticity between output and inflation is needed to account for the episode. Gilchrist et al. (2016) argue that financial frictions explain why firms did not cut prices more during the recession. Linde and Trabandt (2017) provides an alternative explanation in an environment in which deflation is avoided due to a state-dependent sensitivity of firms prices to demand. It would be of interest to extend our work in these dimensions.

Finally, a key challenge for macro models at use in central banks following the crisis is to provide a framework where the central bank can use both conventional monetary policy (manipulating short rates) and unconventional policies (large scale asset purchases (LSAPs) and QE to affect term premiums) to affect the economy. We have provided a reduced form approach in which both term-premiums and short-term interest rates matter for equilibrium allocations. Even so, a more serious treatment of unconventional monetary policy in policy models seems to imply that we have to tackle one old key-challenge in macro modeling, namely the failure of the expectations hypothesis (see e.g. Campbell and Shiller [24]), in favor of environments where the expectations hypothesis does not necessarily hold. One theoretical framework consistent with the idea that large scale asset purchases can reduce term premiums for different maturities and put downward pressure on long-term yields is the theory of preferred habit, see e.g. Andrés, López-Salido, and Nelson [10] and Vayanos and Vila [93].

Appendix A. Linearized Model Representation

In this appendix, we summarize the log-linear equations of the basic SW-model stated in Section 2. The complete model also includes the eight exogenous shocks $\varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^i, \varepsilon_t^p, \varepsilon_t^w, \varepsilon_t^r, \varepsilon_t^{tp}$ and g_t, but their processes are not stated here as they were already shown in the main text. Consistent with the notation of the log-linearized *endogenous* variables $\hat{x}_t = dx_t/x$, the exogenous shocks are denoted with a 'hat', i.e. $\hat{\varepsilon}_t = \ln \varepsilon_t$.

First, we have the consumption Euler equation:

$$\widehat{c}_t = \tfrac{1}{(1+\varkappa/\gamma)} \mathrm{E}_t \widehat{c}_{t+1} + \tfrac{\varkappa/\gamma}{(1+\varkappa/\gamma)} \widehat{c}_{t-1} - \tfrac{1-\varkappa/\gamma}{\sigma_c(1+\varkappa/\gamma)} (\widehat{\widetilde{R}}_t - \mathrm{E}_t \widehat{\pi}_{t+1} + \widehat{\varepsilon}_t^b) - \tfrac{(\sigma_c-1)(w_*^h L/c_*)}{\sigma_c(1+\varkappa/\gamma)} (\mathrm{E}_t \widehat{L}_{t+1} - \widehat{L}_t), \quad (A.1)$$

where $\varkappa$ is the external habit parameter, σ_c the reciprocal of the intertemporal substitution elasticity, $w_*^h L/c_*$ the steady state nominal labor earnings to consumption ratio, and the effective interest rate $\widehat{\widetilde{R}}_t$ is given by

$$\widehat{\widetilde{R}}_t = (1-\kappa)\, \hat{R}_t + \kappa \hat{R}_t^G, \quad (A.2)$$

where the government yield $\hat{R}_t^G$, in turn, is determined by the central bank policy rate $\hat{R}_t$ plus a term-premium shock $\hat{\varepsilon}_t^{tp}$:

$$\hat{R}_t^G = \hat{R}_t + \hat{\varepsilon}_t^{tp}. \quad (A.3)$$

Next, we have the investment Euler equation:

$$\widehat{i}_t = \tfrac{1}{(1+\overline{\beta}\gamma)} \left(\widehat{i}_{t-1} + \overline{\beta}\gamma \mathrm{E}_t \widehat{i}_{t+1} + \tfrac{1}{\gamma^2 \varphi} \widehat{Q}_t^k \right) + \widehat{\varepsilon}_t^q, \quad (A.4)$$

where $\bar{\beta} = \beta\gamma^{-\sigma_c}$, φ is the investment adjustment cost, and the investment specific technology shock $\widehat{\varepsilon}_t^q$ has been re-scaled so that it enters linearly with a unit coefficient. Additionally $i_1 = 1/(1+\beta)$ and $i_2 = i_1/\psi$, where β is the discount factor and ψ is the elasticity of the capital adjustment cost function.

The price of capital is determined by:

$$\widehat{Q}_t^k = -(\widehat{\widetilde{R}}_t - \mathrm{E}_t \widehat{\pi}_{t+1} + \widehat{\varepsilon}_t^b) + q_1 \mathrm{E}_t r_{t+1}^k + (1-q_1) \mathrm{E}_t Q_{t+1}^k, \quad (A.5)$$

where $q_1 \equiv r_*^k/(r_*^k + (1-\delta))$ in which r_*^k is the steady state rental rate to capital, δ the depreciation rate.

Fourth, we have the optimal condition for the capital utilization rate $\widehat{u}_t$:

$$\widehat{u}_t = (1-\psi)/\psi \widehat{r}_t^k, \tag{A.6}$$

where ψ is the elasticity of the capital utilization cost function and capital services used in production $(\widehat{k}_t)$ is defined as:

$$\widehat{k}_t \ = \ \widehat{u}_t \ + \ \widehat{\bar{k}}_{t-1}, \tag{A.7}$$

where $\widehat{\bar{k}}_{t-1}$ is the physical capital stock which evolves according to the capital accumulation equation:

$$\widehat{\bar{k}}_t = \ \kappa_1 \ \widehat{\bar{k}}_{t-1} \ + (1-\kappa_1)\widehat{i}_t + \kappa_2 \widehat{\varepsilon}_t^q \tag{A.8}$$

with $\kappa_1 = (1-(i_*/\bar{k}_*)$ and $\kappa_2 = (i_*/\bar{k}_*)\gamma^2\varphi$.

The following optimal capital/labor input condition also holds:

$$\widehat{k}_t \ = \ \widehat{w}_t \ - \ \widehat{r}_t^k \ + \ \widehat{L}_t, \tag{A.9}$$

where $\widehat{w}_t$ is the real wage.

The log-linearized production function is given by:

$$\widehat{y}_t = \phi_p \ (\ \alpha\widehat{k}_t \ + \ (1-\alpha)\widehat{L}_t + \widehat{\varepsilon}_t^a), \tag{A.10}$$

in which ϕ_p is the fixed costs of production corresponding to the gross price markup in the steady state, and $\widehat{\varepsilon}_t^a$ is the exogenous TFP process.

Aggregate demand must equal aggregate supply:

$$\widehat{y}_t \ = \ \frac{c_*}{y_*}\widehat{c}_t \ + \ \frac{i_*}{y_*}\widehat{i}_t \ + \ g_t \ + \frac{r_*^k k_*}{y_*}\widehat{u}_t, \tag{A.11}$$

where g_t represents the exogenous demand component.

Next, we have the following log-linearized price-setting equation with dynamic indexation ι_p:

$$\widehat{\pi}_t - \iota_p\widehat{\pi}_{t-1} = \pi_1 \left(\mathrm{E}_t\widehat{\pi}_{t+1} - \iota_p\widehat{\pi}_t\right) - \pi_2\widehat{\mu}_t^p + \hat{\varepsilon}_t^p, \tag{A.12}$$

where $\pi_1 = \beta$, $\pi_2 = (1-\xi_p\beta)(1-\xi_p)/[\xi_p(1+(\phi_p-1)\epsilon_p)]$, $1-\xi_p$ is the probability of each firm being able to re-optimize the price each period, ϵ_p is the curvature of the aggregator function (eq. (2.2)), and the markup shock $\hat{\varepsilon}_t^p$ has been re-scaled to enter with a unit coefficient. The price markup $\widehat{\mu}_t^p$ equals the inverse of the real marginal cost, $\widehat{\mu}_t^p = -\ \widehat{mc}_t$, which in turn is given by:

$$\widehat{mc}_t = \ (1-\alpha) \ \widehat{w}_t^{real} + \alpha \ \widehat{r}_t^k - \widehat{\varepsilon}_t^a. \tag{A.13}$$

We also have the following wage-setting equation allowing for dynamic indexation of wages for non-optimizing households:

$$(1+\overline{\beta}\gamma)\widehat{w}_t^{real} \; - \widehat{w}_{t-1}^{real} - \overline{\beta}\gamma \mathrm{E}_t \widehat{w}_{t+1}^{real} = \tag{A.14}$$
$$\tfrac{(1-\xi_w\overline{\beta}\gamma)(1-\xi_w)}{[\xi_w(1+(\phi_w-1)\epsilon_w)]}\left(\tfrac{1}{1-\varkappa/\gamma}\widehat{c}_t - \tfrac{\varkappa/\gamma}{1-\varkappa/\gamma}\widehat{c}_{t-1} \; + \sigma_l \widehat{L}_t - \widehat{w}_t\right) - (1+\overline{\beta}\gamma\iota_w)\widehat{\pi}_t + \iota_w\widehat{\pi}_{t-1} + \overline{\beta}\gamma \mathrm{E}_t\widehat{\pi}_{t+1} + \hat{\varepsilon}_t^w,$$

where ϕ_w the gross wage markup, $1-\xi_p$ is the probability of each household being able to re-optimize its wage each period, ϵ_w is the curvature of the aggregator function (eq, 2.7), and σ_l determines the elasticity of labor supply given σ_c (see equation (2.9)). The exogenous wage markup shock $\hat{\varepsilon}_t^w$ has been re-scaled to enter linearly with a unit coefficient.

Finally, we have the monetary policy rule:

$$\widehat{R}_t \;=\; \rho_R \widehat{R}_{t-1} \;+\; (1-\rho_R)\Big(r_\pi \widehat{\pi}_t \;+\; r_y \hat{y}_t^{gap} + r_{\Delta y}\Delta\hat{y}_t^{gap} + r_s\varepsilon_t^{tp} + r_b\varepsilon_t^b\Big) \;+ \hat{\varepsilon}_t^r, \tag{A.15}$$

where $\hat{y}_t^{gap} = \widehat{y}_t - \widehat{y}_t^{pot}$, or in words: the difference between actual output and the output prevailing in the flexible price and wage economy in absence of the inefficient price and wage markup shocks. We solve for $\widehat{y}_t^{pot}$ by setting $\xi_p = \xi_w = 0$ (or arbitrary close to nil) and removing $\hat{\varepsilon}_t^w$ and $\hat{\varepsilon}_t^p$ from the system of equations given by (A.1) – (A.15). Note that when we impose the ZLB on the model, equation (A.15) is replaced by equation (4.1) whenever the ZLB binds.

Appendix B. Data

In this appendix, we provide the sources on the data we use in the analysis.

B.1. Benchmark Model

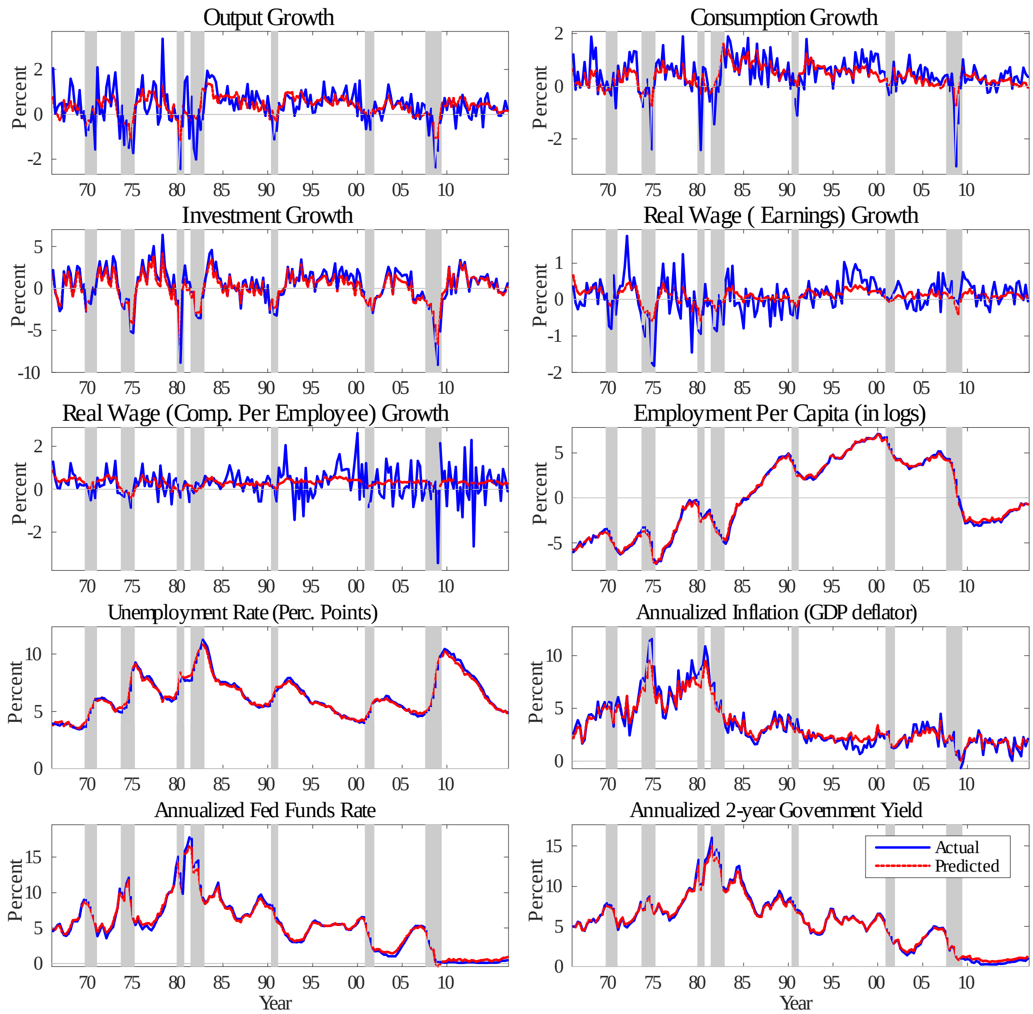
The benchmark model is estimated using eight key macro-economic time series: real GDP, consumption, investment, hours worked, real wages, prices, a short-term interest rate, and a two-year government yield. The Bayesian estimation methodology is extensively discussed by Smets and Wouters [91]. GDP, consumption and investment were taken from the U.S. Department of Commerce – Bureau of Economic Analysis data-bank – on February 25, 2017. Real gross domestic product is expressed in billions of chained 2009 dollars. Nominal personal consumption expenditures and fixed private domestic investment are deflated with the GDP-deflator. Inflation is the first difference of the log of the implicit price deflator of GDP. Hours and wages come from the BLS (hours and hourly compensation for the non-farm business, NFB, sector for all persons). Hourly compensation is divided by the GDP price deflator in order to get the real wage variable. Hours are adjusted to take into account the limited coverage of the NFB sector compared to GDP (the index of average hours for the NFB sector is multiplied with the Civilian Employment (16 years and over) . The aggregate real variables are expressed per capita by dividing with the population size aged 16 or older. All series are seasonally adjusted. The interest rate is the Federal Funds Rate. Consumption, investment, GDP, wages, and hours are expressed in 100 times log. The interest rate and inflation rate are expressed on a quarterly basis during the estimation (corresponding with their appearance in the model), but in the figures the series are reported on an annualized (400 times first log difference) or yearly (100 times the four-quarter log difference) basis.

B.2. Model with Unemployment

When we estimate the GSW model, we match 10 variables. Six of the eight data series used to estimate the SW model are also used here: real GDP, consumption, investment, prices, the federal funds rate, and the two-year government yield, with the first three expressed in per capita terms and log differenced. As this model is reformulated in terms of employment, we use per capita employment rather than hours worked, measured as civilian employment over civilian noninstitutional population in logs, i.e. 100ln(CE16OV/CNP16OV) using BLS mnemonics. In addition, we use two wage concepts. The first one is total compensation per employee obtained from the BLS Productivity and Costs Statistics. The second one is "average weekly earnings" from the Current Employment Statistics. These labor series is matched to the same real wage in the model, but are allow to have differ through i.i.d. measurement errors. Note that in the benchmark model, we use compensation per hour instead, in a way consistent with this model specification. Finally, we use the unemployment rate as an additional observable variable. This set of observables facilitate identification of both labor supply and wage markup shocks when estimating this model.

All variables are scaled the same way as in the benchmark model. The actual data and one-sided one step ahead predicted variables for the posterior mode estimates of this model are shown in Figure B.1. As can be seen from the figures, the one-step ahead variables generally track well their actual counterparts with the exception of the two real wage growth series, emphasizing the need of concurrent shocks to explain the evolution of these two series (Figure 3.1 shows the same tendency for the real wage growth series as in the benchmark model).

Figure B.1: Actual and Predicted Data in Model with Unemployment.



Appendix C. Additional Results for Model with Unemployment

In this appendix, we present additional results for the Galí, Smets and Wouters [64] model referred to in the main text.

C.1. Estimation Results

In Table C.1, we report the posteriors and log-marginal likelihoods for our variant of the Gali, Smets and Wouters (2011) model. In the table, we report results for three variants of model; in the left column, the middle column estimation results which impose the ZLB using the Sigma filter, and finally the right column in which we impose the ZLB The data used to estimate the model is shown in Figure B.1, along with the one-sided predicted values from the basic model which does not impose the ZLB in the estimation.

Table C.1: Posterior Distributions in Model with Unemployment.

Parameter		No ZLB Posterior mode	No ZLB Posterior std.dev. Hess.	ZLB – SIGMA filter Posterior mode	ZLB – SIGMA filter Posterior std.dev. Hess.	ZLB – Regime Switching Posterior mode	ZLB – Regime Switching Posterior std.dev. Hess.
Calvo prob. wages	ξ_w	0.65	0.041	0.64	0.040	0.65	0.035
Calvo prob. prices	ξ_p	0.65	0.046	0.63	0.029	0.67	0.036
Indexation wages	ι_w	0.11	0.049	0.11	0.132	0.10	0.045
Indexation prices	ι_p	0.27	0.098	0.28	0.056	0.20	0.079
Gross price markup	ϕ_p	1.93	0.098	1.93	0.265	1.97	0.093
Gross wage markup	ϕ_w	1.26	0.038	1.30	0.000	1.27	0.034
Capital production share	α	0.15	0.014	0.15	0.019	0.15	0.015
Capital utilization cost	ψ	0.68	0.110	0.62	0.117	0.62	0.119
Investment adj. cost	φ	3.41	0.585	2.87	0.047	3.73	0.695
Habit formation	$\varkappa$	0.65	0.049	0.64	0.034	0.69	0.035
Labor supply elast.	σ_l	3.84	0.500	4.64	0.035	4.16	0.488
Log hours worked in S.S.	$\bar{l}$	-4.32	1.167	-6.182	0.047	-5.97	0.800
Discount factor	$100(\beta^{-1}-1)$	0.16	0.061	0.15	0.123	0.13	0.053
Quarterly growth in S.S.	$\bar{\gamma}$	0.29	0.020	0.27	0.050	0.31	0.018
Real wage growth in S.S.	$\bar{\gamma}_w$	0.08	0.025	0.08	0.092	0.09	0.027
Labor supply wealth effect	υ	0.01	0.003	0.01	0.028	0.01	0.004
Response of a_t to ε_t^l	ρ_{al}	0.24	0.035	0.20	0.016	0.21	0.028
Stationary tech. shock	ρ_a	0.97	0.006	0.97	0.018	0.98	0.006
Risk premium shock	ρ_b	0.85	0.026	0.83	0.000	0.81	0.030
Invest. spec. tech. shock	ρ_i	0.94	0.036	0.96	0.029	0.93	0.039
Gov't cons. shock	ρ_g	0.97	0.010	0.97	0.030	0.97	0.010
Price markup shock	ρ_p	0.78	0.073	0.81	0.050	0.77	0.065
Wage markup shock	ρ_w	0.95	0.021	0.92	0.025	0.94	0.021
Response of g_t to ε_t^a	ρ_{ga}	0.95	0.094	0.95	0.114	0.94	0.093
Stationary tech. shock	σ_a	0.30	0.019	0.30	0.051	0.30	0.018
Risk premium shock	σ_b	0.82	0.200	0.74	0.093	0.87	0.167
MA(1) risk premium shock	ϑ_b	0.54	0.108	0.48	0.046	0.46	0.097
Invest. spec. tech. shock	σ_i	0.33	0.052	0.37	0.087	0.30	0.039
Gov't cons. shock	σ_g	0.42	0.022	0.42	0.001	0.42	0.022
Price markup shock	σ_p	0.10	0.038	0.09	0.028	0.12	0.033
MA(1) price markup shock	ϑ_p	0.66	0.107	0.69	0.088	0.63	0.102
Wage markup shock	σ_w	0.05	0.043	0.04	0.012	0.03	0.014
Labor supply shock	σ_l	1.03	0.141	1.24	0.099	1.11	0.134
Meas. err. - earn. comp.	σ_{w_1}	0.66	0.034	0.65	0.036	0.66	0.035
Meas. err. - ave. week earn.	σ_{w_2}	0.33	0.024	0.32	0.066	0.33	0.022
MA(1) wage markup shock	ϑ_w	0.59	0.285	0.42	0.103	0.41	0.177
Quarterly infl. rate. in S.S.	$\bar{\pi}$	0.64	0.084	0.54	0.287	0.50	0.082
Inflation response	r_π	1.70	0.141	1.82	0.331	1.88	0.160
Output gap response	r_y	0.10	0.023	0.14	0.056	0.16	0.028
Diff. output gap response	$r_{\Delta y}$	0.15	0.021	0.18	0.061	0.18	0.023
Mon. pol. shock std	σ_r	0.16	0.012	0.18	0.019	0.18	0.011
Mon. pol. shock pers.	ρ_r	0.04	0.031	0.05	0.063	0.08	0.042
Interest rate smoothing	ρ_R	0.87	0.019	0.84	0.030	0.82	0.026
Termp spread response	r_s	0.13	0.051	0.31	0.036	0.36	0.095
Risk spread response	r_b	0.13	0.047	0.11	0.017	0.09	0.029
Gov't yield weight.	κ	0.17	0.119	0.15	0.065	0.12	0.084
Average term spread	$\bar{r}^G$-$\bar{r}$	0.13	0.055	0.15	0.049	0.22	0.062
Term premium pers.	ρ_{tp}	0.88	0.028	0.85	0.052	0.88	0.021
Term premium shock	σ_{tp}	0.16	0.014	0.16	0.036	0.14	0.014
Trans. Prob. – R1 to R2	p_{12}					0.01	0.005
Trans. Prob. – R2 to R1	p_{21}					0.23	0.042
Log marginal likelihood		Laplace	-1156.47	Laplace	-1135.94	Laplace	-1144.22

Note: See notes to Table 4.1. The "No ZLB model" neglects the presence of the zero lower bound in the estimations, whereas the "ZLB-SIGMA filter" imposes the ZLB with the Sigma filter and allows the duration of the ZLB to be endogenous as described in the main text. Finally, the "RS-approach" assumes that the central bank follows the Taylor rule (4.1) in Regime 1 and while $R_t = 0$ in Regime 2. All priors are adopted from Tables 2.2 and 4.1. The priors for the parameters specific to the GSW model are as follows: ϕ_w – average wage markup – same as for ϕ_p; υ – parameter governing the wealth effect on labor suppy – follows a beta distribution with mean 0.5 and standard error 0.2; $\bar{\gamma}_w$ – the average real wage growth – follows a normal with mean 0.2 and standard error 0.1; ρ_{al} – governing the response of a_t innovations in the labor supply shock ε_t^l – follows a beta distribution with mean 0.5 and standard error 0.2; σ_l – standard deviation of the unit labor supply shock – same as for all other shocks; σ_{w_1}

and σ_{w_2} – real wage growth measurement errors – same as for σ_l.

C.2. Historical Shock Estimates

In Figure C.1, we report the historical shock decompositions for the GSW [64] model. Confirming the results in the SW model, the impact of the ZLB on all fundamental shocks are very modest. [**Remains to be written.**]

C.3. Influence on Forecasts

[**Remains to be written.**]

C.4. Influence on Impulse Responses

[**Remains to be written.**]

C.5. The Macroeconomic Costs of the ZLB

In Figure C.7, we report the results of the same exercise as in the benchmark model discussed in the main text. [**Remains to be written.**]

Appendix D. The ZLB Algorithm and the Likelihood Function

This appendix provides some details on the ZLB algorithm we use and how the likelihood function takes the ZLB into account. For more details on the ZLB algorithm we refer to Hebden, Lindé, and Svensson [72].

D.1. The ZLB Algorithm

The DSGE model can be written in the following practical state-space form,

$$\begin{bmatrix} X_{t+1} \\ Hx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}. \tag{D.1}$$

Here, X_t is an n_X-vector of *predetermined* variables in period t (where the period is a quarter) and x_t is a n_x-vector of *forward-looking* variables. The i_t is generally a n_i-vector of (policy) *instruments*

Figure C.1: Filtered Shocks With and Without the ZLB in Model with Unemployment.

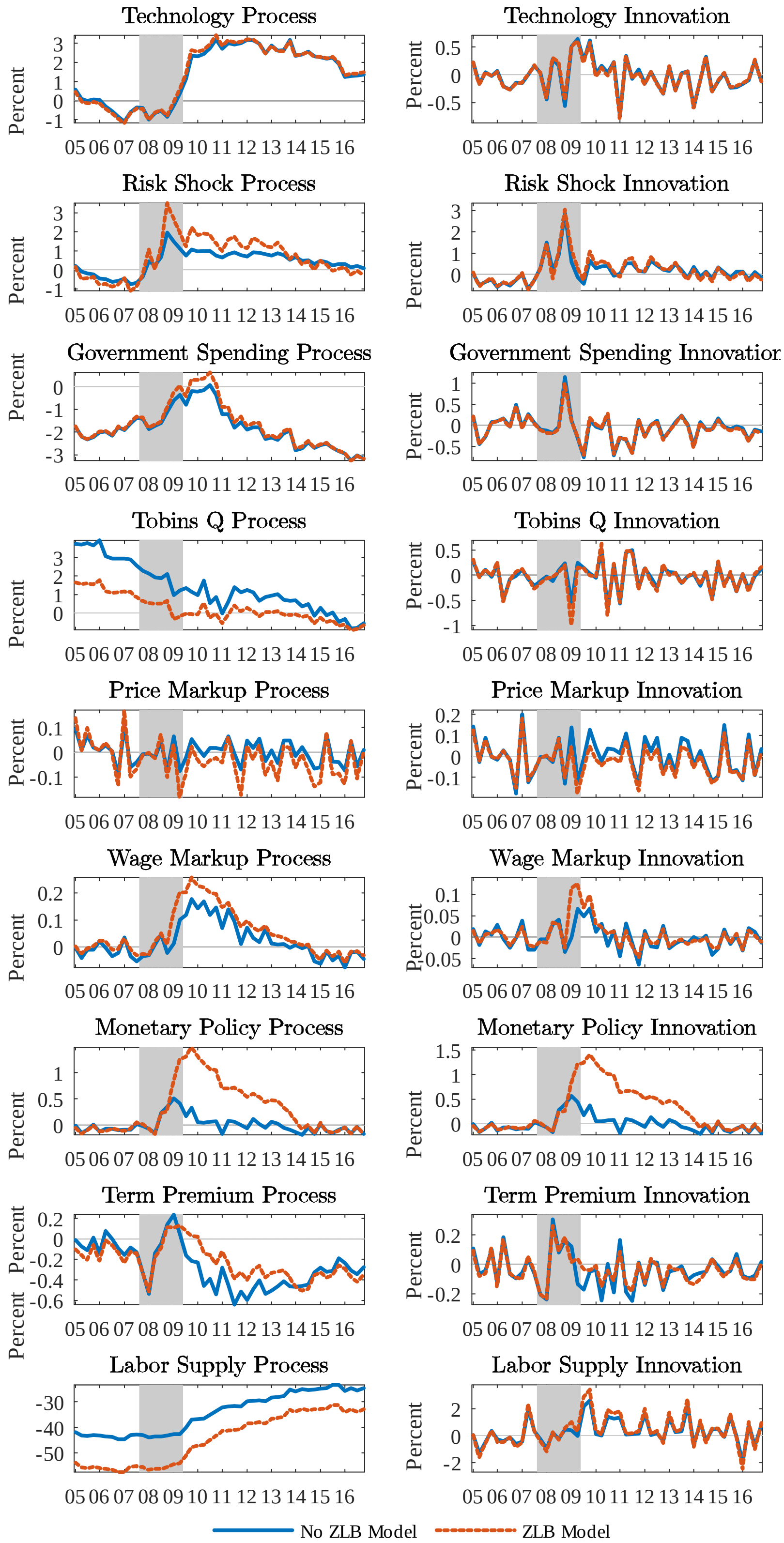


Figure C.2: Actual Outcomes and Rolling 12-Quarter Ahead Forecasts With and Without the ZLB in Model with Unempoloyment.

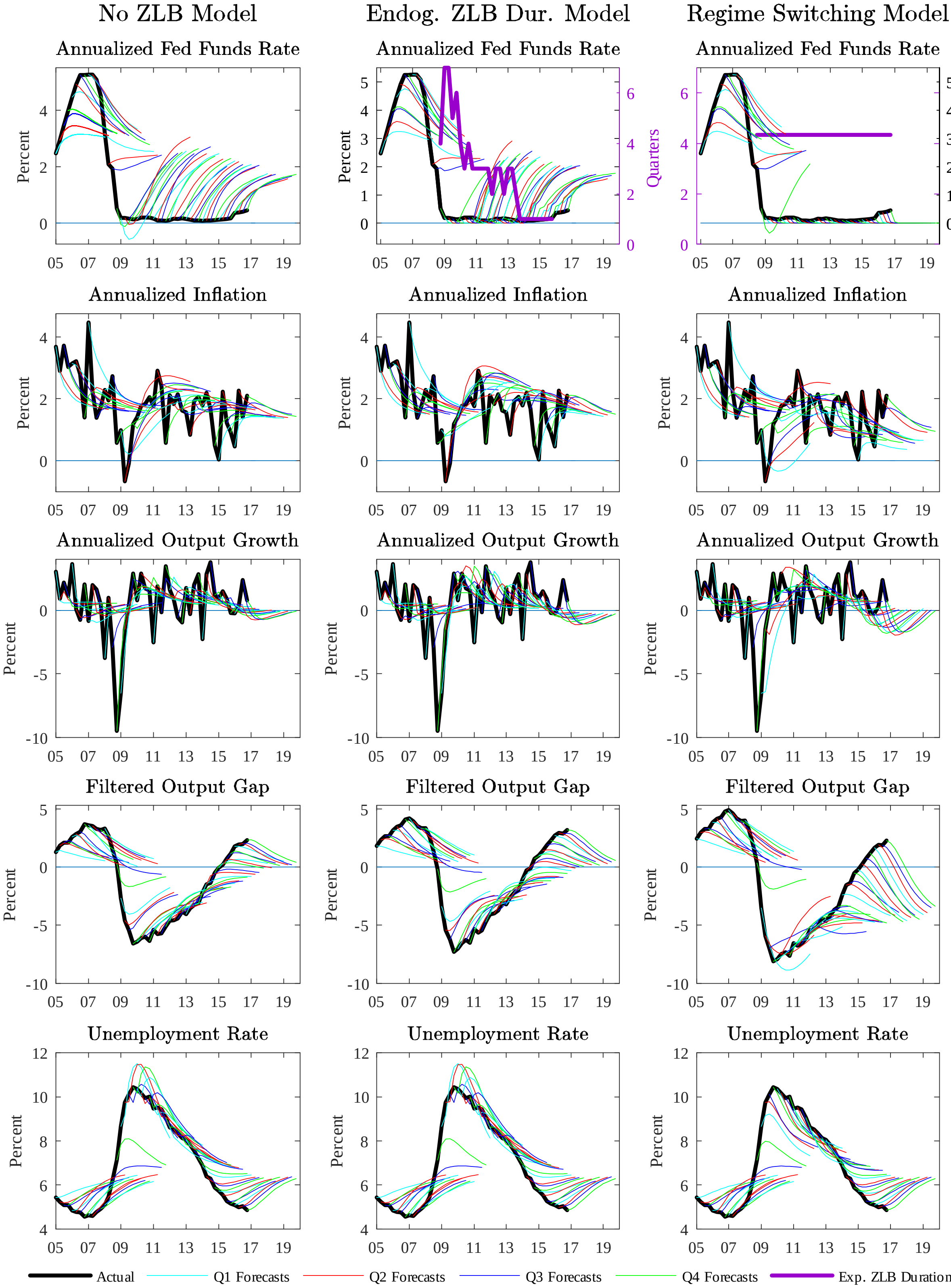


Figure C.3: Assesing the Impact of ZLB on Predictive Densisites 2008Q4 in Model with Unemployment.

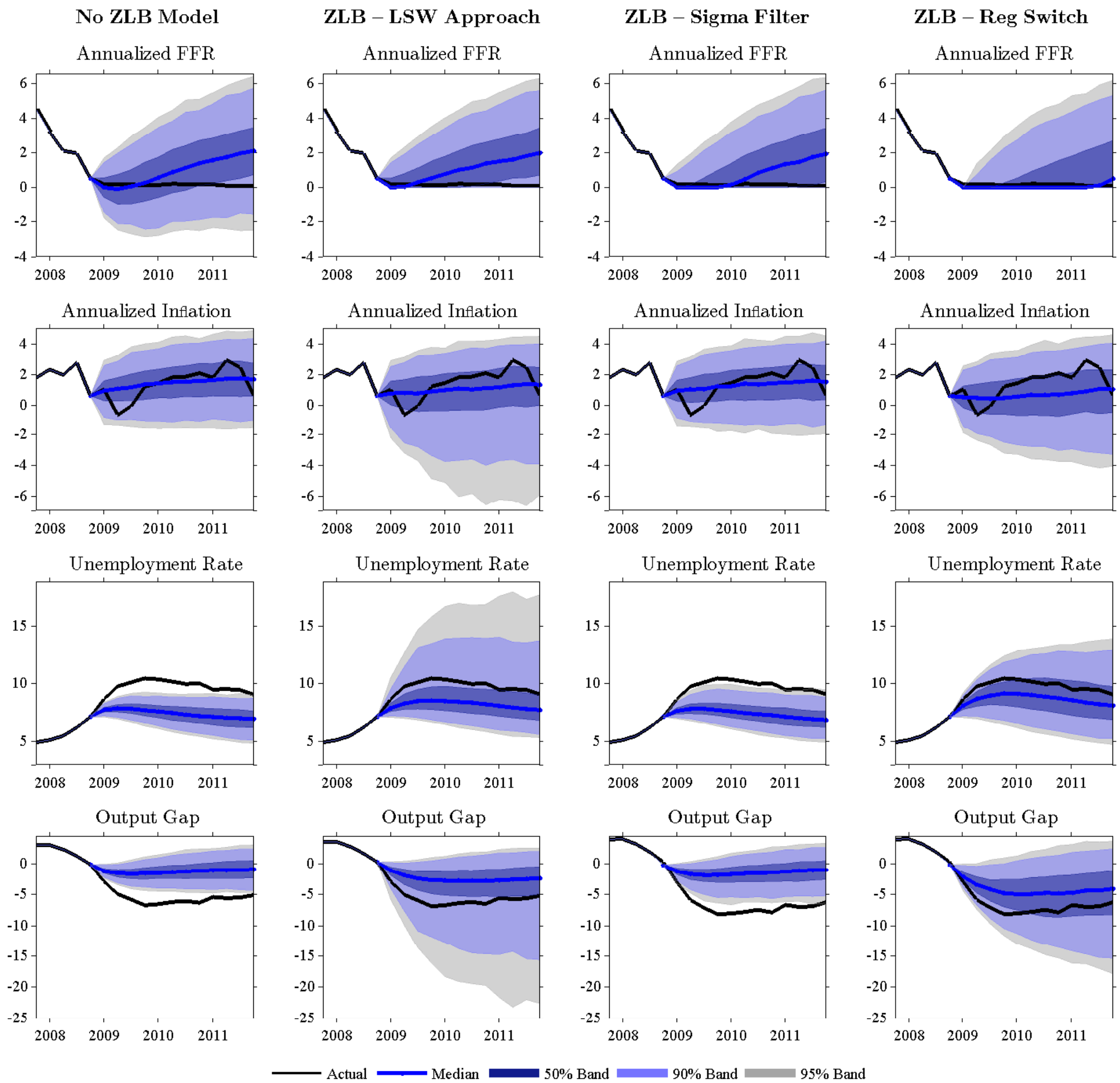


Figure C.4: Assesing the Impact of ZLB on Predictive Densisites 2009Q1 in Model with Unemployment.

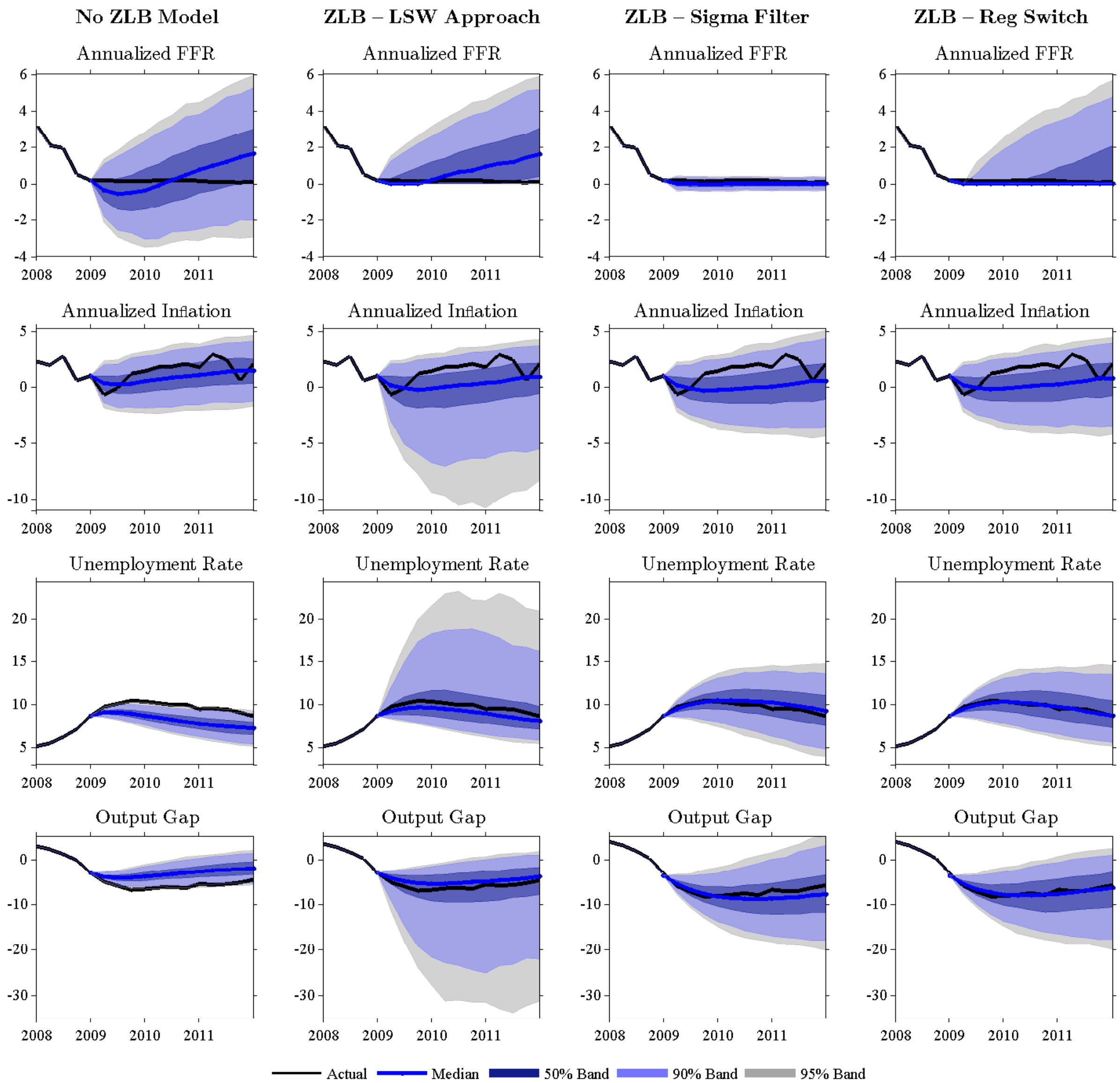


Figure C.5: Impulse Responses to Technology and Government Spending Shocks in Model with Unemployment.

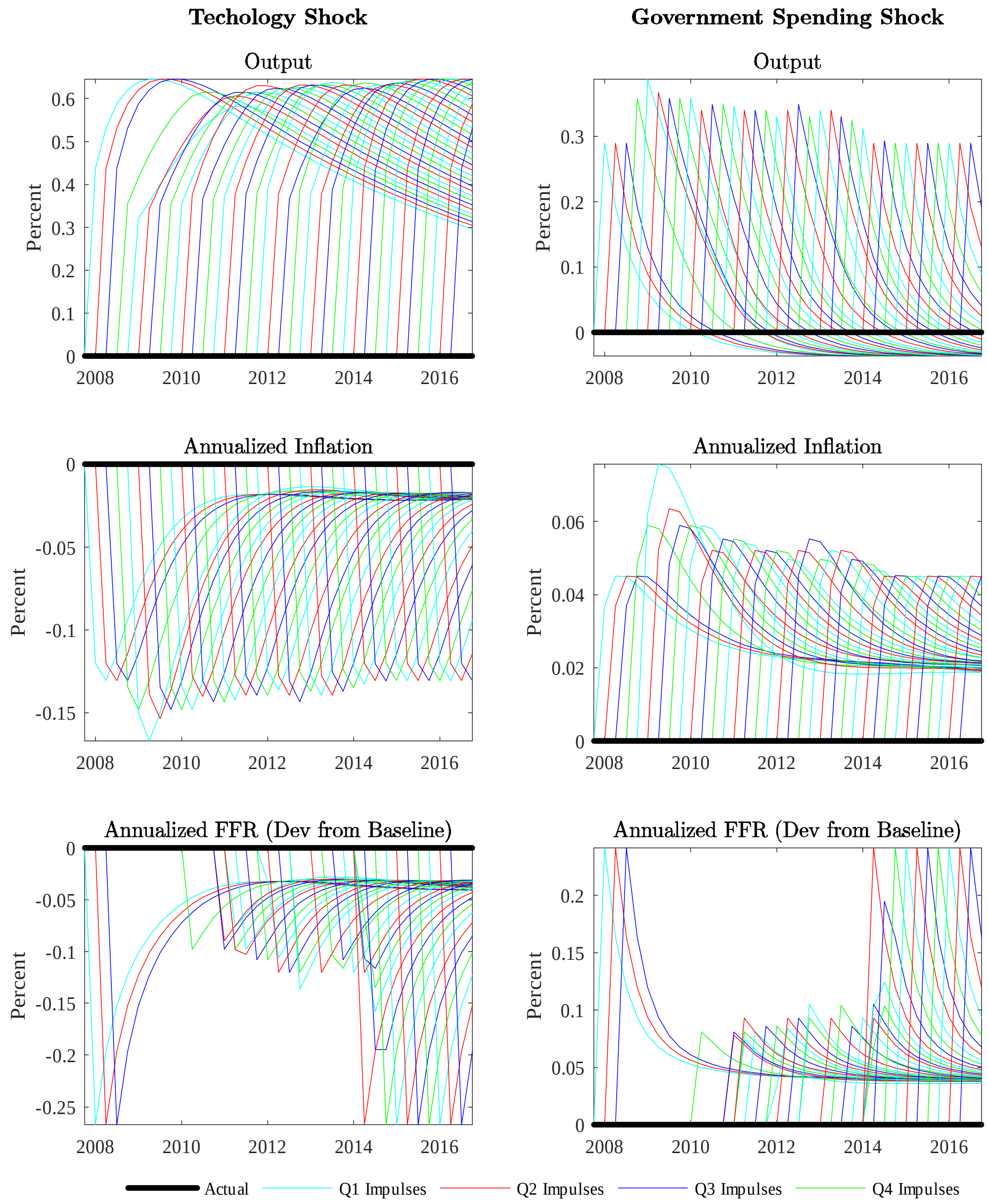


Figure C.6: Impulse Responses to Wage Markup and Risk-Premium Shocks in Model with Unemployment.

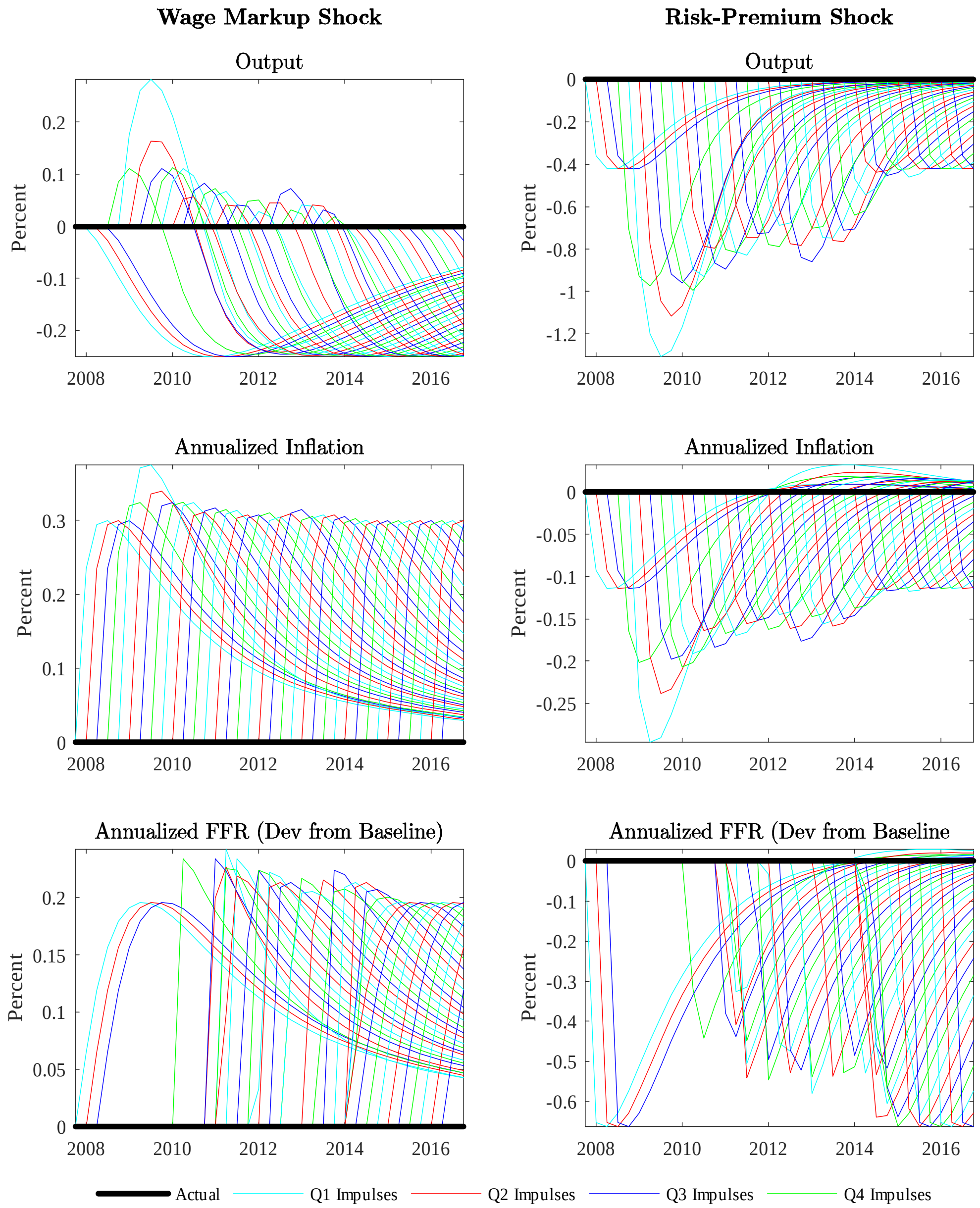
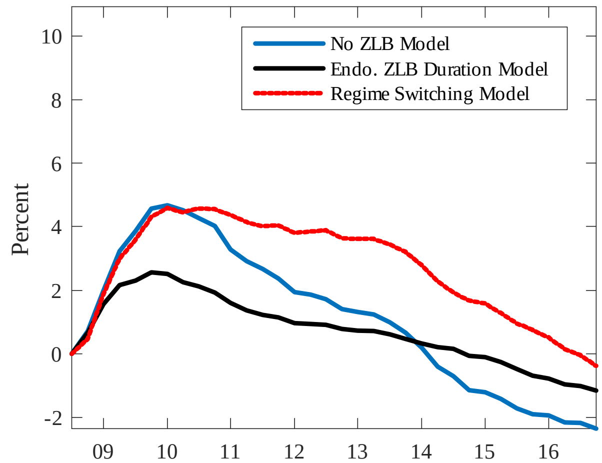


Figure C.7: Output Costs of the Interest Rate Lower Bound in Alternative Variants of the Model with Unemployment.



but in the cases examined here it is a scalar – the central bank's policy rate – giving $n_i = 1$. The ε_t is an n_ε-vector of independent and identically distributed shocks with mean zero and covariance matrix I_{n_ε}, while A, B, C, and H are matrices of the appropriate dimension. Lastly $x_{t+\tau|t}$ denotes $\mathrm{E}_t x_{t+\tau}$, i.e. the rational expectation of $x_{t+\tau}$ conditional on information available in period t. The forward-looking variables and the instruments are the *non-predetermined* variables.[D.1]

The variables are measured as differences from steady-state values, in which case their unconditional means are zero. In addition, the elements of the matrices A, B, C, and H are considered fixed and known.

We let i_t^* denote the policy rate when we disregard the ZLB. We call it the *unrestricted* policy rate. We let i_t denote the actual or *restricted* policy rate that satisfies the ZLB,

$$i_t + \bar{\imath} \geq 0,$$

where $\bar{\imath} > 0$ denotes the steady-state level of the policy rate and we use the convention that i_t and i_t^* are expressed as deviations from the steady-state level. The ZLB can therefore be written as

$$i_t + \bar{\imath} = \max\{i_t^* + \bar{\imath}, 0\}. \tag{D.2}$$

We assume the unrestricted policy rate follows the (possibly reduced-form) unrestricted linear policy rule,

$$i_t^* = f_X X_t + f_x x_t, \tag{D.3}$$

[D.1] A variable is predetermined if its one-period-ahead prediction error is an exogenous stochastic process (Klein [75]). For (D.1), the one-period-ahead prediction error of the predetermined variables is the stochastic vector $C\varepsilon_{t+1}$.

where f_X and f_x are row vectors of dimension n_X and n_x respectively. From (D.2) it then follows that the restricted policy rate is given by:

$$i_t + \bar{\imath} = \max\{f_X X_t + f_x x_t + \bar{\imath}, 0\}. \tag{D.4}$$

Consider now a situation in period $t \geq 0$ where the ZLB may be binding in the current or the next finite number T periods but not beyond period $t+T$. That is, the ZLB constraint

$$i_{t+\tau} + \bar{\imath} \geq 0, \quad \tau = 0, 1, ..., T \tag{D.5}$$

may be binding for some $\tau \leq T$, but we assume that it is not binding for $\tau > T$,

$$i_{t+\tau} + \bar{\imath} > 0, \quad \tau > T.$$

We will implement the ZLB with anticipated shocks to the unrestricted policy rule, using the techniques of Laséen and Svensson [78]. Thus, we let the restricted and unrestricted policy rate in each period t satisfy

$$i_{t+\tau,t} = i^*_{t+\tau,t} + z_{t+\tau,t}, \tag{D.6}$$

for $\tau \geq 0$. The ZLB policy rule in (D.4) – as we explain in further detail below – implies that all current and future anticipated shocks $z_{t+\tau,t}$ in (D.6) must be non-negative, and that $z_{t,t}$ is strictly positive in periods when the ZLB is binding.

Disregarding for the moment when z_t are non-negative, we follow Laséen and Svensson [78] and call the stochastic variable z_t the deviation and let the $(T+1)$-vector $z^t \equiv (z_{t,t}, z_{t+1,t}, ..., z_{t+T,t})'$ denote a projection in period t of future realizations $z_{t+\tau}$, $\tau = 0, 1, ..., T$, of the deviation. Furthermore, we assume that the deviation satisfies

$$z_t = \eta_{t,t} + \sum_{s=1}^{T} \eta_{t,t-s}$$

for $T \geq 0$, where $\eta^t \equiv (\eta_{t,t}, \eta_{t+1,t}, ..., \eta_{t+T,t})'$ is a $(T+1)$-vector realized in the beginning of period t. For $T = 0$, the deviation is given by $z_t = \eta_t$. For $T > 0$, the deviation is given by the moving-average process

$$z_{t+\tau,t+1} = z_{t+\tau,t} + \eta_{t+\tau,t+1}$$

$$z_{t+\tau+T+1,t+1} = \eta_{t+T+1,t+1},$$

where $\tau = 1, ..., T$. It follows that the dynamics of the projection of the deviation can be written more compactly as

$$z^{t+1} = A_z z^t + \eta^{t+1}, \tag{D.7}$$

where the $(T+1) \times (T+1)$ matrix A_z is defined as

$$A_z \equiv \begin{bmatrix} 0_{T\times 1} & I_T \\ 0 & 0_{1\times T} \end{bmatrix}.$$

Hence, z^t is the projection in period t of current and future deviations, and the innovation η^t can be interpreted as the new information received in the beginning of period t about those deviations.

Let us now combine the model, (D.1), the dynamics of the deviation, (D.7), the unrestricted policy rule, (D.3), and the relation (D.6). Taking the starting period to be $t = 0$, we can then write the combined model as

$$\begin{bmatrix} \tilde{X}_{t+1} \\ \tilde{H}\tilde{x}_{t+1|t} \end{bmatrix} = \tilde{A} \begin{bmatrix} \tilde{X}_t \\ \tilde{x}_t \end{bmatrix} + \begin{bmatrix} C & 0_{n_X\times(T+1)} \\ 0_{(T+1)\times n_\varepsilon} & I_{T+1} \\ 0_{(n_x+2)\times n_\varepsilon} & 0_{(n_x+2)\times(T+1)} \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1} \\ \eta^{t+1} \end{bmatrix} \tag{D.8}$$

for $t \geq 0$, where

$$\tilde{X}_t \equiv \begin{bmatrix} X_t \\ z^t \end{bmatrix}, \quad \tilde{x}_t \equiv \begin{bmatrix} x_t \\ i_t^* \\ i_t \end{bmatrix}, \quad \tilde{H} \equiv \begin{bmatrix} H & 0_{n_x\times 1} & 0_{n_x\times 1} \\ 0_{1\times n_x} & 0 & 0 \\ 0_{1\times n_x} & 0 & 0 \end{bmatrix}.$$

Under the standard assumption of the saddle-point property (that the number of eigenvalues of $\tilde{A}$ with modulus larger than unity equals the number of non-predetermined variables, here $n_x + 2$), the system of difference equations (D.8) has a unique solution and there exist unique matrices M and F returned by the Klein [75] algorithm such that the solution can be written:

$$\tilde{x}_t = F\tilde{X}_t \equiv \begin{bmatrix} F_x \\ F_{i^*} \\ F_i \end{bmatrix} \tilde{X}_t\tilde{X}_{t+1} = M\tilde{X}_t + \begin{bmatrix} C\varepsilon_{t+1} \\ \eta^{t+1} \end{bmatrix} \equiv \begin{bmatrix} M_{XX} & M_{Xz} \\ 0_{(T+1)\times n_X} & A_z \end{bmatrix} \begin{bmatrix} X_t \\ z^t \end{bmatrix} + \begin{bmatrix} C\varepsilon_{t+1} \\ \eta^{t+1} \end{bmatrix},$$

for $t \geq 0$, and where X_0 in $\tilde{X}_0 \equiv (X_0', z^{0\prime})'$ is given but the projections of the deviation z^0 and the innovations η^t for $t \geq 1$ (and thereby z^t for $t \geq 1$) remain to be determined. They will be determined such that the ZLB is satisfied, i.e. equation (D.4) holds. Thus, the *policy-rate projection* is given by

$$i_{t+\tau,t} = F_i M^\tau \begin{bmatrix} X_t \\ z^t \end{bmatrix} \tag{D.9}$$

for $\tau \geq 0$ and for given X_t and z^t.

We will now show how to determine the $(T+1)$-vector $z^t \equiv (z_t, z_{t+1,t}, ..., z_{t+T,t})'$, i.e. the projection of the deviation, such that policy-rate projection satisfies the ZLB restriction (D.5) and the policy rule (D.4).

When the ZLB restriction (D.5) is disregarded or not binding, the policy-rate projection in period t is given by

$$i_{t+\tau,t} = F_i M^\tau \begin{bmatrix} X_t \\ 0_{(T+1)\times 1} \end{bmatrix}, \quad \tau \geq 0. \tag{D.10}$$

The policy-rate projection disregarding the ZLB hence depends on the initial state of the economy in period t, represented by the vector of predetermined variables X_t. If the ZLB is disregarded, or not binding for any $\tau \geq 0$, the projections of the restricted and unrestricted policy rates will be the same,

$$i_{t+\tau,t} = i^*_{t+\tau,t} = f_X X_{t+\tau,t} + f_x x_{t+\tau,t}, \quad \tau \geq 0.$$

Assume now that the policy-rate projection according to (D.10) violates the ZLB for one or several periods, that is,

$$i_{t+\tau,t} + \bar{\imath} < 0, \quad \text{for some } \tau \text{ in the interval } 0 \leq \tau \leq T. \tag{D.11}$$

In order to satisfy the ZLB, we then want to find a projection of the deviation z^t such that the policy-rate projection satisfies (D.5) and

$$i_{t+\tau,t} + \bar{\imath} = \max\{i^*_{t+\tau,t} + \bar{\imath}, 0\} = \max\{f_X X_{t+\tau,t} + f_x x_{t+\tau,t} + \bar{\imath}, 0\} \tag{D.12}$$

for $\tau \geq 0$. This requires that the projection of the deviation satisfies a non-negativity constraint

$$z_{t+\tau,t} \geq 0, \quad \tau \geq 0, \tag{D.13}$$

and that the policy-rate projection and the projection of the deviation satisfies the complementary-slackness condition

$$(i_{t+\tau,t} + \bar{\imath})\, z_{t+\tau,t} = 0, \quad \tau \geq 0. \tag{D.14}$$

Notice that the complementary-slackness condition implies that $z_{t+\tau,t} = 0$ if $i_{t+\tau,t} + \bar{\imath} > 0$.

For given X_t, we now proceed under the presumption that there exists a unique projection of the deviation z^t that satisfies (D.9) and (D.12)–(D.14).[D.2] We call this projection of the deviation and the corresponding policy-rate projection the *equilibrium* projection. This projection of the deviation either has all elements equal to zero (in which case the ZLB is not binding for any period) or has some elements positive and other elements zero. Let

$$\mathcal{T}_t \equiv \{0 \leq \tau \leq T \mid z_{t+\tau,t} > 0\}$$

[D.2] This assumption is discussed in further detail in Hebden, Lindé, and Svensson [72].

denote the set of periods for which the projection of the deviation are positive in equilibrium.

For each $\tau \in \mathcal{T}_t$, the solution will satisfy

$$i_{t+\tau,t} + \bar{\imath} = F_i M^{\tau} \begin{bmatrix} X_t \\ z^t \end{bmatrix} + \bar{\imath} = 0 \quad \text{for} \quad \tau \in \mathcal{T}_t. \tag{D.15}$$

Let $n_{\mathcal{T}_t}$ denote the number of elements of $\mathcal{T}_t$, that is, the number of periods that the ZLB binds. The equation system (D.15) then has $n_{\mathcal{T}}$ equations to determine the $n_{\mathcal{T}}$ elements of z^t that are positive. From the system (D.15), it is clear that the solution for z^t and the set $\mathcal{T}_t$ will depend on X_t as well as the initial situation, and thereby also on the initial innovation ε_t. For other periods (that is $\tau \notin \mathcal{T}_t$), the ZLB will not be binding and the elements in z^t will be zero. The equation system (D.15) and the periods in the set $\mathcal{T}_t$ hence refer to the periods where the ZLB is *strictly* binding, that is, when $z_{t+\tau,t}$ is positive. Furthermore, it is important to notice that the set of periods τ in (D.11), for which the policy-rate projection (D.10) violates the ZLB, is not necessarily the same as the set of periods $\mathcal{T}_t$ for which the ZLB is strictly binding *in equilibrium*. That is because the projections of the predetermined and forward-looking variables $X_{t+\tau,t}$ and $x_{t+\tau,t}$, that determine the unrestricted policy rate differ, depending on whether z^t is zero or not. This means that the whole policy-rate path is affected when the ZLB is imposed.

The difficulty in imposing the ZLB is to find the set $\mathcal{T}_t$ for which the ZLB is strictly binding in equilibrium, that is, to find the periods for which the equation system (D.15) applies. Once this is done, solving the equation system (D.15) is trivial. Hebden et al. [72] outline a simple shooting algorithm to find the set $\mathcal{T}_t$.

D.2. Computation of the Likelihood Function

To compute the likelihood function, we follow the general idea outlined by Maih [84]. Maih's algorithm allows us to add anticipated policy shocks (using the algorithm outlined above) to the state space formulation of the model and filter those shocks with the Kalman filter to impose the zero lower bound on policy rates in the estimation. The appealing feature of Maih's algorithm is that it does not require us to include standard deviations for each of the anticipated policy shocks. Thus, the log-marginal likelihood can be directly compared to the models which does not impose the ZLB.

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