

Challenges for Macro Models Used at Central Banks*

Preliminary and Incomplete - Comments Welcome

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Abstract

This paper discusses the current state of play for macroeconomic models used by central bank and other large international organizations. We analyse the key challenges the recent “great recession” in the United States and Europe poses for the future generation of policy models.

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1. Introduction

In this paper, we discuss new challenges for structural macroeconomic used at central banks in light of the Great Recession in United States and other advanced economies. The recession has had widespread implications for economic policy and economic performance, with historically low nominal interest rates and elevated unemployment levels in its aftermath. The fact that the intensification of the crisis in the fall of 2008 was largely unexpected and much deeper than central banks predicted, has raised many questions about the design of macroeconomic models at use in these institutions. Specifically, the models have been criticized for omitting key financial mechanisms and shocks stemming from the financial sector.

In this paper, we analyze the performance of a benchmark macroeconomic model during the Great Recession. The model we use – the well-known Smets and Wouters [77] model – shares many features with the models currently used by central banks. When we analyze this model, we find, confirming previous results in Del Negro and Shorfheide [28], that actual GDP growth was outside the predictive density of the model during the most acute phase of the recession. And to account for the recession, the model thus needs a cocktail of extremely unlikely shocks. We then proceed to document that these shocks are non-Gaussian, and strongly related to some observable financial observable variables. In addition, we show that the predictive density of the benchmark model during the recession can be improved by allowing time-varying volatilities for some of the shocks.

These observations hold up when taking the zero lower bound (ZLB henceforth) into account in the estimation of the model, which we do through two alternative approaches. First, we implement the ZLB as a binding constraint on the policy rule with an expected duration that is consistent with the endogenous model expectations. Second, we impose the expected duration of the ZLB spells during the recession to be consistent with external information derived from overnight index swap rates. Interestingly, we find that the variants of the model estimated subject to the ZLB constraint typically feature a substantially higher degree of nominal stickiness in both prices and wages which helps to understand the inflation dynamics during the recession period and the subsequent slow recovery. In addition, an important characteristic of these variants of the model is a substantially higher response coefficient on the output gap in the policy rule.

Then, we proceed to examine how the performance and properties of the basic model can be improved by introducing a financial accelerator mechanism and shocks stemming from the financial sector. Moreover, because of the non-Gaussian features of the smoothed shocks in the benchmark

model, we examine if the performance of this augmented model can be improved by allowing for regime-switching in the sensitivity of the external finance premium to the leverage ratio, which one may think of as “risk-on”/“risk-off” behavior in the financial sector.

In addition to examining the performance of the models during the crisis, we also discuss new challenges for macroeconomic models. These new challenges stems from the fact that – following the crisis – the models might be asked to be able to address additional policy issues (e.g. macroeconomic prudential policies) in addition to standard monetary policy matters. Another important aspect is that future monetary policy might be conducted with more instruments than the traditional short-term interest rate. In addition to discussing these new challenges, we also bring up some key “old”, yet unresolved, challenges for macroeconomic models used for policy analysis.¹

The outline of the paper is as follows: Section 2 provides an incomplete survey of the macroeconomic models used by central banks and other international organizations. Following this survey, Section 3 presents the prototype model – the estimated model of Smets and Wouters [76]. This model shares many features of models in use by central banks. The section also discusses the data and the estimation of this model on pre-crisis data. In Section 4, we use this model estimated on pre-crisis data to analyze the crisis episode, which gives us many valuable insights into the workings of the model. We also compare the performance of our structural model to a reduced-form benchmark VAR, which is estimated with Bayesian priors. As this analysis points to some important shortcomings of this benchmark model, we augment the baseline model to allow for the zero lower bound on policy rates, financial frictions (allow for the possibility that the influence of financial frictions is time-varying through regime-switching), and a working capital channel in Section 5, and examine how these alterations improve the empirical performance of the model, with a particular focus on the “Great Recession”. Section 6 discusses some other new and old challenges for structural macro models used in policy analysis. Some of these challenges involve features that are particularly relevant when this type of models are used in an open economy context. Finally, section 7 presents a summary and some conclusions. An appendix contains some technical details on the model, methods and data used in the analysis.

¹ Notably, these challenges include the expectations hypothesis of interest rates, the uncovered interest rate parity condition, and lack of comovement of international business cycles.

2. Common features of central bank models

In this section, we provide an incomplete survey of the key policy models currently at use at central banks and other key policy institutions like the IMF, European Commission and the OECD. We aim at determining the similarity in models used, and assess if – and how – they have been changed in response to the Recession and development since then.

A good starting point for the discussion is the paper by Coenen et al. [23], who studies the effects of fiscal stimulus in the key policy models in use at the Bank of Canada (BoC-GEM), the Board of Governors of the Federal Reserve System (with two models, FRB-US and SIGMA), the European Central Bank (NAWM), the European Commission (QUEST), the International Monetary Fund (GIMF), and the OECD (OECD Fiscal). Of the seven models, six of the models are dynamic stochastic general equilibrium (DSGE) models, while FRB-US is based on the polynomial adjustment cost (PAC) framework. Hence, an overwhelming majority of key policy institutions today use DSGE models as the core policy tool. Other prominent institutions that have adopted estimated DSGE model as their core policy tool include Bank of England (COMPASS), Norges Bank (NEMO), Sveriges Riksbank (RAMSES), Federal Reserve Bank of New York [31], and the Federal Reserve Bank of Chicago [12].

As outlined in detail in Tables 1 and 2 in Coenen et al. [23], the DSGE models share many similarities to the seminal models of Christiano, Eichenbaum and Evans [17] and Smets and Wouters [76], [77]. They typically feature imperfect competition in product and labor markets as vehicles to introduce sticky prices and wages. They also include important real rigidities like habit formation, costs of adjusting investment and variable capital utilization. Monetary policy is generally determined by a simple Taylor-type policy rule which allows for interest rate smoothing. But although they share many similarities with the academic benchmark models of CEE and SW07, an important difference is that they have a significant share of financially constrained households, ranging between 20 and 50 percent. In some models these are hand-to-mouth households, who take their labor income as given and determine consumption residually from a period-by-period budget constraint. In other models these are liquidity-constrained households, who face the same period-by-period budget constraint, but who solve an intratemporal decision problem between consumption and work effort. An additional difference between the policy models and the academic style models, is that the former generally has a much more detailed fiscal sector with many distortionary taxes, types of government spending and various transfers from the government to the households.

Another interesting observation is that neither the CEE nor the SW07 model include frictions in financial markets or a detailed banking sector.² 4 or out 7 of the policy models included financial frictions prior to the crisis. By asking the policy institutions that were part of this study about their development efforts since then, it is clear that efforts have been made towards better integration of financial markets, with a focus on the interaction between banks and the firms in the economy. For instance, following the crisis, financial frictions following the approach of Bernanke, Gertler and Gilchrist [9] have been introduced in (at least) 2 of the 3 models that did not feature them before.

The key lesson we draw from this is that while the crisis has had some impact on improving the modeling of the financial sector in DSGE models, it has not so far had a material impact on the type of models used at key policy institutions, which share many features of the basic model developed by Christiano, Eichenbaum and Evans [17].

3. A benchmark model

In this section, we show the benchmark model environment, which is the model of Smets and Wouters [76], SW07 henceforth. The SW07 model builds on the framework in Christiano, Eichenbaum and Evans [17], but allow for a richer set of stochastic shocks. In Section 3.4, we describe how we estimate it on aggregate times series for the United States.

3.1. Firms and Price Setting

Final Goods Production The single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball [63], the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G_Y \left(\frac{Y_t(f)}{Y_t} \right) df = 1. \quad (3.1)$$

Following Dotsey and King [33], we assume that $G_Y(\cdot)$ is given by a strictly concave and increasing function:

$$G_Y \left(\frac{Y_t(f)}{Y_t} \right) = \left(\frac{\phi_t^p}{1 - (\phi_t^p - 1)\epsilon_p} \left[\left(\frac{\phi_t^p + (1 - \phi_t^p)\epsilon_p}{\phi_t^p} \right) \frac{Y_t(f)}{Y_t} + \frac{(\phi_t^p - 1)\epsilon_p}{\phi_t^p} \right]^{\frac{1 - (\phi_t^p - 1)\epsilon_p}{\phi_t^p - (\phi_t^p - 1)\epsilon_p}} + \left[1 - \frac{\phi_t^p}{1 - (\phi_t^p - 1)\epsilon_p} \right] \right), \quad (3.2)$$

² CEE, but not the SW07 model, includes a working capital – or cost channel – of monetary policy whereby firms have to borrow at the policy rate to finance the wage bill. This channel allows the CEE model to account for the “Price-puzzle” generally present for monetary policy shocks in identified VAR models.

where $\phi_t^p \geq 1$ denotes the gross markup of the intermediate firms. The parameter ϵ_p governs the degree of curvature of the intermediate firm's demand curve. When $\epsilon_p = 0$, the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When ϵ_p is positive the firms instead face a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a large fall in demand. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost. Finally, notice that $G_Y(1) = 1$, implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both the product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index Y_t , taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price P_t , and hence solve the following problem:

$$\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f) Y_t(f) df, \quad (3.3)$$

subject to the constraint (3.1). The first order conditions for this problem can be written

$$\begin{aligned} \frac{Y_t(f)}{Y_t} &= \frac{\phi_t^p}{\phi_t^p - (\phi_t^p - 1)\epsilon_p} \left(\left[\frac{P_t(f)}{P_t} \frac{1}{\Lambda_t^p} \right]^{-\frac{\phi_t^p - (\phi_t^p - 1)\epsilon_p}{\phi_t^p - 1}} + \frac{(1 - \phi_t^p)\epsilon_p}{\phi_t^p} \right), \\ P_t \Lambda_t^p &= \left[\int P_t(f)^{-\frac{1 - (\phi_t^p - 1)\epsilon_p}{\phi_t^p - 1}} df \right]^{-\frac{\phi_t^p - 1}{1 - (\phi_t^p - 1)\epsilon_p}}, \\ \Lambda_t^p &= 1 + \frac{(1 - \phi_t^p)\epsilon_p}{\phi_t^p} - \frac{(1 - \phi_t^p)\epsilon_p}{\phi_t^p} \int \frac{P_t(f)}{P_t} df, \end{aligned} \quad (3.4)$$

where Λ_t^p denotes the Lagrange multiplier on the aggregator constraint (3.1). Note that for $\epsilon_p = 0$ and $\Lambda_t^p = 1$ in each period t , the demand and pricing equations collapse to the usual Dixit-Stiglitz expressions

$$\frac{Y_t(f)}{Y_t} = \left[\frac{P_t(f)}{P_t} \right]^{-\frac{\phi_t^p}{\phi_t^p - 1}}, P_t = \left[\int P_t(f)^{\frac{1}{1 - \phi_t^p}} df \right]^{1 - \phi_t^p}. \quad (3.5)$$

Intermediate Goods Production A continuum of intermediate goods $Y_t(f)$ for $f \in [0, 1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces the demand schedule in eq. (3.4) from the final goods firms through the solution to the problem in (3.3), which varies inversely with its output price $P_t(f)$ and directly with aggregate demand Y_t .

Each intermediate goods producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = \varepsilon_t^a K_t(f)^\alpha [\gamma^t L_t(f)]^{1-\alpha} - \gamma^t \Phi, \quad (3.6)$$

where γ^t represents the labour-augmenting deterministic growth rate in the economy, Φ denotes the fixed cost (which is related to the gross markup ϕ_t^p so that profits are zero in the steady state), and ε_t^a is total factor productivity which follows the process

$$\ln \varepsilon_t^a = (1 - \rho_a) \ln \varepsilon^a + \rho_a \ln \varepsilon_{t-1}^a + \eta_t^a, \eta_t^a \sim N(0, \sigma_a). \quad (3.7)$$

Firms face perfectly competitive factor markets for renting capital and hiring labor. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital R_{Kt} and the aggregate wage index W_t (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo [13] and Yun [80] style staggered nominal contracts. In each period, each firm f faces a constant probability, $1 - \xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to re-optimize its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price by a weighted combination of the lagged and steady-state rate of inflation, i.e., $P_t(f) = (1 + \pi_{t-1})^{\iota_p} (1 + \pi)^{1-\iota_p} P_{t-1}(f)$ where $0 \leq \iota_p \leq 1$ and π_{t-1} denotes net inflation in period $t-1$, and π the steady-state net inflation rate. A positive value of ι_p introduces structural inertia into the inflation process. All told, this leads to the following optimization problem for the intermediate firms

$$\max_{\tilde{P}_t(f)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \frac{\Xi_{t+j} P_t}{\Xi_t P_{t+j}} \left[\tilde{P}_t(f) \left(\Pi_{s=1}^j (1 + \pi_{t+s-1})^{\iota_p} (1 + \pi)^{1-\iota_p} \right) - MC_{t+j} \right] Y_{t+j}(f), \quad (3.8)$$

where $\tilde{P}_t(f)$ is the newly set price. Notice that with our assumptions all firms that re-optimize their prices actually set the same price.

As noted previously, we assume that the gross price-markup is time-varying, and given by $\phi_t^p = \phi^p \varepsilon_t^p$, for which the exogenous component ε_t^p is given by an exogenous ARMA(1,1) process:

$$\ln \varepsilon_t^p = \rho_p \ln \varepsilon_{t-1}^p + \eta_t^p - \vartheta_p \eta_{t-1}^p, \eta_t^p \sim N(0, \sigma_p). \quad (3.9)$$

3.2. Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household's labor services $L_t(h)$, $h \in [0, 1]$, as imperfect substitutes for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The aggregated labor index L_t has the Kimball [63] form:

$$L_t = \int_0^1 G_L \left(\frac{L_t(h)}{L_t} \right) dh = 1, \quad (3.10)$$

where the function $G_L(\cdot)$ has the same functional form as (3.2), but is characterized by the corresponding parameters ϵ_w (governing convexity of labor demand by the aggregator) and a time-varying gross wage markup ϕ_t^w . The aggregator minimizes the cost of producing a given amount of the aggregate labor index L_t , taking each household's wage rate $W_t(h)$ as given, and then sells units of the labor index to the intermediate goods sector at unit cost W_t , which can naturally be interpreted as the aggregate wage rate. From the FOCs, the aggregator's demand for the labor hours of household h – or equivalently, the total demand for this household's labor by all goods-producing firms – is given by

$$\frac{L_t(h)}{L_t} = G_L'^{-1} \left[\frac{W_t(h)}{W_t} \int_0^1 G_L' \left(\frac{L_t(h)}{L_t} \right) \frac{L_t(h)}{L_t} dh \right], \quad (3.11)$$

where $G_L'(\cdot)$ denotes the derivative of the $G_L(\cdot)$ function in eq. (3.10).

The utility function of a typical member of household h is

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{1 - \sigma_c} (C_{t+j}(h) - \varkappa C_{t+j-1}) \right]^{1 - \sigma_c} \exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} L_{t+j}(h)^{1 + \sigma_l} \right), \quad (3.12)$$

where the discount factor β satisfies $0 < \beta < 1$.³ The period utility function depends on household h 's current consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for external habit persistence. The period utility function also depends inversely on hours worked $L_t(h)$.

Household h 's budget constraint in period t states that its expenditure on goods and net pur-

³ Note that we deviate slightly from the notation in SW by using h to index households and using \varkappa to denote the degree of habit formation.

chases of financial assets must equal its disposable income:

$$\begin{aligned} P_t C_t(h) + P_t I_t(h) + \frac{B_{t+1}(h)}{\varepsilon_t^b R_t} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h) \\ = B_t(h) + W_t(h) L_t(h) + R_t^k Z_t(h) K_t^p(h) - a(Z_t(h)) K_t^p(h) + \Gamma_t(h) - T_t(h). \end{aligned} \quad (3.13)$$

Thus, the household purchases part of the final output good (at a price of P_t), which it chooses either to consume $C_t(h)$ or invest $I_t(h)$ in physical capital. Following Christiano, Eichenbaum, and Evans [17], investment augments the household's (end-of-period) physical capital stock $K_{t+1}^p(h)$ according to

$$K_{t+1}^p(h) = (1 - \delta) K_t^p(h) + \varepsilon_t^i \left[1 - S \left(\frac{I_t(h)}{I_{t-1}(h)} \right) \right] I_t(h). \quad (3.14)$$

The extent to which investment by each household h turns into physical capital is assumed to depend on an exogenous shock ε_t^i and how rapidly the household changes its rate of investment according to the function $S \left(\frac{I_t(h)}{I_{t-1}(h)} \right)$, which we specify as

$$S(x_t) = \frac{\varphi}{2} (x_t - \gamma)^2. \quad (3.15)$$

Notice that this function satisfies $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) = \varphi$. The stationary investment-specific shock ε_t^i follows

$$\ln \varepsilon_t^i = \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim N(0, \sigma_i). \quad (3.16)$$

In addition to accumulating physical capital, households may augment their financial assets through increasing their government nominal bond holdings (B_{t+1}), from which they earn an interest rate of R_t . The return on these bonds is also subject to a risk-shock, ε_t^b , which follows

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim N(0, \sigma_b). \quad (3.17)$$

Fisher [43] shows that this shock can be given a structural interpretation.

We assume that agents can engage in frictionless trading of a complete set of contingent claims to diversify away idiosyncratic risk. The term $\int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of these state-contingent domestic bonds, with $\xi_{t,t+1}$ denoting the state-dependent price, and $B_{D,t+1}(h)$ the quantity of such claims purchased at time t .

On the income side, each member of household h earns after-tax labor income $W_t(h) L_t(h)$, after-tax capital rental income of $R_t^k Z_t(h) K_t^p(h)$, and pays a utilization cost of the physical capital equal to $a(Z_t(h)) K_t^p(h)$ where $Z_t(h)$ is the capital utilization rate, so that capital services provided

by household h , $K_t(h)$, equals $Z_t(h) K_t^p(h)$. The capital utilization adjustment function $a(Z_t(h))$ is assumed to be given by

$$a(Z_t(h)) = \frac{r^k}{\tilde{z}_1} [\exp(\tilde{z}_1(Z_t(h) - 1)) - 1], \quad (3.18)$$

where r^k is the steady state net real interest rate (\bar{R}_t^K/\bar{P}_t). Notice that the adjustment function satisfies $a(1) = 0$, $a'(1) = r^k$, and $a''(1) \equiv r^k \tilde{z}_1$. Following SW, we want to write $a''(1) = z_1 = \psi/(1 - \psi) > 0$, where $\psi \in [0, 1)$ and a higher value of ψ implies a higher cost of changing the utilization rate. Our parameterization of the adjustment cost function then implies that we need to set $\tilde{z}_1 \equiv z_1/r^k$. Finally, each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (regarded as taxes net of any transfers).

In every period t , each member of household h maximizes the utility function (3.12) with respect to its consumption, investment, (end-of-period) physical capital stock, capital utilization rate, bond holdings, and holdings of contingent claims, subject to its labor demand function (3.11), budget constraint (3.13), and transition equation for capital (3.14).

Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described previously. Thus, the probability that a household receives a signal to re-optimize its wage contract in a given period is denoted by $1 - \xi_w$. In addition, SW07 specify the following dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to re-optimize: $W_t(h) = \gamma(1 + \pi_{t-1})^{\iota_w}(1 + \pi)^{1-\iota_w} W_{t-1}(h)$. All told, this leads to the following optimization problem for the households

$$\max_{\tilde{W}_t(h)} E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\Xi_{t+j} P_t}{\Xi_t P_{t+j}} \left[\tilde{W}_t(h) \left(\prod_{s=1}^j \gamma(1 + \pi_{t+s-1})^{\iota_w} (1 + \pi)^{1-\iota_w} \right) - W_{t+j}(h) \right] L_{t+j}(h), \quad (3.19)$$

where $\tilde{W}_t(h)$ is the newly set wage; notice that with our assumptions all households that reoptimize their wages will actually set the same wage.

Following the same approach as with the intermediate-goods firms, we introduce a shock ε_t^w to the time-varying gross markup, $\phi_t^w = \phi^w \varepsilon_t^w$, where ε_t^w is assumed to be given by an exogenous ARMA(1,1) process

$$\ln \varepsilon_t^w = \rho_w \ln \varepsilon_{t-1}^w + \eta_t^w - \vartheta_w \eta_{t-1}^w, \eta_t^w \sim N(0, \sigma_w). \quad (3.20)$$

3.3. Market Clearing Conditions and Monetary Policy

Government purchases G_t are exogenous, and the process for government spending relative to trend output, i.e. $g_t = G_t / (\gamma^t Y)$, is given by the following exogenous AR(1) process:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g (\ln g_{t-1} - \rho_{ga} \ln \varepsilon_{t-1}^a) + \varepsilon_t^g, \varepsilon_t^g \sim N(0, \sigma_g). \quad (3.21)$$

Government purchases have no effect on the marginal utility of private consumption, nor do they serve as an input into goods production. The consolidated government sector budget constraint is

$$\frac{B_{t+1}}{R_t} = G_t - T_t + B_t. \quad (3.22)$$

By comparing the debt terms in the household budget constraint in eq. (3.13) with the equation above, one can see that receipts from the risk shock are subject to iceberg costs, and hence do not add any income to the government.⁴

The conduct of monetary policy is assumed to be approximated by a Taylor-type policy rule (here stated in linearized form)

$$R_t = R_{t-1}^{\rho_R} \Pi_t^{r_\pi(1-\rho_R)} \left(\frac{Y_t}{Y_t^{pot}} \right)^{r_y(1-\rho_R)} \left(\frac{Y_t}{Y_t^{pot}} / \frac{Y_{t-1}}{Y_{t-1}^{pot}} \right)^{r_{\Delta y}(1-\rho_R)} \varepsilon_t^r, \quad (3.23)$$

where Π_t denotes the gross inflation rate and Y_t^{pot} is defined the level of output that would prevail if prices and wages were flexible. The policy shock ε_t^r is supposed to follow an AR(1) process in logs:

$$\ln \varepsilon_t^r = \rho_r \ln \varepsilon_{t-1}^r + \eta_t^r, \eta_t^r \sim N(0, \sigma_r).$$

Total output of the final goods sector is used as follows:

$$Y_t = C_t + I_t + G_t + a(Z_t) \bar{K}_t, \quad (3.24)$$

where $a(Z_t) \bar{K}_t$ is the capital utilization adjustment cost.

Finally, we need to specify the aggregate production constraint. To do that, we note that the unweighted sum of the intermediate firms' output equals

$$Y_t^{sum} = \int_0^1 Y_t(f) df, \quad (3.25)$$

⁴ But even if they did, it would not matter as the government balances its expenditures each period through lump-sum taxes, $T_t = G_t + B_t - B_{t+1}/R_t$, so that government debt $B_t = 0$ in equilibrium. Furthermore, as Ricardian equivalence holds in the model, it does not matter for equilibrium allocations whether the government balances its debt or not in each period.

which from eq. (3.6) can be rewritten as

$$\begin{aligned} Y_t^{sum} &= \int_0^1 \left[\varepsilon_t^a K_t(f)^\alpha [\gamma^t L_t(f)]^{1-\alpha} - \gamma^t \Phi \right] df \\ &= \varepsilon_t^a \left(\frac{K_t}{\gamma^t L_t} \right)^\alpha \int_0^1 \gamma^t L_t(f) df - \gamma^t \Phi, \end{aligned} \quad (3.26)$$

where the second equality follows from the fact that every firms capital-labor ratio will be the same in equilibrium.

From the first-order conditions to the final goods aggregator problem (3.4), it follows that

$$Y_t^{sum} = Y_t \int_0^1 \frac{\phi_t^p}{\phi_t^p - (\phi_p - 1)\epsilon_p} \left(\left[\frac{P_t(f)}{P_t} \frac{1}{\Lambda_t^p} \right]^{-\frac{\phi_t^p - (\phi_p - 1)\epsilon_p}{\phi_t^p - 1}} + \frac{(1 - \phi_t^p)\epsilon_p}{\phi_t^p} \right) df, \quad (3.27)$$

so that

$$\varepsilon_t^a \left(\frac{K_t}{\gamma^t L_t} \right)^\alpha \gamma^t \int_0^1 L_t(h) dh - \gamma^t \Phi = Y_t \int_0^1 \frac{\phi_t^p}{\phi_t^p - (\phi_t^p - 1)\epsilon_p} \left(\left[\frac{P_t(f)}{P_t} \frac{1}{\Lambda_t^p} \right]^{-\frac{\phi_t^p - (\phi_t^p - 1)\epsilon_p}{\phi_t^p - 1}} + \frac{(1 - \phi_t^p)\epsilon_p}{\phi_t^p} \right) df.$$

By inserting the expression for the unweighted sum of labor, $\int_0^1 \gamma^t L_t(h) dh$, into this last expression, we can finally derive the aggregate production constraint which depends on aggregate technology, capital, labor, fixed costs, as well as the price and wage dispersion terms.

3.4. Estimation on Pre-Crisis Data

We now proceed to discuss how the model is estimated. To begin with, we limit the sample to the period 1965Q1-2007Q4 to see how a model estimated on pre-crisis data fares during the recession. Subsequently, we will estimate the model on data spanning the crisis.

3.4.1. Solving the model

Before estimating the model, we log-linearize all the equations of the model. The log-linearized representation is provided in Appendix A. To solve the system of log-linearized equation, we use the code packages Dynare and RISE which provides an efficient and reliable implementation of the method proposed by Blanchard and Kahn [10].

3.4.2. Data

We use seven key macro-economic quarterly US time series as observable variables: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log

difference of the GDP deflator and the federal funds rate. A full description of the data used is given in C.1. The solid blue line in Figure 3.1 show the data for the full sample, which spans 1965Q1-2014Q2.⁵ From the figure, we see the extraordinary large fall in private consumption, which exceeded the fall during the recession in the early 1980s. The strains in the labor market are also evident, with hours worked per capita falling to a post-war bottom low in early 2010. Finally, we see that the Federal reserve cut the federal funds rate to near zero in 2009Q1 (the FFR is measured as an average of daily observations in each quarter). Evidently, the zero bound was perceived as an effective lower bound by the FOMC committee, and they kept it at this level during the crisis and adopted alternative tools to make monetary policy more accommodative (see e.g. Bernanke [8]). Meanwhile, inflation fell to record lows and into deflationary territory by late 2009. Since then, inflation has rebounded close to the new target (2 percent) announced by the Federal Reserve in January 2012.

The measurement equation, relating the variables in the model to the various variables we match in the data, is given by:

$$Y_t^{obs} = \begin{bmatrix} \Delta \ln GDP_t \\ \Delta \ln CONS_t \\ \Delta \ln INVE_t \\ \Delta \ln W_t^{\text{real}} \\ \ln HOURS_t \\ \Delta \ln PGDP_t \\ FFR_t \end{bmatrix} = \begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln I_t - \ln I_{t-1} \\ \ln (W/P)_t - \ln (W/P)_{t-1} \\ \ln L_t \\ \ln \Pi_t \\ \ln R_t \end{bmatrix} \approx \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t^{\text{real}} - \hat{w}_{t-1}^{\text{real}} \\ l_t \\ \pi_t \\ \hat{R}_t \end{bmatrix} \quad (3.28)$$

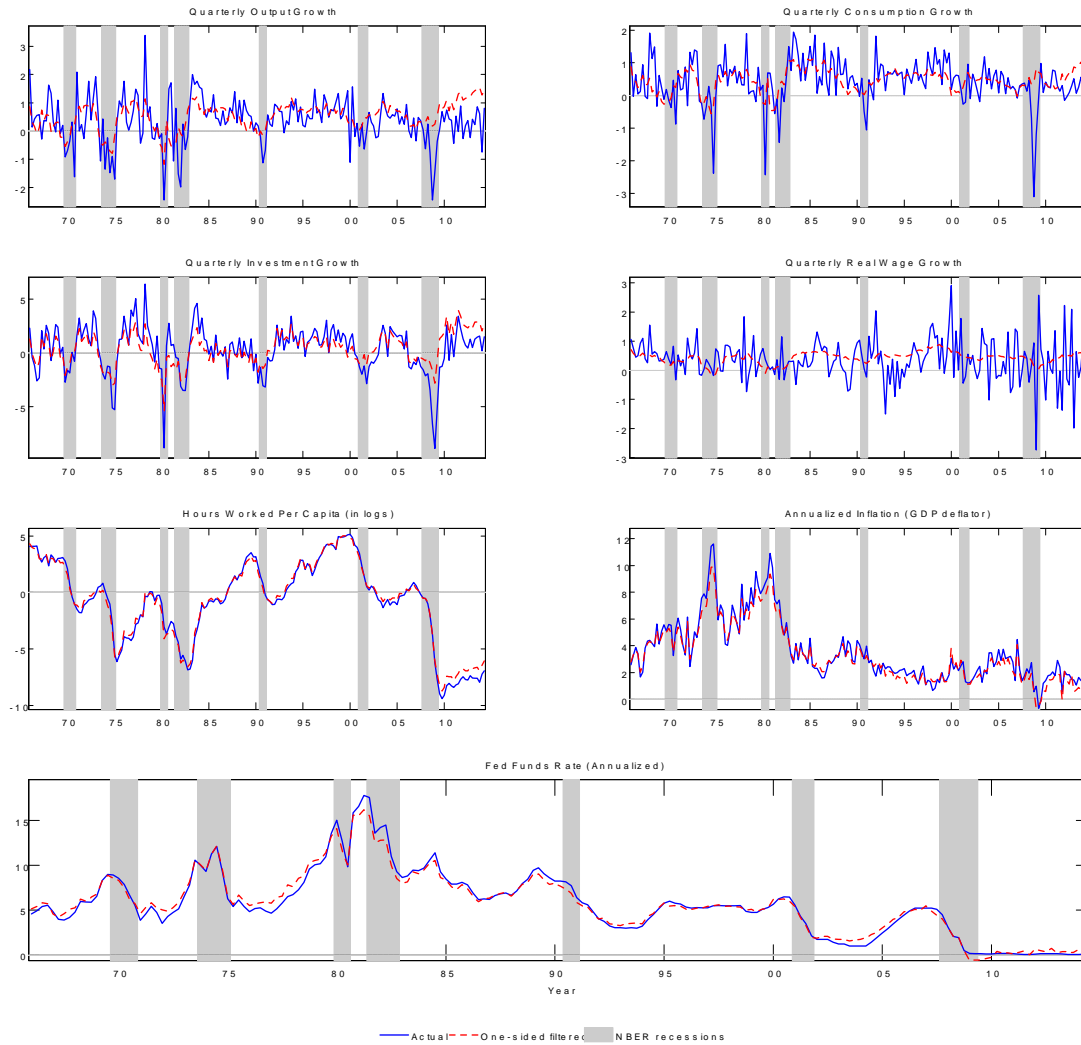
where \ln and $\Delta \ln$ stand for log and log-difference respectively, $\bar{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate to real GDP, consumption, investment and wages, $\bar{\pi} = 100\pi$ is the quarterly steady-state inflation rate and $r = 100(\beta^{-1}\gamma^{\sigma_c}(1 + \pi) - 1)$ is the steady-state nominal interest rate. Given the estimates of the trend growth rate and the steady-state inflation rate, the latter will be determined by the estimated discount rate. Finally, \bar{l} is steady-state hours-worked, which is normalized to be equal to zero.

3.4.3. Estimation Methodology

Following SW07, Bayesian techniques are adopted to estimate the parameters using the 7 U.S. macroeconomic variables in eq. (3.28) during the period 1965Q1–2007Q4. Bayesian inference starts out from a prior distribution that describes the available information prior to observing the data used in the estimation. The observed data is subsequently used to update the prior, via Bayes'

⁵ The figure also includes a red-dashed line, whose interpretation will be discussed in further detail in Section 4.

Figure 3.1: Actual and Filtered Data in Model Estimated on Pre-Crisis Data.



theorem, to the posterior distribution of the model's parameters which can be summarized in the usual measures of location (e.g. mode or mean) and spread (e.g. standard deviation and probability intervals).⁶

Some of the parameters in the model are kept fixed throughout the estimation procedure (i.e., having infinitely strict priors). We choose to calibrate the parameters we think are weakly identified by the variables included in \tilde{Y}_t in eq. (3.28). In Table 3.1 we report the parameters we have chosen to calibrate. These parameters are calibrated to the same values as in SW07.

⁶ We refer the reader to Smets and Wouters [76] for a more detailed description of the estimation procedure.

Table 3.1: Calibrated parameters

Parameter	Description	Calibrated Value
δ	Depreciation rate	0.025
ϕ_w	Gross wage markup	1.50
g_y	Government G/Y ss-ratio	0.18
ϵ_p	Kimball Elast. GM	10
ϵ_w	Kimball Elast. LM	10

Note: The calibrated parameters are adapted from SW07.

The remaining 36 parameters, which mostly pertain to the nominal and real frictions in the model as well as the exogenous shock processes, are estimated. The first three columns in Table 3.2 shows the assumptions for the prior distribution of the estimated parameters. The location of the prior distribution is identical to that in SW07. We use the beta distribution for all parameters bounded between 0 and 1. For parameters assumed to be positive, we use the inverse gamma distribution, and for the unbounded parameters, we use the normal distribution. The exact location and uncertainty of the prior can be seen in Table 3.2, but for a more comprehensive discussion of our choices regarding the prior distributions we refer the reader to SW07.

3.4.4. Prior distributions of the estimated parameters

Given these calibrated parameters in 3.1, we obtain the joint posterior distribution mode for the estimated parameters in Table 3.2 on pre-crisis data in two steps. First, the posterior mode and an approximate covariance matrix, based on the inverse Hessian matrix evaluated at the mode, is obtained by numerical optimization on the log posterior density. Second, the posterior distribution is subsequently explored by generating draws using the Metropolis-Hastings algorithm. The proposal distribution is taken to be the multivariate normal density centered at the previous draw with a covariance matrix proportional to the inverse Hessian at the posterior mode; see Schorfheide [75] and Smets and Wouters [76] for further details. The results in 3.2 shows the posterior mode of all the parameters along with the approximate posterior standard deviation obtained from the inverse Hessian at the posterior mode. In addition, it shows the mean along with the 5th and 95th percentiles of the posterior distribution. Finally, the last column in the table reports the posterior mode in the SW07 paper.

As can be seen from Table 3.2, the obtained posterior mode for our pre-crisis sample are very similar to those originally obtained by SW07, reflecting a largely overlapping estimation sample (SW07 used data for the 1965Q1-2004Q4 period to estimate the model).

Table 3.2: Prior and posterior distributions: 1966Q1-2007Q4.

Parameter		Prior distribution			Posterior distribution					SW07 results Posterior mode
		type	mean	std.dev. /df	Optimization mode	std.dev. Hess.	Metropolis Chain mean	5%	95%	
Calvo prob. wages	ξ_w	beta	0.50	0.10	0.79	0.055	0.75	0.61	0.82	0.73
Calvo prob. prices	ξ_p	beta	0.50	0.10	0.69	0.051	0.69	0.60	0.76	0.65
Indexation wages	ι_w	beta	0.50	0.15	0.63	0.136	0.58	0.36	0.79	0.59
Indexation prices	ι_p	beta	0.50	0.15	0.23	0.093	0.26	0.13	0.44	0.22
Gross price markup	ϕ_p	normal	1.25	0.12	1.64	0.076	1.64	1.52	1.77	1.61
Capital production share	α	normal	0.30	0.05	0.21	0.018	0.20	0.18	0.24	0.19
Capital utilization cost	ψ	beta	0.50	0.15	0.60	0.100	0.59	0.43	0.75	0.54
Investment adj. cost	φ	normal	4.00	1.50	5.50	1.019	5.69	4.23	7.65	5.48
Habit formation	κ	beta	0.70	0.10	0.67	0.042	0.69	0.62	0.76	0.71
Inv subs. elast. of cons.	σ_c	normal	1.50	0.37	1.53	0.138	1.44	1.23	1.69	1.59
Labor supply elast.	σ_l	normal	2.00	0.75	2.15	0.584	2.03	1.13	2.99	1.92
Log hours worked in S.S.	\bar{l}	normal	0.00	2.00	1.56	0.985	1.15	-0.56	2.72	-0.10
Discount factor	$100(\beta^{-1} - 1)$	gamma	0.25	0.10	0.13	0.052	0.16	0.08	0.25	0.16
Quarterly Growth in S.S.	\bar{y}	normal	0.40	0.10	0.43	0.014	0.43	0.41	0.45	0.43
Stationary tech. shock	ρ_a	beta	0.50	0.20	0.96	0.008	0.96	0.93	0.97	0.95
Risk premium shock	ρ_b	beta	0.50	0.20	0.18	0.081	0.22	0.10	0.38	0.18
Invest. spec. tech. shock	ρ_i	beta	0.50	0.20	0.71	0.053	0.71	0.61	0.80	0.71
Gov't cons. shock	ρ_g	beta	0.50	0.20	0.97	0.008	0.97	0.96	0.98	0.97
Price markup shock	ρ_p	beta	0.50	0.20	0.90	0.038	0.89	0.80	0.95	0.90
Wage markup shock	ρ_w	beta	0.50	0.20	0.98	0.010	0.97	0.94	0.98	0.97
Response of g_t to ε_t^a	ρ_{ga}	beta	0.50	0.20	0.52	0.086	0.49	0.38	0.67	0.52
Stationary tech. shock	σ_a	invgamma	0.10	2.00	0.44	0.026	0.45	0.40	0.49	0.45
Risk premium shock	σ_b	invgamma	0.10	2.00	0.24	0.022	0.24	0.19	0.27	0.24
Invest. spec. tech. shock	σ_i	invgamma	0.10	2.00	0.41	0.041	0.41	0.34	0.48	0.45
Gov't cons. shock	σ_g	invgamma	0.10	2.00	0.50	0.028	0.51	0.46	0.57	0.52
Price markup shock	σ_p	invgamma	0.10	2.00	0.12	0.015	0.13	0.10	0.15	0.14
MA(1) price markup shock	ϑ_p	beta	0.50	0.20	0.74	0.080	0.72	0.46	0.83	0.74
Wage markup shock	σ_w	invgamma	0.10	2.00	0.31	0.025	0.30	0.25	0.34	0.24
MA(1) wage markup shock	ϑ_w	beta	0.50	0.20	0.95	0.030	0.92	0.77	0.95	0.88
Quarterly infl. rate. in S.S.	$\bar{\pi}$	gamma	0.62	0.10	0.79	0.114	0.82	0.65	1.01	0.81
Inflation response	r_π	normal	1.50	0.25	2.01	0.174	2.07	1.75	2.33	2.03
Output gap response	r_y	normal	0.12	0.05	0.10	0.023	0.10	0.05	0.13	0.08
Diff. output gap response	$r_{\Delta y}$	normal	0.12	0.05	0.23	0.026	0.23	0.18	0.27	0.22
Mon. pol. shock std	σ_r	invgamma	0.10	2.00	0.23	0.014	0.24	0.21	0.26	0.24
Mon. pol. shock pers.	ρ_r	beta	0.50	0.20	0.12	0.062	0.15			0.12
Interest rate smoothing	ρ_R	beta	0.75	0.10	0.82	0.022	0.82			0.81
Log marginal likelihood					Laplace	-961.81	MCMC	-960.72		

Note: Data for 1965Q1-1965Q4 are used as pre-sample to form a prior for 1966Q1, and the log-likelihood is evaluated for the period 1966Q1-2007Q4. A posterior sample of 250,000 post burn-in draws was generated in the Metropolis-Hastings chain. Convergence was checked using standard diagnostics such as CUSUM plots and the potential scale reduction factor on parallel simulation sequences. The MCMC marginal likelihood was numerically computed from the posterior draws using the modified harmonic estimator in Geweke [49].

There are two issues to notice with regards to the parameters in Table 3.2. First, the policy and deep parameters are generally very similar to those estimated by SW07. The only noticeable difference relative to SW07 is that the estimated degree of wage and price stickiness is somewhat more pronounced (posterior mode for ξ_w is 0.79 instead of 0.73 in SW07, and the mode for ξ_p has increased from 0.65 in SW07 to 0.69). The tendency of an increased degree of price and wage stickiness in the extended sample is supported by Del Negro, Giannoni and Schorfheide [26], who argues that a New Keynesian model similar to ours augmented with financial frictions points towards a high degree of price and wage stickiness to fit the behavior of inflation during the Great

Recession. Second, the estimated variances of the shocks are somewhat lower (apart from the wage markup shock) relative to the estimation done in SW07. Given that SW07 ended their estimation in 2004, and the so-called “Great Moderation” was still in effect from 2005 into the first half of 2007, the finding of reduced shock variances is not surprising.

4. Empirical performance of benchmark models during the Great Recession

We will now assess the performance of our benchmark DSGE model during the great recession in a number of dimensions. First and foremost, we study the forecasting performance of the model in detail during the most intense phase of the recession, the third and fourth quarters of 2008. In addition, we look into what the model has to say about the speed of recovery in the economy during the post-crisis period. In this exercise, we benchmark the performance of the DSGE model against a standard Bayesian VAR, which includes the same set of variables.

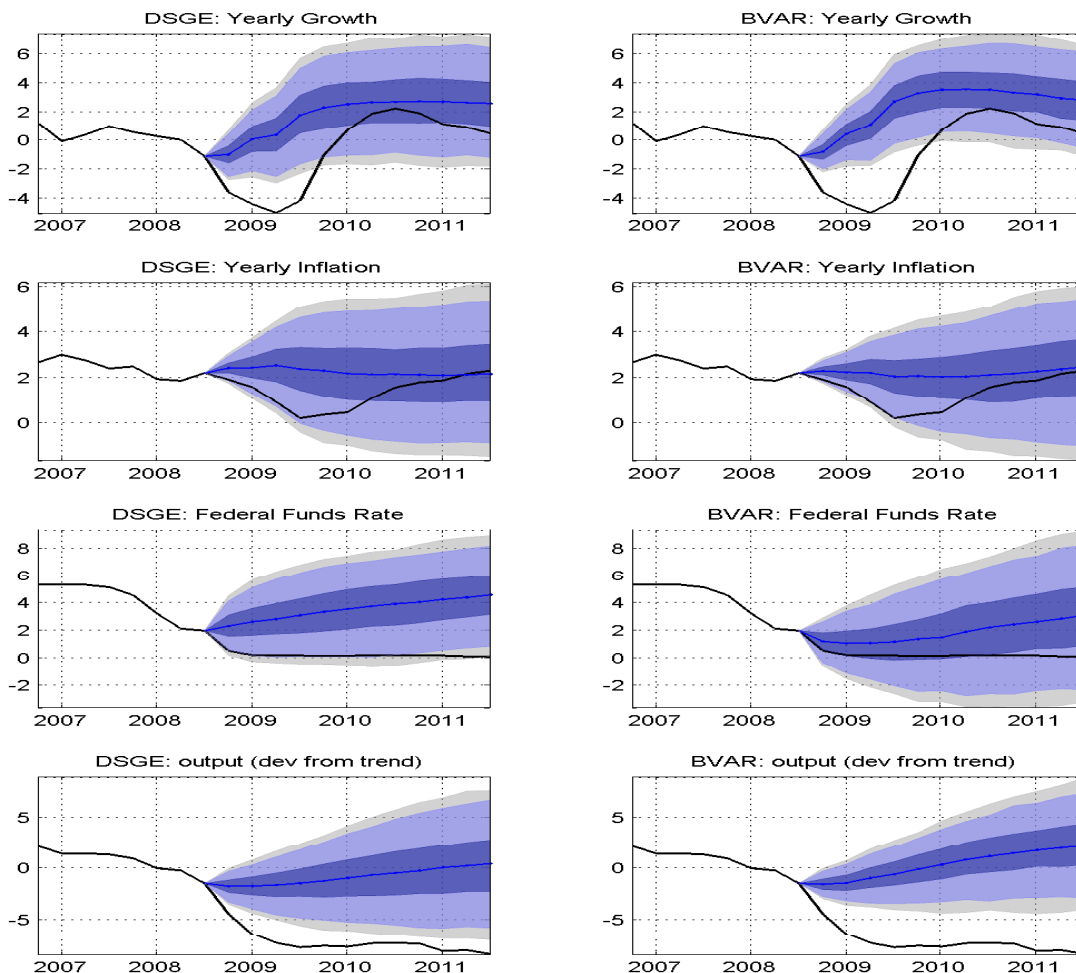
Second, we examine how the model interprets the “Great Recession”, and assess the plausibility of the shocks the model needs to explain it. We do this from both an statistical and economic viewpoint. Third, we allow for regime shifts in the shock variances and examine to what extent do so improves the fit of the model.

4.1. Forecasting performance of benchmark models during the recession

We now use the DSGE model estimated on data up to 2007Q4 to forecast out-of-sample. We start to make forecasts for 1,2,...,12 quarters ahead in the third and fourth quarter of 2008, conditional on observing data up to and including 2008Q3 and 2008Q4, respectively.⁷ Forecasts starting in these quarters are of particular interest as output plummeted in 2008Q4 (about -9.75 percent at an annualized quarterly rate) and in 2009Q1 (roughly -5.75 percent at an annualized rate). To provide a benchmark to the DSGE forecasts, we also report the forecasts of a Bayesian vector autoregressive (BVAR) model estimated on the same sample on the same seven observables. The BVAR uses the standard Doan-Litterman-Sims [32] prior on the dynamics and an informative prior on the steady state following the procedure outlined in Villani [78]. We select the priors on the steady state in the BVAR to be consistent with those used in the DSGE model, which facilitates comparison between the two models. In both the DSGE and the BVAR, the median projections and 50, 90 and 95 percent uncertainty bands are based on 10,000 simulations of respective model

⁷ We perform these forecasts on ex post data, collected September 25th 2014. However, the points we make remain unaffected if we had used real-time data.

Figure 4.1: Forecast 2008Q4-2011Q3 conditional on state in 2008Q3.



in which we allow for both shock and parameter uncertainty.⁸

In Figure 4.1, the left column shows the forecasts in the DSGE conditional on observing data up to 2008Q3. As can be seen in the upper left panel, the endogenous DSGE model forecast predicted yearly GDP growth (four quarter change of log-output) to be about unchanged, whereas actual economic activity fell dramatically in the fourth quarter. Moreover, the 95-percent uncertainty band suggests that the large drop in output was completely unexpected from the point of the view of the DSGE model. Thus, in line with Del Negro and Schorfheide [28], our estimated model carries the implication that the “Great Recession” as late as of observing the outcome in 2008Q3 was a highly unlikely tail event. Turning to yearly inflation and the federal funds rate in the middle and bottom left panels, we also see that they fell considerably more than predicted by the model, but

⁸ For an extensive comparison of the forecasting performance of the Smets and Wouters model along with a comparison to a BVAR and Greenbook forecasts on real-time data, see Edge and Gürkaynak [35].

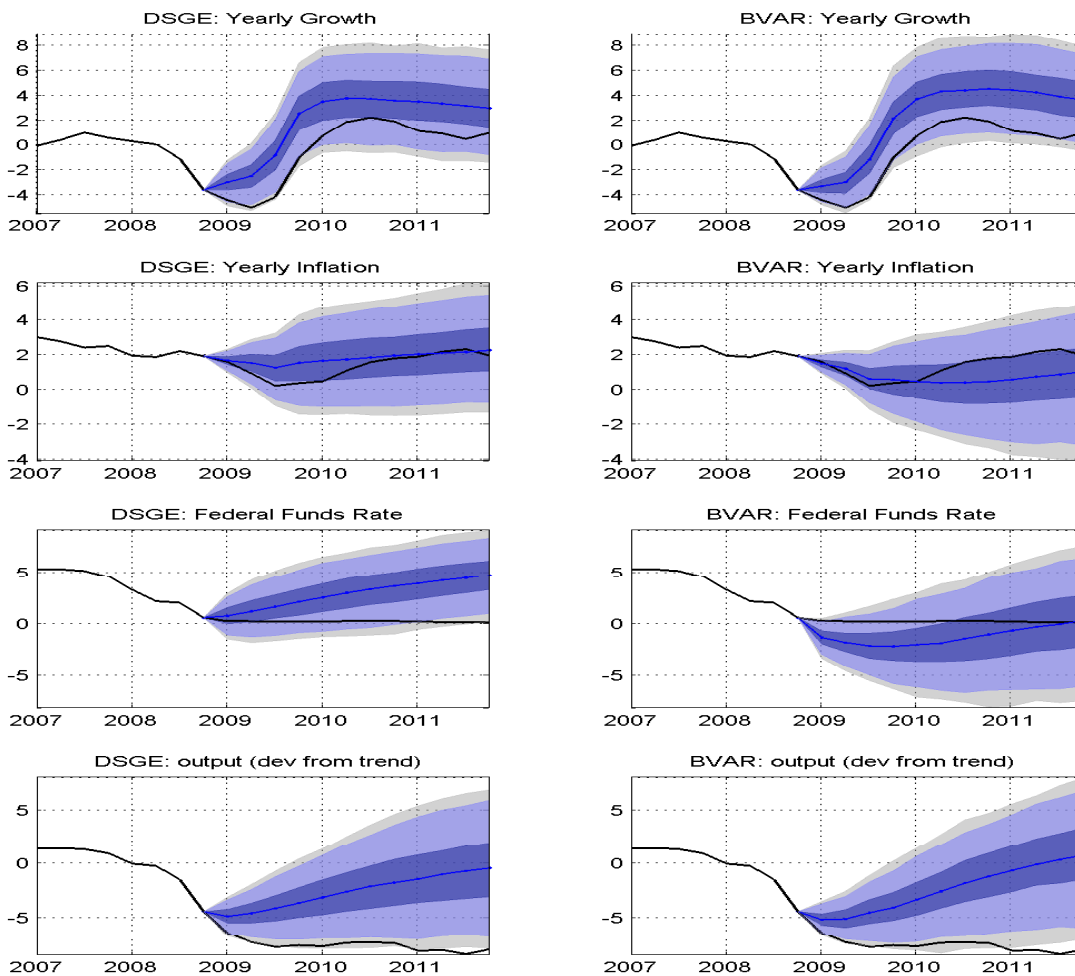
their decline are within or close to the 95-percent uncertainty bands of the linearized DSGE model and cannot hence be considered as a tail event to the same extent.

Turning to the results for the BVAR, which are reported in the right column in Figure 4.1, we see that the forecast distribution in the BVAR for yearly GDP growth is both quantitatively and qualitatively very similar to that in the DSGE model. Hence, the “Great Recession” was also a highly unlikely tail event according to the BVAR model. Given that the BVAR and the DSGE both are linearized models, the relatively high degree of similarity of the two model forecasts is not completely surprising. However, we also see that the uncertainty bands for output are roughly equally-sized in the DSGE as those in the BVAR model. This finding is neither obvious nor trivial as the DSGE model does not have a short-lag BVAR representation. On the other hand, the BVAR does not impose nearly as many cross-restrictions on the parameter space as the DSGE model; hence, allowing for parameter uncertainty will tend to increase the uncertainty bands considerably more in the BVAR relative to the DSGE model (the BVAR has around free 190 parameters, whereas the DSGE has 36). On net, these two forces appears to cancel out.

Moreover, as is clear from Figure 4.2, the high degree of coherence between the DSGE and BVAR output growth forecasts also holds up when conditioning on the state in 2008Q4 and using the estimated models to make predictions for 2009Q1, 2009Q2, ..., 2011Q4. For yearly inflation and the federal funds rate, the forecasts conditional on the state in 2008Q3 are very similar, as can be seen in the middle rows in Figure 4.1. However, for the forecast conditional on the state in 2008Q4 (Figure 4.2), the DSGE and BVAR forecasts differ substantially, at least qualitatively. In this period, the BVAR predicts a pro-longed period with near-zero inflation and a federal funds rate well below zero for two years, whereas the modal outlook in the DSGE model is that inflation would quickly return to near 2 percent and that the federal funds rate should therefore be increased steadily throughout the forecast horizon. The zero lower bound is not much of a concern in the DSGE model, whereas the BVAR suggests that it should be a binding constraint for nearly two years.

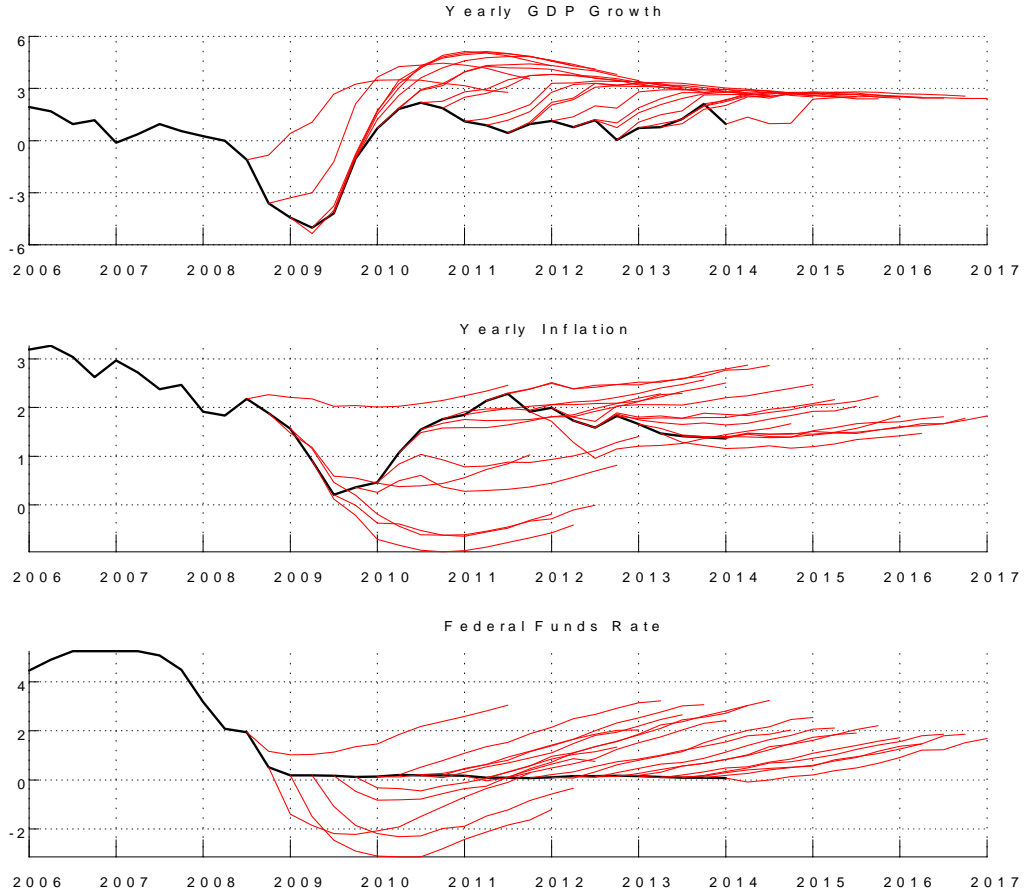
Apart from failing to predict the crisis in the first place, both the BVAR and the DSGE model also have a clear tendency to forecast a quick recovery. For the benchmark DSGE model, this feature is evident already from Figure 3.1. In this figure, the red-dotted line shows the one-sided filtered Kalman projections of the observed variables; that is, the projection for period t given all available information in period $t - 1$. By comparing the one-sided filtered Kalman projections against the outcome, the blue-solid line, it is evident that the benchmark DSGE model predicts that growth

Figure 4.2: Forecast 2009Q1-2011Q4 conditional on state in 2008Q4.



in output, consumption and investment would pick up much quicker than they did following the recession. Hence, consistent with the findings in Chung, Laforte, and Reifschneider the benchmark DSGE model consistently suggests a V-shaped recovery and that better times were just around the corner, whereas the outcome is consistent with a much more slower recovery out of the recession as is evident from Figures 4.1 and 4.2. Figure 4.3 shows sequential BVAR forecasts 1,2,...,12 quarters ahead for the period 2008Q3-2014Q1 conditional on observing the state up to the date in which the forecasts start. In line with the results for the DSGE model, the results in this figure indicate that the BVAR also tends to predict a quick recovery of economic activity. Consistent with this reasoning, the forecasts for the level of output (as deviation from the deterministic trend), shown in the bottom row in Figures 4.1 and 4.2, display that both the DSGE and the BVAR models

Figure 4.3: Sequential BVAR forecasts 2008Q3-2014Q1.



overestimate the speed of recovery out of the recession.⁹

The slow recovery following the recession is consistent with the work by Reinhart and Rogoff [73] and Jordà, Schularick and Taylor [59], who suggest that recoveries from financial crises are slower than recoveries from other recessions. The empirical observation by Reinhart and Rogoff has also been corroborated in subsequent theoretical work by Queralto [72].¹⁰ As our benchmark equilibrium model does not include the mechanisms in Queralto, it has a hard time accounting for the slow recovery following the recession, both in terms of the level and the growth rate of GDP. Our benchmark models – both the DSGE and the BVAR – rely on significant influence of adverse

⁹ For both the BVAR and the DSGE model, the series for detrended output is the smoothed estimate from the DSGE model. When we construct the forecast of detrended output in the BVAR, we use accumulate the projected quarterly growth rate of output after subtracting estimated the steady state growth rate in each period.

¹⁰ Notwithstanding these results, Howard, Martin and Wilson [56] argues out that the finding pertains to the level of economic activity, and not the growth rate (which is what we focused on in Figure 3.1).

exogenous shocks which weighs on economic activity during the recovery. While this might be deemed a significant weakness of these models, it should be noted that some major negative events may have contributed to hold back the recovery; e.g. the European debt crisis which intensified in May 2010, and the showdown between the Republicans and democrats in the congress with created significant uncertainty in the U.S. economy according to estimates by Fernandez-Villaverde et al [42]. With this events in mind, it is not entirely implausible that the models need some adverse shocks during the recovery to fit the data.

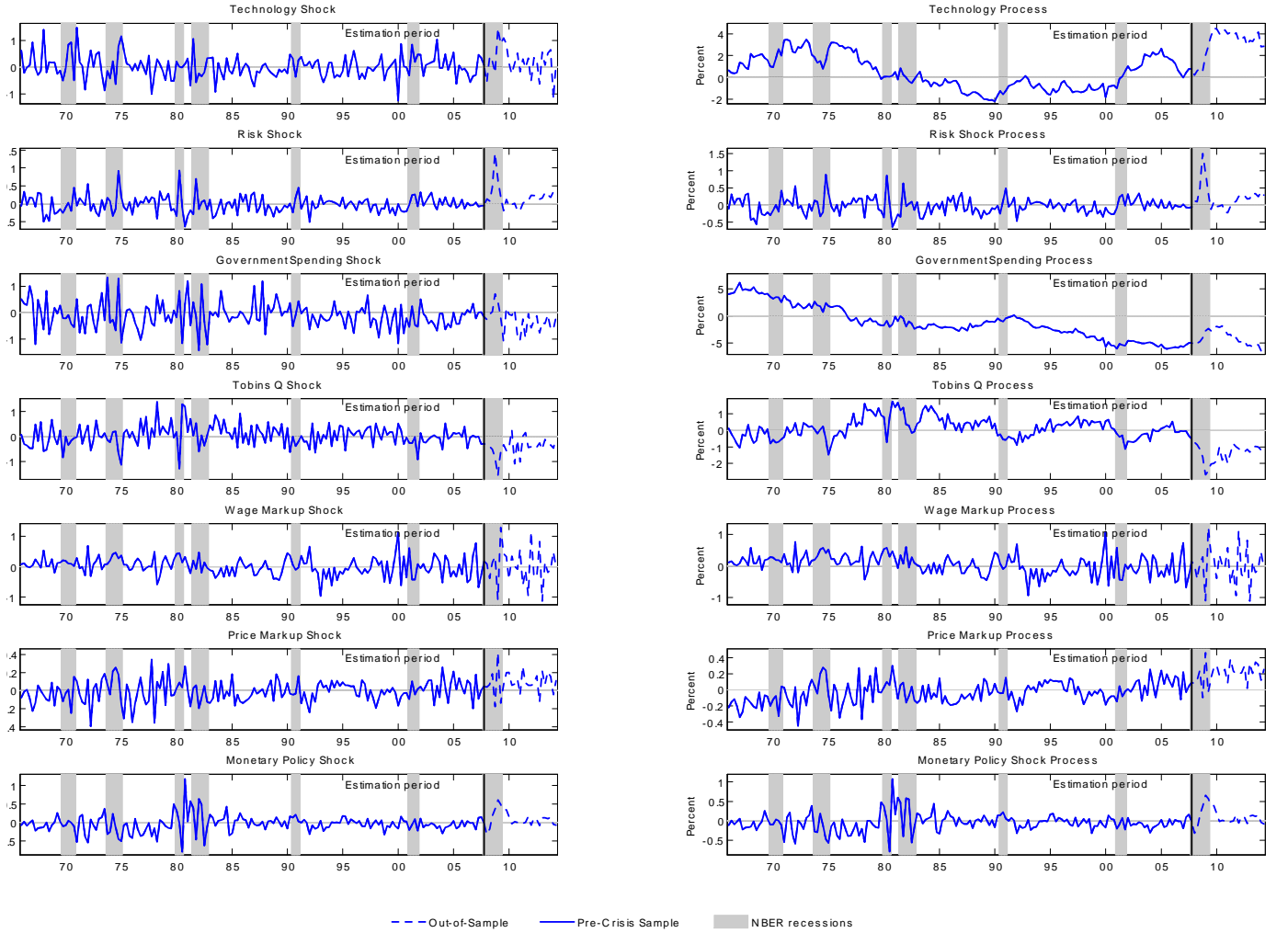
4.2. Economic interpretation of the recession

As indicated in the previous section, the DSGE model is dependent on adverse shocks to account for the recession. In this section, we examine what shocks the model filters out as the drivers of the recession. We will focus on the benchmark DSGE entirely, as it would be hard to identify all the shocks in the BVAR model. We extract the smoothed shocks through the Kalman filter by using the model estimated on the pre-crisis period for the full sample (without re-estimating the parameters).

In Figure 4.4, the left column shows the two-sided smoothed Kalman filtered innovations (e.g. η_t^a for the technology shock in eq. 3.7) for the seven shocks processes in the model using the posterior mode parameters. In the right column, we show the two-sided smoothed shock processes in levels (e.g. ε_t^a for the technology shock in eq. 3.7). The blue solid-line indicates the in-sample period, and the blue-dotted line the out-of-sample period. The Grey bars are NBER dated recessions.

As is clear from the figure, the key innovations happened to technology, investment specific technology (the Tobin's Q-shock), and the risk-premium shock during the most intense phase of the recession. More specifically, the model filters out a very large positive shock to technology (about 1.5 percent, as shown in the upper left panel, which corresponds to a 3.4 standard error shock) in 2009Q1. In 2008Q4 and 2009Q1, the model also filters out two negative investment specific technology shocks (about -1 and -1.5 percent, or 2.0 and 3.7 standard errors, respectively). Moreover, the model filters out a large positive risk shocks in 2008Q3-Q4, and in 2009Q1 (0.5, 1.5 and 0.5 percent, respectively, equivalent to 1.9, 6.0 and 2.8 standard errors). These smoothed shocks account for the bulk of the sharp decline in output, consumption and investment during the acute phase of the crisis at the end of 2008 and the beginning of 2009. Our finding of a large positive technology shock in the first quarter of 2009 may at first glance be puzzling, but can be understood from Figures 3.1 and 4.2. From these figures, we see that output (as deviation from

Figure 4.4: Smoothed Innovations and Shocks in Model Estimated on Pre-Crisis Data.



trend) fell less than hours worked per capita did during the recession. Hence, labor productive rose sharply during the most acute phase of the recession. The model replicates this feature of the data by filtering out a sequence of positive technology shocks. These technology shocks will be stimulate for output, consumption and investment. Hence, the model needs some really adverse shocks that depresses these quantities even more and causes hours worked per capita to fall, and this is where the positive risk premium and investment specific technology shocks come into play. These shocks cause consumption (risk premium) and investment (investment specific) – and thereby GDP – to fall. Lower consumption and investment also causes firms to hire less labor, and hours worked per capita falls.

Another shock that helps account for the collapse in activity at the end of 2008 is the smoothed monetary policy shock shown the bottom left panel (expressed at a quarterly rate). This shock becomes quite positive in 2008Q4 and 2009Q1; in annualized terms it equals roughly 150 (1.6 s.e.) and 250 (2.8 s.e.) basis points in each of these quarters, respectively. As the actual observations for the annualized federal funds rate is about 50 and 20 basis points, these sizeable policy shocks suggest that the zero lower bound is likely to have been a binding constraint, at least in these quarters. This finding is somewhat different than those in Del Negro and Schorfheide [28] and Del Negro, Giannoni, and Schorfheide [26], who argued that the zero lower bound was not a binding constraint in their estimated models.

The large smoothed innovations translate into very persistent movements in some of the smoothed shock processes, reported in the right column in Figure 4.4. For the simple AR(1) shock processes, the degree of persistence is governed by the posterior for ρ . As can be seen from Table 3.2, the posterior for ρ_a (ρ_b) is very high (low), whereas the posterior for ρ_i is somewhere in between. It is therefore not surprising that the technology process is almost permanently higher following the crisis, whereas the risk shock process quickly recedes towards steady state. Our finding of a very persistent rise in the exogenous component of total factor productivity is seemingly at odds with Christiano, Eichenbaum and Trabandt [18], who reports that TFP fell in the aftermath of the recession. In line with Christiano et al., Gust, Lopez-Salido and Smith [54] also report negative innovations to technology in 2008 (see Figure 5 in their paper). While a closer examination behind the differences in the results would take us too far, we note that our findings aligns very well with Fernald [40]. Specifically, our smoothed innovations to technology are highly correlated with the two TFP measures computed by Fernald [40], as can be seen from Table 4.1. The table shows the correlations between our technology innovations η_t^a , shown in the left column in Figure 4.4, and the period-by-period change in the raw and utilization-corrected measure of TFP in Fernald. From the first column in the Table, we learn that the correlation between our innovations and his raw measure is almost 0.5 for the estimation sample period. As we are studying first differences and innovations, this correlation must be considered quite high. Even more reassuring for our model is that the correlation between our smoothed innovation series and Fernald's utilization adjusted series is as high as 0.6. When extending the sample to include the crisis and post-crisis period, we see that these correlations remains high; if anything, they become slightly higher. We believe this lends support for our basic result that weak TFP growth was not a key contributing factor to the crisis.

Table 4.1: Correlations between Smoothed and Actual TFP shocks.

TFP measure	Sample Period	
	Pre-Crisis: 66Q1-07Q4	Full: 66Q1-14Q2
$\text{corr}(\Delta\text{Raw}, \eta_t^a)$	0.483	0.522
$\text{corr}(\Delta\text{Corrected}, \eta_t^a)$	0.602	0.608

Note: “ ΔRaw ” denotes the first difference of the quarterly unadjusted measure in Fernald [40], whereas “ $\Delta\text{Corrected}$ ” is the first difference Fernald’s capacity utilization adjusted TFP measure. In the model, the smoothed estimates of the innovations η_t^a (see eq. 3.7) are used. This series is depicted in the upper left column of Figure 4.4.

For the two markup shocks, we notice that they are not nearly as highly correlated as the technology shock, although the estimated AR(1) coefficients for these processes are quite high (0.89 for the price markup shock, and 0.97 for the wage markup shock, see Table 3.2). The reason why their correlation is so low is the estimated MA(1) coefficients, ϑ_p and ϑ_w in equations (3.9) and (3.20) are rather high, 0.72 and 0.92, respectively. Despite the generally low correlation of the price shock process during the pre-crisis period, we see that its outcome is unusually large and positive during the crisis period. This finding is in line with Fratto and Uhlig [44], who found that price markup shocks played an important role to avoid an even larger fall in inflation during the crisis, and contributed to the slow decline in employment during the post-crisis recovery.¹¹ The wage markup shock process does not display any clear pattern after the pre-crisis period, but it is clear that its variance has increased since the end of the 1990s, suggesting that the model provides a less accurate description of wage-setting behavior in the U.S. labor market since then. However, it should be kept in mind that this finding may not necessarily obtain for alternative wage series.¹²

While the smoothed shocks the model needs to explain the crisis period are not too surprising given its specification, it is nevertheless clear that the benchmark model needs a highly unlikely combination of adverse shocks in 2008Q4 and 2009Q1 to account for the most intense phase of the recession. Therefore, we now discuss the statistical properties of the shocks and examine if they correlate with some key observable financial variables not included in our set of observables.

4.3. Statistical properties of the innovations and their relation to financial indicators

Table 4.2 provides an overview of the statistical properties of the estimated structural shocks and of the forecast errors for the seven observed macro variables. Most of the forecast errors display

¹¹ The prominent role of the price and wage markup for explaining inflation and behavior of real wages in the SW07 model have been criticized by Chari, Kehoe and McGrattan [15] as implausibly large. Galí, Smets and Wouters [48], however, shows that the size of the markup shocks can be reduced substantially by allowing for preference shocks to household preferences.

¹² Because of potential measurement problems pertaining to wages, Justiniano, Primiceri and Tambalotti [61] use two series for real wage growth when estimating their DSGE model.

a significant amount of kurtosis, a feature that they inherit from the underlying macro variables. For the structural shocks, the problems are mostly concentrated in two shocks – the monetary policy and the risk premium shock – that display highly significant deviations from the underlying Gaussian assumption. The structural innovations in the policy rate and the risk premium are characterized by a highly skewed and fat-tailed distribution.¹³ We identified the large disturbances in these shocks already in the previous section as crucial drivers of the recent recession, but Table 4.2 illustrates that both processes were already affected by non-Gaussian innovations in the pre-crisis model as well. As observed in Figure 4.4, these negative outliers occur mostly during the recession periods.

Table 4.2: Statistical Distribution of Innovations.

Innovations in	Sample Period							
	Pre-Crisis: 66Q1-07Q4				Full Sample: 66Q1-14Q2			
	mean	std	skew	kurt	mean	std	skew	kurt
Technology	0.04	0.44	0.43*	4.09*	0.04	0.46	0.32	3.76
Risk premium	0.00	0.24	0.74**	5.12**	0.00	0.19	1.03**	7.08**
Inv.spec.techn.	0.02	0.42	0.09	3.95*	0.02	0.37	0.09	3.73
Exog.spending	-0.07	0.50	0.30	3.66	-0.07	0.49	0.25	3.65
Price markup	0.00	0.12	-0.14	3.49	0.00	0.12	0.01	3.62
Wage markup	0.01	0.31	0.10	3.89	0.01	0.37	0.03	4.48**
Monetary policy	-0.03	0.23	0.76**	8.09**	-0.04	0.23	0.80**	8.45**

Forecast errors in								
Output growth	-0.04	0.66	0.38*	5.05**	0.01	0.69	0.12	5.10**
Consumption growth	0.01	0.56	-0.42*	4.50**	0.08	0.62	-0.89**	6.77**
Investment growth	0.25	1.62	0.14	5.24**	0.25	1.73	-0.02	5.43**
Hours per capita	-0.04	0.53	0.03	4.25**	-0.02	0.55	-0.03	3.96*
Inflation	0.05	0.26	0.22	4.05*	0.04	0.25	0.30	4.14**
Real wage growth	-0.05	0.63	0.14	3.89	-0.04	0.73	-0.03	4.72**
Short rate	-0.01	0.24	1.29**	12.25**	-0.02	0.22	1.80**	15.31**

Note: *, ** indicate significance at respectively 5% and 1% probability.

This feature implies that the predictive density of linear Gaussian DSGE models underestimates systematically the probability of these large recession events. This observation is important because

¹³ The innovations in the structural shocks are also characterized by a significant ARCH effect illustrating the systematic time-varying volatility structures.

it means that the model considers the strong economic downturns that we typically observe during recession periods as extremely unlikely tail events.¹⁴ Linear Gaussian models may therefore be inappropriate instruments for analyzing policy questions related to risk scenario's or stress test exercises.

Table 4.3: Correlation between Innovations and Financial Indicators.

Innovations								
Innovations in	σ_a	σ_b	σ_i	σ_g	σ_p	σ_w	σ_r	
Technology	σ_a	1.00						
Risk premium	σ_b	-0.11	1.00					
Inv.spec.techn.	σ_i	-0.19	-0.08	1.00				
Exog.spending	σ_g	0.01	0.27	-0.06	1.00			
Price markup	σ_p	-0.03	0.18	0.05	0.13	1.00		
Wage markup	σ_w	0.00	-0.01	-0.07	-0.21	-0.09	1.00	
Monetary policy	σ_m	0.09	-0.17	-0.05	0.17	-0.05	-0.04	1.00

Innovations								
Financial Indicator	σ_a	σ_b	σ_i	σ_g	σ_p	σ_w	σ_r	
Baa-Aaa	-0.10	0.39	-0.21	0.28	0.04	-0.02	0.04	
Term spread	0.11	0.33	-0.11	-0.04	-0.07	0.10	-0.46	
Ted Spread	-0.20	-0.13	0.13	0.18	0.14	-0.02	0.34	
return S&P	0.14	-0.24	0.18	-0.20	-0.13	0.02	-0.13	
return Fin	0.02	0.03	0.01	-0.05	-0.14	0.02	-0.10	
return HP	-0.07	-0.07	0.25	-0.06	0.00	0.02	-0.14	
VOX	-0.12	0.10	0.03	0.13	0.09	0.01	-0.05	

Note: The data sources are provided in Appendix C.

It is also interesting to note that the two structural shocks that generate most of the extreme events are directly related to the intertemporal decisions and to the developments in the monetary and the financial sector of the economy. The non-Gaussian nature of financial returns, spreads and risk premiums is widely documented in the financial literature. Therefore, it appears like a natural hypothesis to assume that the non-Gaussian shocks that are identified in our macro model reflect the influence – or the feedback – from financial disruptions to the rest of the economy. To support this argument, we calculate the correlations between our estimated structural innovations and a

¹⁴ This observation is consistent with the findings presented in Chung et al. [22] This paper also evaluates the implications for the probability of hitting the ZLB.

set of popular financial returns and spreads. We selected seven measures related to the different segments of the financial sector and for which long time series are available: the Baa-Aaa spread, the term spread, the Ted spread, the return on the S&P index, the return on the Fama-French financial sector portfolio, the change in the Shiller house price index and the VOX index. Table 4.3 summarizes the correlation between these seven financial indicators and our seven structural innovations. The strongest correlations in this table – exceeding 0.3 in absolute terms – are observed between our identified risk premium innovation and the Baa-Aaa and Term spreads, and between the monetary policy innovation and the Term and Ted spreads.

Table 4.4: Regression Analysis of Innovations and Financial Indicators.

Innovations in	Pre-crisis sample			Full sample		
	σ_b	σ_i	σ_r	σ_b	σ_i	σ_r
Contemporaneous impact from financial indicator on innovations						
BAA-AAA	0.29*	-0.57*	0.09	0.28*	-0.26*	-0.02
Term spread	0.10*	-0.05	-0.18*	0.09*	-0.02	-0.18*
Ted Spread	-0.09*	0.16*	0.15*	-0.08*	0.12*	0.14*
return S&P	-0.64	1.51*	-0.27	-0.70*	1.37*	-0.45
return Fin	0.46*	-0.24	-0.05	0.33*	-0.22	-0.01
return HP	1.35	5.41*	-3.52	0.10	4.67*	-2.63
VOX	0.38	0.00	-0.44	0.34	0.67	-0.61
F / p-value	7.00/0.00	4.45/0.00	15.80/0.00	11.79/0.00	4.86/0.00	14.31/0.00
Skew/kurt resid	0.04/2.97	0.17/3.22	0.60/4.39	0.15/3.11	0.14/3.15	0.57/4.08
Granger Causality regressions						
F / p-value	1.73/0.06	1.53/0.11	2.11/0.01	1.67/0.06	2.05/0.02	1.62/0.07

Note: * indicate significance at 5% probability. The financial indicators do not have a significant effect on the other non-reported innovations.

To see the strong linkages between some of the smoothed shocks and the financial variables in an alternative way, we regress the structural innovations on this set of financial observables. In contemporaneous regressions, the significant coefficients are again only apparent in the risk premium, monetary policy and – at a slightly weaker significance level – for the investment specific technology innovation. The most interesting feature of the regression results is that the remaining unexplained variation (i.e. the regression residuals) are basically normally distributed. Thus, shock outliers seem to coincide with periods of clear financial stress as measured by our observed financial indicators. Also noteworthy is that in Granger causality regression tests, none of the

financial indicators carry significant predictive power for the structural innovations. But because financial variables can essentially be observed in real time, they can still provide timely indications of big structural innovations and including these variables in our list of observables can therefore be very useful to improve the model now-cast and the conditional forecast performance.¹⁵ Even so, this strategy will probably not improve the out-of-sample prediction performance of our linearized models *ex ante* to the observation of financial stress signals. It might also require non-Gaussian and non-linear models to exploit this information from financial variables more efficiently in our macromodels.

5. Augmenting the benchmark model

As the analysis in Section 4 suggested that the benchmark model suffers from some important shortcomings, we study in this section if its can be improved by allowing for zero lower bound on policy rates and time-varying volatility of the shocks. Finally, we introduce financial frictions and a cost-channel into the model. We start by assessing the impact of the zero lower bound, and then analyze a variant of the model which allows for financial frictions following the seminal work of Bernanke, Gertler and Gilchrist (1999). In contrast to the analysis in Section 3.4, we estimate the model on data including crisis period when we impose zero lower bound for the federal funds rate.

5.1. Assessing the impact of the zero lower bound

We assess the impact of imposing the zero lower bound, ZLB henceforth, in the estimation in two alternative ways. These procedures differ in the way the duration of the ZLB spells is determined. In our first approach, the incidence and duration of the ZLB spells are endogenous and consistent with the model expectations, In the second approach, we model them as “exogenous” and require the model to match information from the market-based overnight index swap rates following Del Negro et al. [26]. In both approaches, we make use of the same linearized model equations (stated in Appendix A), except that we impose the non-negativity constraint on the federal funds rate. To do this, we adopt the following policy rule for the federal funds rate

$$\begin{aligned}\hat{R}_t^* &= \rho_R \hat{R}_{t-1} + (1 - \rho_R)(r_\pi \hat{\pi}_t + r_y(\widehat{ygap}_t) + r_{\Delta y} \Delta(\widehat{ygap}_t)) , \\ \hat{R}_t &= \max\left(-\bar{r}, \hat{R}_t^* + \hat{\varepsilon}_t^r\right).\end{aligned}\tag{5.1}$$

¹⁵See Del Negro and Schorfheide [28] for strong evidence in this direction.

The policy rule (5.1) assumes that the interest rate set by the bank, \widehat{R}_t , if unconstrained by the ZLB equals $\widehat{R}_t^* + \widehat{\varepsilon}_t^r$. \widehat{R}_t^* , in turn, is a shadow interest rate that is not subject to the policy shock $\widehat{\varepsilon}_t^r$. Note that since \widehat{R}_t in the policy rule (5.1) is measured as percentage point deviation of the federal funds rate from its quarterly steady state level (\bar{r}), restricting \widehat{R}_t not to fall below $-\bar{r}$ is equivalent of imposing the ZLB on the nominal policy rate.¹⁶ In its setting of the shadow, or notional, rate, we assume the Fed is smoothing over the actual interest rate, as opposed to the notional rate \widehat{R}_t^* . We made this assumption to preserve the property that $\widehat{\varepsilon}_t^r$ is close to white noise. Smoothing over the notional rate in (5.1) would cause the policy shock to be highly persistent, with an AR(1) coefficient roughly equal to ρ_R .¹⁷

To impose the policy rule (5.1) when we estimate the model, we use the method outlined in Hebden, Lindé and Svensson [55]. This method is convenient, because it is quick even when the model contains many state variables, and we provide further details about the algorithm in Appendix A.¹⁸ In a nutshell, the algorithm imposes the ZLB on the policy rate through current and anticipated shocks (addfactors) to the policy rule. More specifically, if the projection of \widehat{R}_t in (5.1) in any of the periods $t, t+1, \dots, t+T$ for some sufficiently large non-negative integer T is below $-\bar{r}$, the algorithm adds a sequence of policy shocks $\widehat{\varepsilon}_{t+h|t}^r$ such that $E_t \widehat{R}_{t+h} \geq 0$ for all $h = \tau_1, \tau_1 + 1, \dots, \tau_2$. If the added policy shocks put enough downward pressure on the economic activity and inflation, the duration of the ZLB spell can be extended both backwards (τ_1 shrinks) and forward (τ_2 increases) in time. Moreover, as we think about the ZLB as a constraint on monetary policy, we further require all current and anticipated policy shocks to be *positive* whenever $\widehat{R}_t^* < -\bar{r}$. The non-negativity requirement on the current and anticipated policy shocks for each possible state and draw from the posterior forces the posterior itself to move into a part of parameter space where the model can account for long ZLB spells which are contractionary to the economy, and without this requirement DSGE models with endogenous lagged state variables may experience sign switches for the policy shocks (so that the ZLB has a stimulative rather than contractionary impact on the economy) even for fairly short ZLB spells as documented in Carlstrom, Fuerst and Paustian [14]. As discussed in further detail in Hebden et al., the non-negativity assumption for all states and draws from the posterior also mitigates the possibility of multiple equilibria (indeterminacy).

¹⁶ See (3.28) for the definition of \bar{r} . Had we written the policy rule in levels, the first part of (5.1) would have been replaced by (3.23) (omitting the policy shock), and the ZLB part would have been $R_t = \max(1, R_t^*)$.

¹⁷ To see this, replace \widehat{R}_{t-1} with \widehat{R}_{t-1}^* in the first equation in (5.1) and then substitute $\widehat{R}_t = \widehat{R}_t^* + \widehat{\varepsilon}_t^r$ from the second equation to write the unconstrained policy rule with the actual policy rate \widehat{R}_t , the residual will be $\widehat{u}_t^r \equiv \widehat{\varepsilon}_t^r - \rho_R \widehat{\varepsilon}_{t-1}^r$. Hence, the residual \widehat{u}_t^r will be roughly white noise in this case if $\widehat{\varepsilon}_t^r$ has an AR(1)-root ρ_R .

¹⁸ Iacoviello and Guerrieri [58] have subsequently shown how this method can be applied to solve DSGE models with other forms of asymmetry constraints.

Finally, it is important to point out that when the ZLB is not a binding constraint, we assume the policy shock $\widehat{\varepsilon}_t^r$ can be either negative or positive.

However, a potentially serious shortcoming of the method we adapt to assess the implications of the ZLB is that it relies on perfect foresight and hence does not explicitly account for the role of future shock uncertainty as in the work of Adam and Billi [1] and Gust, Lopez-Salido and Smith [54]. Even so, we implicitly allow for parameter and shock uncertainty by requiring that the filtered current and anticipated policy shocks in each point time are positive for all parameter and shock draws from the posterior whenever the ZLB binds. More specifically, when we evaluate the likelihood function and find that $E_t \widehat{R}_{t+h} < 0$ in the modal outlook for some period t and horizon h conditional on the parameter draw and associated filtered state, we draw a large number of sequences of fundamental shocks for $h = 0, 1, \dots, 12$ and verify that the policy rule (5.1) can be implemented for all possible shock realizations through positive shocks only. For those parameter draws this is not feasible, we add a smooth penalty to the likelihood which is set large enough to ensure that the posterior will satisfy the constraint.^{19,20} As we document below, the non-negativity constraint on the anticipated policy shocks in the face of parameter and fundamental shock uncertainty has considerable implications for the estimation of the model, and in effect shock and parameter uncertainty is therefore accounted for in our estimation procedure, at least implicitly.

To provide a reference point for the ZLB estimations, we start out by estimating the model for the full sample period, but disregarding the existence of the ZLB. The posterior mode and standard deviation in this case are shown in the first two columns in Table 5.1, and labeled “No ZLB model”. The only difference between these results and those reported in Table 3.2 is that the sample period has been extended from 2007Q4 to 2014Q2. By comparing the results, a noteworthy difference is that the estimated degree of wage and price stickiness has increased even further relative to the pre-crisis sample. The posterior mode for the sticky wage parameter (ξ_w) has increased from 0.79 to 0.83, and the sticky price parameter (ξ_p) from 0.69 to 0.75. Relative to the SW07 posterior

¹⁹ For example, it turns out that the model in 2008Q4 implies that the ZLB would be a binding constraint in 2009Q1 through 2009Q3 in the modal outlook. For this period we generated 1,000 shock realizations for 2009Q1, 2009Q2, ..., 2011Q4 and verified that we could implement the policy rule (5.1) for all forecast simulations of the model through non-negative current and anticipated policy shocks. For the draws with adverse shocks, the duration of the ZLB was prolonged substantially during the forecast horizon, with simulated states with expected ZLB spells close to 4 years occurring. We provide further details in Appendix B how the likelihood function is constructed when we impose the ZLB in the estimations.

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mode, ξ_w has increased from 0.73 to 0.83 and ξ_p from 0.65 to 0.75. These increases are substantial, considering that the sample has been expanded with less than 10 years and that these parameters affect the slope of the wage and price pricing curves in a non-linear fashion, implying an even sharper reduction in the slope coefficients for the forcing variables (wage markup and marginal costs, respectively) in the linearized price and wage equations. Evidently, the much higher degree of price and wage stickiness is only partly driven by the fact that inflation and real wage growth fell modestly during the Great Recession (as can be seen in Figure 3.1); even before the recession materialized there was already a strong trend in the data towards higher stickiness parameters, consistent with the findings by Del Negro, Giannoni and Schorfheide [26]. Even so, we note that our estimated full sample model without the ZLB still features a much lower degree of price and wage stickiness than the policy models recently estimated by Brave et al. [12] and Del Negro et al. [31].

In Figure 5.1, we plot conditional forecast distributions for selected variables for the “No ZLB model” posterior in Table 5.1. In the left column, the forecast is conditional on the state in 2008Q3, whereas in the other two columns, it is conditional on the filtered state in 2008Q4. Similarly to the results for the pre-crisis models in Figure 4.1, the results in the left column shows that the severe drop in economic activity in 2008Q4 was outside the 95th percent uncertainty bands, even though the model is estimated on the full sample and this should hence be considered as an in-of-sample exercise. However, the median forecast conditional on the state in 2008Q4 is very accurate for yearly output growth and output (as deviation from trend) and the actual outcome is well within the uncertain bands for these variables, even disregarding the ZLB. For the federal funds rate, we see that the median forecast for the federal funds rate falls only slightly below nil for three quarters (2009Q1-2009Q3). This seemingly suggest that the ZLB was not much of a binding constraint during the Great Recession, consistent with the finding and interpretation in Del Negro, Giannoni and Schorfheide [26]. However, this interpretation ignores the fact that the forecast distribution for the federal funds rate has considerable mass below nil. And shifting this part of the distribution to 0 and above may therefore change the median outlook considerably.

Table 5.1: Posterior distributions in SW Model: 1966Q1-2014Q2.

Parameter		No ZLB model		Endogenous ZLB duration		OIS-based ZLB duration	
		mode	std.dev. Hess.	mode	std.dev. Hess.	mode	std.dev. Hess.
Calvo prob. wages	ξ_w	0.83	0.040	0.85	0.026	0.86	0.035
Calvo prob. prices	ξ_p	0.75	0.039	0.83	0.032	0.89	0.023
Indexation wages	ι_w	0.69	0.122	0.57	0.120	0.56	0.122
Indexation prices	ι_p	0.22	0.081	0.25	0.085	0.38	0.106
Gross price markup	ϕ_p	1.60	0.073	1.46	0.073	1.39	0.072
Capital production share	α	0.19	0.015	0.16	0.018	0.14	0.016
Capital utilization cost	ψ	0.80	0.075	0.73	0.094	0.60	0.120
Investment adj. cost	φ	4.58	0.941	4.61	0.61	5.84	1.095
Habit formation	\varkappa	0.62	0.054	0.62	0.031	0.68	0.041
Inv subs. elast. of cons.	σ_c	1.49	0.138	1.02	0.105	0.80	0.080
Labor supply elast.	σ_l	1.81	0.555	2.03	0.465	2.06	0.576
Hours worked in S.S.	\bar{l}	-0.40	1.178	-0.18	1.024	0.25	0.844
Discount factor	$100(\beta^{-1} - 1)$	0.10	0.042	0.13	0.056	0.12	0.054
Quarterly Growth in S.S.	$\bar{\gamma}$	0.41	0.014	0.42	0.026	0.43	0.016
Stationary tech. shock	ρ_a	0.96	0.008	0.97	0.014	0.97	0.018
Risk premium shock	ρ_b	0.40	0.104	0.85	0.055	0.97	0.008
Invest. spec. tech. shock	ρ_i	0.84	0.039	0.85	0.057	0.78	0.075
Gov't cons. shock	ρ_g	0.97	0.007	0.98	0.010	0.97	0.009
Price markup shock	ρ_p	0.92	0.030	0.88	0.046	0.86	0.048
Wage markup shock	ρ_w	0.97	0.010	0.98	0.024	0.99	0.005
Response of g_t to ε_t^a	ρ_{ga}	0.51	0.077	0.52	0.063	0.52	0.069
Stationary tech. shock	σ_a	0.46	0.025	0.48	0.032	0.50	0.029
Risk premium shock	σ_b	0.19	0.026	0.10	0.015	0.08	0.007
Invest. spec. tech. shock	σ_i	0.36	0.032	0.31	0.028	0.30	0.044
Gov't cons. shock	σ_g	0.49	0.025	0.48	0.026	0.48	0.025
Price markup shock	σ_p	0.12	0.013	0.13	0.011	0.14	0.012
MA(1) price markup shock	ϑ_p	0.80	0.058	0.79	0.070	0.80	0.071
Wage markup shock	σ_w	0.37	0.022	0.36	0.020	0.36	0.021
MA(1) wage markup shock	ϑ_w	0.96	0.013	0.96	0.025	0.98	0.007
Quarterly infl. rate. in S.S..	$\bar{\pi}$	0.81	0.102	0.76	0.106	0.70	0.103
Inflation response	r_π	1.69	0.153	1.86	0.159	2.15	0.165
Output gap response	r_y	0.05	0.016	0.10	0.013	0.16	0.027
Diff. output gap response	$r_{\Delta y}$	0.24	0.027	0.24	0.020	0.23	0.024
Mon. pol. shock std	σ_r	0.23	0.013	0.22	0.012	0.21	0.011
Mon. pol. shock pers.	ρ_r	0.21	0.070	0.10	0.058	0.06	0.041
Interest rate smoothing	ρ_R	0.80	0.028	0.83	0.016	0.85	0.018
Log marginal likelihood		Laplace	-1146.69	Laplace	-1151.99	Laplace	-1175.24

Note: See notes to Table 3.2. The “No ZLB model” neglects the presence of the zero lower bound in the estimations, whereas the “Endogenous ZLB duration” allows the duration of the ZLB to be endogenous as described in the main text. Finally, the “OIS-based ZLB duration” imposes the duration of the ZLB in each point in time according to OIS rates for the federal funds rate between 2008Q4 and 2011Q2.

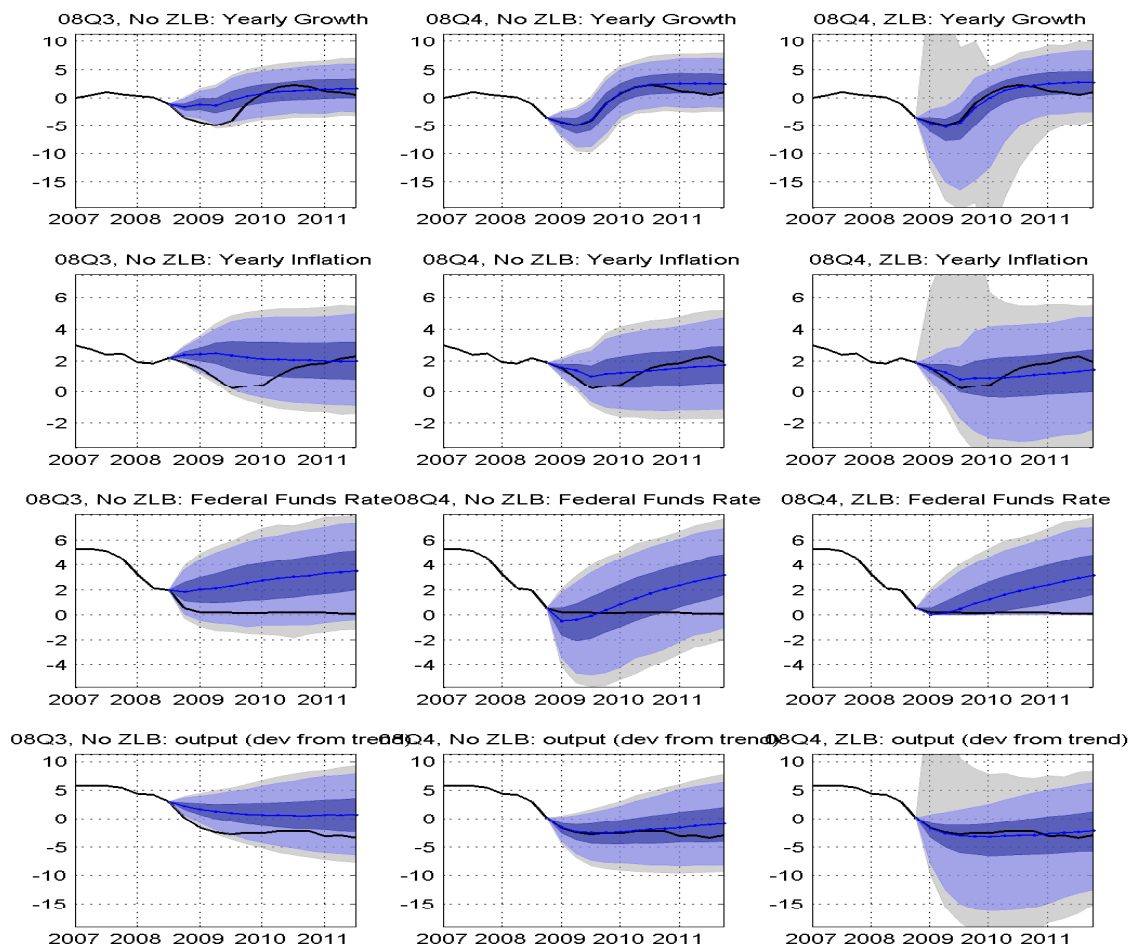
To examine this possibility, the third column in the figure reports the forecast distribution when sampling parameters and shocks from the posterior distribution for the “No ZLB model” in Table 5.1, but with the unconstrained policy rule replaced by the policy rule (5.1). This means that the actual and expected federal funds rate will respect the ZLB during the forecast horizon. Importantly, the 1,000 different shock realizations used to construct the forecast distribution in the ZLB case are identical to those used to construct the unconstrained forecast distribution; given the state in 2008Q4 the only difference between the results in the second and third column is thus that the federal funds rate is constrained from falling below zero. As can be seen from the panels for

output growth and output as deviation from trend, imposing the ZLB on the federal funds rate widens their uncertainty bands downwards quite notably. For output as deviation from trend, the lower 95th percentile shifts down from roughly -10 percent to nearly -20 percent in 2010. Hence, in the absence of unconventional monetary policies and coordination between monetary and fiscal policy (i.e. fiscal stimulus when the economy enters a long-lived liquidity trap), the baseline model suggests that the ZLB may be associated with large economic costs.

On the other hand, the upper-95th percent bands for these variables are also much higher when the federal funds rate is constrained to fall below zero conditional on the given state in 2008Q4. For detrended output, the upper 95th percentile is above 10 percent in 2009. For yearly inflation, the upper 95th percentile is above 6 percent. Despite these elevated upper uncertainty bands for output growth, detrended output and inflation, the upper 95th percentile for the federal funds rate is lower than the corresponding percentile in the unconstrained policy rate distribution. This seemingly goes against the specification of the policy rule (5.1), as the systematic part of the policy rule governing \hat{R}_t^* calls for a high policy rate whenever inflation, output growth and the output gap is high. The reason why this does not happen in the conditional ZLB distribution in Figure 5.1 is that the model estimated without the ZLB constraint imposed needs large *negative* current and anticipated policy shocks $\hat{\varepsilon}_{t+h|t}^r$ to satisfy $E_t \hat{R}_{t+h} \geq 0$. In essence, when the economy is hit by some really adverse shocks in these simulations and the policy rate is constrained to respond to these shocks for a sufficiently long period, inflation expectations and economic activity fall to such a large extent that a sequence of *negative* instead of *positive* policy shocks $\hat{\varepsilon}_{t+h|t}^r$ for $h = 0, 1, \dots, \tau_2$ are needed to prevent the federal funds rate to fall below nil. As discussed in Hebden et al. [55] and Carlstrom et al. [14], the switch in signs of the policy shocks only happens in the relatively few draws for which the policy rate is expected to be constrained by the lower bound for a very pro-longed period of time (i.e. τ_2 is large). This also explains why the upper 95th percentiles for inflation and output shifts up so much whereas the 90th percentile is roughly unchanged relative to the unconstrained distribution. The 90th percentile is associated with simulations of favorable fundamental shocks and parameter draws for which no large negative policy shocks are needed to prevent the policy rate to fall below nil.

We believe this result – that the ZLB can trigger adverse shocks to have sharply expansionary effects on the economy – is an unpalatable feature of the model. Therefore, when we re-estimate the model subject to the ZLB constraint on the federal funds rate, we believe it is crucial to impose the additional constraint – discussed in the beginning of this section – that the parameters of the

Figure 5.1: Forecast 2009Q1-2011Q4 conditional on state in 2008Q4 for model estimated through 2014Q2 without imposing the ZLB.



economy have to be such that all current and expected policy shocks used to impose the policy rule (5.1) are positive whenever the ZLB binds. By imposing this constraint, we ensure that the re-estimated model does not feature any sign reversals of the policy shocks even for the most long-lived liquidity traps in our forecast distributions.

The estimation results for this variant of the model are reported in Table 5.1 and labeled “Endogenous ZLB duration”. We use this label because both the incidence and duration of the ZLB spells are endogenous estimation outcomes in the model, and do not necessarily conform with other commonly used measures of the expected future path of the federal funds rate, such as overnight index swap (OIS, henceforth) rates. By comparing the results with the “No ZLB model”, we see that imposing the ZLB in the estimations have quite important implications for the

posterior distribution. First of all, the degree of price and wage stickiness is elevated even further, and the estimated parameters imply a slope of the New Keynesian Phillips curve of 0.006. This is somewhat lower than the median estimates of literature, which cluster in the range of about 0.009 – 0.014, but well within standard confidence intervals provided by empirical studies (see e.g. Adolfson et al. [3], Altig et al. [7], Galí and Gertler [46], Galí, Gertler, and López-Salido [47] and Lindé [67]). In addition, the higher degree of nominal wage stickiness makes marginal costs even more sticky in the ZLB model. Together, these features makes inflation and inflation expectations more slow to react to various shocks, and therefore allow the model to cope with long spells at the ZLB without triggering indeterminacy problems (i.e. switches in signs for the policy shocks). This finding is consistent with Erceg and Lindé [38], who argued that a low slope of the Phillips curve is consistent with the development during the recent crisis where inflation and inflation expectations have fallen very moderately despite large contractions in output. It is also consistent with many recent papers which have estimated similar DSGE models, see e.g. Brave et al. [12], Del Negro et al. [31], and Del Negro, Giannoni and Schorfheide [26].

In addition to the higher degree of wage and price stickiness, there are two other important differences. First, the coefficient on the output gap in policy rule, r_y , is about twice as high as in the “No ZLB model”. To the extent the output gap becomes significantly negative during the Great Recession, this will tend to push down the path of the federal funds rate and extend the duration of the ZLB. Second, the persistence coefficient for the risk premium, ρ_b , increases sharply, from 0.40 to 0.85. Even so, since the standard deviation of the innovations, σ_b , is reduced from 0.19 to 0.10 the unconditional variance for the risk-premium shock nevertheless falls slightly (from 0.044 to 0.039) in the ZLB model. Therefore, the higher persistence does not imply a significantly larger role for the risk-premium shocks (apart from expectational effects). [Provide more intuition here why ρ_b (σ_b) rises (falls).]

Figure 5.2 show the forecast distribution (given the state in 2008Q4) in the “Endogenous ZLB duration” variant of the model. The left column shows the results when the ZLB is counterfactually neglected, whereas the right column shows the results when the ZLB is imposed. As expected, we see that the forecast distribution in the variant of the model which counterfactually neglects the ZLB features symmetric uncertainty bands around the modal outlook, and is a little bit too optimistic about the outlook for output relative to the model which imposes the ZLB (right column). But by and large, and perhaps surprisingly, the modal outlook for 2008Q4 in the model estimated and imposing ZLB constraint (right column in Figure 5.2) differs very little to the modal outlook in the

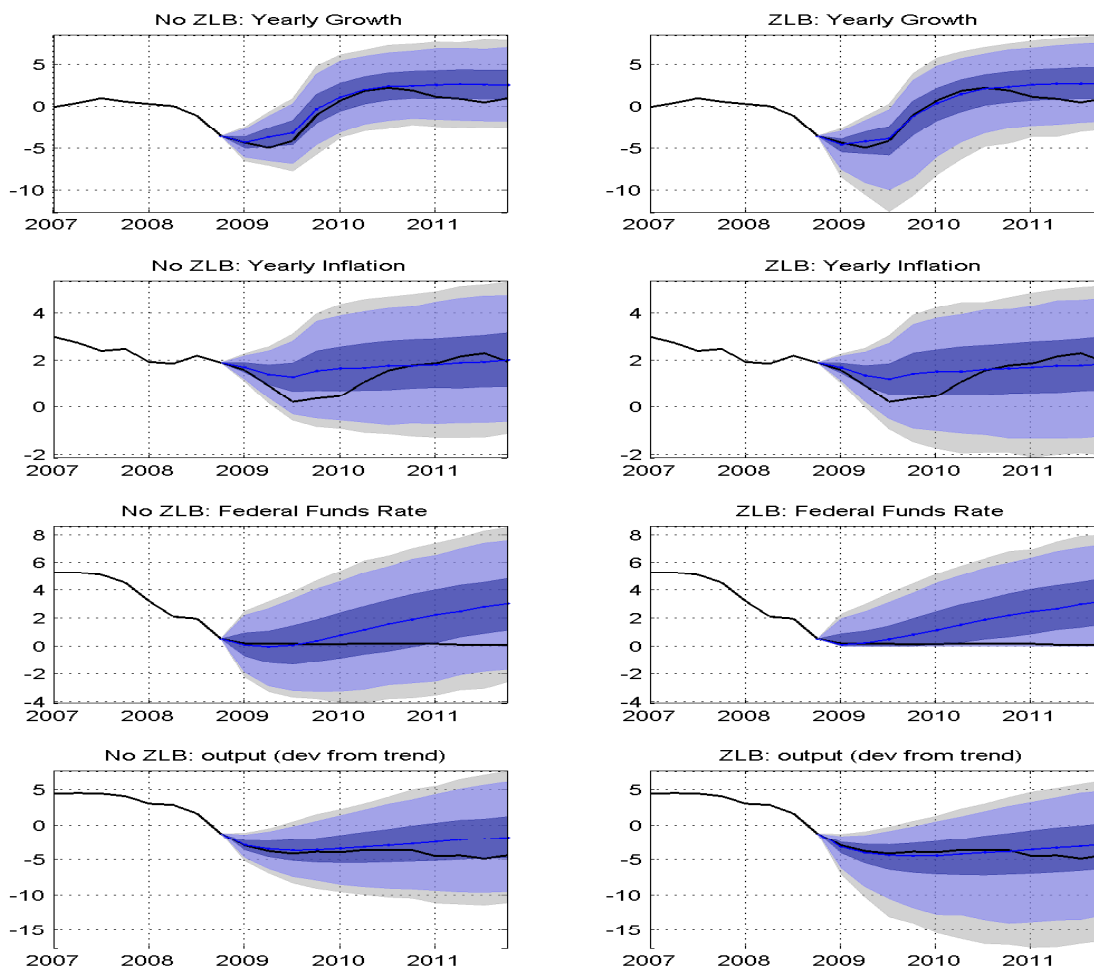
“No ZLB model” which completely neglects the ZLB (the middle column in Figure 5.1). Obviously, a key difference is that the median path of the federal funds rate is constrained by the lower bound in 2009, and below nil in the unconstrained version of the model. Still, the quantitative difference for the median projection is small. Instead, the most noticeable difference between the No ZLB model and the model estimated under the ZLB is the uncertainty bands: they are wider and downward skewed in the model that imposes the ZLB constraint (the right column of Figure 5.2) compared to the No ZLB model that neglects the presence of the ZLB constraint.

However, the forecast distributions in the “No ZLB model” which enforces the ZLB ex post (the right column in Figure 5.1) differs dramatically to the forecast distributions in the model estimated under the ZLB constraint (the right column in Figure 5.2). The higher degree of wage and price stickiness in the model estimated under the ZLB constraint insulate the economy from the disaster scenarios and the indeterminate equilibria and therefore shrink the uncertainty bands considerably. Overall, this suggests that taking the ZLB into account in the estimation stage may be of key importance in assessing its economic consequences, and that it is not evident that models estimated on pre-crisis data can be useful for policy analysis when the economy enters into a long-lived liquidity trap. In such situations, the pre-crisis policy models may feature too much flexibility in price and wage setting, and e.g. yield implausibly large fiscal multipliers as noted in e.g. Erceg and Lindé [38].

Another interesting feature of the model which neglects the ZLB and the variant of the model which is constrained to impose (5.1) through positive current and anticipated policy shocks is that the former has a higher log-marginal likelihood (-1146.7 vs. -1152). This implies that imposing the ZLB on the model is somewhat costly in terms of data coherence. However, as suggested by the small differences in the conditional forecast distributions in Figures 5.1 (middle column) and 5.2 (right column), is not evident if this difference in log marginal likelihood is important from an economic viewpoint, although it is large enough to be sizeable in terms of a Bayesian posterior odds ratio.

As the model is endogenously determining the incidence and duration of the ZLB spells, it is interesting to note that according to the model, the ZLB is expected in 2008Q4 to be a binding constraint from 2009Q1 through 2009Q3 in the modal outlook. The expected positive policy shocks we use to impose the ZLB substitute partially for the exceptionally huge risk premium shocks that drive the economy to the ZLB in the first place. The constraint is then expected to be binding during 2009, with a maximum duration of five quarters given the state in 2009Q1, and from

Figure 5.2: Forecast 2009Q1-2011Q4 conditional on state in 2008Q4 for model estimated through 2014Q2 when imposing the ZLB.



2010Q2 onwards, the model expects the interest rate to lift off already in the next quarter. The short duration of the ZLB spells is consistent with the findings in Chung et al. [22]. The fact that the federal funds rate has remained at the ZLB since then is by model explained either as a result of expansionary monetary policy actions – forward guidance – or as standard policy reactions to unexpected headwinds. The filtered shocks suggest a dominant role for the second interpretation.

As noted previously, an alternative to letting the DSGE model determine the expected duration of the ZLB in each time period is to use OIS data for the federal funds rate as observables when estimating the model. By doing so, we follow Del Negro et al. [26] and require that the expected federal funds rate in the model matches the OIS data in each point in time when the ZLB is binding, i.e. from 2008Q4 and onwards. We use OIS data (acquired from the Federal Reserve Board) for

1, 2, ..., 12 quarters ahead expected federal funds rates, and require the model to match those rates exactly through anticipated policy shocks following the general idea outlined in Maih [70]. The appealing feature of Maih’s algorithm is that it does not require us to include standard deviations for each of the anticipated policy shocks we use to fit the OIS data and that the log-marginal likelihood can be compared to the models which does not condition on OIS data.

Before we turn to the results in Table 5.1, there are two additional important pieces of information. First, as we interpret the OIS data as expected means of future federal funds rates, we set them equal to nil in each point in time whenever they are lower than 50 basis points. We do this as our OIS estimation procedure does not explicitly account for future shock uncertainty, and the projected path of the interest rate from the model should therefore be viewed as a modal outlook (which will be notably lower than the mean of the forecast distribution when the ZLB binds). Second, because the Federal Reserve did not use explicit time-dependent forward guidance until August 2011, we restrict all anticipated policy shocks to be positive prior to this date. After this date, we do not impose any signs on the anticipated policy shocks, because credible forward guidance, or a “Lower for longer policy” in the spirit of Eggertsson and Woodford [36] which extends the duration of the ZLB is better viewed as expansionary than contractionary policy. Specifically, we allow the model to explain sharp flattening of the OIS curve between the second and third quarter in 2011 with negative policy shocks, and do not impose this flattening to be associated with a noticeable deterioration in the economic outlook.

The results when imposing the incidence and duration of the ZLB to adhere with OIS rates are shown in the left panel in Table 5.1, labeled “OIS-based ZLB duration”. Relative to the “Endogenous ZLB duration” posterior for which the incidence and duration of the ZLB is determined endogenously in the model, we see that the degree of price stickiness is elevated further (from 0.83 to 0.89), and now implies a slope of the Phillips curve (i.e. direct sensitivity of current inflation to marginal cost) of 0.003. This is substantially lower than e.g. the estimate in Altig et al. [7], but still higher than Brave et al. [12]. To square this estimate with the microliterature is a challenge, and probably require a combination of firm-specific capital (as in Altig et al.), firm-specific labor (as in Woodford [79]), and a higher sensitivity of demand to relative prices (i.e. higher Kimball parameter ε_p). Apart from the higher stickiness, we also see an elevated role for risk-premium shock in this model (ρ_b rises sharply from 0.85 to 0.97, whereas the std of the innovations only falls moderately from 0.10 to 0.08), and that the degree of habit formation consumption (\varkappa) and investment adjustment costs (φ) rises somewhat. Finally, the response coefficient for the output

gap in the policy rule is increased further, and is now three times higher than in the model which neglects the presence of the ZLB.

The reason why these parameters are further changed relative to the “No ZLB model” is that the OIS data generally imposes longer-lived ZLB episodes than the model endogenously produces. In order to be able to explain those episodes with *positive* anticipated policy shocks through 2011Q2 the model needs to make dynamics more sluggish and explain the rebound in inflation during 2010 with temporary shocks. However, enforcing this sluggish dynamics on the model is rather costly in terms of log-marginal likelihood, which falls from -1152 in the model with endogenous ZLB duration to -1175.2 for the OIS-based ZLB duration. This is sizeable drop, and one possible interpretation is that the SW model, despite imposing the ZLB constraint, was more optimistic about the recovery than market participants during this episode. Other possibilities are that; (i) the model mis-measures the size and persistence of the relevant output gap, (ii) the model-consistent or rational expectation hypothesis fails to capture the stickiness and persistence in expectations that might be caused by learning dynamics or information filtering issues, or (iii) that the Federal Reserve changed policy behavior since the outset of the Great Recession, with a larger emphasis on the output gap. Yet other possibilities is that our model above misses out on time-varying volatilities of the shocks and omits financial frictions and the cost-channel of monetary policy. We explore these latter possibilities below.

5.2. Allowing for Time-Varying Volatility

As documented above, the prototype linear Gaussian model with constant volatility does not provide a realistic predictive density for the forecast in particular around severe recession periods or periods of high financial and monetary stress. A large share of the research effort on DSGE models since the financial crisis and the Great recession has tried to overcome these weaknesses of the basic DSGE setup. By now, most models used in academia and in policy institutions contain financial frictions and financial shocks in an effort to introduce stronger amplification mechanisms in the model. But as we will discuss in the next section, to the extent that these models belong to the class of Gaussian linear approaches, they still depend on extremely large shocks to predict important recessions. The explicit modelling of the non-linear macro-finance interactions is complex and ambitious and the research in that direction has not yet been integrated in empirical macro models. A mechanical solution to improve the predictive densities of linear DSGE model exists in allowing for a more complicated stochastic structure. Here we illustrate this approach by considering a Markov

Switching stochastic structure following Liu, Waggoner and Zha [68].²¹

Low frequency changes in the shock variances have been analyzed by Fernandez-Villaverde and Rubio-Ramirez [41] and Justiniano and Primiceri [60] via stochastic volatility processes. Chib and Ramamurthy [16] and Curdia, Del Negro and Greenwald [24] show that a Student's t-distribution for the innovations is also strongly favoured by the data as it allows for rare large shocks. The latter paper makes the point that the time-variation in shock variances should contain both a low and a high frequency component.

To capture these insights, we consider a version of the benchmark model in which we allow for two independent Markov Switching processes in the shock variances. Each Markov process can switch between a low and a high volatility regime. One process affects the volatility of all the structural innovations with exception of the wage markup shock that follows an a-typical dynamic profile. The second Markov process is restricted to the non-Gaussian structural shocks as identified in Section 4.3: this process affects the volatility in the monetary policy, the risk premium and the investment specific innovations. The volatility in these three shocks is scaled by both the common (σ_c) and the monetary/financial volatility factor (σ_{mf}). The typical process for these three shocks is now written as follows:

$$\ln \varepsilon_t = \rho \ln \varepsilon_{t-1} + \sigma_{mf}(s_{mf}) \cdot \sigma_c(s_c) \cdot \sigma \cdot \eta_t, \quad \eta_t \sim N(0, 1)$$

The estimated transition probabilities are summarized by the following matrices:

$$Q_c \begin{pmatrix} low \\ high \end{pmatrix} = \begin{bmatrix} 0.95 & 0.07 \\ 0.05 & 0.93 \end{bmatrix} \quad Q_{mf} \begin{pmatrix} low \\ high \end{pmatrix} = \begin{bmatrix} 0.92 & 0.46 \\ 0.08 & 0.54 \end{bmatrix}$$

The relative volatilities of the two regimes are estimated as:

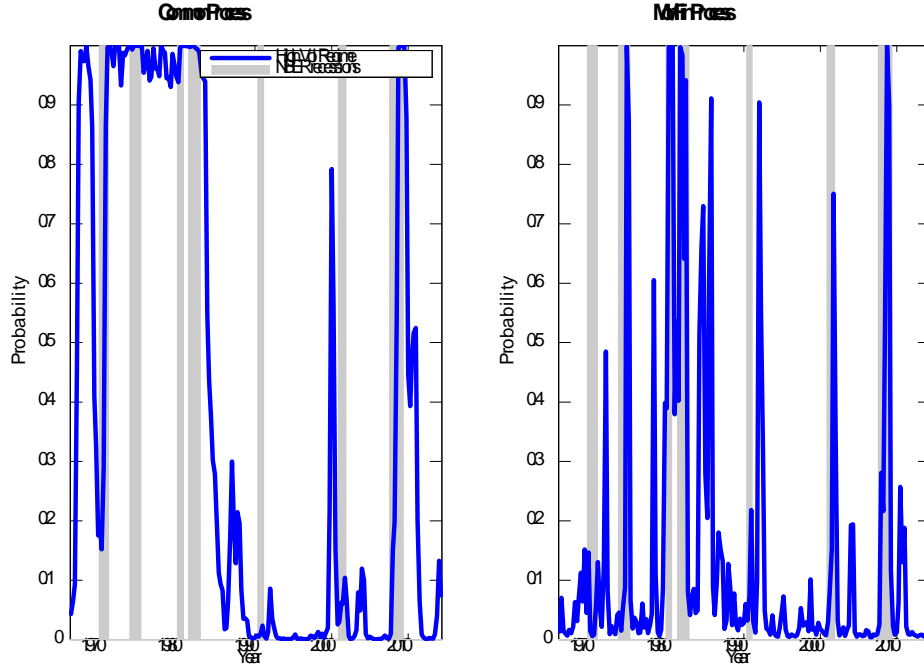
$$\sigma_c \begin{pmatrix} low \\ high \end{pmatrix} = \begin{bmatrix} 1 \\ 1.74 \end{bmatrix} \quad \text{and} \quad \sigma_{mf} \begin{pmatrix} low \\ high \end{pmatrix} = \begin{bmatrix} 1 \\ 2.33 \end{bmatrix}$$

In figure 5.3, we plot the smoothed regime probabilities for the model estimated over the complete sample.

The common volatility process captures the great moderation phenomena. The high volatility regime is typically preferred during most of the seventies and the first half of the eighties, while the low volatility regime is active during the great moderation and is interrupted by the financial crisis and the resulting Great Recession. Both regimes are estimated to be persistent and the

²¹ We use the RISE toolbox to implement this exercise, see Maih [71].

Figure 5.3: Smoothed probabilities of the two volatility Markov processes



relative volatility during the high volatility regime is almost twice as high as in the low volatility regime. The monetary/financial volatility process captures the increase in the volatility during most of the recession periods and in the late seventies-early eighties episode of increased monetary policy uncertainty. The expected duration of this high volatility/financial stress regime is relatively short-lived with a quarterly transition probability of 0.46 percent. The estimated parameters that describe the regimes and the regime probabilities are very stable when estimating the model for the pre-crisis period or for the complete sample.

The estimated log marginal likelihood of our model with switching volatility outperforms by far the log marginal likelihood of the homoscedastic Gaussian models. In this sense, our results confirms the results in the literature based stochastic volatility or t-distributed shocks. In contrast with Liu et al. [68] and in support of the results of Curdia et al. [24], we find strong evidence in favour of a setup that allows for multiple sources of volatility changes. The time-varying volatility structure requires sufficient flexibility to account for a common low frequency trend on the one hand, and a more cyclical high frequency process that controls mainly the monetary and financial shocks on the other hand.²²

²² Our restrictive setup of two processes improve the log marginal likelihood by 115 for the complete sample. More flexible structures could easily improve this result but this goes with a cost because these setups are less robust, are computational much more intensive and lack an intuitive interpretation of the regimes. Curdia et al report a gain of 154 in the log marginal likelihood for a setup that contains a combination of shock specific stochastic volatility and t-distributed innovations.

Table 5.2: Log marginal likelihood of alternative regime switching specifications

	Sample Period	
	Pre-Crisis: 66Q1-07Q4	Full sample: 66Q1-14Q2
No Regime Switching (RS)	-961.8	-1146.7
RS in common process	-894.6	-1060.9
RS in mon/fin process	-911.8	-1082.1
RS in common and mon/fin process	-881.7	-1046.0

Note: None of the models in this table are estimated subject to the ZLB on policy rates.

Accounting for the non-Gaussian stochastic structure improves drastically the log marginal likelihood of our models, but leaves the estimated parameters, the central forecasts and identified innovations relatively unaffected. Most of the gains are realized because the predictive densities attribute appropriate probabilities to the extreme tail events: the large downturns in recessions and the corresponding sharp responses in policy rates. To illustrate this property, we consider the predictive forecast distribution with the pre-crisis model conditional on data up to 2008Q3, and we calculated the percentile interval that contains the 2008Q4 realized output growth observation. For our baseline pre-crisis model, the realized 2008Q4 growth rate falls completely outside of the simulated predictive densities based on 10,000 draws with parameter and shock volatility (see Figure 4.1). In contrast, in the model with Markov Switching volatility, almost 1 percent of the simulated forecasts fall below the 2008Q4 realization. The Markov Switching volatility structure, by allowing for a mixture of normal distributions, gives more probability to the tails in general. In addition, the probability of the high volatility regimes in both the high and the low frequency Markov processes increased already by 2008Q3 because the magnitude of the realized shocks preceding the fourth quarter observation were relatively large.

5.3. Augmenting the Model with Financial Frictions and a Cost-Channel

We incorporate a financial accelerator mechanism into the benchmark model in Section 3 following the basic approach of Bernanke, Gertler and Gilchrist [9]. Thus, the intermediate goods producers rent capital services from entrepreneurs rather than directly from households. Entrepreneurs purchase physical capital from competitive capital goods producers (and resell it back at the end of each period), with the latter employing the same technology to transform investment goods into finished capital goods as described by equations 3.14) and 3.15). To finance the acquisition of physical capital, each entrepreneur combines his net worth with a loan from a bank, for which

the entrepreneur must pay an external finance premium (over the risk-free interest rate set by the central bank) due to an agency problem. We follow Christiano, Motto and Rostagno [19] by assuming that the debt contract between entrepreneurs and banks is written in nominal terms (rather than real terms as in Bernanke, Gertler and Gilchrist [9]). Banks, in turn, obtain funds to lend to the entrepreneurs by issuing deposits to households at the interest rate set by the central bank, with households bearing no credit risk (reflecting assumptions about free competition in banking and the ability of banks to diversify their portfolios). In equilibrium, shocks that affect entrepreneurial net worth – i.e., the leverage of the corporate sector – induce fluctuations in the corporate finance premium.²³

When estimating the model with the financial friction mechanism embedded, we add one more observable variable, the widely-used Baa-Aaa corporate credit spread (see Appendix C for exact definition and data sources). This spread plays a key role in the BGG framework. Since we also want to learn about the importance of shocks originating in the financial sector, and because we need as many shocks as observables to avoid stochastic singularity, we also add a “net worth” shock to the set of estimated shocks. Formally, we derive this shock by allowing the survival probability of the entrepreneurs to vary over time. Hence, this shock will enter in the accumulation equation for the entrepreneurs net worth. An alternative would have been to allow for a shock directly in the equation which relates the spread (or equivalently, the external finance premium) to the entrepreneurs leverage ratio following e.g. Del Negro and Schorfheide [28] or Christiano, Motto and Rostagno [19]. We preferred, however, not to add a shock directly in the spread equation in an attempt to elevate the endogenous propagation of the financial accelerator mechanism.²⁴ Even so, our external finance premium equation,

$$E_t \hat{R}_{t+1}^e - \hat{R}_t^b = \chi \left(\hat{Q}_t^k + \hat{k}_t - \widehat{NW}_t^e \right), \quad (5.2)$$

still contains a shock because we assume that the financing rate of the banks, \hat{R}_t^b , is not the risk-free rate, but rather the sum of the policy rate \hat{R}_t and the risk-premium shock $\hat{\varepsilon}_t^b$.

Because recent research by Christiano, Eichenbaum and Trabandt [18] and Gilchrist et al. [52] emphasize the importance of firms financing conditions for their price setting behavior, we also embed a cost channel into the model. Specifically, we assume that firms have to borrow short

²³ For further details about the setup, see Bernanke, Gertler and Gilchrist, and Christiano, Motto and Rostagno. Excellent expositions are also provided in Christiano, Trabandt and Walentin [21] and Gilchrist, Ortiz and Zakrajsek [50].

²⁴ Christiano, Motto and Rostagno [19] embed a complete banking sector into their model and estimate it using 17 time series and an equal number of shocks. A benefit, however, of our more modest perturbation of the model size and number of observables matched is that it allows for a straightforward comparison with the findings in the benchmark SW model.

to finance their wage bill following CEE [17]. As shown in the CEE paper, the working capital channel can cause inflation to rise following tightening of monetary policy if firms financing costs rise sufficiently. To allow for sharp increases in firms financing costs, we assume that the relevant financing rate is the expected nominal return on capital for the entrepreneurs as opposed to the risk-free policy rate. However, instead of imposing that all firms borrow to finance their entire wage bills as in CEE, we estimate a parameter, ν , which determines the share of firms that are subject to working capital, so that the expression for log-linearized marginal costs becomes

$$\widehat{mc}_t = (1 - \alpha) \left(\widehat{w}_t + \hat{R}_t^f \right) + \alpha \hat{r}_t^k - \widehat{\varepsilon}_t^a,$$

where \hat{R}_t^f is the effective working capital interest rate, given by

$$\hat{R}_t^f = \frac{\nu R}{\nu R + 1 - \nu} E_t \hat{R}_{t+1}^e, \quad (5.3)$$

where $E_t \hat{R}_{t+1}^e$ is the nominal expected return on capital for the entrepreneurs. From eq. (5.3), we notice that $\hat{R}_t^f = E_t \hat{R}_{t+1}^e$ when $\nu = 1$.

The SW model embedded with the financial friction mechanism and the cost-channel thus include five additional estimated parameters; ν , the two parameters for the AR(1) process for net worth (ρ_{nw} and σ_{nw}), the monitoring cost parameter μ which indirectly determines the sensitivity of the external finance premium to the entrepreneurs leverage ratio (χ in eq. 5.2), and a constant (\bar{c}_{sp}) which captures the mean of the credit spread. Estimation results for three specifications of the model are provided in Table 5.3; first we have the “Pre-crisis sample” (sample 1966Q1-2007Q4 without the ZLB), second, the full sample (66Q1-14Q2) when imposing the ZLB constraint with endogenous duration, and third we study a variant of the model with the ZLB which allows the key parameter μ to switch stochastically between a high and low value. The adopted priors for the five new parameters are provided in the notes to the table. The priors for the other parameters are the same as before (and already stated in Table 3.3).

Table 5.3: Posterior distributions in SW Model with Financial Frictions.

Parameter		Pre-crisis sample		Endogenous ZLB duration		Endog. ZLB dur. with regime switch	
		mode	std.dev. Hess.	mode	std.dev. Hess.	mode	std.dev. Hess.
Calvo prob. wages	ξ_w	0.72	0.082	0.83	0.009	0.86	0.017
Calvo prob. prices	ξ_p	0.68	0.045	0.84	0.024	0.83	0.029
Indexation wages	ι_w	0.67	0.129	0.63	0.125	0.60	0.130
Indexation prices	ι_p	0.21	0.084	0.23	0.081	0.23	0.085
Gross price markup	ϕ_p	1.61	0.077	1.45	0.062	1.43	0.063
Capital production share	α	0.21	0.018	0.17	0.016	0.17	0.016
Capital utilization cost	ψ	0.44	0.114	0.50	0.100	0.64	0.096
Investment adj. cost	φ	4.71	0.845	4.61	0.564	4.00	0.560
Habit formation	κ	0.77	0.037	0.67	0.018	0.63	0.025
Inv subs. elast. of cons.	σ_c	1.27	0.110	0.97	0.100	1.04	0.084
Labor supply elast.	σ_l	1.50	0.565	1.58	0.437	1.85	0.459
Hours worked in S.S.	\bar{l}	0.85	1.082	-0.48	0.804	-0.23	0.768
Discount factor	$100(\beta^{-1} - 1)$	0.13	0.051	0.12	0.049	0.12	0.049
Quarterly Growth in S.S.	$\bar{\gamma}$	0.43	0.015	0.42	0.015	0.42	0.017
Stationary tech. shock	ρ_a	0.96	0.011	0.96	0.012	0.97	0.012
Risk premium shock	ρ_b	0.26	0.083	0.83	0.022	0.85	0.029
Invest. spec. tech. shock	ρ_i	0.80	0.055	0.84	0.040	0.88	0.035
Gov't cons. shock	ρ_g	0.96	0.010	0.97	0.009	0.97	0.009
Price markup shock	ρ_p	0.92	0.034	0.89	0.039	0.89	0.040
Wage markup shock	ρ_w	0.98	0.013	0.98	0.007	0.97	0.001
Response of g_t to ε_t^a	ρ_{ga}	0.49	0.076	0.53	0.068	0.53	0.068
Stationary tech. shock	σ_a	0.47	0.029	0.49	0.027	0.49	0.027
Risk premium shock	σ_b	0.21	0.021	0.11	0.010	0.10	0.010
Invest. spec. tech. shock	σ_i	0.35	0.036	0.31	0.020	0.32	0.013
Gov't cons. shock	σ_g	0.47	0.029	0.47	0.024	0.47	0.024
Price markup shock	σ_p	0.12	0.015	0.12	0.013	0.13	0.013
MA(1) price markup shock	ϑ_p	0.75	0.079	0.79	0.070	0.79	0.071
Wage markup shock	σ_w	0.31	0.025	0.37	0.020	0.37	0.021
MA(1) wage markup shock	ϑ_w	0.92	0.049	0.96	0.008	0.96	0.001
Quarterly infl. rate. in S.S.	$\bar{\pi}$	0.78	0.105	0.73	0.097	0.76	0.093
Inflation response	r_π	1.91	0.170	1.78	0.119	1.83	0.133
Output gap response	r_y	0.07	0.022	0.10	0.008	0.11	0.012
Diff. output gap response	$r_{\Delta y}$	0.24	0.028	0.24	0.014	0.24	0.015
Mon. pol. shock std	σ_r	0.23	0.014	0.22	0.012	0.22	0.011
Mon. pol. shock pers.	ρ_r	0.14	0.068	0.10	0.047	0.09	0.047
Interest rate smoothing	ρ_R	0.81	0.026	0.84	0.006	0.84	0.009
Net worth shock pers.	ρ_{nw}	0.25	0.080	0.30	0.088	0.30	0.084
Net worth shock std	σ_{nw}	0.27	0.031	0.19	0.024	0.23	0.032
Working capital share	ν	0.34	0.120	0.64	0.228	0.60	0.251
Credit spread in S.S.	\bar{c}_{sp}	1.51	0.292	1.28	0.285	0.97	0.059
Monitoring cost	μ	0.03	0.004	0.06	0.007		
Monitoring cost - Regime 1	μ_1					0.03	0.004
Monitoring cost - Regime 2	μ_2					0.08	0.011
Trans. Prob. - R1 to R2	p_{12}					0.04	0.015
Trans. Prob. - R2 to R1	p_{21}					0.16	0.055
Log marginal likelihood		Laplace	-897.80	Laplace	-1112.00	Laplace	-1063.00

Note: For the financial frictions parameters, we use the same prior as for the other exogenous shocks (stated in Table 3.2). For μ and \bar{c}_{sp} , we use a normal distribution with means 0.25 and 1.00 and standard deviations 0.10 and 0.50, respectively. Finally, for ν we use a beta distribution with mean 0.50 and standard deviation 0.20. The “Pre-crisis sample” neglects the presence of the zero lower bound in the estimations and is estimated on data up to 2007Q4, whereas the “Endogenous ZLB duration” allows the duration of the ZLB to be endogenous as described in Section 5.1 and is estimated up to 2014Q2. “Endog. ZLB dur. with regime switch” does same thing as the preceding case, but allows μ to vary stochastically between a low (μ_1) and high (μ_2) value. For μ_1 and μ_2 , we use a normal distribution with means 0.025 and 0.25 and standard deviations 0.01 and 0.10, respectively. For the transition probabilities p_{12} and p_{21} , we use a beta distribution with means 0.10 and 0.30 and standard deviations 0.05 and 0.10, respectively.

In the pre-crisis model, the external finance premium delivers only a very modest amplification of the standard shocks. The estimated elasticity of the spread to the net worth ratio is small (with

$\mu = 0.033$, χ in eq. 5.2 equals 0.012, implying an annualized spread sensitivity of 0.048), a result that is in line with the estimates reported in Gilchrist, Ortiz, and Zakrajsek [50]. The exogenous risk-premium shock and – to a lower degree – the monetary policy shock are most impacted by the introduction of the FA mechanism because they have the biggest impact on the price of capital and net worth. The net worth channel tends to support the persistence in the response of investment to these shocks. The low sensitivity of the spread to the traditional shocks also implies that most of the fluctuations in the external finance premium are generated by the new exogenous shock that is assumed to hit directly the net worth of the entrepreneurs. This highly volatile shock explains up to 70 percent of the variance in the spread and one-third of the variance in investment. As such, the net worth shock substitutes for the exogenous risk premium and for the investment-specific technology shock – the latter also captures financial frictions as suggested by Justiniano, Primiceri and Tambalotti [62]. Overall, the impact of the net worth shock on the macrodynamics remains modest and one important reason for this is that the net worth shock typically crowds out private consumption and this clashes with the observed strong comovement between consumption and investment over the business cycle.²⁵

The direct comparison of the marginal likelihood with the baseline model is complicated because the FF model has an additional observable in the form of the Baa-Aaa spread. When we estimate the FF-model without this additional observable, the log marginal likelihood improves by a factor of 10 when no additional shock is considered and by a factor of 20 when the net worth shock is retained. With a posterior mode for $\mu = 0.2$ in this variant of the model, the estimated sensitivity of the spread to the net worth ratio in this model is much higher 0.08, or 0.32 in annualized terms. This result is more supportive for an important endogenous amplification effect of the standard shocks through the net worth channel (see also De Graeve [25] for a similar result). This observation suggests that the use of the Baa-Aaa spread as an observable for the external finance premium in the model can be too restrictive. Baa-Aaa spread is only one specific measure for default risk, and the cost of credit for firms is determined by various risks and constraints in the financial sector.²⁶

Not surprisingly, when we evaluate the performance of the FF-model for the complete sample including the 2008Q4-2009Q1 crisis period, the monitoring cost parameter μ and the implied elasticity of the spread to the net-worth ratio doubles. Perhaps surprisingly, the standard error of the exogenous net-worth shock is substantially lower, 0.19 versus 0.27 in the model estimated on

²⁵ This crowding out problem is not present for our reduced-form risk-premium shock ε_t^b in eq. (3.17), see Fisher [43] for a structural interpretation of this risk-premium shock.

²⁶ Gilchrist et al. [50] and Gilchrist and Zakrajsek [51] presents alternative indicators of the default spread that have a stronger predictive power for economic activity than the Baa-Aaa spread.

pre-crisis data. We interpret this finding to imply that the endogenous amplification becomes more important when including the crisis period in the estimation sample. As we also impose the ZLB constraint in the estimation of this model, the estimated nominal wage and price stickiness is again very high (0.83 and 0.84, respectively) so that all the expected policy shocks that are required for the model to respect the ZLB constraint are positive. It is also striking that in this full-sample model, the estimated fraction of the wage bill that requires external financing is substantially higher than in the pre-crisis version, supporting the argument in Christiano et al. [18] that this channel was important during crisis. The magnitude of this cost channel increases from 0.33 to 0.64, but in both models the uncertainty in the posterior distribution for this parameter is very high. These two observations, the time-variation in the role of financial frictions and the potential role of the cost channel for the inflation dynamics, are discussed in more detail below.

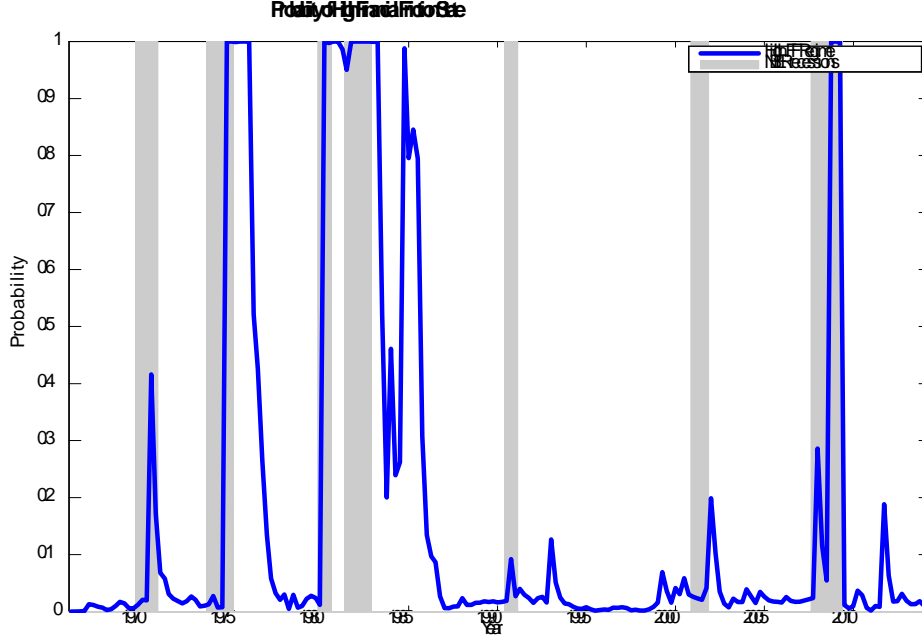
5.3.1. A regime switching model with occasionally binding financial frictions

Pre-crisis DSGE models typically neglected the role of financial frictions. This additional transmission mechanism was considered non-vital for forecasting output and inflation during the great moderation period, and by Occam's razor arguments this mechanism were typically left out. However, as our discussion of the in-sample innovations illustrated, there was already strong evidence in our estimated pre-crisis model for occasionally big disturbances that seemed to be highly correlated with financial spreads and return indicators. When looking at these results from a broader perspective that also gives appropriate attention to the potential risks around the central banks forecast, these outliers should not be disregarded. A linear Gaussian approach is not the most efficient framework for handling these issues. The instability in the estimated parameters of our FF-model depending on the estimation sample clearly illustrates these limitations. To capture more efficiently the time-varying relevance of the financial frictions in our model, we therefore consider here a Markov switching setup in which the constraints from the financial frictions can become much more binding occasionally.

In our Regime Switching Financial Friction model (RS-FF), we allow for two possible regimes: one regime (high-FF) with a high monitoring costs - implying a high sensitivity of the spread to the net worth position - and another regime (low-FF) with a low monitoring costs - low sensitivity of spread to leverage.²⁷ The estimation results for this model is reported last in Table 5.3, and

²⁷ Christiano Motto Rostagno[20] focus instead on the distribution of the idiosyncratic productivity risk as the source for time-varying financial frictions. Levin, Natalucci and Zakrasjeck [66] identify the time variation in the bankruptcy cost parameter, the equivalent of our monitoring cost, as the source for the countercyclical external

Figure 5.4: Smoothed probabilities of the Markov process in the financial frictions model.



the data prefer this RS-FF-setting compared to the linear FF-model as shown by the gain in the log marginal likelihood of more than 30 in the pre-crisis context (not shown) and around 50 in the sample with the recent crisis. The transition probabilities and the regime-specific μ parameter are given by:

$$Q_{FF} \begin{pmatrix} \text{low} \\ \text{high} \end{pmatrix} = \begin{bmatrix} 0.96 & 0.16 \\ 0.04 & 0.84 \end{bmatrix} \quad \mu_{FF} \begin{pmatrix} \text{low} \\ \text{high} \end{pmatrix} = \begin{bmatrix} 0.029 \\ 0.084 \end{bmatrix}$$

The estimation results indicate that the elasticity of the spread to the leverage ratio varies between the two regimes by a factor of 2.7. The high-FF regime is active mainly around the two recession periods in the seventies, and its probability increases slightly during all recessions. When evaluated over the more recent period, the probability of the high-FF regime starts to rise early in 2008 and remains active during the financial crisis in 2009 but quickly return to the low-FF regime after 2009. The higher marginal likelihood is due to the time-varying volatility in the spread: in the high-FF regime, the financial friction is strongly binding and the spread reacts more than twice as strong to the leverage ratio. The impact of shocks on investment is also higher but the magnitude of the amplification is moderate up to a factor of 1.5 maximum. The expected period-by-period persistence of the high-FF regime is limited (0.84) and this reduces the impact of spread increases on the discounted value of future expected returns on investment.

premium behavior.

The central forecast of the single-regime pre-crisis FF-model, conditional on data up to 2008Q3 is missing completely the magnitude of the 2008Q4 downturn just as the benchmark SW model without financial frictions. The distribution around the FF-forecast is however more dispersed due to the extra volatility that is generated by the spread and the additional net worth shock. As a result, the extreme negative output growth realization of 2008Q4 now falls within the 0.25 percent interval of the predictive density, which already is some improvement relative to the baseline model. The pre-crisis RS-FF model further improves on this result because the probability of being in the high friction regime increased by 2008Q3 (56 percent against an unconditional probability of 20 percent) and this introduces a high degree of skewness in the predictive density of the spread. While the pre-crisis FF-model predicts a 1 percent upper tail for the expected spread above 2.3 percentage points in 2008Q4, this becomes as high as 3 percentage points in the RS-FF model. The probability of the observed 2008Q4 output growth outcome lies now around the 0.5 percent tail interval, which is still small but at least the ex post realized event obtains some non-zero probability in the predictive density. This result indicates that when we can integrate appropriately the non-linear accelerator dynamics from financial frictions in our DSGE models we obtain a more realistic predictive density that resembles these from the reduced form time-varying volatility models such as our RS-volatility example in Section 5.2.

Given the important role of the spread in the short run forecast, it is also informative to show how a conditional forecast, conditional on the timely observation of the spread, performs in the crisis period. Therefore, we make a forecast conditional on the 2008Q3 state of the economy as filtered by the pre-crisis FF-model but now we also provide the model with the information that the spread increased to the exceptionally high observed level of 3.02 percentage points in 2008Q4 (from 1.55 percentage points in 2008Q3). The forecast conditional on the timely information from the spread displays a median prediction for annual gdp growth of -2.11 percent in 2008Q4 and -1.92 percent in 2009Q1 which should be compared to the observed -3.61 and -4.42 percent in the actual data and an unconditional forecast of -1.05 and 0.06 percent. In the RS-FF model, the result depends very much on the regime in which the economy is finding itself in 2008Q3: the impact of conditioning on the spread is most disturbing when the economy is in the low friction regime. Extreme high spreads are very difficult to reconcile with the low friction regime, with its low elasticity of spread to leverage, and therefore the spreads are translated in really huge negative shocks in net worth and/or risk premiums which then also result in worse output growth predictions

of -2.53 and -3.01 percent in 2008Q4 and 2009Q1.²⁸

The real-time information on the spread and the presence of the additional transmission mechanism allow the FF-model to improve considerably the accuracy of the central forecast in the crisis period. Our results confirm the findings of Del Negro and Schorfheide [28], who also compare the predictive performance of a standard SW setup with an augmented SW-FF model. They observe that the relative performance of the two models changes over time. On average the model without financial frictions generates more accurate forecasts, but during the recent financial crisis, a SW-FF model - that also exploits the timely information on spread and interest rate - produces better forecasts for output and inflation. Del Negro et al. [27] built on these results and develop a new method for combining predictive densities from recursively estimated models using time-varying weights. As in our RS-approach, this dynamic linear prediction pooling relies on weights that follow an exogenous process. The next step in this research agenda would be to endogenize the occurrence of financial stress periods during which constraints are reinforced and additional feedback mechanisms activated.²⁹

5.3.2. The cost channel of financial spreads and inflation dynamics

In section 4.2, when we discussed the economic interpretation of the great recession through the lense of the baseline SW model, we observed that the model requires a series of positive mark up shocks to explain the maintained inflation rate during the period of slow recovery and persistent negative output gap. These positive mark up shocks are necessary despite the high estimate of nominal stickiness in price and wage setting. This trend towards more nominal stickiness was already present in the subsample estimates presented in SW07. The high nominal stickiness also plays a crucial role in the explanation of the recent inflation dynamics by Del Negro et al. [26] and Fratto and Uhlig [44]. These positive markup shocks disappear completely in our version of SW in which we implement the ZLB and that feature an even higher degree of nominal stickiness. The question arises whether this estimated stickiness parameter should be interpreted effectively as a sign of pure nominal stickiness in the price setting practice or whether it reflects some other mechanism that lowered the responsiveness of inflation to the slack in production capacity.

²⁸ This somewhat counter-intuitive result of the RS-FF model is related to the nature of the conditional forecast exercise: conditioning on a given spread observation has larger effects when that observation deviates more from the baseline unconditional forecast. The gain from the RS-FF model is precisely that the unconditional forecast will show larger dispersion in the high-FF regime and lower dispersion in the low-FF regime.

²⁹ Various approaches have been developped in this context: Iacoviello and Guerrieri [53] with occasionally binding constraints, Dewachter and Wouters [30] with third order non-linear approximations and Bocola [11] for a combination of occasionally binding constraints and non-linear risk premiums.

As noted by Christiano et al. [18], one mechanism that might contribute to this inflation resilience, in particular during periods of increased financial constraints and high financing costs, is the cost channel. Firms that are financially constrained and that must finance their operations with expensive external financing can experience an increase in their marginal production costs if these financing costs dominate the influence of the other cost components. Related to this cost channel, firms can have other arguments to keep their prices high during periods of financial constraints: high markups can be necessary for firms to generate sufficient cash flow or firms might be forced by their financing constraints to give up on market share (see Gilchrist et al. [52]). Note that this cost channel also plays a crucial role in the explanation of the inflation inertia following a monetary policy shock in CEE [17].

Our FF-model contains a parameter that controls the strength of the cost channel. This parameter reflects the fraction of the wage bill that firms have to finance with credit. In this setup, we assume that the external finance premium is also affecting the cost for these intratemporal loans of the firms. In the pre-crisis model this fraction of the wage bill on which the financial cost applies, is estimated to be quite low 0.33 and the posterior distribution has a large uncertainty margin around this mode. This parameter increases to 0.63 in the complete sample estimation, still with a large uncertainty, but at least there is some indication that the cost channel was more relevant during the recent crisis. To examine the potency of this channel in our model, Figure 5.5 plots the irfs of the three shocks that affect directly the external financing costs – the monetary policy shock, the exogenous risk premium shock and the wealth shock – on the marginal cost and inflation for the two extreme values - zero and one - of the cost channel parameter. Given the large estimation uncertainty around the magnitude of the cost channel parameter, these two extreme values are not completely unlikely and their relevance can probably change depending on the nature of the financial shocks and the constraints. We plot the results for both the pre-crisis model, with a moderate degree of nominal stickiness, and the full sample ZLB model with a high degree of stickiness.

In both model versions and for all three shocks, it is obvious that marginal cost behaves quite different if the cost channel is fully active compared to a situation in which the cost channel is completely absent. The presence of the cost channel implies that the marginal cost increases at least during the first quarters following each of these shocks. The persistence of this positive effect depends on the type of the shock and tends to be shorter for the risk-premium shock and most persistent for the net-worth shock.

The impact on inflation can differ substantially depending on the volatility of the cost shock

and on the persistence of the shock relative to the degree of nominal stickiness which determines the degree of forward-lookingness in price setting. In the pre-crisis model, the exogenous risk-premium shock is highly volatile, but short lived. Combined with the moderate degree of stickiness, the cost channel drastically changes the response of inflation to this shock. Inflation rises on impact due to the high risk-premium component in the financing costs, but the effect is very short lived. In the model with ZLB constraint – with more stickiness – the price setting is more forward looking and the persistence of the shock is crucial. In such a context, the smooth inflation process is dependent on the long-run expected marginal cost. In this case, only the net worth shock has a sufficiently persistent effect on the financing cost to exert a positive impact on inflation; the temporarily high risk free rate and risk premium shock are missing sufficient persistence to have a substantial impact on the inflation dynamics.

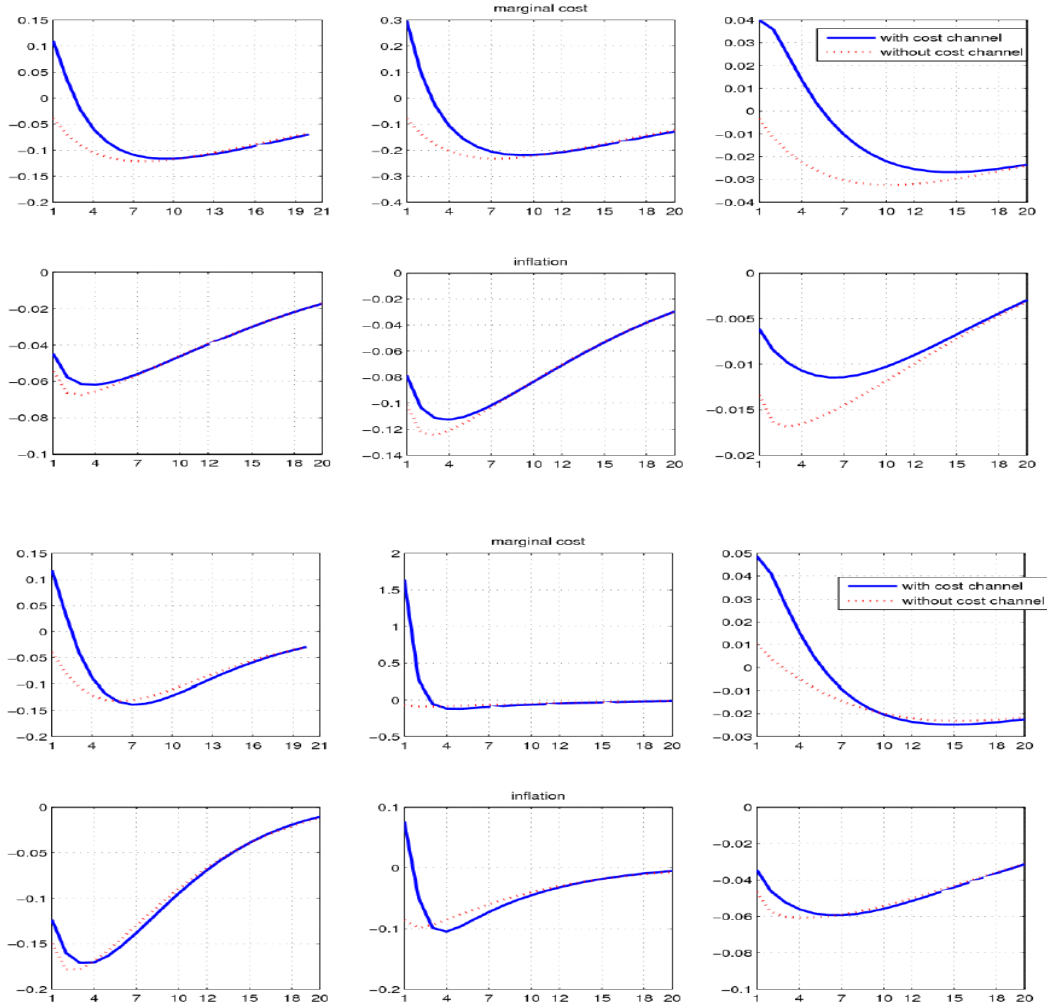
From this impulse response analysis, it follows that the cost channel can contribute to the slow response of inflation in a financial crisis context. When the external finance shock for firms are sufficiently high and/or sufficiently persistent, as it is the case for a net worth shock that is expected to have long lasting effects on the financing costs, this inflationary pressure from the cost channel can be quantitatively important. These results illustrate that the financial crisis should not necessarily be viewed as a purely negative aggregate demand shock, without an impact on the supply side of the economy. With both aggregate demand and aggregate supply shifting inward by the financial shock, inflation should not necessarily be expected to react that much in a financial crisis situation.

[Remains to be written.]

6. Challenges for macroeconomic models

In this section, we discuss both “new” and “old” challenges for macroeconomic models. As evidenced above, the financial crisis has generated new challenges for macroeconomic models used at central banks. The old challenges were known prior to the financial crisis, and they have not been mitigated by the evidence brought forward by the crisis.

Figure 5.5: The transmission of financial shocks: monetary policy (left column); risk premium (middel column) and net-worth (right column) shock. Upper Panel: Pre-crisis model; Lower Panel; Endogenous ZLB model.



6.1. New Challenges

6.1.1. Models with more monetary policy instruments

Building models where the central bank has more than one tool to affect the economy, i.e. a framework where the central bank can use both conventional monetary policy (manipulating short rates) and unconventional policies (LSAPs/QE).

6.1.2. Integrating financial frictions and endogenous financial risk

Building models that can integrate analysis of financial markets into general equilibrium models. Both on the household side and on the firms side. Iacoviello (2005), and Liu, Wang and Zha (2013).

Innovating models in terms of endogenous risk and feedback mechanisms between the real and the financial sectors: Brunnermeier and Sannikov, He and Krishnamurthy, Mendoza etc.

[Remains to be written.]

6.1.3. Integrating financial stability considerations into monetary policy model

This involves stress testing exercises and role for various macroprudential tools. More realistic modelling of the interbank market as in Boissy, Collard and Smets (2013) paper, the 3-D ECB-model.

[Remains to be written.]

6.2. Old Challenges

[Remains to be written.]

6.2.1. More than one interest rate relevant - the failure of the EH hypothesis

[Remains to be written.]

6.2.2. Open economy aspects - Comovement puzzle

[Remains to be written.]

6.2.3. Open economy aspects - Failure of the UIP condition

[Remains to be written.]

7. Conclusions

Our most important tentative conclusions are:

- The basic prototype DSGE model we analyze, which shares many key features with policy models at central banks, imply too strong mean reversion of output towards trend.
- The model displays a strong tendency of more stickiness in price and wage setting when the sample is extended through the crisis period.
- When we impose the ZLB as a constraint on the federal funds rate, price and wage stickiness is elevated even further.
- These models rely on exceptionally large exogenous shocks to generate the Great Recession (and other recessions too), a feature that is common with other linear Gaussian macromodels.
- The typical financial frictions that are considered in the policy models allow to improve the forecast quality by conditioning on timely information from financial spreads and other indicators of financial stress.
- As the models are linearized for the empirical implementation, the potential non-linear amplification and feedback from the financial frictions is not exploited and the time-varying volatility is still dependent on exogenous innovations in the financial observables.
- High nominal stickiness in the model helps to understand the inflation dynamics during the financial crisis and the slow recovery with persistent under-utilization of capacity and high unemployment.
- It remains an open issue whether this low inflation response reflect pure price stickiness or whether this stickiness substitute for a working capital or a cost channel that might have affected a large share of the firms during this particular episode of the financial crisis.
- It is also conceivable that the higher degree of nominal stickiness compensates for the absence of learning dynamics in the way agents form their expectations or filtering issues in the interpretation of the fundamental state of the economy.
- **[Remains to be written.]**

Appendix

A. Linearized model representation

In this appendix, we summarize the log-linear equations of the basic SW07 model stated in Section 3. The complete model also includes the seven exogenous shocks, but they are shown in the main text. **[Remains to be done: integrate SW utility function and growth γ into the equations.]**

- Consumption Euler equation:

$$\widehat{c}_t = c_1 E_t [\widehat{c}_{t+1}] + (1 - c_1) \widehat{c}_{t-1} - c_2 (\widehat{R}_t - E_t [\widehat{\pi}_{t+1}] - \widehat{\varepsilon}_t^b)$$

with $c_1 = 1/(1 + \varkappa)$, $c_2 = c_1(1 - \varkappa)$ where \varkappa is the external habit parameter. $\widehat{\varepsilon}_t^b$ is the exogenous AR(1) risk premium process.

- Investment Euler equation:

$$\widehat{i}_t = i_1 \widehat{i}_{t-1} + (1 - i_1) \widehat{i}_{t+1} + i_2 \widehat{Q}_t^k + \widehat{\varepsilon}_t^q$$

with $i_1 = 1/(1 + \beta)$, $i_2 = i_1/\psi$ where β is the discount factor, and ψ is the elasticity of the capital adjustment cost function. $\widehat{\varepsilon}_t^q$ is the exogenous AR(1) process for the investment specific technology.

- Value of the capital stock:

$$\widehat{Q}_t^k = -(\widehat{R}_t - E_t [\widehat{\pi}_{t+1}] - \widehat{\varepsilon}_t^b) + q_1 E_t [r_{t+1}^k] + (1 - q_1) E_t [Q_{t+1}^k]$$

with $q_1 = r_*^k/(r_*^k + (1 - \delta))$ where r_*^k is the steady state rental rate to capital, and δ the depreciation rate.

- Aggregate demand equals aggregate supply:

$$\begin{aligned} \widehat{y}_t &= \frac{c_*}{y_*} \widehat{c}_t + \frac{i_*}{y_*} \widehat{i}_t + \widehat{\varepsilon}_t^g + \frac{r_*^k k_*}{y_*} \widehat{u}_t \\ &= \phi_p (\alpha \widehat{k}_t + (1 - \alpha) \widehat{L}_t + \widehat{\varepsilon}_t^a) \end{aligned}$$

with ϕ_p reflecting the fixed costs in production which corresponds to the price markup in steady state. $\widehat{\varepsilon}_t^g$ and $\widehat{\varepsilon}_t^a$ are the AR(1) processes representing exogenous demand components and the TFP process.

- Price-setting under the Calvo model with indexation:

$$\widehat{\pi}_t - \gamma_p \widehat{\pi}_{t-1} = \pi_1 (E_t [\widehat{\pi}_{t+1}] - \gamma_p \widehat{\pi}_t) - \pi_2 \widehat{\mu}_t^p + \widehat{\varepsilon}_t^p$$

with $\pi_1 = \beta$, $\pi_2 = (1 - \xi_p\beta)(1 - \xi_p)/[\xi_p(1 + (\phi_p - 1)\varepsilon_p)]$, with ξ_p and ι_p are, respectively, the probability and indexation of the Calvo model, and ε_p the curvature of the aggregator function. The price markup $\widehat{\mu}_t^p$ is equal to the inverse of the real marginal $\widehat{mc}_t = (1 - \alpha) \widehat{w}_t + \alpha \widehat{r}_t^k - \widehat{\varepsilon}_t^a$.

- Wage-setting under the Calvo model with indexation:

$$\begin{aligned} & (1 + \bar{\beta}\gamma)\widehat{w}_t - \widehat{w}_{t-1} - \bar{\beta}\gamma E_t[\widehat{w}_{t+1}] \\ = & \frac{(1 - \zeta_w\bar{\beta}\gamma)(1 - \zeta_w)}{[\xi_w(1 + (\phi_w - 1)\varepsilon_w)]} \left[\frac{1}{1 - \varkappa/\gamma} \widehat{c}_t - \frac{\varkappa/\gamma}{1 - \varkappa/\gamma} \widehat{c}_{t-1} + \nu_l \widehat{L}_t - \widehat{w}_t \right] \\ & - (1 + \bar{\beta}\gamma\iota_w)\widehat{\pi}_t + \iota_w \widehat{\pi}_{t-1} + \bar{\beta}\gamma E_t[\widehat{\pi}_{t+1}] + \widehat{\varepsilon}_t^w \end{aligned}$$

where ξ_p and ι_p , respectively, are the probability and indexation of the Calvo model, and ε_w the curvature of the aggregator function.

- Capital accumulation equation:

$$\widehat{k}_t = \kappa_1 \widehat{k}_{t-1} + (1 - \kappa_1) \widehat{i}_t + \kappa_2 \widehat{\varepsilon}_t^q$$

with $\kappa_1 = (1 - (i_*/\bar{k}_*))$, $\kappa_2 = (i_*/\bar{k}_*)(1 + \beta)\psi$. Capital services used in production is defined as:
 $\widehat{k}_t = \widehat{u}_t + \widehat{k}_{t-1}$

- Optimal capital utilisation condition:

$$\widehat{u}_t = (1 - \psi)/\psi \widehat{r}_t^k$$

with ψ is the elasticity of the capital utilisation cost function.

- Optimal capital/labor input condition:

$$\widehat{k}_t = \widehat{w}_t - \widehat{r}_t^k + \widehat{L}_t$$

- Monetary policy rule:

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1 - \rho_R)(r_\pi \widehat{\pi}_t + r_y(\widehat{ygap}_t) + r_{\Delta y} \Delta(\widehat{ygap}_t) + \widehat{\varepsilon}_t^r)$$

with $ygap_t = \widehat{y}_t - \widehat{y}_t^{flex}$, the difference between actual output and the output in the flexible price

and wage economy in absence of distorting price and wage markup shocks.

B. The ZLB algorithm and the likelihood function

First, we describe the algorithm we use to impose the ZLB, then we briefly comment on how the likelihood function takes the ZLB into account. For more details on the ZLB algorithm we refer to

Hebden et al [55], whereas more details on the computation of the likelihood is provided in Lindé, Maih and Wouters ??.

B.1. The ZLB algorithm

The model above can be written in the following practical state-space form,

$$\begin{bmatrix} X_{t+1} \\ Hx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}. \quad (\text{B.1})$$

Here, X_t is an n_X -vector of *predetermined* variables in period t (where the period is a quarter); x_t is an n_x -vector of *forward-looking* variables; i_t is generally an n_i -vector of (policy) *instruments* but in the cases examined here it is a scalar, the central bank's policy rate, so $n_i = 1$; ε_t is an n_ε -vector of i.i.d. shocks with mean zero and covariance matrix I_{n_ε} ; A , B , C , and H are matrices of the appropriate dimension; and $y_{t+\tau|t}$ denotes $E_t y_{t+\tau}$ for any variable y_t , the rational expectation of $y_{t+\tau}$ conditional on information available in period t . The forward-looking variables and the instruments are the *non-predetermined* variables.³⁰

The variables are measured as differences from steady-state values, in which case their unconditional means are zero. In addition, the elements of the matrices A , B , C , and H are considered fixed and known.

We let i_t^* denote the policy rate when we disregard the ZLB. We call it the *unrestricted* policy rate. We let i_t denote the actual or *restricted* policy rate that satisfies the ZLB,

$$i_t + \bar{i} \geq 0. \quad (\text{B.2})$$

where $\bar{i} > 0$ denotes the steady-state level of the policy rate and we use the convention that i_t and i_t^* are expressed as deviations from the steady-state level. The ZLB can therefore be written as

$$i_t + \bar{i} = \max\{i_t^* + \bar{i}, 0\}, \quad (\text{B.3})$$

We assume the unrestricted policy rate follows the (possibly reduced-form) unrestricted linear policy rule,

$$i_t^* = f_X X_t + f_x x_t, \quad (\text{B.4})$$

where f_X and f_x are row vectors of dimension n_X and n_x , respectively. From (B.3) it then follows that the restricted policy rate is given by:

$$i_t + \bar{i} = \max\{f_X X_t + f_x x_t + \bar{i}, 0\}. \quad (\text{B.5})$$

³⁰ A variable is predetermined if its one-period-ahead prediction error is an exogenous stochastic process (Klein [64]). For (B.1), the one-period-ahead prediction error of the predetermined variables is the stochastic vector $C\varepsilon_{t+1}$.

Consider now a situation in period $t \geq 0$ where the ZLB may be binding in the current or the next finite number T periods but not beyond period $t + T$. That is, the ZLB constraint

$$i_{t+\tau} + \bar{i} \geq 0, \quad \tau = 0, 1, \dots, T \quad (\text{B.6})$$

may be binding for some $\tau \leq T$, but we assume that it is not binding for $\tau > T$,

$$i_{t+\tau} + \bar{i} > 0, \quad \tau > T.$$

We will implement the ZLB with anticipated shocks to the unrestricted policy rule, using the techniques in Laséen and Svensson [69]. Thus, we let the restricted and unrestricted policy rate in each period t satisfy

$$i_{t+\tau,t} = i_{t+\tau,t}^* + z_{t+\tau,t}, \quad (\text{B.7})$$

for $\tau \geq 0$. The ZLB policy rule (B.5) - as we explain in further detail below - implies that all current and future anticipated shocks $z_{t+\tau,t}$ in (B.7) must be nonnegative, and that $z_{t,t}$ is strictly positive in periods when the ZLB is binding.

Disregarding for the moment that z_t are nonnegative, we follow Laséen and Svensson [69] and call the stochastic variable z_t the deviation and let the $(T+1)$ -vector $z^t \equiv (z_{t,t}, z_{t+1,t}, \dots, z_{t+T,t})'$, denote a projection in period t of future realizations $z_{t+\tau}$, $\tau = 0, 1, \dots, T$, of the deviation. Furthermore, we assume that the deviation satisfies

$$z_t = \eta_{t,t} + \sum_{s=1}^T \eta_{t,t-s}$$

for $T \geq 0$, where $\eta^t \equiv (\eta_{t,t}, \eta_{t+1,t}, \dots, \eta_{t+T,t})'$ is a $(T+1)$ -vector realized in the beginning of period t . For $T = 0$, the deviation is given by $z_t = \eta_t$. For $T > 0$, the deviation is given by the moving-average process

$$\begin{aligned} z_{t+\tau,t+1} &= z_{t+\tau,t} + \eta_{t+\tau,t+1}, \quad \tau = 1, \dots, T, \\ z_{t+\tau+T+1,t+1} &= \eta_{t+T+1,t+1}. \end{aligned}$$

It follows that the dynamics of the projection of the deviation can be written more compactly as

$$z^{t+1} = A_z z^t + \eta^{t+1}, \quad (\text{B.8})$$

where the $(T+1) \times (T+1)$ matrix A_z is defined as

$$A_z \equiv \begin{bmatrix} 0_{T \times 1} & I_T \\ 0 & 0_{1 \times T} \end{bmatrix}.$$

Hence, z^t is the projection in period t of current and future deviations, and the innovation η^t can be interpreted as the new information received in the beginning of period t about those deviations.

Let us now combine the model, (B.1), the dynamics of the deviation, (B.8), the unrestricted policy rule, (B.4), and the relation (B.7). Taking the starting period to be $t = 0$, we can then write the combined model as

$$\begin{bmatrix} \tilde{X}_{t+1} \\ \tilde{H}\tilde{x}_{t+1|t} \end{bmatrix} = \tilde{A} \begin{bmatrix} \tilde{X}_t \\ \tilde{x}_t \end{bmatrix} + \begin{bmatrix} C & 0_{n_X \times (T+1)} \\ 0_{(T+1) \times n_\varepsilon} & I_{T+1} \\ 0_{(n_x+2) \times n_\varepsilon} & 0_{(n_x+2) \times (T+1)} \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1} \\ \eta^{t+1} \end{bmatrix} \quad (\text{B.9})$$

for $t \geq 0$, where

$$\tilde{X}_t \equiv \begin{bmatrix} X_t \\ z^t \end{bmatrix}, \quad \tilde{x}_t \equiv \begin{bmatrix} x_t \\ i_t^* \\ i_t \end{bmatrix}, \quad \tilde{H} \equiv \begin{bmatrix} H & 0_{n_x \times 1} & 0_{n_x \times 1} \\ 0_{1 \times n_x} & 0 & 0 \\ 0_{1 \times n_x} & 0 & 0 \end{bmatrix}, \quad (\text{B.10})$$

Under the standard assumption of the saddlepoint property (that the number of eigenvalues of \tilde{A} with modulus larger than unity equals the number of non-predetermined variables, here $n_x + 2$), the system of difference equations (B.9) has a unique solution and there exist unique matrices M and F returned by the Klein [64] algorithm such that the solution can be written

$$\tilde{x}_t = F\tilde{X}_t \equiv \begin{bmatrix} F_x \\ F_{i^*} \\ F_i \end{bmatrix} \tilde{X}_t, \quad (\text{B.11})$$

$$\tilde{X}_{t+1} = M\tilde{X}_t + \begin{bmatrix} C\varepsilon_{t+1} \\ \eta^{t+1} \end{bmatrix} \equiv \begin{bmatrix} M_{XX} & M_{Xz} \\ 0_{(T+1) \times n_X} & A_z \end{bmatrix} \begin{bmatrix} X_t \\ z^t \end{bmatrix} + \begin{bmatrix} C\varepsilon_{t+1} \\ \eta^{t+1} \end{bmatrix}, \quad (\text{B.12})$$

for $t \geq 0$, where X_0 in $\tilde{X}_0 \equiv (X'_0, z^0)'$ is given but the projections of the deviation z^0 and the innovations η^t for $t \geq 1$ (and thereby z^t for $t \geq 1$) remain to be determined. They will be determined such that the ZLB is satisfied, i.e. equation (B.5) holds. Thus, the *policy-rate projection* is given by

$$i_{t+\tau,t} = F_i M^\tau \begin{bmatrix} X_t \\ z^t \end{bmatrix} \quad (\text{B.13})$$

for $\tau \geq 0$ and for given X_t and z^t .

We will now show how to determine the $(T+1)$ -vector $z^t \equiv (z_t, z_{t+1,t}, \dots, z_{t+T,t})'$, the projection of the deviation, such that policy-rate projection satisfies the ZLB restriction (B.6) and the policy rule (B.5).

When the ZLB restriction (B.6) is disregarded or not binding, the policy-rate projection in period t is given by

$$i_{t+\tau,t} = F_i M^\tau \begin{bmatrix} X_t \\ 0_{(T+1) \times 1} \end{bmatrix}, \quad \tau \geq 0. \quad (\text{B.14})$$

The policy-rate projection disregarding the ZLB hence depends on the initial state of the economy in period t , represented by the vector of predetermined variables, X_t . If the ZLB is disregarded, or not binding for any $\tau \geq 0$, the projections of the restricted and unrestricted policy rates will be the same,

$$i_{t+\tau,t} = i_{t+\tau,t}^* = f_X X_{t+\tau,t} + f_x x_{t+\tau,t}, \quad \tau \geq 0. \quad (\text{B.15})$$

Assume now that the policy-rate projection according to (B.14) violates the ZLB for one or several periods, that is,

$$i_{t+\tau,t} + \bar{i} < 0, \quad \text{for some } \tau \text{ in the interval } 0 \leq \tau \leq T. \quad (\text{B.16})$$

In order to satisfy the ZLB, we then want to find a projection of the deviation, z^t ; such that the policy-rate projection satisfies (B.6) and

$$i_{t+\tau,t} + \bar{i} = \max\{i_{t+\tau,t}^* + \bar{i}, 0\} = \max\{f_X X_{t+\tau,t} + f_x x_{t+\tau,t} + \bar{i}, 0\} \quad (\text{B.17})$$

for $\tau \geq 0$. This requires that the projection of the deviation satisfies a nonnegativity constraint,

$$z_{t+\tau,t} \geq 0, \quad \tau \geq 0; \quad (\text{B.18})$$

and that the policy-rate projection and the projection of the deviation satisfies the complementary-slackness condition,

$$(i_{t+\tau,t} + \bar{i}) z_{t+\tau,t} = 0, \quad \tau \geq 0. \quad (\text{B.19})$$

Notice that the complementary-slackness condition implies that $z_{t+\tau,t} = 0$ if $i_{t+\tau,t} + \bar{i} > 0$.

For given X_t , we now proceed under the presumption that there exists a unique projection of the deviation z^t that satisfies (B.13) and (B.17)-(B.19).³¹ We call this projection of the deviation and the corresponding policy-rate projection the *equilibrium* projection. This projection of the deviation either has all elements equal to zero (in which case the ZLB is not binding for any period) or has some elements positive and other elements zero. Let

$$\mathcal{T}_t \equiv \{0 \leq \tau \leq T \mid z_{t+\tau,t} > 0\}$$

³¹ This assumption is discussed in further detail in Hebden et al. [55].

denote the set of periods for which the projection of the deviation are positive in equilibrium.

For each $\tau \in \mathcal{T}_t$, the solution will satisfy

$$i_{t+\tau,t} + \bar{i} = F_i M^\tau \begin{bmatrix} X_t \\ z^t \end{bmatrix} + \bar{i} = 0 \quad \text{for } \tau \in \mathcal{T}_t. \quad (\text{B.20})$$

Let $n_{\mathcal{T}_t}$ denote the number of elements of \mathcal{T}_t , that is, the number of periods that the ZLB binds. The equation system (B.20) then has $n_{\mathcal{T}}$ equations to determine the $n_{\mathcal{T}}$ elements of z^t that are positive. From the system (B.20), it is clear that the solution for z^t and the set \mathcal{T}_t will depend on X_t , the initial situation, and thereby on the initial innovation, ε_t . For other periods, that is, $\tau \notin \mathcal{T}_t$, the ZLB will not be binding and the elements in z^t will be zero for those periods. The equation system (B.20) and the periods in the set \mathcal{T}_t hence refer to the periods where the ZLB is *strictly* binding, that is, when $z_{t+\tau,t}$ is positive. Furthermore, it is important to notice that the set of periods τ in (B.16) for which the policy-rate projection (B.14) violates the ZLB is not necessarily the same as the set of periods \mathcal{T}_t for which the ZLB is strictly binding in equilibrium. That is because the projections of the predetermined and forward-looking variables, $X_{t+\tau,t}$ and $x_{t+\tau,t}$, that determine the unrestricted policy rate differ depending on whether z^t is zero or not. This means that the whole policy-rate path is affected when the ZLB is imposed.

The difficulty in imposing the ZLB is to find the set \mathcal{T}_t for which the ZLB is strictly binding in equilibrium, that is, to find the periods for which the equation system (B.20) applies. Once this is done, solving the equation system (B.20) is trivial. Hebden et al. [55] outline a simple shooting algorithm to find the set \mathcal{T}_t .

B.2. Computation of the Likelihood Function

[Remains to be written.]

C. Data

C.1. Benchmark Model

The benchmark model is estimated using seven key macro-economic time series: real GDP, consumption, investment, hours worked, real wages, prices and a short-term interest rate. The Bayesian estimation methodology is extensively discussed in Smets and Wouters [76]. GDP, consumption and investment were taken from the US Department of Commerce - Bureau of Economic Analysis databank on September 25, 2014. Real Gross Domestic Product is expressed in Billions of Chained

2010 Dollars.[**Check this!**] Nominal Personal Consumption Expenditures and Fixed Private Domestic Investment are deflated with the GDP-deflator. Inflation is the first difference of the log of the Implicit Price Deflator of GDP. Hours and wages come from the BLS (hours and hourly compensation for the NFB sector for all persons). Hourly compensation is divided by the GDP price deflator in order to get the real wage variable. Hours are adjusted to take into account the limited coverage of the NFB sector compared to GDP (the index of average hours for the NFB sector is multiplied with the Civilian Employment (16 years and over) . The aggregate real variables are expressed per capita by dividing with the population over 16. All series are seasonally adjusted. The interest rate is the Federal Funds Rate. Consumption, investment, GDP, wages and hours are expressed in 100 times log. The interest rate and inflation rate are expressed on a quarterly basis during the estimation (corresponding with their appearance in the model), but in the figures the series are reported on an annualized (400 times first log difference) or yearly (100 times the four-quarter log difference) basis.

C.2. Model with financial frictions

The first seven variables are exactly those used to estimate the benchmark model, and are described in C.1. In addition to those series, this model features an interest rate spread. Following [9], this spread is measured as the difference between the BAA corporate interest rate and the U.S. 10-year government yield.

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