Professor Easley
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Econ 6090

First Exam

Part I (30 points) Provide a brief answer for each of the following questions.

1. A consumer has the expenditure function $e(p_1, p_2, u) = \frac{(u^2 p_1 p_2)}{(p_1+p_2)}$. Find this consumer's Walrasian demand function for good 1.
2. A Consumer has preferences $\succeq$ on $x \in R^1_+$ which can be represented by the utility function $u(x)$ where $u(x) = x^{1/2}$ for $x \in [0, 1]$ and $u(x) = x$ for $x > 1$. Is $\succeq$ a convex preference relation? Explain.
3. A consumer has indirect utility function $V(w, p_1, p_2) = \ln(w) + \alpha \ln(p_1) + \beta \ln(p_2) + K$, where K is a constant. Compute the Slutsky matrix for this consumer.

Part II (30 points) A consumer makes choices of the amounts of three goods, $x = (x_1, x_2, x_3)$, to purchase at prices, $p = (p_1, p_2, p_3)$, using wealth w. You observe the choices of goods 1 and 2, all prices, and wealth. You do not observe the quantity of good 3 that the consumer purchases. You do know that the consumer's demands satisfy homogeneity of degree 0 and Walras Law.

1. In observation a, prices are $p^a = (1, 1, 2)$, wealth is $w^a = 13$ and you observe $(x_1^a, x_2^a) = (2, 3)$. In observation b, prices are $p^b = (2, 1, 1)$, wealth is $w^b = 12$ and you observe $(x_1^b, x_2^b) = (1, 2)$. Are these choices consistent with WARP? Explain.
2. Now you can no longer observe purchases of good 2. You only observe prices, wealth and the purchases of good 1. In observation a, prices are $p^a = (1, 1, 1)$, wealth is $w^a = 20$ and you observe $x_1^a = 10$. In observation b, prices are $p^b = (2, 1, 2)$, wealth is $w^b = 30$ and you observe $x_1^b = 5$. What restrictions on the purchases of goods 2 (in observations a and b) must be satisfied for the information you have to be consistent with WARP?

Part III (40 points) A consumer has $W > 0$ to spend on consumption over T years. Let $y_t \geq 0$ be the amount spent on consumption in year t and let $y = (y_1, \ldots, y_T)$ be the expenditure stream (the amounts spent on consumption in each year). The consumer's budget constraint is $\sum_{t=1}^{T} y_t \leq W$. The consumer's utility from expenditure stream y is $U(y)$ where $U(\cdot)$ is strictly increasing, strictly concave and smooth. The consumer's objective is to pick a feasible expenditure stream that maximizes utility. [You can view this setup as one in which the price of consumption is 1 in every year, so expenditure on consumption in any year equals the amount of consumption in that year.]

Let $V(W)$ be the value of the consumer's decision problem, i.e.the consumer's indirect utility of wealth W.

1. Show that the value of the decision problem is strictly increasing in W.
2. Let $y^*(W)$ be the solution to the decision problem. Assume that the value of the decision problem and the solution are differentiable, and that $y^*(W) >> 0$. We are interested in how the value of the decision problem changes as wealth changes. Compute $\frac{dV(W)}{dW}$. Is this derivative equal to $\frac{\partial U(y^*(W))}{\partial y_1}$? Explain carefully.
3. Suppose that $U(y) = \sum_{t=1}^{T} u(y_t)$ where $u(\cdot)$ is strictly increasing, strictly concave and smooth. Now suppose that the price of consumption in year one increases from one to $p > 1$ and no other prices change. So the year one utility of expenditure function can be represented by $u(y_1/p)$ while all other utility of expenditure functions are unchanged. What happens to consumption (y_1/p) in year one? Explain.