ECON 6130 - Problem Set # 2

Julien Manuel Neves

December 1, 2017

Part (a)

Note that the equations for the New Keynesian business cycle can be listed as follows

$$
\begin{aligned}
\pi_t - \beta \mathbb{E}_t(\pi_{t+1}) - \kappa \tilde{y}_t &= 0 \\
\tilde{y}_t - \mathbb{E}_t(\tilde{y}_{t+1}) + \frac{1}{\sigma}[(i_t - \rho) - \mathbb{E}_t(\pi_{t+1}) - (r_t^n - \rho)] &= 0 \\
(i_t - \rho) - \phi_\pi \pi_t - \phi_y \tilde{y}_t - \phi_y \psi_{ya} a_t - v_t &= 0 \\
(r_t^n - \rho) + \sigma(1 - \rho_a)\psi_{ya} a_t - (1 - \rho_z) z_t &= 0 \\
v_t &= \rho_v v_{t-1} + \epsilon_t^v \\
a_t &= \rho_a a_{t-1} + \epsilon_t^a \\
z_t &= \rho_z z_{t-1} + \epsilon_t^z \\
\tilde{y}_t &= \mathbb{E}_{t-1}(\tilde{y}_t) + \eta_t^{\tilde{y}_t} \\
\pi_t &= \mathbb{E}_{t-1}(\pi_t) + \eta_t^{\tilde{n}}
\end{aligned}
$$

where η_t are the forecast errors.

Thus, we define the following state space for our model

$$S_t = [\tilde{y}_t, \pi_t, r_t^n - \rho, i_t - \rho, v_t, a_t, z_t, \mathbb{E}_t(\tilde{y}_{t+1}), \mathbb{E}_t(\pi_{t+1})]$$

Note that the gensys.m code solves the following linear equation

$$\Gamma_0 S_t = \Gamma_1 S_{t-1} + B + \Psi \epsilon_t + \Pi \eta_t$$

where $E(\epsilon_t \epsilon_t') = I$.

Therefore, by rewriting the set of equations for the New Keynesian business cycle in matrix form, we get

$$\Gamma_0 = \begin{bmatrix} -\kappa & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta \\ 1 & 0 & -\frac{1}{\sigma} & \frac{1}{\sigma} & 0 & 0 & 0 & -1 & -\frac{1}{\sigma} \\ -\phi_y & -\phi_\pi & 0 & 1 & -1 & -\phi_y\psi_{ya} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \sigma(1-\rho_a)\psi_{ya} & -(1-\rho_z) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_v & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_z & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0 & 0 & 0 & 0 & \sigma_v & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_z & 0 & 0 \end{bmatrix}'$$

$$\Pi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}'$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$$

Then the gensys.m will ouput the coefficients of an $VAR(1)$ process of the following form

$$S_t = AS_{t-1} + C\epsilon_t$$

where $\epsilon_t \sim N(0, I)$.

Given the parameters of set in the problem set, we have

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & -0.2684 & -0.4039 & 0.1331 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0974 & -0.3561 & 0.0795 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.2723 & 0.2100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2196 & -0.3956 & 0.1858 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.7000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1342 & -0.3231 & 0.0932 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0487 & -0.2849 & 0.0556 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.0054 & -0.0040 & 0.0057 \\ -0.0019 & -0.0036 & 0.0034 \\ 0 & -0.0027 & 0.0090 \\ 0.0044 & -0.0040 & 0.0080 \\ 0.0100 & 0 & 0 \\ 0 & 0.0080 & 0 \\ 0 & 0 & 0.0300 \\ -0.0027 & -0.0032 & 0.0040 \\ -0.0010 & -0.0028 & 0.0024 \end{bmatrix}$$

Part (b)

The impulse responses for inflation, output, output gap, interest rates and labor in responses to productivity, monetary policy and demand shocks are plotted in Fig.1, Fig.2 and Fig.3.

Note that the impulses responses are consistent with the results shown in class.

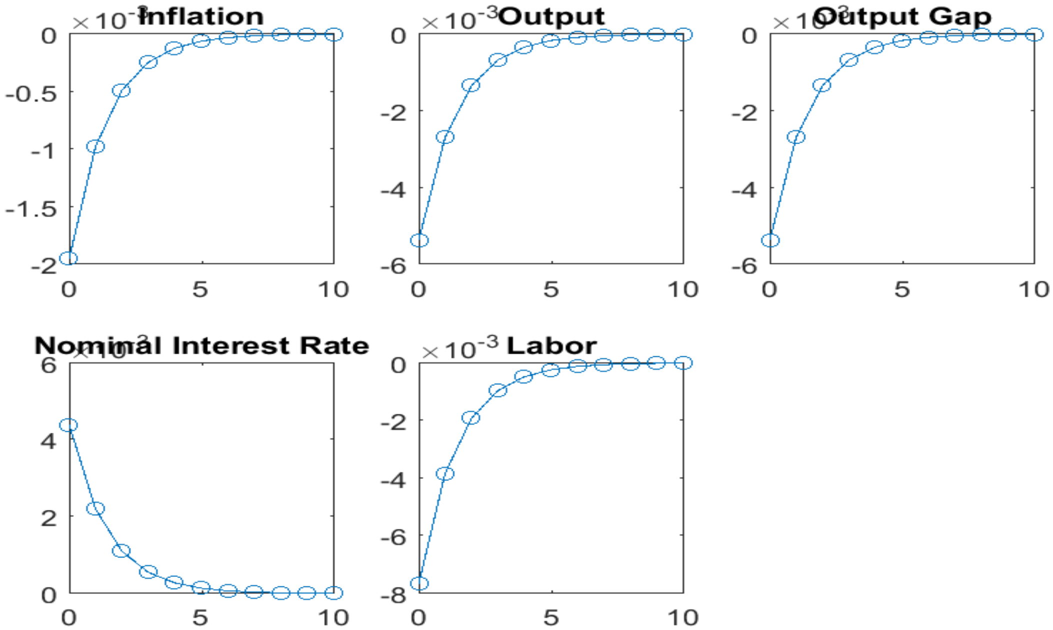

Figure 1: Impulse response to monetary shocks

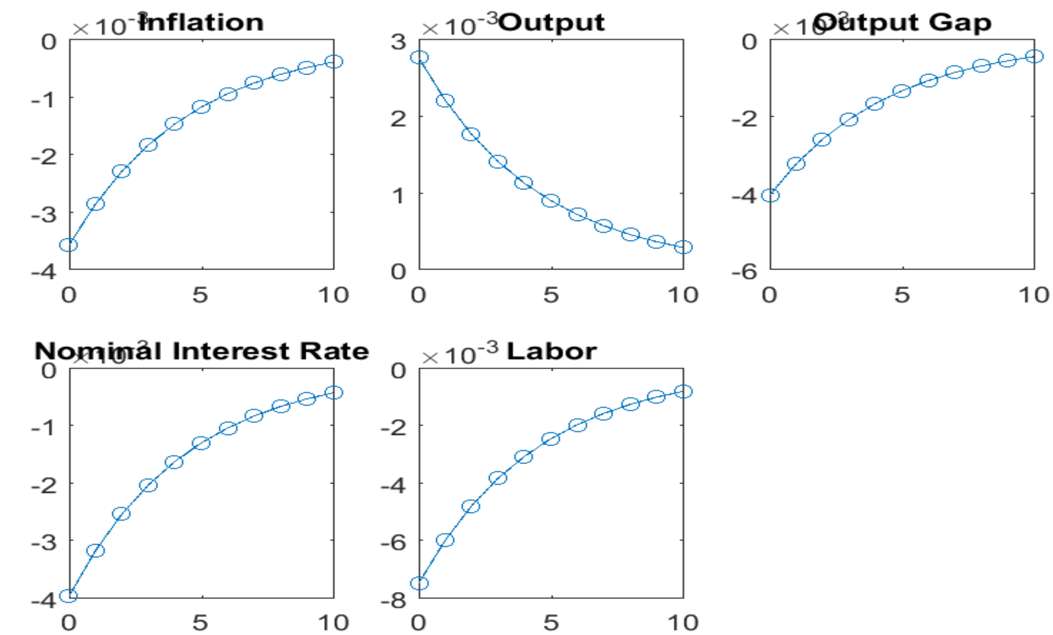

Figure 2: Impulse response to productivity shocks

Figure 3: Impulse response to demand shocks

Part (c)

Note that the nominal interest rates and inflation are included in our states S_t. As for output, we need to compute it using the following formula

$$\begin{aligned}\hat{y}_t &= y_t - y \\ &= (y_t - y_t^n) + (y_t^n - y) \\ &= \tilde{y}_t + \hat{y}_t^n \\ &= \tilde{y}_t + \psi_{ya} a_t\end{aligned}$$

where y^n, y, ψ_{ya} and a_t are defined as in Gali's book.

Then, we can write inflation, output and nominal interest rates in the following matrix form

$$Z(t) = DX(t)$$

where

$$D = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & \psi_{ya} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The simulated inflation, output and nominal interest rates are plotted in Fig.4. The standard deviations are reported in part (d) and Tab.1.

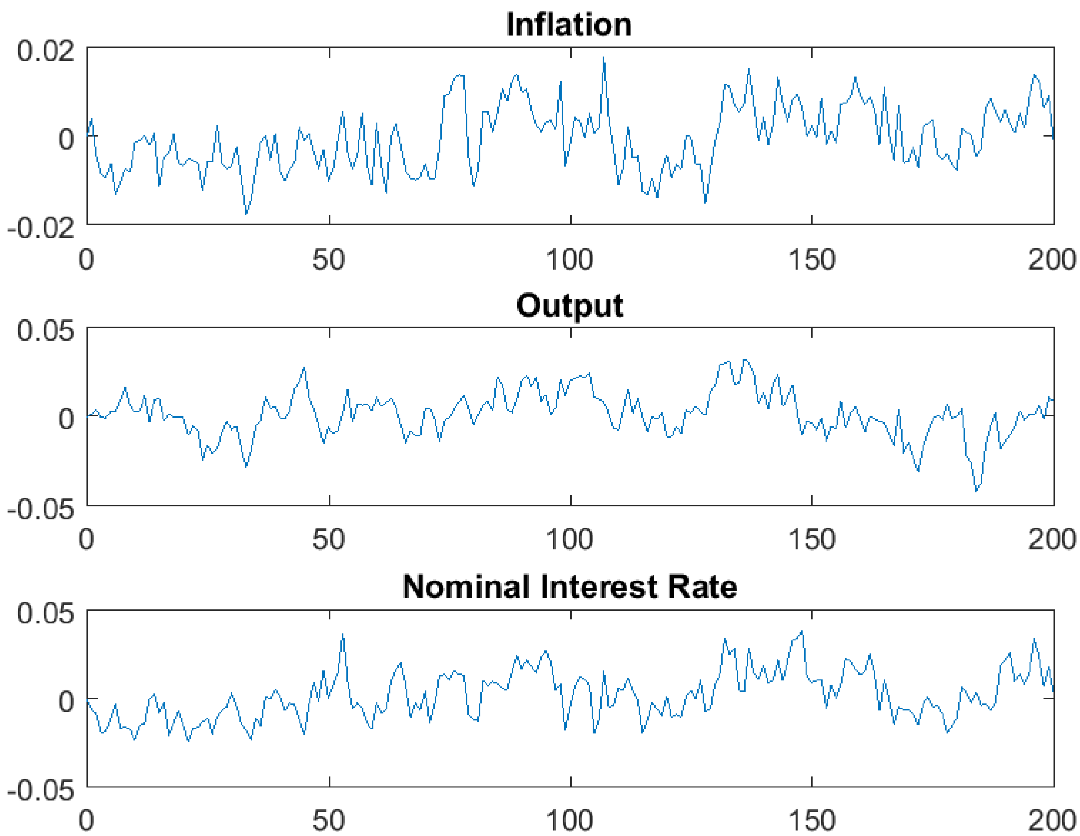

Figure 4: Simulation of New Keynesian business cycle model

Part (d)

We use the following FRED series for our data:

(i) Consumer Price Index of All Items in United States (USACPIALLQINMEI)

(ii) Real Gross Domestic Product (GDPC1)

(iii) Immediate Rates: Less than 24 Hours: Federal Funds Rate for the United States (IRSTFR01USQ156N)

Note that to compare the data to our model with need to log-linearized the series and detrend them using the Hodrick–Prescott filter. While the correspondence with the model states for GDP and the Federal Funds Rate is straightforward, it is important to note that the Consumer Price Index represents are price level. Therefore, after taking the log of the CPI, it is important to differentiate it before detrending it since

$$\pi_t = \log(P_t) - \log(P_{t-1})$$

where P_t is the price level (CPI) at time t.

The sample inflation, output and nominal interest rates are plotted in Fig.4. The standard deviations are reported in Tab.1.

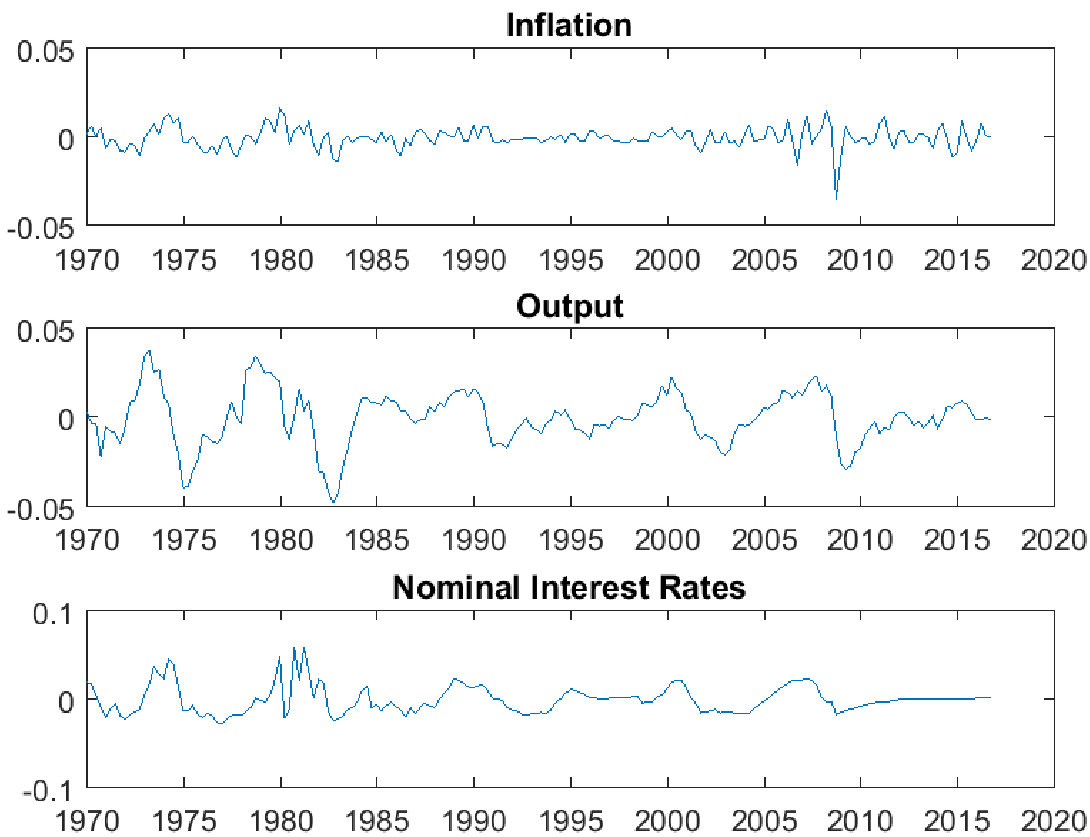

Figure 5: FRED Detrended Data

Note that the values for σ for our simulation somewhat matches the data. This might imply reasonable parameters values for our model.

	Simulation	Data
σ_π	0.0070	0.0060
σ_y	0.0113	0.0150
σ_i	0.0147	0.0154

Table 1: Standard deviation of inflation, output and interest rate

Part (e)

Note that we can write our model in the following state space form

Then, we can write inflation, output and nominal interest rates in the following matrix form

$$S_t = AS_{t-1} + C\epsilon_t$$
$$Z(t) = DX(t)$$

where A and C are defined as previously, $\epsilon_t \sim N(0, I)$, and

$$D = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & \psi_{ya} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Using the Kalman filter code from the last problem set we can compute S_t. In Fig.6, we report both the ouput and the output gap.

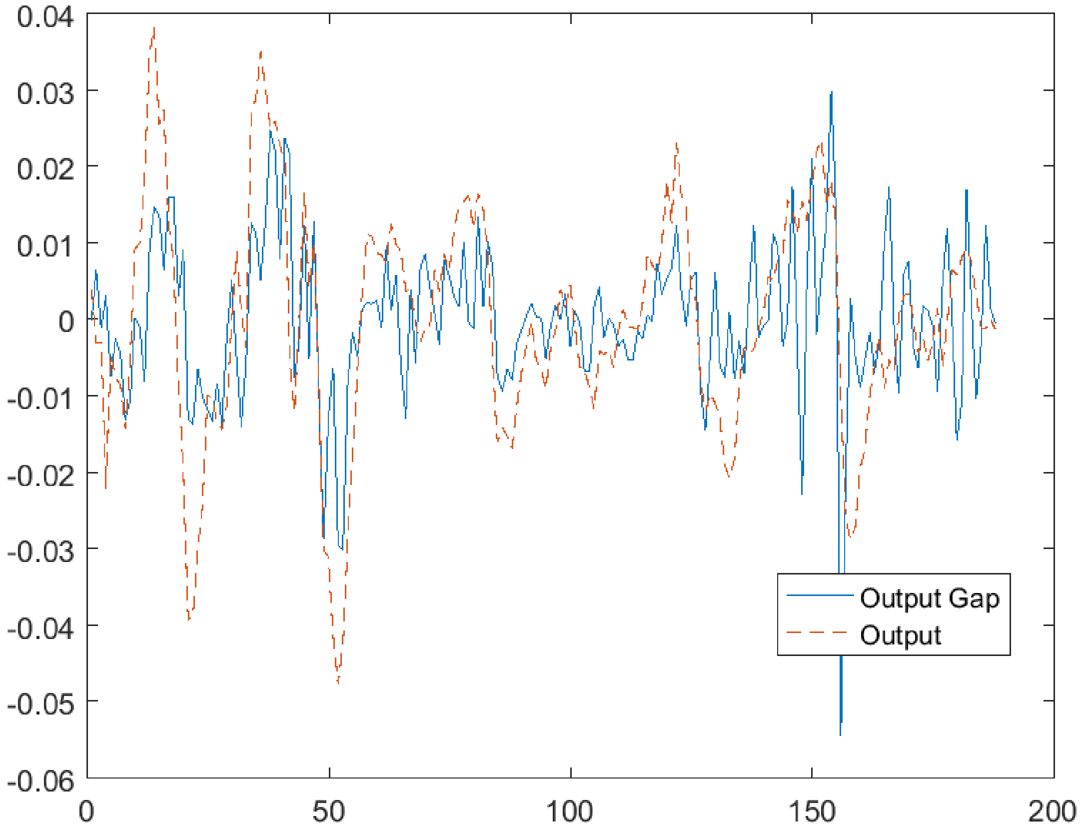

Figure 6: Kalman Filter estimate of output and output gap

As seen in Fig.6, the output gap follows output closely. This implies that $y_t^n - y$ is fairly close to 0 for all t. Is it reasonable? It seems unlikely that the natural level of output should always be close to the trend as we expect shocks in productivity to push y_t^n.

Code

```
%% Housekeeping
close all;
clear all;
% Code based on Tilahun's code for NKBC model.

%% Part (a)
% Calibration
sigma     = 2; % CRRA parameter.
beta      = 0.99; % discount factor
phi       = 3; % inverse of elastiicity of labor supply
eps       = 5; % elasticity of substitution between goods i and j
phi_pi    = 1.5; % taylor rule parameter
phi_y     = 0.5; % taylor rule parameter
theta     = 0.75; % degree of price stickiness
alpha     = 0.3; % production function parameter
rho_a     = 0.8; % persistence parameter
rho_v     = 0.5; % persistence parameter
rho_z     = 0.7; % persistence parameter
sigma_a   = 0.008; % standard deviation
sigma_v   = 0.01; % standard deviation
sigma_z   = 0.03; % standard deviation

% Compute the coefficients
rho = -log(beta);
lambda = (1-theta)*(1-beta*theta)*(1-alpha)/(theta*(1-alpha+alpha*
    eps));
kappa = lambda*(sigma + (phi+alpha)/(1-alpha));
psi_ya = (1+phi)/(sigma*(1-alpha)+phi+alpha);

% State: 'y'; 'pi'; 'r^n'; 'i'; 'v'; 'a'; 'z'; 'E(y)'; 'E(pi)'
Gamma0 = [-kappa 1 0 0 0 0 0 0 -beta;
    1 0 -1/sigma 1/sigma 0 0 0 -1 -1/sigma;
    -phi_y -phi_pi 0 1 -1 -phi_y*psi_ya 0 0 0;
    0 0 1 0 0 sigma*(1-rho_a)*psi_ya -(1-rho_z) 0 0;
    0 0 0 0 1 0 0 0 0;
    0 0 0 0 0 1 0 0 0;
    0 0 0 0 0 0 1 0 0;
    1 0 0 0 0 0 0 0 0;
    0 1 0 0 0 0 0 0 0];

Gamma1 = [0 0 0 0 0 0 0 0 0;
    0 0 0 0 0 0 0 0 0;
```

```
      0 0 0 0 0 0 0 0 0;
      0 0 0 0 0 0 0 0 0;
      0 0 0 0 rho_v 0 0 0 0;
      0 0 0 0 0 rho_a 0 0 0;
      0 0 0 0 0 0 rho_z 0 0;
      0 0 0 0 0 0 0 1 0;
      0 0 0 0 0 0 0 0 1];

Psi     = [0 0 0 0 sigma_v 0 0 0 0;
      0 0 0 0 0 sigma_a 0 0 0;
      0 0 0 0 0 0 sigma_z 0 0;]';

Pi      = [0 0 0 0 0 0 0 1 0;
      0 0 0 0 0 0 0 0 1]';

Cons    = [0 0 0 0 0 0 0 0 0]';

% Solve New Keynesian Model
[A,~,C]=gensys(Gamma0,Gamma1,Cons,Psi,Pi);

%% Part (b)
% Set up measurement matrix Z(t)=D*S(t)
D = [0 1 0 0 0 0 0 0 0; % inflation
      1 0 0 0 0 psi_ya 0 0 0; % output
      1 0 0 0 0 0 0 0 0;  % output gap
      0 0 0 1 0 0 0 0 0;  % nominal interest rate
      1/(1-alpha) 0 0 0 0 (psi_ya-1)/(1-alpha) 0 0 0]; % labor

% Compute impulse responses
time = 0:10;    % set time horizon
col = C;     % start impulse matrix
for j=1:size(time,2)
      resp(:,:,j)=D*col; % compute observations
      col=A*col;  % compute next period states
end

for i = 1:3
     % Extract impulse responses for observations
      resp_pi(:,i)=squeeze(resp(1,i,:));
      resp_y(:,i)=squeeze(resp(2,i,:));
      resp_yg(:,i)=squeeze(resp(3,i,:));
      resp_i(:,i)=squeeze(resp(4,i,:));
      resp_n(:,i)=squeeze(resp(5,i,:));

     % Plot Impulse Responses
```

```
    figure(i)
    subplot(2,3,1)
    plot(time,resp_pi(:,i),'-O')
    title('Inflation')
    subplot(2,3,2)
    plot(time,resp_y(:,i),'-O')
    title('Output')
    subplot(2,3,3)
    plot(time,resp_yg(:,i),'-O')
    title('Output Gap')
    subplot(2,3,4)
    plot(time,resp_i(:,i),'-O')
    title('Nominal Interest Rate')
    subplot(2,3,5)
    plot(time,resp_n(:,i),'-O')
    title('Labor')
end

% Print Impulse Responses - Monetary Shock
figure(1)
saveas(gcf, 'impulse_monetary.png')
% Print Impulse Responses - Productivity Shock
figure(2)
saveas(gcf, 'impulse_prod.png')
% Print Impulse Responses - Demand Shock
figure(3)
saveas(gcf, 'impulse_demand.png')

%% Part (c)
% State space model:
% X(t) = A*X(t-1) + C*eps(t)
% Z(t) = D*X(t)

% Set up measurement matrix Z(t) = D*X(t)
D = [0 1 0 0 0 0 0 0 0;
    1 0 0 0 0 psi_ya 0 0 0;
    0 0 0 1 0 0 0 0 0];

time = 0:200;   % set time horizon
X(:,1)= zeros(9,1); % starting value for states

error = randn(3,size(time,2)); % generate epsilon

% Compute state space model recursively
for t=1:size(time,2)-1
```

```
    X(:,t+1)=A*X(:,t)+C*error(:,t+1); % X(t+1) = A*X(t) + C*eps(t+1)
    Z(:,t+1)= D*X(:,t+1);    % Z(t+1) = D*X(t+1)
end

% Compute model standard deviations
sig_model = sqrt(var(Z'))

% Plot simulated state space model
figure(4)
subplot(3,1,1)
plot(time,Z(1,:))
title('Inflation')
subplot(3,1,2)
plot(time,Z(2,:))
title('Output')
subplot(3,1,3)
plot(time,Z(3,:))
title('Nominal Interest Rate')
saveas(gcf, 'simulation.png')

%% Part (d)
% Set handle for FRED data
url = 'https://fred.stlouisfed.org/';
c = fred(url);

% Set dates for sample period
startdate = '01/01/1970';
enddate = '12/01/2016';

CPI = fetch(c,'USACPIALLQINMEI','09/01/1969',enddate); % fetch CPI from FRED
GDP = fetch(c,'GDPC1',startdate,enddate); % fetch GDP from FRED
INT = fetch(c,'IRSTFR01USQ156N',startdate,enddate); % fetch rate from FRED

pi_data = diff(log(CPI.Data(:,2))); % log[price(t)] - log[price(t-1)]
gdp_data = log(GDP.Data(:,2)); % log of GDP
i_data = log(1+INT.Data(:,2)/100); % log of nominal interest rate

[~, pi_data] = hpfilter(pi_data,1600); % extract cyclical component of pi
[~,gdp_data] = hpfilter(gdp_data,1600); % extract cyclical component of y
```

```
[~,i_data] = hpfilter(i_data,1600); % extract cyclical component
    of i

% Combine data
Z=[pi_data';gdp_data'; i_data'];

% Compute model standard deviations
sig_data = sqrt(var(Z'))

% Plot FRED detrended data
figure(5)
subplot(3,1,1)
plot(INT.Data(:,1),Z(1,:))
datetick('x','yyyy')
title('Inflation')
subplot(3,1,2)
plot(INT.Data(:,1),Z(2,:))
datetick('x','yyyy')
title('Output')
subplot(3,1,3)
plot(INT.Data(:,1),Z(3,:))
datetick('x','yyyy')
title('Nominal Interest Rates')
saveas(gcf, 'data.png')

%% Part (e)
% State space model:
% X(t) = A*X(t-1) + C*eps(t)
% Z(t) = D*X(t)

% Set up measurement matrix Z(t) = D*X(t)
D = [0 1 0 0 0 0 0 0 0;
     1 0 0 0 0 psi_ya 0 0 0;
     0 0 0 1 0 0 0 0 0];

% Set starting values
X0 = zeros(9,1);    % set starting X
P0 = dlyap(A,C*C'); % set starting for the variance

% Compute the Kalman Filter
[ X_post, P_post, X_prior, P_prior, K] = kfilter(Z, A, C, D, 0, X0
    , P0 );

% Plot output and output gap
figure(6)
```

```
plot(X_post(1,:))
hold
plot(Z(2,:),'--')
legend('Output Gap', 'Output', 'Location', 'best')
saveas(gcf, 'kalman.png')
```

Kalman Filter (kfilter.m)

```
function [ X_post, P_post, X_prior, P_prior, K] = kfilter(Z, A, C,
    D, S_vv, X0, P0 )
%KFILTER Compute the Kalman Filter for a VAR(1) process
%       X[t+1] = AX[t] + Cu[t+1]
%   Z[t+1] = DX[t+1] + v[t+1]

% Get size of observations
[~,T] = size(Z);

% Allocate space for estimates
X_prior = cell(1,T);
X_post = cell(1,T);
K = cell(1,T);

% Set starting values
X_post{1} = X0;
P_post = P0;

for t = 1:T-1
    % Compute X_{t+1|t}
    X_prior{t+1}  = A*X_post{t} ;

    % Compute P_{t+1|t}
    P_prior = A * P_post * A' + C * C';

    % Compute K_{t+1}
    K{t+1} = P_prior*D'*(D*P_prior*D'+S_vv)^-1;

    % Compute X_{t+1|t+1}
    X_post{t+1}  = A*X_post{t}  + K{t+1}  * (Z(:,t) - D*X_prior{t
        +1} );

    % Compute P_{t+1|t+1}
    P_post = P_prior- K{t+1}*D*P_prior;

end

```

```
% Convert post and prior estimates to matrices
X_prior = cell2mat(X_prior);
X_post = cell2mat(X_post);

end
```