ECON 6130 - Problem Set # 3

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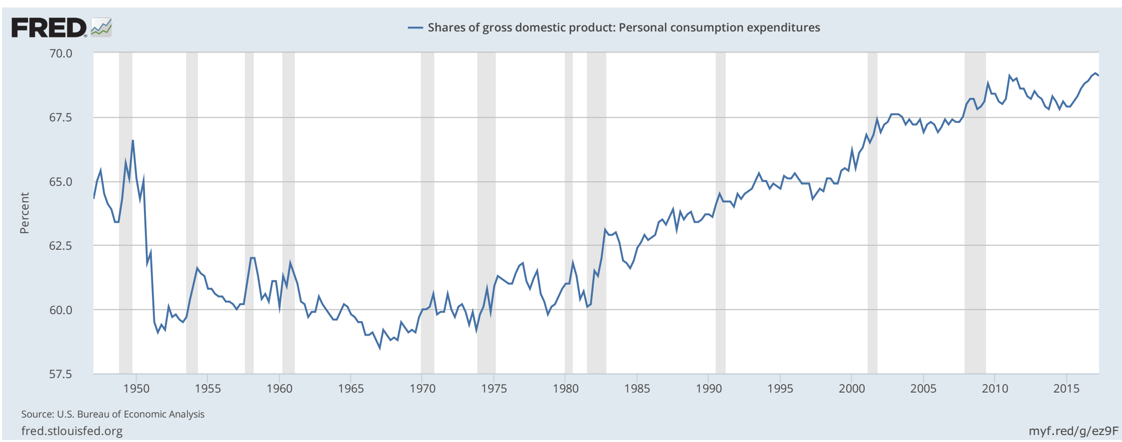
Problem 1

1.



(i) On average 65% and closer to 68% in the last few years.

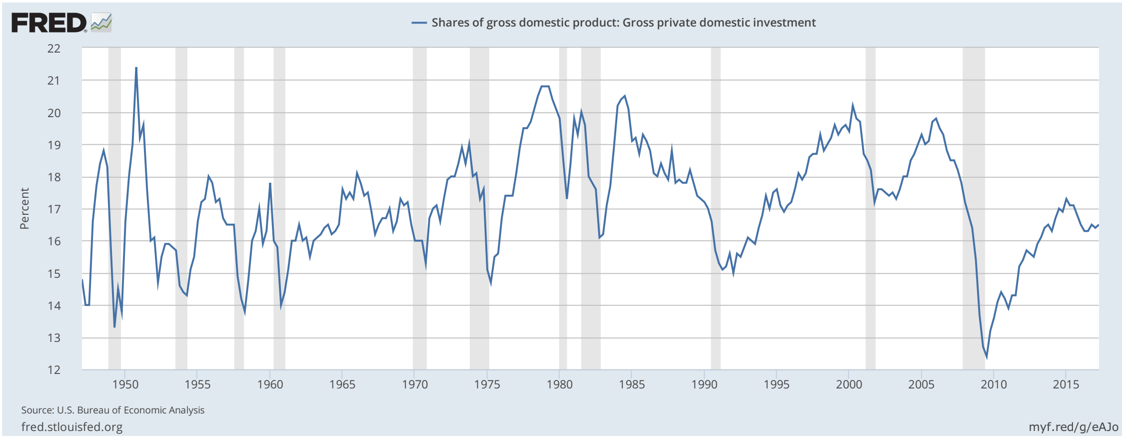
(ii) Stationary.

(iii) Trending upward. Accordingly, it steadily increased by roughly 10% over the course of the last 50 years.

(iv) Small and rapid fluctuations around the trend, (less than 1%).

(v) Stable even during the recessions.

2.



(i) On average 17%.

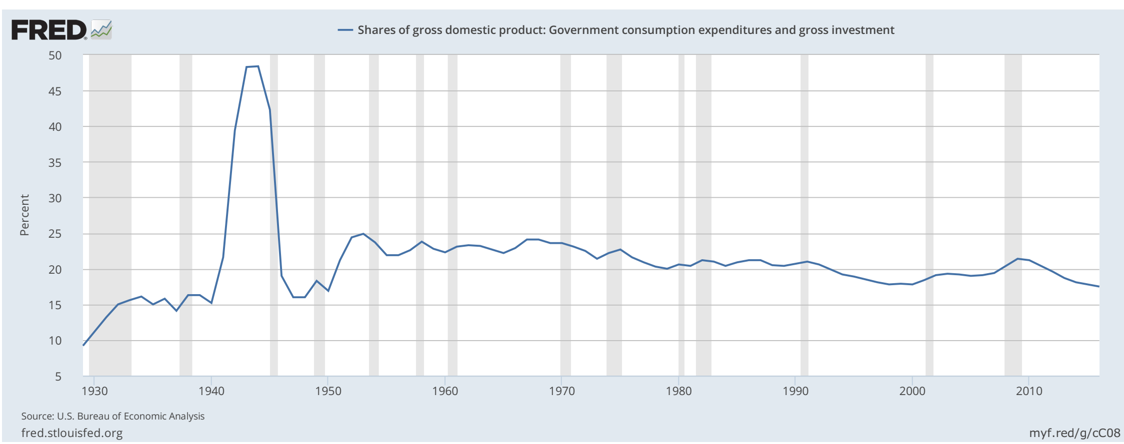
(ii) Stationary.

(iii) No trend. It fluctuates around its mean.

(iv) Big and rapid fluctuations around the trend (up to 8%). This contrasts with consumption.

(v) Investment seem to drop enormously during recessions. Again, this is in opposition to the previous figure.

3.



(i) On average 20%.

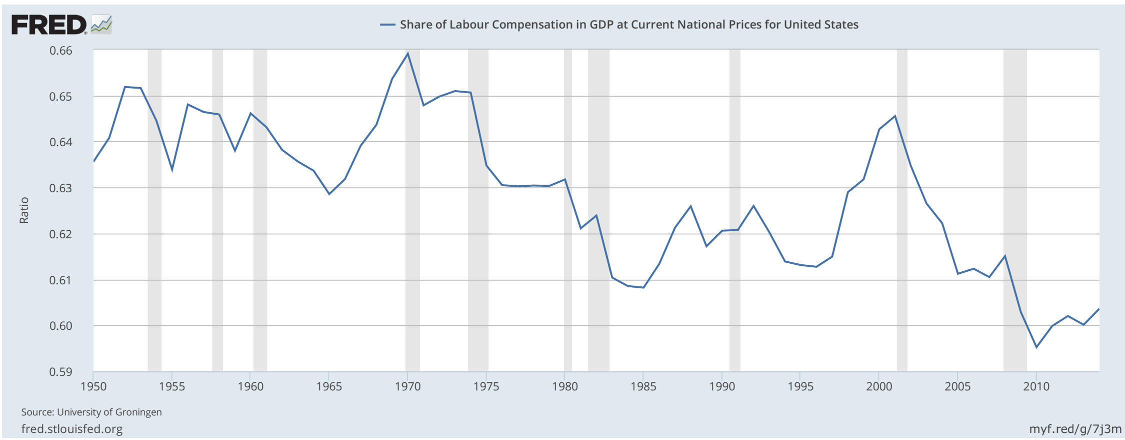
(ii) Stationary.

(iii) No trend. It fluctuates around its mean.

(iv) Really smooth curve around the trend after World War II. This comes from the fact that plans for government spending are usually adopted once a year. Moreover, the increase during World War II comes from the major military spending the U.S. government incurred during that period.

(v) Slightly increasing during recessions.

4.



(i) On average 63%.

(ii) Stationary.

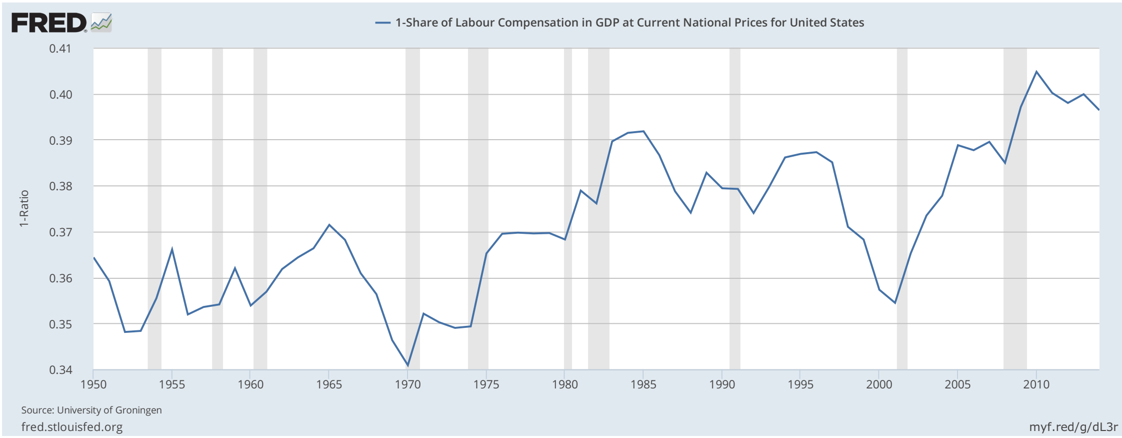
(iii) Stable before 1975 and slightly declining after 1975 (drop by roughly 6% over the course of the last 30 years).

(iv) Small and slow fluctuations around the trend (around 1%).

(v) Sharp declines during recessions. In the recent one, payments to labor as share of GDP drop by 2%.

5.

Note this figure was constructed by taking the complement of the previous series.



(i) On average 37%.

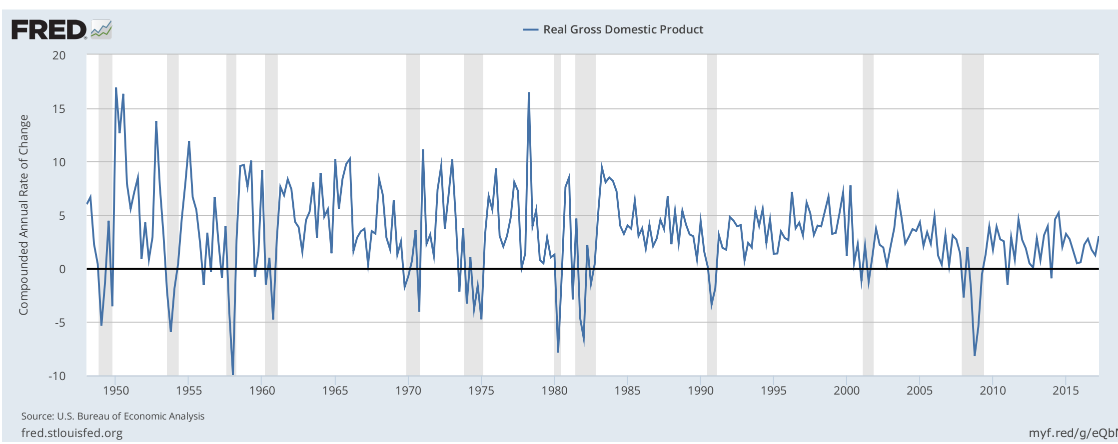
(ii) Stationary.

(iii) Stable before 1975 and slightly increasing after 1975 (drop by roughly 6% over the course of the last 30 years).

(iv) Small and slow fluctuations around the trend (around 1%).

(v) Sharp increases during recessions. In the recent one, payments to capital as share of GDP jump by 2%.

6.



(i) On average 2-3 %.

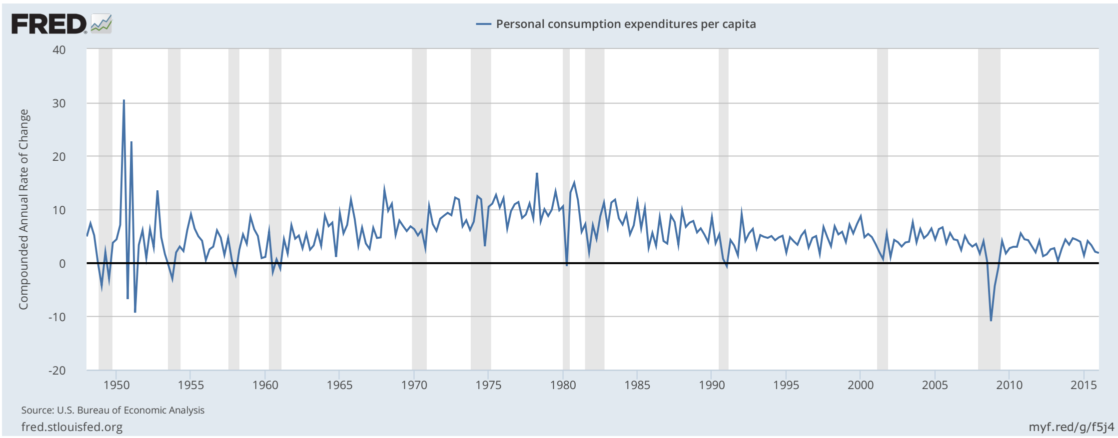
(ii) Stationary.

(iii) Stable around the mean.

(iv) Rapid and large fluctuations around the trend (around 5%). More volatile before 1980.

(v) Sharp decline during the recessions. In the recent recession, $\hat{Y}_i$ drop by 8%.

7.



(i) On average 3 %.

(ii) Stationary.

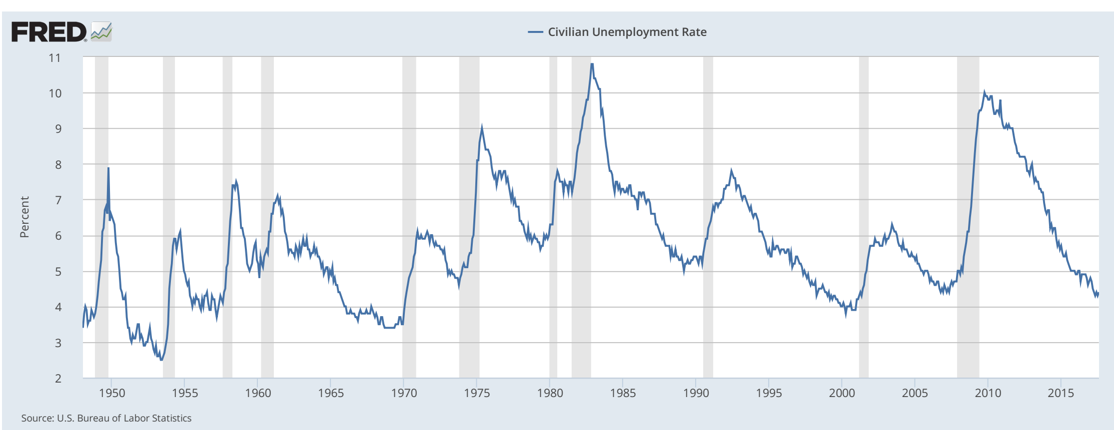
(iii) Stable around the mean.

(iv) Rapid and small fluctuations around the trend (around 2%). More volatile before 1955.

(v) Sharp decline during the recessions. In the recent recession, $\hat{c}_i$ drop by 8%.

(vi) Follows the growth rate of GDP pretty closely.

8.



(i) On average 6 %.

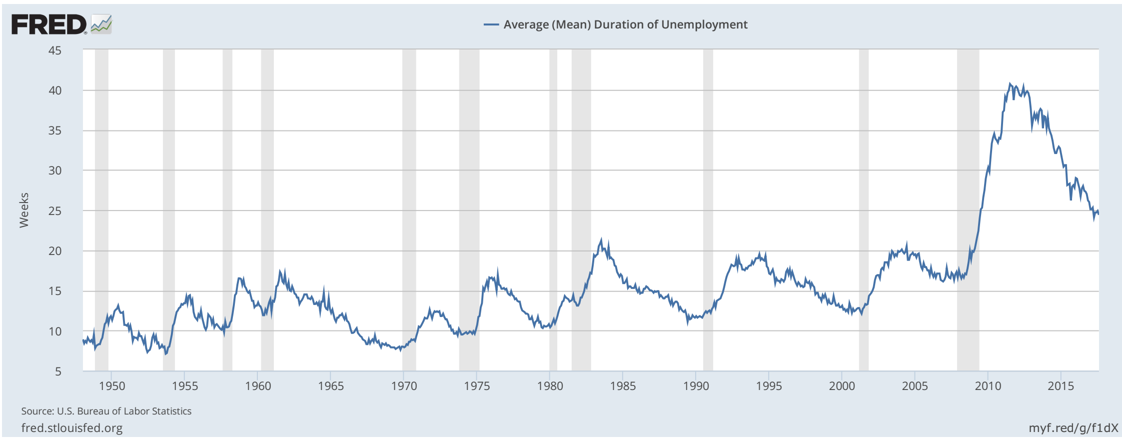
(ii) Stationary.

(iii) Overall trend: sharp increase during the recessions followed by slow recovery.

(iv) Small fluctuation around overall trend.

(v) In recent recession, unemployment went up by around 6% and only just now is it back to its original level.

9.



(i) On average 13 weeks before 2000 and 30-35 weeks in the last decade.

(ii) Non-stationary.

(iii) Overall trend: sharp increase during the recessions followed by slow recovery. Unlike the unemployment rate, the support for the average duration of unemployment seems to be increasing at an exponential rate.

(iv) Small fluctuations around the trend.

(v) In recent recession, average duration increase up to 40 weeks and is only just now back to 25 weeks. This contrast with previous recessions where average duration would consistently increase to roughly 15 weeks and go back down to 10 weeks afterwards.

Problem 2

1.

Planner's optimization problem:

$$v(k) = \max_{0 \leq k' \leq f(k)} \{\log(f(k) - k') + \beta v(k')\}$$

where $f(k) = F(k, 1) + (1 - \delta)k = k^\alpha + (1 - \delta)k$.

For simplicity, this problem can be reformulated as such

$$v(k) = \max_{0 \leq k' \leq k^\alpha + (1-\delta)k} \{\log(k^\alpha + (1 - \delta)k - k') + \beta v(k')\}$$

where

(i) State Variable $\Rightarrow k$.

We can interpret k as the level of capital today. For the planner, it is taken as given, and its value influence the state of the world. In the setting of this problem, k constrains the amount of goods available today by $f(k)$.

(ii) Control Variable $\Rightarrow k'$.

While the planner is constraint by the current level of capital, he is free to choose the level of capital in the next period, i.e. k'. In this problem, the planner is maximizing today's utility and tomorrow's future value of utility by setting k'.

Note that in this dynamic setting there's no need for an aggregate law of motion for the state variables since the planner is already at the aggregate level.

2.

Note that instead of looking at a large grid, we can narrow our search by first finding the steady state, i.e. $k = k_t = k_{t+1}$.

Using the Euler equation of the sequential problem we have

$$\begin{aligned}
u'(f(k_t) - k_{t+1}) &= \beta u'(f(k_{t+1}) - k_{t+2}) f'(k_t) \\
\frac{1}{f(k_t) - k_{t+1}} &= \beta \frac{1}{f(k_{t+1}) - k_{t+2}} (\alpha k_t^{\alpha-1} + (1-\delta)) \\
\frac{1}{f(k) - k} &= \beta \frac{1}{f(k) - k} (\alpha k^{\alpha-1} + (1-\delta)) \\
\frac{1}{\beta} &= \alpha k^{\alpha-1} + (1-\delta) \\
k &= \left[\frac{1 - \beta + \beta\delta}{\alpha\beta} \right]^{\frac{1}{\alpha-1}}
\end{aligned}$$

Given $\alpha = 0.3$, $\beta = 0.6$ and $\delta = 0.8$, then

$$\begin{aligned}
k &= \left[\frac{1 - \beta + \beta\delta}{\alpha\beta} \right]^{\frac{1}{\alpha-1}} \\
&= \left[\frac{1 - 0.6 + 0.6 \cdot 0.8}{0.3 \cdot 0.6} \right]^{\frac{1}{0.3-1}} \\
&\approx 0.1036
\end{aligned}$$

Thus, we can build a grid around 0.1036 to capture the policy and value functions around the steady state.

With a grid of size $n = 1000$, we have the following numerical solution:

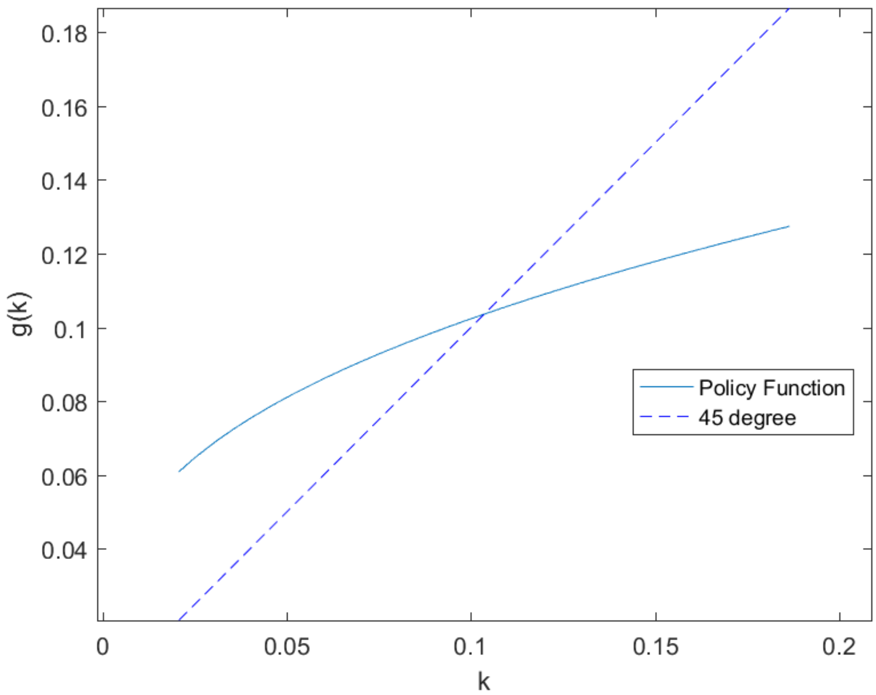


Figure 1: Policy function

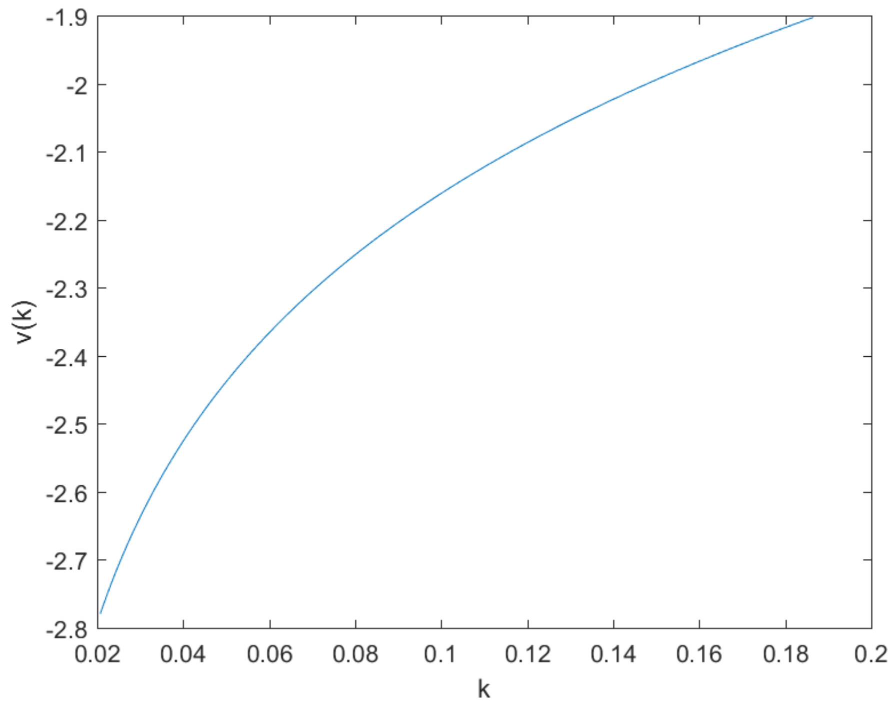


Figure 2: Value function

```matlab
%% Value Function Iteration
clear, clc;

%% Set parameters
n = 1000;        % Size of grid

delta = 0.8;     % Depreciation rate
alpha = 0.3;     % Capital share of income
beta = 0.6;      % Discount factor

crit = 1;        % Initialize convergence criterion
tol = 1e-10;     % Convergence tolerance

%% Grid
k_star = ((1/beta+delta-1)/alpha)^(1/(alpha-1)); % Steady state
k_max = k_star + 0.8*k_star;     % Upper bound
k_min = k_star - 0.8*k_star;     % Lower bound
k_grid = linspace(k_min,k_max,n)';       % Create grid

% Empty
val_temp = zeros(n,1);  % Initialize temporary value function vector
val_fun = zeros(n,1);   % Initialize value function vector
pol_fun = zeros(n,1);   % Initialize policy function vector

ite = 0;     % Initialize iteration counter

%% Value function iteration
while crit>tol ;
    % Iterate on k
    for i=1:n
        c = k_grid(i)^alpha + (1-delta)*k_grid(i) - k_grid; % Compute consumption for kt
        utility_c = log(c); % Compute utility for every ct
        utility_c(c<=0) = -Inf; % Set utility to -Inf for c<=0
        [val_fun(i),pol_fun(i)] = max(utility_c + beta*val_temp); % Solve bellman equation
    end
    crit = max(abs(val_fun-val_temp));  % Compute convergence criterion
    val_temp = val_fun; % Update value function
    ite = ite + 1 % Print iteration number
end

%% Plot figures
```

```
% Value function
figure
plot(k_grid,val_fun)

xlabel('k')
ylabel('v(k)')

print('plot_value','-dpng')

% Policy function
figure
plot(k_grid,k_grid(pol_fun))
axis equal
hold on
plot(k_grid,k_grid, '--b')

xlabel('k')
ylabel('g(k)')
legend('Policy Function','45 degree', 'Location', 'best')

print('plot_policy','-dpng')
```