Lab Report #2: Equity Returns & Certainty Equivalents

Revised: August 20, 2015

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Use Matlab where possible and attach your code to your answer.

1. *Properties of US equity returns.* Gene Fama is the 2013 recipient of the Nobel Prize in economics and one of the giants of modern finance. His focus has largely been on asset returns — why some assets have higher returns, on average, than others. His long-time coauthor Ken French kindly posts the "Fama-French datasets" on his his website. I've attached some of their data to this document in spreadsheet form (download the pdf, open in the Adobe Reader or the equivalent, click on pushpin):  The variables are

   - Column 1: year
   - Column 2: excess return on equity (return on a broad-based portfolio of stocks minus the riskfree rate)
   - Column 3: one-year riskfree rate of return

   Returns are expressed as percentages.

   Your first task is to read the data into Matlab. Once you've done that:

   (a) Use the command `hist` to plot a histogram for the excess return.

   (b) Compute the mean, standard deviation, skewness, and excess kurtosis for the excess return series.

   (c) In what respects do excess returns look normal? In what respects not?

   (d) Why do you think the mean excess return is positive?

2. *Concave and convex functions.* Consider the following functions defined over positive values of x:

   (a) $f(x) = \log x$

   (b) $f(x) = x^\alpha$ for $\alpha = 2$

   (c) $f(x) = x^{1-\alpha}/(1-\alpha)$ for $\alpha = 2$

   For each one:

   - State whether it's concave, convex, or something else.
   - Given your answer to (a), what does Jensen's inequality say about the relative magnitudes of $E[f(x)]$ and $f[E(x)]$?
   - Verify your answer to (b) by computing $E[f(x)]$ and $f[E(x)]$ for the Bernoulli random variable

$$x = \begin{cases} 100 & \text{with probability } 1/2 \\ 200 & \text{with probability } 1/2. \end{cases}$$

3. *Certainty equivalents with power utility.* Consider an agent with power utility,

$$u(c) \;=\; c^{1-\alpha}/(1-\alpha),$$

facing Bernoulli consumption risk,

$$c \;=\; \begin{cases} a & \text{with probability } 1-\omega \\ a+b & \text{with probability } \omega. \end{cases}$$

Here a and b are both positive, ω is between zero and one, and $\alpha = 5$.

(a) If $\omega = 0.1$, what values of (a, b) give c a mean of 150 and a standard deviation of 50?

(b) With these numbers, what is the agent's expected utility?

(c) What is the agent's certainty equivalent? Risk penalty?

(d) How do your answers change if $\omega = 0.9$?

(e) Why do the certainty equivalents differ between the two cases?