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Bayesian Model Averaging and exchange rate forecasts

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ABSTRACT

Exchange rate forecasting is hard and the seminal result of Meese and Rogoff [Meese, R., Rogoff, K., 1983. Empirical exchange rate models of the seventies: Do they fit out of sample? Journal of International Economics 14, 3–24] that the exchange rate is well approximated by a driftless random walk, at least for prediction purposes, still stands despite much effort at constructing other forecasting models. However, in several other macro and financial forecasting applications, researchers in recent years have considered methods for forecasting that effectively combine the information in a large number of time series. In this paper, I apply one such method for pooling forecasts from several different models, Bayesian Model Averaging, to the problem of pseudo out-of-sample exchange rate predictions. For most currency–horizon pairs, the Bayesian Model Averaging forecasts using a sufficiently high degree of shrinkage, give slightly smaller out-of-sample mean square prediction error than the random walk benchmark. The forecasts generated by this model averaging methodology are however very close to, but not identical to, those from the random walk forecast.

1. Introduction

Out-of-sample forecasting of exchange rates is hard. Meese and Rogoff (1983) argued that all exchange rate models do less well in out-of-sample forecasting exercises than a simple driftless random walk. Although this finding was heresy to many at the time that Meese and Rogoff wrote their paper, it has now become the conventional wisdom. Mark (1995) claimed that a monetary fundamentals model can generate better out-of-sample forecasting performance at long horizons, but that result has been found to be very sensitive to the sample period (Groen, 1999; Faust et al., 2003). Claims that a particular variable has predictive power for exchange rates crop up frequently, but these results typically apply just to a particular exchange rate and a particular subsample. As such, they are by now met with justifiable skepticism and are thought of by many as the result of data-mining exercises.

However, in many contexts, researchers have recently made substantial progress in the econometrics of forecasting using large datasets (i.e. a large number of predictors). The trick is to combine the information in these different variables in a judicious way that avoids the estimation of a large number of unrestricted parameters. Bayesian VARs have been found to be useful in forecasting: these often use many time series, but impose a prior that many of the coefficients in the VAR are close to zero. Approaches in which the researcher estimates a small number of factors from a large dataset and forecasts using these estimated factors have also been shown to be capable of superior predictive performance (see for example Stock and Watson (2002) and Bernanke and Boivin (2003)). Stock and Watson (2001, 2004) obtained good results in out-of-sample prediction of international output growth and inflation by taking forecasts from a large number of different simple models and just averaging them. They found the good performance of simple model averaging to be remarkably consistent across subperiods and across countries. The basic idea that forecast combination outperforms any individual forecast is part of the folklore of economic forecasting, going back to Bates and Granger (1969). It is of course crucial to the result that the researcher just averages the forecasts (or takes a median or trimmed mean). It is, in particular, tempting to run a forecast evaluation regression in which the weights on the different forecasts are estimated as free parameters. While this leads to a better in-sample fit, it gives less good out-of-sample prediction.

Bayesian Model Averaging (BMA) is another method for forecasting with large datasets that has received considerable recent attention in both the statistics and econometrics literature. The idea is to take forecasts from many different models, and to assume that one of them is the true model, but that the researcher does not know which this is. The researcher starts from a prior about which model is true and computes the posterior probabilities that each model is the true one. The forecasts from all the models are then weighted by these posterior probabilities. It has been used in a number of econometric applications, including

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output growth forecasting (Min and Zellner, 1993; Koop and Potter, 2003), cross-country growth regressions (Doppelhofer et al., 2000; Fernandez et al., 2001) and stock return prediction (Avramov, 2002; Cremers, 2002). Avarmov and Cremers both report improved pseudo-out-of-sample predictive performance from BMA.

The contribution of this paper is to argue that BMA is useful for out-of-sample forecasting of exchange rates in the last fifteen years although the magnitude of the improvement that it offers relative to the random walk benchmark is quite small.

One does not have to be a subjectivist Bayesian to believe in the usefulness of BMA, or of Bayesian shrinkage techniques more generally. A frequentist econometrician can interpret these methods as pragmatic devices that may be useful for out-of-sample forecasting in the face of model and parameter uncertainty.[1] The plan for the remainder of the paper is as follows. Section 2 describes the idea of BMA. The out-of-sample exchange rate prediction exercise is described in Section 3. Section 4 concludes.

2. Bayesian model averaging

The idea of Bayesian Model Averaging was set out by Leamer (1978), and has recently received a lot of attention in the statistics literature, including in particular Raftery et al. (1997), Hoeting et al. (1999) and Chipman et al. (2001).

Consider a set of n models $M_1, \ldots, M_n$. The ith model is indexed by a parameter vector θ_i. The researcher knows that one of these models is the true model, but does not know which one.[2] The researcher has prior beliefs about the probability that the ith model is the true model which we write as $P(M_i)$, observes data D, and updates her beliefs to compute the posterior probability that the ith model is the true model:

$$P(M_i|D) = \frac{P(D|M_i)P(M_i)}{\sum_{j=1}^{n} P(D|M_j)P(M_j)} \tag{1}$$

where

$$P(D|M_i) = \int P(D|\theta_i, M_i)P(\theta_i|M_i)\mathrm{d}\theta_i \tag{2}$$

is the marginal likelihood of the ith model, $P(\theta_i|M_i)$ is the prior density of the parameter vector in this model and $P(D|\theta_i, M_i)$ is the likelihood. Each model implies a forecast. In the presence of model uncertainty, our overall forecast weights each of these forecasts by the posterior for that model. This gives the minimum mean square error forecast. The researcher needs only to specify the set of models, the model priors, $P(M_i)$, and the parameter priors, $P(\theta_i|M_i)$.[3]

The models do not have to be linear regression models, but I shall henceforth assume that they are. The ith model then specifies that

$$y = X_i\beta_i + \varepsilon \tag{3}$$

where y is a $T \times 1$ vector of observations on a variable that the researcher wishes to forecast, X_i is a $T \times p_i$ matrix of predictors, β_i is a $p_i \times 1$ parameter vector, ε is the disturbance vector, the disturbances are i.i.d. with mean zero and variance σ^2, and $\theta_i = (\beta_i', \sigma^2)$. To obtain a closed form expression for the model likelihood, I assume that the regressors are strictly exogenous. This assumption is clearly false in the application that I am considering in this paper, but many methods for combining a large number of variables in forecasting exercises (including bagging and empirical Bayes methods) likewise have a theoretical justification that relies on strict exogeneity of the regressors. And, any of these methods may still have good forecasting power even if the premise of strict exogeneity underlying their theoretical justification is false (see Stock and Watson (2005)). In this spirit, the goal of this paper is simply to assess the power of BMA in forecasting exchange rates.

For the parameter priors, I shall take the natural conjugate g-prior specification for β_i, so that the prior for β_i conditional on σ^2 is $N(0, \sigma^2(X_i'X_i)^{-1})$. For σ^2, I assume the improper prior that is proportional to $1/\sigma^2$. It is well known (see, for example, Zellner (1971)) that one can then calculate the required likelihood of the model analytically as:

$$p(D|M_i) = \frac{1}{2}\frac{\Gamma(T/2)}{\pi^{T/2}}(1+\varphi)^{-p_i/2}S_i^{-T} \tag{4}$$

where $S_i^2 = Y'Y - Y'X_i(X_i'X_i)^{-1}X_i'\,Y\frac{\varphi}{1+\varphi}$, while the posterior mean for β_i is

$$\tilde{\beta}_i = E(\beta_i|D, M_i) = \frac{\varphi}{\varphi+1}(X_i'X_i)^{-1}X_i'\,Y. \tag{5}$$

The prior for β_i is a proper prior. In BMA, one cannot use improper priors for model-specific parameters, because improper priors are unique only up to an arbitrary multiplicative constant and so their use would lead to an indeterminacy of the model posterior probabilities (Kass and Raftery, 1995). Besides, I want to have an informative prior for β_i. The prior for β_i is centered around zero and so each model is shrunk towards a lack of forecastability. The extent of this shrinkage is governed by φ. A smaller value of φ means more shrinkage and makes the prior more informative, but this may help in out-of-sample forecasting. Researchers often try to make the prior as uninformative as possible, but at least in the exchange rate forecasting problem considered in this paper, a more informative prior turns out to give better predictive performance. One way of thinking about the role of φ is that it controls the relative weight of the data and our prior beliefs in computing the posterior probabilities of different models. If $\varphi = 0$, then $P(D|M_i)$ is equal for all models and so the posterior probability of each model being true is equal to the prior probability. The larger is φ, the more we are willing to move away from the model priors in response to what we observe in the data. The prior for σ^2 is an inverse-gamma (0,0) prior, and is improper. An improper prior can be used for σ^2, because this parameter is common to all models, and thus the arbitrary multiplicative constant in this prior does not affect the posterior probabilities for the different models.

This parameter prior has been used by Fernandez et al. (2001) and many others in the BMA literature.[4] The model priors will be discussed in the context of the application in Section 3.

[1] For recent work considering frequentist approaches to model averaging, see Hjort and Claeskens (2003).

[2] For recent work considering Bayesian Model Averaging when none of the models is in fact true, see Key et al. (1998) and Fernández-Villaverde and Rubio-Ramírez (2004). The Bayesian Model Averaging procedure is consistent in the sense that the weight on the model that is best in the Kullback–Leibler metric (which is the true model if that exists, but is still well defined if none of the models is true) goes to one asymptotically as the sample size increases.

[3] The distinction between model uncertainty and parameter uncertainty is a little artificial in the sense that one could write all the models as nested by a sufficiently general model, and, within this nesting model, the only uncertainty is then parameter uncertainty. The BMA approach would however imply very particular and rather peculiar priors for these parameters.

[4] Alternatively, a conjugate inverse-gamma (c_0, d_0) prior could have been used for σ^2 with $c_0 > 0$ and $d_0 > 0$. This is a proper prior, but has the disadvantage that is not invariant under scale transformations. Moreover it seems appropriate to have a prior for σ^2 that is as uninformative as possible.

3. Exchange rate forecasting

I consider prediction of the bilateral exchange value of the Canadian dollar, pound, yen and mark/euro, relative to the US dollar, using both direct forecasting and iterated multistep forecasting. Iterated forecasts are more efficient if the model is correctly specified, but direct forecasts may be more robust to model misspecification (see Marcellino et al. (2006)).

3.1. Direct forecasts

Turning first to direct forecasts, the ith model for forecasting exchange rates that I consider is of the form

$$e_{t+h} - e_t = \beta_i' X_{i,t} + \varepsilon_t \tag{6}$$

where e_t denotes the log exchange rate (foreign currency per dollar), h is the forecasting horizon, $X_{i,t}$ is the vector of regressors in time period t for model i, and ε_t is the error term. Each model implies a forecast $\tilde{\beta}_i' X_{i,t}$ where $\tilde{\beta}_i$ denotes the posterior mean of β_i. The proposed overall forecast then weights each of these models by their posterior probabilities, so that the forecast is $\sum_{i=1}^{n} P(M_i|D)\tilde{\beta}_i' X_{i,t}$.

The models will consist of all possible permutations of a set of λ potential predictor variables, including all of these predictors and none of them, making a total of 2^λ candidate models. Each of these models will also include a constant, except for the model with no predictor variables at all which is simply the driftless random walk

$$e_{t+h} - e_t = \varepsilon_t. \tag{7}$$

Following Cremers (2002) and Koop and Potter (2003), among others, I specify that the prior probability for any model with κ predictors (excluding any intercept) is

$$P(M_i) = \rho^{\kappa}(1-\rho)^{\lambda-\kappa}. \tag{8}$$

If $\rho = 0.5$, then all the models get equal weight, but assigning equal prior probability to each model means that models with a small number of predictors may receive too little prior weight. A smaller value of the hyperparameter ρ favors smaller models. The probability that the true model has no predictors (i.e. the driftless random walk) is $(1-\rho)^\lambda$. The expected number of predictors is $\rho\lambda$.

The posterior model probabilities are calculated using Eqs. (1), (4) and (8). One issue that arises is that the forecasts are overlapping h-step ahead forecasts and so forecast errors less than h periods apart are likely to be serially correlated. Meanwhile the derivation of the model likelihood only applies with serially uncorrelated errors. I circumvent this problem by the simple device of using only every hth observation in computing the posterior mean $\tilde{\beta}_i$ and the model likelihood.

3.2. Iterated multistep forecasts

Turning now to iterated multistep forecasts, the ith model for forecasting exchange rates is instead of the form

$$e_{t+1} - e_t = \beta_i' X_{i,t} + \varepsilon_t. \tag{9}$$

In each model, I consider an auxiliary equation for $X_{i,t}$, which is

$$X_{i,t} = A_i X_{i,t-1} + u_t \tag{10}$$

where u_t is i.i.d.. Estimating the vector autoregression (10) by OLS gives an estimate $\hat{A}_i$ and treating $\hat{A}_i^j X_{i,t}$ as a forecast of $X_{i,t+j}$, the forecast of $e_{t+h} - e_t$ in the ith model is given by $\sum_{j=0}^{h-1} \tilde{\beta}_i' \hat{A}_i^j X_{i,t}$ where $\tilde{\beta}_i$ denotes the posterior mean of β_i in (9). The proposed overall forecast then weights each of these models by their posterior probabilities, so that the forecast is $\sum_{i=1}^{n} P(M_i|D) \sum_{j=0}^{h-1} \tilde{\beta}_i' \hat{A}_i^j X_{i,t}$ where the posterior model probabilities are again calculated using Eqs. (1), (4) and (8). Since Eq. (9) does not have overlapping errors, it is not necessary to drop any observations when using iterated multistep forecasting.

3.3. Monthly financial dataset

I first consider pseudo-out-of-sample exchange rate prediction using a dataset of financial variables as the possible predictors. These data are available at a monthly frequency, are available in real-time and are never revised. Data vintage issues can substantially affect the results of exchange-rate forecasting exercises, as noted by Faust et al. (2003). The predictors are (i) the relative stock prices (foreign–US) (logs), (ii) the relative annual stock price growth rate, (iii) the relative dividend yield, (iv) relative short term interest rates, (v) the relative term spread (long minus short term rates, motivated by the finding of Bekaert and Hodrick (1992) and Clarida (2003) that term structure variables are useful for exchange rate prediction), (vi) exchange rate returns over the previous month, and (vii) the sign of exchange rate returns over the previous month. Data sources are given in Appendix A. The data cover the months 1973:01–2005:12. The models considered use as regressors all possible permutations of these 7 predictors (including all 7 and none at all). A constant is included in each model, except for the one with no predictors which is simply the driftless random walk. This gives a total of $2^7 = 128$ models.

The pseudo-out-of-sample prediction exercise involves forecasting the exchange rate for 1990:01–2005:12 as of h months previously, for $h = 3, 6, 9, 12$. For example, the first 3-month ahead forecast is the prediction of the exchange rate in 1990:01 that was made in 1989:10. Of course, this forecast is constructed only using data from 1989:10 and earlier.

3.4. Quarterly macro and financial dataset

Although the monthly financial dataset has some advantages, it is missing a great many of the variables that might have some predictive power for exchange rates. To include these, I switch to quarterly data, and give up on the "real-time" feature of the monthly asset price dataset.

This larger dataset contains all the same variables as the monthly data, aggregated to quarterly frequency. In addition it includes (i) relative real GDP (foreign–US) (logs), (ii) relative money supply (logs), (iii) the relative price level (logs), (iv) relative annual growth rates, (v) relative annual inflation rates, (vi) relative annual money growth rates, (vii) the relative ratio of current account to GDP and (viii) relative labor productivity (logs).

The data cover the quarters 1973:1–2005:4. The models considered use as regressors all possible permutations of these 15 basic predictor variables (including all 15 and none at all). Again, a constant is included in each model, except for the one with no predictors which is simply the driftless random walk. This gives a total of $2^{15} = 32{,}768$ models. Many standard models for forecasting exchange rates, such as the monetary fundamentals model of Mark (1995) and the sticky price monetary fundamentals model of Dornbusch (1976) are effectively included in the set of models as these are just particular permutations of the 15 regressors (for example, the Mark model includes relative prices, relative money supply and relative output).[5]

3.5. Basic results for Bayesian model averaging

Table 1 shows the relative out-of-sample relative mean square prediction error (RMSPE) of the forecasts obtained by BMA using the monthly dataset, relative to the forecasts assuming that the exchange rate is a driftless random walk, for various values of the hyperparameters φ and ρ. A number greater than 1 means

[5] The Mark fundamentals cannot however be included as one of the basic predictor variables because this would induce a problem of perfect multicollinearity.

Table 1
Out-of-sample RMSPE for Bayesian model averaging—Monthly financial data

Currency	Horizon	$\varphi = 20$			$\varphi = 5$			$\varphi = 1$			$\varphi = 0.5$		
		$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$
Direct forecasts													
Canadian $	3 months	1.012	1.006	1.003	1.006	1.004	1.003	0.995	0.998	0.999	0.997	0.999	0.999
	6 months	1.020	1.007	1.003	1.017	1.008	1.004	0.996	0.999	0.999	0.996	0.999	0.999
	9 months	1.128	1.036	1.008	1.091	1.035	1.010	0.992	0.994	0.996	0.991	0.995	0.998
	12 months	1.049	1.022	1.011	1.046	1.025	1.015	1.002	1.003	1.003	0.998	1.000	1.001
Mark/Euro	3 months	1.026	1.028	1.024	0.987	0.992	0.997	0.974	0.981	0.987	0.987	0.991	0.994
	6 months	1.072	1.056	1.036	1.012	1.014	1.010	0.972	0.980	0.986	0.984	0.989	0.993
	9 months	1.111	1.069	1.039	1.060	1.042	1.028	0.991	0.993	0.996	0.992	0.994	0.997
	12 months	1.255	1.135	1.101	1.152	1.070	1.053	0.985	0.988	0.996	0.986	0.992	0.996
Yen	3 months	0.991	0.989	0.989	0.981	0.982	0.985	0.983	0.987	0.991	0.991	0.994	0.996
	6 months	0.966	0.949	0.951	0.950	0.945	0.948	0.962	0.969	0.977	0.982	0.987	0.991
	9 months	1.004	0.978	0.982	0.995	0.973	0.974	0.973	0.976	0.983	0.984	0.988	0.992
	12 months	0.935	0.943	0.964	0.914	0.929	0.948	0.935	0.957	0.973	0.967	0.980	0.988
Pound	3 months	1.045	1.020	1.009	1.039	1.022	1.013	1.006	1.005	1.004	1.002	1.002	1.001
	6 months	1.094	1.028	1.013	1.094	1.037	1.018	1.012	1.009	1.006	1.003	1.003	1.002
	9 months	1.029	1.010	1.004	1.049	1.022	1.010	1.010	1.011	1.006	1.004	1.005	1.003
	12 months	1.444	1.097	1.027	1.322	1.133	1.045	1.029	1.020	1.011	1.005	1.006	1.004
Iterated multi-step forecasts													
Canadian $	3 months	1.004	1.002	1.003	0.988	0.991	0.995	0.984	0.991	0.995	0.992	0.996	0.998
	6 months	1.027	1.023	1.025	0.997	1.002	1.008	0.980	0.991	0.997	0.990	0.996	0.999
	9 months	1.022	1.021	1.027	0.985	0.995	1.005	0.970	0.986	0.995	0.985	0.993	0.997
	12 months	1.029	1.028	1.034	0.988	0.999	1.009	0.969	0.986	0.995	0.984	0.993	0.998
Mark/Euro	3 months	1.025	1.025	1.021	0.988	0.989	0.989	0.974	0.977	0.980	0.986	0.989	0.991
	6 months	1.102	1.100	1.088	1.027	1.027	1.024	0.975	0.979	0.982	0.985	0.988	0.991
	9 months	1.180	1.176	1.157	1.069	1.069	1.061	0.979	0.982	0.985	0.986	0.989	0.991
	12 months	1.226	1.219	1.196	1.100	1.097	1.087	0.987	0.989	0.990	0.989	0.991	0.993
Yen	3 months	0.968	0.967	0.968	0.961	0.961	0.962	0.975	0.976	0.977	0.988	0.988	0.989
	6 months	1.044	1.047	1.052	1.016	1.019	1.024	0.992	0.995	0.998	0.995	0.996	0.998
	9 months	1.006	1.019	1.032	0.977	0.987	0.999	0.972	0.979	0.984	0.985	0.989	0.991
	12 months	0.995	1.008	1.022	0.967	0.975	0.988	0.966	0.972	0.978	0.982	0.985	0.988
Pound	3 months	1.073	1.069	1.065	1.036	1.035	1.034	0.998	1.000	1.000	0.998	0.999	0.999
	6 months	1.128	1.114	1.107	1.080	1.071	1.068	1.010	1.013	1.013	1.003	1.004	1.004
	9 months	1.186	1.157	1.143	1.124	1.104	1.094	1.021	1.021	1.021	1.007	1.008	1.008
	12 months	1.255	1.208	1.181	1.180	1.143	1.124	1.036	1.032	1.029	1.012	1.012	1.011

Notes: This table reports the out-of-sample mean square prediction error from the forecasts taken by Bayesian Model Averaging, relative to the mean square prediction error from a driftless random walk forecast, using monthly financial data. A number less than 1 means that Bayesian Model Averaging predicts better than the random walk benchmark. The models used in the Bayesian Model Averaging procedure are described in the text.

that BMA is forecasting less well than a random walk. Results are shown for both direct and iterated multistep forecasts. For the pound, the out-of-sample RMSPE is almost uniformly above 1 indicating that the random walk gives better forecasts. But, for small values of φ, for the other three currencies, the RMSPE is generally slightly below 1 in the monthly dataset, indicating that BMA gives better forecasts. For larger values of φ, corresponding to less shrinkage, BMA typically underperforms the random walk. And, while BMA often performs better than the random walk with the smallest values of φ in the monthly data, the improvements are quite small. For example, in the case $\varphi = 1$ and $\rho = 0.2$, the improvement from using BMA is at most a 4% reduction in mean square prediction error. As to the comparison between the direct and iterated multistep forecasts, both appear to give similar performance and neither dominates the other in mean square prediction error.

The corresponding results for the quarterly dataset are shown in Table 2. The addition of the macro variables in the quarterly dataset in many cases improves the predictive performance of BMA. The relative performance of BMA is again poor for larger values of φ. But, with small values of φ, the BMA procedure gives better forecasts than the random walk for all currency–horizon pairs, using either the direct or iterated forecasting methods, except for the pound with either forecasting method at any horizon, and the Canadian dollar with the direct forecasting method at a 4-quarter horizon. The improvement is still modest. For example, with $\varphi = 1$ and $\rho = 0.2$, BMA represents at best a 5% improvement in mean square prediction error over the random walk forecast.

Bootstrap *p*-values of the hypothesis that the population RMSPE is equal to one in the monthly dataset are shown in Table 3. In each bootstrap sample, an artificial dataset is generated in which the exchange rate is by construction a driftless random walk, using the bootstrap methodology described in Appendix B. The *p*-values in Table 3 represent the proportion of bootstrap samples for which the RMSPE is smaller than that which was actually observed in the data. These are therefore one-sided *p*-values, testing the null of equal predictability against the alternative that BMA gives a significant improvement over the driftless random walk. Using the indirect forecasts, with small values of φ, the null hypothesis is rejected at the 10% significance level or better for the mark/euro, yen and Canadian dollar at the three-month horizon, and for the mark/euro it is rejected at the six-month horizon too. In several other cases, the bootstrap *p*-values are between 0.1 and 0.2 which is at least somewhat noteworthy, given the renowned difficulty of exchange rate forecasting. In the case $\varphi = 1$ and $\rho = 0.2$, the *p*-values using indirect forecasts are 0.22 or better at all horizons for all currencies except the pound. And in this case, the *p*-values using direct forecasts are 0.25 or better using direct forecasts for the mark/euro and yen. Even though the improvements in mean-square prediction error reported in Table 1 are modest, we see in Table 3 that they are still often statistically significant or at least close to being significant, using the bootstrap *p*-values. This is not surprising because overfitting typically tends to inflate out-of-sample mean square prediction errors, so if the exchange rate truly were a driftless random walk, then we would expect to obtain RMSPEs greater than one. I did not simulate bootstrap *p*-values for the quarterly dataset because of the computational cost with 2^{15} quarterly models.

Researchers are rightly suspicious of significant *p*-values in a test of the hypothesis that a particular model forecasts the exchange rate better than a random walk. The key reason is that these *p*-values ignore the data mining that was implicit in choosing the particular model to use. Researchers publish the results of these tests only if they find a model which forecasts the exchange rate significantly better than a random walk, and thus "significant" results can be expected to crop up from time to time even if the exchange rate is in fact totally unpredictable. But to the extent that the BMA approach is starting out with a set of models that spans the space of all models researchers would ever want to consider, the results and specifically the *p*-values in the forecast comparison test are then immune to any such data-mining critique. Clearly, the set of models considered here does not span the space of all conceivable models, but still BMA does mitigate the very legitimate concern about data mining.

BMA forecasts are not necessarily very different from random walk forecasts. The driftless random walk forecast is of course for no change in the exchange rate. Thus, the root mean square forecast exchange rate change in BMA gives a metric for how different this forecast is from a random walk forecast. I report this root mean square forecast exchange rate change in Tables 4 and 5 for the monthly and quarterly datasets, respectively. In general, the higher is φ and the longer is the forecast horizon, the larger is the magnitude of the forecast exchange rate changes. But, for $\rho = 0.2$ and $\varphi = 1$, where the BMA procedure outperforms the random walk for most currency–horizon pairs, the root mean square forecast exchange rate change at a one-year horizon is small: at most 2%. Since most economists believe that the exchange rate is very well approximated by a random walk, this is a reassuring feature of the BMA procedure.

One question of some interest is whether BMA predicts the *sign* of the exchange rate change correctly. For one thing, this metric is robust to the possibility of outliers that artificially improve or degrade predictive performance. Tables 6 and 7 report the proportion of times that it predicts the correct sign of the exchange rate change in the monthly and quarterly datasets, respectively. In nearly every case, BMA predicts the correct sign more than half the time for the Canadian dollar, mark/euro and yen. Typically, BMA predicts the correct sign of the returns less than half the time for the pound. For a few currency pair–horizon–hyperparameter combinations, the proportion of times that BMA predicts the sign of the exchange rate change correctly is significantly greater than 0.5. Cheung et al. (2002) also considered forecasting exchange rates over the 1990s, but not using Bayesian methods. They found that they were able to predict the direction of exchange rate changes more than half the time, but nonetheless typically underperformed the random walk in mean square error. These results are very much consistent with the idea that model and parameter uncertainty are the stumbling blocks to exchange rate forecasting (given that the exchange rate is so close to being a random walk), and that the researcher who wants to get good out-of-sample prediction, rather than in-sample fit, should use shrinkage methods, such as BMA.

3.6. *Forecasting at longer horizons*

In this paper, I have reported exchange rate forecasting at short to medium horizons, up to one year. These are the horizons considered by Meese and Rogoff (1983). Some more recent authors have found that exchange rates are more forecastable at longer horizons, although others like Kilian (1999) have argued convincingly that there is no evidence of higher predictability at longer horizons once the issues of statistical inference in long-horizon regressions are taken into account. In Tables 1 and 2, the performance of BMA relative to the random walk generally deteriorates slightly as the horizon increases from three months to one year. I have also experimented with using BMA to forecast exchange rates at horizons longer than one year, but found that its performance relative to the random walk degrades further at these longer horizons.

Table 2
Out-of-sample RMSPE for Bayesian model averaging—Quarterly data

Currency	Horizon	$\varphi = 20$			$\varphi = 5$			$\varphi = 1$			$\varphi = 0.5$		
		$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$
Direct forecasts													
Canadian $	1 quarter	1.040	1.014	1.001	0.975	0.967	0.973	0.951	0.968	0.979	0.976	0.986	0.991
	2 quarters	1.186	1.067	1.032	1.088	1.032	1.014	0.974	0.985	0.992	0.984	0.991	0.996
	3 quarters	1.097	1.063	1.045	1.041	1.020	1.020	0.974	0.984	0.992	0.984	0.990	0.995
	4 quarters	1.301	1.103	1.054	1.204	1.077	1.047	1.020	1.007	1.005	1.003	1.001	1.001
Mark/Euro	1 quarter	1.043	1.053	1.064	0.990	0.989	0.995	0.965	0.964	0.969	0.980	0.981	0.985
	2 quarters	1.185	1.165	1.143	1.082	1.051	1.044	0.976	0.969	0.976	0.982	0.983	0.988
	3 quarters	1.186	1.122	1.087	1.069	1.033	1.017	0.961	0.964	0.974	0.975	0.981	0.988
	4 quarters	1.530	0.984	0.988	1.379	0.973	0.965	0.992	0.966	0.978	0.984	0.984	0.990
Yen	1 quarter	1.102	1.085	1.083	1.029	1.019	1.019	0.974	0.977	0.981	0.984	0.987	0.990
	2 quarters	1.203	1.082	1.001	1.078	1.016	0.979	0.966	0.968	0.975	0.978	0.983	0.989
	3 quarters	2.452	1.444	1.030	1.752	1.233	1.000	0.985	0.961	0.971	0.972	0.978	0.986
	4 quarters	2.016	1.094	1.007	1.634	1.082	1.002	1.014	0.978	0.984	0.987	0.985	0.991
Pound	1 quarter	1.311	1.205	1.113	1.209	1.147	1.085	1.040	1.020	1.009	1.013	1.005	1.003
	2 quarters	1.809	1.245	1.046	1.639	1.268	1.078	1.132	1.054	1.024	1.045	1.020	1.010
	3 quarters	2.150	1.113	1.020	1.914	1.217	1.049	1.155	1.058	1.023	1.054	1.022	1.010
	4 quarters	2.142	1.498	1.103	1.589	1.327	1.127	1.096	1.049	1.025	1.033	1.017	1.010
Iterated multi-step forecasts													
Canadian $	1 quarter	1.040	1.014	1.001	0.975	0.967	0.973	0.951	0.968	0.979	0.976	0.986	0.991
	2 quarters	1.142	1.126	1.077	1.030	1.030	1.025	0.954	0.976	0.988	0.976	0.988	0.994
	3 quarters	1.163	1.133	1.050	1.024	1.018	0.999	0.932	0.961	0.976	0.965	0.982	0.990
	4 quarters	1.249	1.208	1.087	1.085	1.072	1.029	0.947	0.971	0.983	0.970	0.985	0.992
Mark/Euro	1 quarter	1.043	1.053	1.064	0.990	0.989	0.995	0.965	0.964	0.969	0.980	0.981	0.985
	2 quarters	1.107	1.110	1.121	1.023	1.015	1.019	0.963	0.963	0.967	0.978	0.980	0.985
	3 quarters	1.163	1.157	1.168	1.049	1.034	1.036	0.956	0.958	0.964	0.973	0.977	0.983
	4 quarters	1.241	1.207	1.208	1.107	1.069	1.061	0.966	0.965	0.969	0.976	0.980	0.985
Yen	1 quarter	1.102	1.085	1.083	1.029	1.019	1.019	0.974	0.977	0.981	0.984	0.987	0.990
	2 quarters	1.265	1.242	1.227	1.118	1.103	1.097	0.980	0.982	0.984	0.984	0.987	0.990
	3 quarters	1.404	1.398	1.371	1.161	1.159	1.150	0.954	0.960	0.965	0.967	0.974	0.981
	4 quarters	1.490	1.509	1.483	1.188	1.205	1.201	0.943	0.954	0.961	0.960	0.970	0.978
Pound	1 quarter	1.311	1.205	1.113	1.209	1.147	1.085	1.040	1.020	1.009	1.013	1.005	1.003
	2 quarters	1.663	1.454	1.258	1.458	1.328	1.194	1.097	1.053	1.027	1.033	1.017	1.009
	3 quarters	2.145	1.845	1.516	1.778	1.586	1.365	1.158	1.087	1.042	1.052	1.026	1.013
	4 quarters	2.550	2.189	1.753	2.043	1.812	1.526	1.210	1.122	1.060	1.069	1.036	1.019

Notes: As for Table 1, except that quarterly data are used.

Table 3
p-values in tests that BMA & random walk have equal out-of-sample RMSPE—monthly financial data

Currency	Horizon	$\varphi = 20$			$\varphi = 5$			$\varphi = 1$			$\varphi = 0.5$		
		$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$
Direct forecasts													
Canadian $	3 months	0.44	0.57	0.64	0.26	0.33	0.41	0.21	0.26	0.30	0.26	0.31	0.33
	6 months	0.44	0.46	0.45	0.33	0.35	0.38	0.27	0.31	0.32	0.30	0.34	0.36
	9 months	0.87	0.77	0.64	0.69	0.61	0.50	0.28	0.27	0.28	0.29	0.31	0.32
	12 months	0.51	0.61	0.65	0.41	0.49	0.56	0.32	0.36	0.39	0.35	0.38	0.39
Mark/Euro	3 months	0.69	0.89	0.95	0.10	0.11	0.13	0.08	0.07	0.07	0.10	0.11	0.12
	6 months	0.75	0.89	0.90	0.25	0.43	0.59	0.13	0.13	0.14	0.18	0.19	0.17
	9 months	0.79	0.88	0.90	0.45	0.63	0.75	0.24	0.25	0.26	0.27	0.28	0.28
	12 months	0.89	0.91	0.94	0.70	0.68	0.80	0.24	0.25	0.29	0.26	0.28	0.29
Yen	3 months	0.08	0.05	0.03	0.06	0.05	0.03	0.10	0.08	0.08	0.15	0.15	0.14
	6 months	0.06	0.02	0.01	0.05	0.02	0.02	0.08	0.07	0.07	0.17	0.15	0.14
	9 months	0.19	0.08	0.07	0.17	0.11	0.08	0.17	0.16	0.16	0.22	0.20	0.20
	12 months	0.06	0.05	0.05	0.06	0.06	0.06	0.09	0.12	0.13	0.17	0.17	0.19
Pound	3 months	0.89	0.86	0.86	0.80	0.83	0.85	0.39	0.52	0.60	0.37	0.43	0.47
	6 months	0.90	0.82	0.81	0.87	0.81	0.80	0.44	0.49	0.55	0.39	0.45	0.47
	9 months	0.41	0.46	0.48	0.46	0.47	0.47	0.37	0.46	0.48	0.39	0.44	0.46
	12 months	0.99	0.89	0.80	0.97	0.92	0.82	0.50	0.51	0.55	0.40	0.43	0.45
Iterated multi-step forecasts													
Canadian $	3 months	0.27	0.32	0.69	0.11	0.11	0.11	0.04	0.07	0.09	0.04	0.09	0.11
	6 months	0.57	0.84	0.94	0.23	0.29	0.64	0.10	0.16	0.24	0.10	0.18	0.25
	9 months	0.43	0.70	0.90	0.19	0.23	0.40	0.09	0.14	0.21	0.09	0.16	0.23
	12 months	0.43	0.73	0.90	0.22	0.27	0.50	0.13	0.19	0.26	0.12	0.21	0.27
Mark/Euro	3 months	0.73	0.89	0.96	0.12	0.10	0.06	0.03	0.02	0.01	0.02	0.02	0.00
	6 months	0.92	0.98	0.99	0.46	0.73	0.86	0.09	0.07	0.05	0.07	0.06	0.04
	9 months	0.95	0.99	1.00	0.70	0.89	0.95	0.15	0.14	0.09	0.11	0.10	0.08
	12 months	0.95	0.99	1.00	0.74	0.91	0.96	0.22	0.20	0.17	0.18	0.17	0.14
Yen	3 months	0.02	0.01	0.00	0.02	0.01	0.00	0.01	0.01	0.00	0.01	0.01	0.00
	6 months	0.69	0.91	0.98	0.31	0.62	0.87	0.22	0.22	0.23	0.20	0.18	0.17
	9 months	0.19	0.67	0.90	0.13	0.15	0.23	0.10	0.10	0.07	0.10	0.08	0.06
	12 months	0.15	0.36	0.81	0.12	0.13	0.14	0.11	0.09	0.06	0.13	0.09	0.06
Pound	3 months	0.98	0.99	1.00	0.89	0.98	0.99	0.31	0.36	0.37	0.22	0.23	0.18
	6 months	0.97	0.98	1.00	0.93	0.98	0.98	0.66	0.90	0.98	0.57	0.83	0.97
	9 months	0.97	0.98	0.99	0.95	0.97	0.98	0.77	0.93	0.98	0.69	0.90	0.97
	12 months	0.97	0.98	0.99	0.96	0.98	0.98	0.86	0.96	0.98	0.82	0.93	0.98

Notes: This table reports the bootstrap p-values for a one-sided test of the hypothesis that the driftless random walk and Bayesian Model Averaging forecasts have equal out-of-sample mean square prediction error. Specifically the entries are the fraction of bootstrap samples in which the RMSPE is below the sample value as reported in Table 1. The bootstrap methodology is described in Appendix B.

Table 4
Root mean square forecast of exchange rate change with Bayesian model averaging—Monthly financial data

Currency	Horizon	$\varphi = 20$			$\varphi = 5$			$\varphi = 1$			$\varphi = 0.5$		
		$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$
Direct forecasts													
Canadian $	3 months	0.37	0.18	0.09	0.39	0.24	0.14	0.16	0.10	0.06	0.07	0.04	0.03
	6 months	0.69	0.34	0.17	0.73	0.46	0.27	0.30	0.20	0.12	0.13	0.09	0.05
	9 months	2.12	1.21	0.64	1.93	1.30	0.81	0.69	0.45	0.28	0.30	0.18	0.11
	12 months	1.12	0.50	0.24	1.32	0.76	0.42	0.56	0.36	0.22	0.25	0.16	0.10
Mark/Euro	3 months	2.05	1.87	1.61	1.56	1.43	1.27	0.51	0.42	0.34	0.21	0.16	0.11
	6 months	3.32	2.75	2.14	2.62	2.30	1.92	0.88	0.71	0.53	0.37	0.27	0.18
	9 months	3.72	2.62	1.70	3.16	2.47	1.80	1.12	0.82	0.55	0.48	0.33	0.20
	12 months	7.02	4.93	3.48	5.84	4.26	3.15	1.82	1.21	0.80	0.71	0.44	0.27
Yen	3 months	1.40	1.03	0.69	1.18	0.97	0.74	0.42	0.34	0.25	0.18	0.14	0.10
	6 months	3.16	2.41	1.64	2.56	2.17	1.68	0.87	0.70	0.51	0.37	0.28	0.19
	9 months	3.10	1.81	1.00	2.92	2.09	1.37	1.10	0.83	0.56	0.49	0.36	0.24
	12 months	4.29	2.46	1.34	4.05	2.85	1.83	1.49	1.09	0.74	0.65	0.47	0.31
Pound	3 months	0.77	0.30	0.13	0.84	0.43	0.21	0.29	0.17	0.09	0.12	0.07	0.04
	6 months	1.83	0.68	0.30	1.94	0.96	0.47	0.63	0.34	0.18	0.25	0.14	0.07
	9 months	0.81	0.19	0.07	1.62	0.43	0.18	0.63	0.26	0.13	0.26	0.12	0.06
	12 months	5.65	2.13	0.77	4.99	2.76	1.22	1.50	0.83	0.43	0.58	0.32	0.16
Iterated multi-step forecasts													
Canadian $	3 months	0.73	0.54	0.43	0.62	0.47	0.37	0.21	0.15	0.11	0.09	0.06	0.04
	6 months	1.33	0.96	0.73	1.13	0.84	0.64	0.39	0.28	0.20	0.16	0.11	0.08
	9 months	1.87	1.35	1.03	1.60	1.18	0.90	0.56	0.39	0.29	0.23	0.16	0.11
	12 months	2.37	1.72	1.32	2.03	1.49	1.15	0.71	0.50	0.37	0.30	0.20	0.14
Mark/Euro	3 months	2.07	2.00	1.90	1.58	1.51	1.42	0.53	0.47	0.41	0.22	0.18	0.15
	6 months	3.80	3.64	3.41	2.88	2.74	2.54	0.95	0.82	0.67	0.39	0.30	0.23
	9 months	5.29	5.08	4.74	4.00	3.81	3.52	1.31	1.12	0.90	0.53	0.41	0.30
	12 months	6.59	6.32	5.89	4.97	4.73	4.37	1.62	1.38	1.10	0.65	0.50	0.36
Yen	3 months	1.61	1.53	1.51	1.27	1.19	1.16	0.45	0.42	0.41	0.20	0.18	0.18
	6 months	2.46	2.28	2.22	1.97	1.78	1.71	0.71	0.64	0.61	0.31	0.28	0.26
	9 months	3.35	3.10	3.02	2.69	2.43	2.33	0.97	0.87	0.84	0.42	0.38	0.36
	12 months	4.25	3.97	3.89	3.41	3.10	3.00	1.23	1.11	1.07	0.54	0.49	0.47
Pound	3 months	1.53	1.41	1.34	1.22	1.09	1.03	0.42	0.37	0.35	0.18	0.16	0.15
	6 months	2.26	1.89	1.69	1.88	1.50	1.31	0.64	0.49	0.42	0.26	0.20	0.18
	9 months	2.91	2.33	2.01	2.46	1.87	1.57	0.83	0.60	0.50	0.34	0.24	0.21
	12 months	3.50	2.72	2.31	2.99	2.21	1.81	1.01	0.70	0.57	0.41	0.28	0.24

Notes: This table reports the root mean square forecast of exchange rate changes from Bayesian Model Averaging using monthly financial data. The exchange rate was transformed by taking logs and then multiplying by 100, so the elements in this table can be interpreted as approximate percentage point forecast changes.

Table 5
Root mean square forecast of exchange rate change with Bayesian model averaging—Quarterly data

Currency	Horizon	$\varphi = 20$			$\varphi = 5$			$\varphi = 1$			$\varphi = 0.5$		
		$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$
Direct forecasts													
Canadian $	1 quarter	1.28	1.06	0.78	1.00	0.83	0.61	0.32	0.22	0.16	0.12	0.08	0.06
	2 quarters	2.06	1.12	0.69	1.74	1.02	0.67	0.52	0.31	0.21	0.20	0.12	0.08
	3 quarters	2.14	1.57	1.16	1.83	1.34	1.05	0.64	0.45	0.33	0.28	0.19	0.13
	4 quarters	2.97	1.65	1.03	2.63	1.54	1.07	0.88	0.52	0.35	0.37	0.22	0.14
Mark/Euro	1 quarter	2.15	2.21	2.20	1.61	1.64	1.63	0.57	0.53	0.48	0.25	0.21	0.17
	2 quarters	4.60	4.44	4.02	3.47	3.32	3.05	1.18	0.99	0.78	0.50	0.38	0.26
	3 quarters	6.14	5.29	4.29	4.83	4.19	3.48	1.70	1.32	0.94	0.73	0.51	0.32
	4 quarters	8.49	3.17	1.57	7.46	3.27	1.89	2.19	1.06	0.62	0.88	0.42	0.25
Yen	1 quarter	2.71	2.55	2.39	2.08	1.94	1.79	0.67	0.55	0.44	0.27	0.21	0.16
	2 quarters	4.80	3.57	2.28	3.68	2.80	2.03	1.08	0.79	0.60	0.44	0.32	0.24
	3 quarters	11.25	6.44	2.48	8.36	5.25	2.66	2.11	1.28	0.85	0.80	0.52	0.36
	4 quarters	10.13	3.51	1.52	7.85	3.79	2.15	2.01	1.29	0.92	0.82	0.56	0.41
Pound	1 quarter	1.55	1.11	0.77	1.21	0.93	0.67	0.37	0.27	0.19	0.15	0.10	0.07
	2 quarters	3.55	1.56	0.39	3.03	1.60	0.60	0.86	0.42	0.23	0.33	0.17	0.09
	3 quarters	6.03	0.78	0.17	5.42	1.45	0.38	1.35	0.46	0.20	0.50	0.19	0.09
	4 quarters	7.93	4.66	1.81	5.20	3.60	2.18	1.32	1.00	0.65	0.54	0.40	0.24
Iterated multi-step forecasts													
Canadian $	1 quarter	1.28	1.06	0.78	1.00	0.83	0.61	0.32	0.22	0.16	0.12	0.08	0.06
	2 quarters	2.32	1.92	1.29	1.83	1.51	1.03	0.60	0.39	0.27	0.23	0.15	0.10
	3 quarters	3.31	2.75	1.78	2.63	2.17	1.44	0.87	0.55	0.37	0.34	0.21	0.14
	4 quarters	4.26	3.57	2.26	3.41	2.82	1.83	1.13	0.71	0.46	0.44	0.27	0.18
Mark/Euro	1 quarter	2.15	2.21	2.20	1.61	1.64	1.63	0.57	0.53	0.48	0.25	0.21	0.17
	2 quarters	4.08	4.17	4.16	3.08	3.10	3.08	1.08	1.00	0.90	0.47	0.40	0.31
	3 quarters	5.88	5.94	5.91	4.49	4.44	4.39	1.59	1.44	1.28	0.69	0.57	0.44
	4 quarters	7.60	7.53	7.49	5.91	5.66	5.56	2.12	1.84	1.62	0.92	0.73	0.56
Yen	1 quarter	2.71	2.55	2.39	2.08	1.94	1.79	0.67	0.55	0.44	0.27	0.21	0.16
	2 quarters	5.25	4.94	4.60	4.02	3.74	3.43	1.29	1.05	0.83	0.52	0.39	0.30
	3 quarters	7.68	7.22	6.71	5.87	5.46	4.99	1.87	1.52	1.19	0.75	0.57	0.44
	4 quarters	9.91	9.33	8.68	7.56	7.04	6.44	2.41	1.96	1.54	0.97	0.73	0.56
Pound	1 quarter	1.55	1.11	0.77	1.21	0.93	0.67	0.37	0.27	0.19	0.15	0.10	0.07
	2 quarters	3.28	2.50	1.80	2.54	2.00	1.45	0.76	0.50	0.33	0.29	0.18	0.12
	3 quarters	5.39	4.40	3.34	4.16	3.42	2.56	1.23	0.77	0.47	0.45	0.26	0.16
	4 quarters	7.27	6.03	4.52	5.59	4.63	3.42	1.63	1.00	0.58	0.58	0.32	0.20

Notes: As for Table 4, except that quarterly data are used.

Table 6
Proportion of times BMA predicts correct sign of exchange rate change—Monthly financial data

Currency	Horizon	$\varphi = 20$			$\varphi = 5$			$\varphi = 1$			$\varphi = 0.5$		
		$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$
Direct forecasts													
Canadian $	3 months	0.54	0.54	0.54	0.53	0.53	0.54	0.53	0.54	0.54	0.53	0.54	0.54
	6 months	0.53	0.53	0.53	0.53	0.53	0.53	0.55	0.54	0.54	0.55	0.55	0.55
	9 months	0.53	0.54	0.55	0.54	0.54	0.55	0.54	0.53	0.55	0.54	0.55	0.54
	12 months	0.56	0.56	0.56	0.58	0.56	0.56	0.57	0.56	0.55	0.57	0.56	0.55
Mark/Euro	3 months	0.58	0.57	0.54	0.59	0.59	0.57	0.58	0.58	0.58	0.56	0.57	0.57
	6 months	0.59	0.60	0.60	0.60	0.61	0.62	0.63	0.64	0.64	0.63	0.64	0.63
	9 months	0.59	0.59	0.59	0.58	0.59	0.59	0.56	0.58	0.56	0.57	0.55	0.53
	12 months	0.53	0.55	0.56	0.53	0.57	0.57	0.54	0.58	0.60	0.54	0.60	0.60
Yen	3 months	0.59	0.54	0.52	0.60	0.53	0.52	0.60	0.54	0.51	0.59	0.52	0.48
	6 months	0.57	0.56	0.55	0.58	0.56	0.55	0.58	0.56	0.55	0.58	0.55	0.52
	9 months	0.60	0.59	0.59	0.63	0.59	0.59	0.62	0.58	0.57	0.60	0.57	0.55
	12 months	0.60	0.57	0.58	0.61	0.57	0.57	0.61	0.57	0.57	0.61	0.56	0.58
Pound	3 months	0.50	0.49	0.49	0.52	0.49	0.49	0.51	0.51	0.50	0.52	0.51	0.49
	6 months	0.48	0.48	0.48	0.48	0.48	0.47	0.48	0.49	0.49	0.48	0.49	0.50
	9 months	0.48	0.46	0.45	0.48	0.46	0.45	0.48	0.46	0.45	0.47	0.47	0.45
	12 months	0.46	0.45	0.45	0.48	0.45	0.45	0.46	0.44	0.44	0.47	0.44	0.44
Iterated multi-step forecasts													
Canadian $	3 months	0.57	0.58	0.58	0.58	0.58	0.58	0.59	0.58	0.58	0.58	0.58	0.56
	6 months	0.59	0.60	0.59	0.59	0.60	0.59	0.60	0.59	0.60	0.60	0.59	0.60
	9 months	0.59	0.59	0.58	0.59	0.59	0.58	0.60	0.59	0.58	0.60	0.58	0.58
	12 months	0.58	0.58	0.58	0.58	0.58	0.57	0.60	0.57	0.58	0.59	0.57	0.57
Mark/Euro	3 months	0.57	0.55	0.55	0.59	0.56	0.57	0.59	0.61	0.60	0.59	0.60	0.57
	6 months	0.57	0.56	0.54	0.58	0.57	0.55	0.56	0.57	0.58	0.58	0.59	0.57
	9 months	0.54	0.54	0.52	0.56	0.54	0.54	0.55	0.55	0.55	0.56	0.56	0.54
	12 months	0.50	0.50	0.49	0.51	0.51	0.50	0.51	0.53	0.55	0.54	0.56	0.56
Yen	3 months	0.58	0.56	0.56	0.58	0.58	0.56	0.58	0.58	0.56	0.58	0.58	0.56
	6 months	0.52	0.48	0.48	0.55	0.51	0.48	0.55	0.51	0.48	0.54	0.50	0.48
	9 months	0.62	0.59	0.59	0.62	0.59	0.58	0.62	0.59	0.57	0.61	0.58	0.56
	12 months	0.61	0.58	0.58	0.61	0.59	0.57	0.61	0.58	0.56	0.61	0.58	0.57
Pound	3 months	0.48	0.49	0.48	0.48	0.49	0.48	0.48	0.47	0.48	0.48	0.47	0.46
	6 months	0.52	0.53	0.47	0.52	0.53	0.49	0.52	0.52	0.46	0.52	0.51	0.48
	9 months	0.49	0.47	0.45	0.51	0.47	0.46	0.49	0.47	0.43	0.48	0.45	0.43
	12 months	0.47	0.45	0.44	0.46	0.45	0.45	0.45	0.45	0.43	0.46	0.45	0.44

Notes: Asymptotic standard errors can be obtained from the formula for the variance of a binomial distribution, adjusting for the overlapping forecasts. The standard errors so obtained vary, but are approximately 0.06, 0.09, 0.11 and 0.13 at horizons of 3, 6, 9 and 12 months ahead, respectively.

Table 7
Proportion of times Bayesian model averaging predicts correct sign of exchange rate change—Quarterly data

Currency	Horizon	$\varphi = 20$			$\varphi = 5$			$\varphi = 1$			$\varphi = 0.5$		
		$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.2$	$\rho = 0.1$
Direct forecasts													
Canadian $	1 quarter	0.63	0.63	0.64	0.64	0.61	0.59	0.64	0.63	0.59	0.63	0.64	0.63
	2 quarters	0.58	0.64	0.63	0.63	0.63	0.63	0.64	0.64	0.64	0.64	0.66	0.63
	3 quarters	0.67	0.61	0.64	0.66	0.64	0.63	0.67	0.64	0.64	0.66	0.64	0.64
	4 quarters	0.58	0.56	0.59	0.59	0.56	0.59	0.59	0.58	0.59	0.61	0.61	0.58
Mark/Euro	1 quarter	0.55	0.55	0.55	0.55	0.55	0.55	0.56	0.55	0.56	0.58	0.55	0.58
	2 quarters	0.56	0.56	0.56	0.53	0.56	0.56	0.56	0.56	0.58	0.58	0.59	0.59
	3 quarters	0.56	0.58	0.64	0.55	0.58	0.64	0.56	0.61	0.64	0.56	0.63	0.64
	4 quarters	0.52	0.55	0.55	0.48	0.55	0.56	0.52	0.53	0.59	0.52	0.53	0.56
Yen	1 quarter	0.64	0.64	0.64	0.64	0.64	0.64	0.63	0.64	0.59	0.63	0.59	0.58
	2 quarters	0.61	0.61	0.58	0.63	0.61	0.59	0.61	0.59	0.58	0.61	0.59	0.58
	3 quarters	0.67	0.59	0.59	0.67	0.59	0.59	0.63	0.59	0.61	0.61	0.59	0.55
	4 quarters	0.58	0.53	0.55	0.59	0.55	0.55	0.59	0.58	0.55	0.59	0.56	0.55
Pound	1 quarter	0.53	0.56	0.59	0.53	0.56	0.58	0.55	0.59	0.61	0.58	0.58	0.56
	2 quarters	0.39	0.44	0.45	0.38	0.42	0.45	0.41	0.45	0.45	0.41	0.45	0.45
	3 quarters	0.45	0.41	0.42	0.45	0.44	0.42	0.41	0.45	0.42	0.42	0.45	0.44
	4 quarters	0.42	0.44	0.47	0.39	0.45	0.47	0.42	0.48	0.50	0.44	0.50	0.50
Iterated multi-step forecasts													
Canadian $	1 quarter	0.63	0.63	0.64	0.64	0.61	0.59	0.64	0.63	0.59	0.63	0.64	0.63
	2 quarters	0.61	0.61	0.59	0.59	0.59	0.63	0.61	0.64	0.63	0.66	0.64	0.63
	3 quarters	0.69	0.67	0.64	0.69	0.67	0.66	0.69	0.69	0.67	0.69	0.69	0.67
	4 quarters	0.67	0.66	0.61	0.67	0.67	0.64	0.67	0.67	0.64	0.67	0.67	0.66
Mark/Euro	1 quarter	0.55	0.55	0.55	0.55	0.55	0.55	0.56	0.55	0.56	0.58	0.55	0.58
	2 quarters	0.55	0.58	0.58	0.55	0.56	0.55	0.56	0.56	0.59	0.59	0.59	0.58
	3 quarters	0.50	0.50	0.48	0.52	0.53	0.48	0.55	0.52	0.53	0.55	0.55	0.55
	4 quarters	0.50	0.48	0.47	0.50	0.53	0.47	0.56	0.52	0.55	0.58	0.55	0.56
Yen	1 quarter	0.64	0.64	0.64	0.64	0.64	0.64	0.63	0.64	0.59	0.63	0.59	0.58
	2 quarters	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.58
	3 quarters	0.69	0.66	0.66	0.69	0.66	0.66	0.69	0.69	0.69	0.69	0.70	0.67
	4 quarters	0.69	0.66	0.64	0.69	0.66	0.64	0.69	0.67	0.69	0.69	0.72	0.67
Pound	1 quarter	0.53	0.56	0.59	0.53	0.56	0.58	0.55	0.59	0.61	0.58	0.58	0.56
	2 quarters	0.44	0.48	0.55	0.42	0.42	0.55	0.42	0.53	0.55	0.45	0.55	0.56
	3 quarters	0.41	0.41	0.52	0.41	0.39	0.52	0.41	0.47	0.52	0.42	0.47	0.52
	4 quarters	0.38	0.38	0.47	0.38	0.36	0.47	0.38	0.42	0.45	0.38	0.42	0.42

Notes: Asymptotic standard errors can be obtained from the formula for the variance of a binomial distribution, adjusting for the overlapping forecasts. The standard errors so obtained vary, but are approximately 0.06, 0.09, 0.11 and 0.13 at horizons of 1, 2, 3 and 4 quarters ahead, respectively.

Table 8
Out-of-sample RMSPE for equal-weighted averaged forecasts

Horizon	Canadian $	Mark/Euro	Yen	Pound
Direct forecasts—monthly financial data				
3 months	1.010	0.969	0.994	1.024
6 months	1.030	0.960	1.006	1.053
9 months	1.055	0.975	1.012	1.077
12 months	1.075	0.983	1.036	1.097
Direct forecasts—quarterly data				
1 quarter	0.989	0.969	0.999	1.026
2 quarter	1.014	0.975	1.016	1.068
3 quarter	1.034	0.986	1.032	1.092
4 quarter	1.068	0.998	1.081	1.111
Iterated multi-step forecasts—monthly financial data				
3 months	1.006	0.970	0.992	1.021
6 months	1.021	0.973	1.020	1.051
9 months	1.029	0.973	1.017	1.082
12 months	1.042	0.983	1.033	1.113
Iterated multi-step forecasts—quarterly data				
1 quarter	0.989	0.969	0.999	1.026
2 quarter	1.009	0.980	1.024	1.065
3 quarter	1.015	0.984	1.032	1.097
4 quarter	1.035	0.999	1.064	1.128

Notes: This table reports the out-of-sample relative mean square prediction error from the forecasts taken by simple equal-weighted averaging of different model forecasts, relative to the mean square prediction error from a driftless random walk forecast. The models each have a constant and just one other predictor, as described in the text.

3.7. Comparison with equal weighted model averaging

The efficacy of BMA is conceptually related to the idea that is very much part of the folklore of the forecasting literature that taking forecasts from several different models and simply averaging them gives better predictions than any one model on its own (Bates and Granger, 1969; Stock and Watson, 2001, 2004).

I therefore experimented with taking all regression models of the form of Eq. (6) with a single right hand side predictor, plus a constant, using all the predictors that I have in the monthly and quarterly datasets, augmented with the monthly long-term interest rate and the quarterly monetary fundamentals of Mark (1995). These last two predictors could not be included in the BMA exercise using all possible permutations of regressors because this would have generated a problem of perfect multicollinearity. This gives a total of 8 models in the monthly dataset and 17 in the quarterly dataset. Needless to say, no *one* of these models gives consistently good forecasting performance. The out-of-sample relative mean square prediction error obtained from simply averaging all of these forecasts, relative to the driftless random walk benchmark, is reported in Table 8. Results from the same exercise using Eqs. (9) and (10) are also reported. A number greater than 1 means that equal-weighted model-averaging is forecasting less well than a random walk. Except for the mark/euro, most entries in the tables are greater than 1. Simple equal-weighted model averaging, that is such an effective strategy in many forecasting contexts, does not seem to buy us so much in exchange rate forecasting, at least not with these models.

4. Conclusion

In this paper I have considered a specific approach to pooling the forecasts from different models, namely Bayesian Model Averaging, and argued that it gives promising results for out-of-sample exchange rate prediction. For most currency–horizon pairs, with a sufficiently high degree of shrinkage, the Bayesian Model Averaging forecasts yield mean square prediction errors modestly lower than one obtains from a random walk forecast. Though the improvements are modest, this nonetheless stands in marked contrast to the results obtained in the exchange rate forecasting literature at horizons of one year or less without using shrinkage methods. The forecasts generated by Bayesian model averaging are very close to those from a random walk forecast.

The sole focus in this paper has been on establishing whether Bayesian Model Averaging methods seem to work in the sense of beating random walk forecasts out-of-sample. One does not have to believe that the model and parameter priors used in this paper are genuine subjective prior beliefs to view Bayesian Model Averaging as practically useful. Moreover, viewing Bayesian Model Averaging as practically useful does not rule out the possibility that other related shrinkage techniques are useful too.

In future work, it would be possible and interesting to include nonlinear models, notably Markov switching and threshold models in the model averaging exercise. Such models have been considered by Engel and Hamilton (1990), Meese and Rose (1991) and Kilian and Taylor (2003) among others. These models may contain information that could make them useful as elements of a forecast pooling exercise such as Bayesian Model Averaging.

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Appendix A. Data sources

Exchange Rates: Monthly average of daily rates, Federal Reserve H-10 Release.

Long Term Interest Rates: 10 year rates from MEI and IMF International Financial Statistics (IFS).

Short Term Interest Rates: 3-month rates, except call money rate for Japan, all from MEI.

Prices*: CPI from MEI. Seasonally adjusted.

GDP*: MEI (real, sa).

LaborProductivity*: GDP (real, sa) divided by total civilian employment (sa), both from MEI.

MoneyStock*: M1 from MEI (sa) except M4 for the United Kingdom.

Monetaryfundamentals*: Computed from money stock, GDP and prices.

Stock Prices: Morgan Stanley Capital International (MSCI) price indices (local currency).

Dividend Yields: MSCI.

CurrentAccount*: MEI (sa).

CurrentAccount/GDPratio*: Current Account (sa) divided by GDP (nominal, sa). both from MEI.

*: included in quarterly dataset only.

Appendix B. Construction of bootstrap samples

To construct bootstrap samples in the monthly dataset, I fitted a VAR(12) to the exchange rate, log relative stock prices, the relative dividend yield, relative short term interest rates, the relative term spread. I estimated this 5-variable VAR but anchored the bootstrap at the bias-adjusted estimates of Kilian (1998), not the OLS estimates. I also imposed that all the coefficients in the exchange rate equation were equal to zero, except for the coefficient on the first lag of the exchange rate which I set to one. So the exchange rate is a driftless random walk by construction (the null hypothesis of no forecastability is imposed). I then generated 500 bootstrap samples of all of the variables in this VAR. All of the predictors in the monthly dataset can be constructed from these 6 variables. So in this way I get a bootstrap sample of the exchange rate and all of the predictors in which the exchange rate is a random walk (but may affect future values of the predictors and so is not strictly exogenous).

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