# Introduction Numerical Methods

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#### Some Info

- ► This course: Provides techniques for the analysis and evaluation of dynamic economic models, e.g. DSGE, firm and industry models
- ► The main emphasis is on learning the computational tools and their implementation
- ► Target group: Students who intend to do quantitative research in macro, applied micro or structural econometrics
- Requires you to use standard computer programming languages (such as Matlab, C or Fortran)
- ▶ Background: Dynamic programming / Recursive macro

# Why Study Numerical Methods?

- "Paper-and-pencil" has its limits
- Often closed form solutions to models are hard or impossible to obtain
- Increasing computational power allows to study complex models
- Techniques also useful for fields that require structural modeling (and estimation), e.g. applied microeconomics
- Macro:
  - Dynamic general equilibrium models: Keystone of modern macroeconomics
  - Provides powerful tool for evaluating macroeconomic theories, macroeconomic policy, welfare
  - "Quantitative": Provides ways in which to evaluate the empirical plausibility of the theory
  - Need to have the tools to analyze and compute dynamic economic models
- ► Drawbacks: Only approximate solution, no theorems/proofs, plenty of room for human error ...

### What About Knowledge of Computers?

- ► In order to implement the techniques we're gonna cover, you need to learn (at least) one programming language
- Interpreted vs compiled languages
  - ▶ Interpreted: Matlab, Mathematica, Python, etc.
  - Compiled: Fortran, C, etc.
- Comparison:
  - Compiled is much faster and easier to spot errors before the code is run
    - ► Check out this comparison: Table 1
  - Interpreted is much more flexible
  - Compiled is much more verbose
- ▶ Basic principle: minimize *total* time

### Getting Things Ready on YOUR Computer

- Interpreted languages:
  - Usually not much to do here; just install them
  - Note: Octave is a software that is essentially compatible with Matlab. Plus, it's free
- Compiled languages:
  - Usually easier to set up on Unix based machines (Mac or Linux)
  - You first need a compiler in order to run things
  - Intel Fortran is fast (and expensive)
  - GNU Fortran (gfortran) is free
  - ▶ Both Intel and the GNU project (gcc) have C compilers
  - Most economists prefer Fortran
  - Good code editor
    - Mac: gedit, Xcode
    - ► Windows: gedit, Notepad++
  - IDE vs command line
    - Open source IDE: Code::Blocks

#### Some Helpful Material

- We're not gonna follow any book in particular, but you may find the following references helpful:
  - Adda and Cooper, Dynamic Economics (Amazon link)
  - Judd, Numerical Methods in Economics (Amazon link)
  - Marimon and Scott, Computational Methods for the Study of Dynamic Economies (Amazon link)
  - Sauer, Numerical Analysis (Amazon link)
- Programming languages:
  - McConnell, Code Complete (Amazon link)
  - Gilat, Matlab: An Introduction (Amazon link)
  - Ellis, Philips and Lahey, Fortran 90 (Amazon link)
  - ► Fortran tutorials: Link here or Grey's slides

# Some Helpful Material (cont.)

- Some economists with helpful stuff online:
  - Grey Gordon: Link
  - Karen Kopecky: Link
  - Makoto Nakajima: Link
- ► Tony Smith's tips: Link
- ► Alan Miller's Fortran software: Link

## Our Running Example for a Good Chunk of the Course

- Standard RBC model
- Representative agent with preferences

$$E\sum_{t=0}^{\infty}\beta^t u(c_t)$$

- ▶ Representative firm with CRS technology subjective to multiplicative TFP shocks:  $Y_t = z_t F(K_t, L_t)$
- ▶ No government
- Competitive equilibrium
- ▶ By the welfare theorems, we'll work with the planner's problem

## Our Example: Recursive Formulation

- Since there's no utility for leisure, let's forget about it and define f(k) = F(k, 1)
- ▶ The (basic) problem we'll be trying to solve can be written as:

$$V(k,z) = \max_{c,k'} u(c) + E\beta V(k',z')$$
s.t.
$$c + k' = zf(k) + (1 - \delta)k$$

Question: how to solve this problem?

#### Different Methods

- We'll discuss several different methods to solve the problem above (and, obviously, a lot more)
- Each method may be more suited for certain types of problems
- Some methods:
  - Value function iteration
  - Projection methods (spectral, finite elements)
  - Perturbation methods (e.g. linearization)
- We'll also need to learn several helpful things:
  - root-finding, interpolation, discretization of stochastic processes, etc.
- We'll also discuss some techniques for some potentially more complicated models:
  - Computing stationary distributions, idiosyncratic and aggregate risk, OLG, calibration, simulation-based estimation, etc.