

Abstract

United States population has increasingly become older. The aging is expected to continue as life expectancy increases, more baby boomers reach retirement age and birth rate remain at today's levels. The demographic changes has impacted labor participation rates as well as the solvency of the Social Security Trust Fund. Social Security Administration projects the fund's insolvency around 2035. Given the "imminent" insolvency, policies such as increasing eligibility age and expanding the tax base have been proposed. These policies are usually analyzed using many period OLG model with a hump-shaped age-dependent productivity profile. We believe this profile is dynamic as the population itself. We incorporate the dynamic age-dependent profile in an OLG framework in a closer look at insolvency of the Social Security Trust Fund.

Keywords: OLG.

JEL Classification:

Sample

1 Introduction

The aging population has hastened the insolvency of Social Security Trust. The downward trend in birth rate, upward trend in life expectancy, and the retirement of the baby boomers have contributed to the population aging. A population that is “older” implies more benefit eligible agents, which increases the expenditures of the fund. The aging population has also changed the labor patterns. The percent of 18-24 employed has been declining while the percent of 65 and over has been increasing. Since Social Security is funded through wage receipts, labor trends will impact Social Security Trust.

We believe the current labor trends can be attributable to increase in life expectancy and the number of college graduates. Social Security replaces only portion of before retirement income. As agents live longer, it is intuitive that agents will work longer to have enough savings to maintain their consumption needs to their expected death. And as production becomes more skill oriented the less educated/experienced workers will earn less. An optimizing agent will therefore substitute labor between older (more productive) and younger years. In this paper we will try to address the implications of the these labor patterns to the solvency of Social Security Trust Fund.

According to the recent projections by Social Security Administration, the insolvency of the fund is likely to occur between 2030 and 2035 (Figure 1). We believe these estimates do not take into account of the labor patterns above. Ceteris paribus the labor patterns will ensure the delay of the insolvency. If older workers never retire then regardless of the composition of the workforce, the Social Security Trust will be sustainable since these workers are not exempt from Social Security tax. And as the older cohorts become more educated than the cohorts before them the employment of 65 and over will continue to increase. However as emphasis on skill and experience increases, the younger cohorts will reduce labor hours for leisure or more schooling, if we assume wages are proportional productivity. We will show that the substitution of labor hours between old (65+) and young (18-24) still delays the insolvency in our analysis.

The layout of the paper is as follow. We will provide evidence that these observed labor patterns can be well explained by the rise in the education. Then using the current estimates of college graduates and future projections of life expectancy we will project

dynamic efficiency weight profile far into the future. These projections will be used to model the insolvency of the Social Security Trust in an OLG framework. Another unique feature in our model is the agent's internalization of the insolvency of Social Security into their choices. As the fund's surplus depletes, agents should expect their benefits to be reduced without any policy interventions. Finally we will compare some policy proposals with respect to the solvency of the fund and welfare.

Literature Review

Results Summary

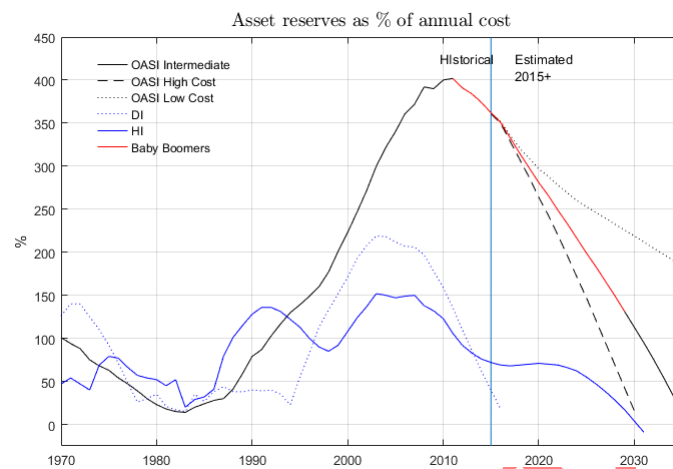


Figure 1: Source: Social Security Bulletin Vol 70 No 3, 2010 and 2015 Annual Report Table IV B4.

2 Empirical

2.1 Social Security

The Social Security Trust Fund is projected to be insolvent in early 2030 (Figure 1). The Social Security Trust Fund is funded through a tax of 9.9% up to \$118,500 on wage receipts and any interest income from the current surplus. The fund pays out benefits that are dependent on past contributions to all eligible beneficiaries who elect into the program¹. The number of beneficiaries have been increasing such that the fund's yearly expenditures exceed receipts. The fund's current surplus will allow the Social Security Administration to fulfill its benefits schedule until its depletion, the insolvent date.

The increase in expenditure is easy to account for. The retirees today are living longer than the retirees before them and the ratio of retirees to workers are at historical highs due to longevity and retirement of baby boomers. Advances in medicine, eradication of many infectious diseases and improved diet have contributed to increases in life expectancy. Figure 2 shows the historical increases in survival probabilities in 5 year increments. The increases have been dramatic. A person 65 years old in 2010 was almost twice as likely to reach the age of 85 than a 65 year old in 1959. These increases are only projected to rise.

Figure 3 shows the projected survival probabilities for each age up to year 2086². According to the projections, in 2085 the probability a 65 year old reaches the age of 85 will be more than 64% almost a three fold increase from 1959. If we assume that the birth rate does not increase (Figure 4) and use the projected survival probabilities, then future age dependency, defined as the ratio of 65+ to the 20-64, will rise above 40% (Figure 5). This ratio is rapidly increasing as baby boomers come into retirement age³. Higher ratio implies there are more beneficiaries and/or decline in the contributors both cases that will increase the budgetary shortfalls.

¹Retirees are able to receive reduced benefits starting at age 62.

²Source: SSA Life Tables.

³The red line in Figure 5 indicates the years when baby boomers are reaching age 65.

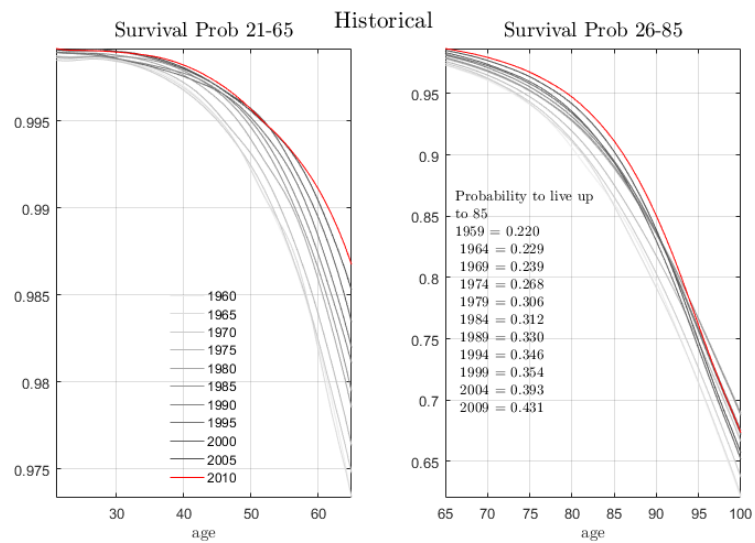


Figure 2: Historical Survival Probabilities. Source: https://www.ssa.gov/oact/NOTES/as116/as116_Tbl_6_1900.html

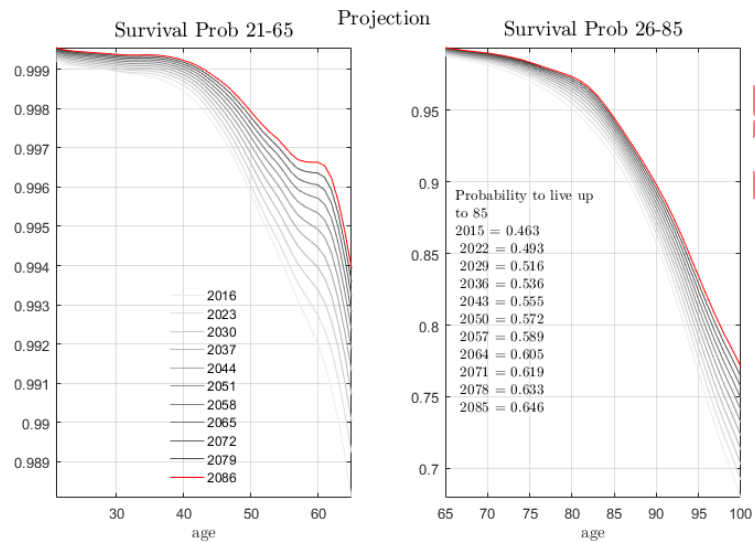


Figure 3: Projected Survival Probabilities by SSA. Source: https://www.ssa.gov/oact/NOTES/as116/as116_Tbl_6_1900.html

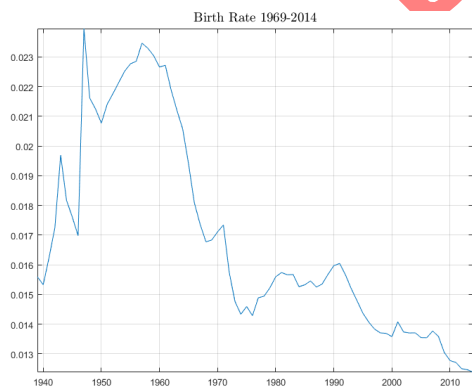


Figure 4: Historical Birth Rate. Source: Census 2010

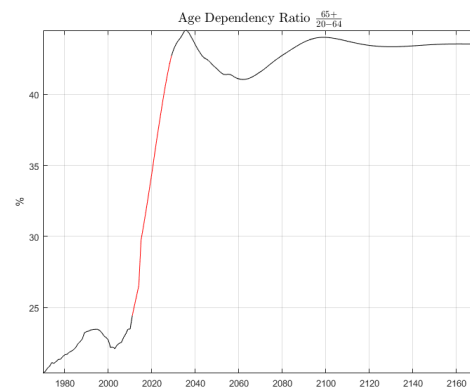


Figure 5: Age Dependency Ratio

The ratio highlights the Social Security Trust's need for younger cohorts as they are the primary wage earners. However in the wake of rapidly rising number of beneficiaries, the population has seen a decline in birth rate and employment percentage. These two trends have adversely affected the growth of the contributors to the fund. The decline in birth rate is significant. Number of births were lower in 2014 than in 1965 when the US population was about 200 million compared to 320 million in 2014. Decline in birth rate imply a slower growth of the workforce who are ready to contribute to the Social Security Trust Fund. The recent declining trend in employed population percentage has further slowed the growth of the workforce.

Figure 6 shows the employment percentage for different age groups, with a cross section of different education levels. First population employment percentage for all age and education has been declining as shown by top left Figure 6. Second employment percentage is approximately the same for age groups 25-54. The 18-24 and 55-64 have lower employment percentage. The oldest age group 65+ have significantly lower employment percentage. For all groups, employed percentage rises for higher levels of education. Jobs associated with higher levels of education are higher paying, making leisure more costly. Manual jobs associated with lower levels of education become more difficult with old age and pay significantly less than skilled jobs. Then cohorts on the verge of retirement will be more likely to delay retirement in professions that are high paying and/or less taxing physically.

Table 1 provides some support for this theory. The professions with higher pay have higher median age. Professions associated with higher degrees are among the largest increases for the oldest age group. Professions associated with manual labor are among the lowest increases with one exception being installation, maintenance and repair occupations. As higher percentage of the population have college degrees or higher, we expect higher proportion of cohorts to be employed for cohorts older than 24. We will further explore the relationship of education and employment in the following section.

Employment by Education Group



Figure 6: Employment Percentage. Source: ASEC CPS.

Profession	Increase in Occupation			Median	
	Δ All	Δ 65	Δ 20-24	Age	Pay
All	3.30	59.39	-0.90	42.4	32278
Management	9.70	63.89	3.12	47.5	87963
Business and financial operations	17.36	88.06	15.64	43.6	58703
Computer and mathematical	23.71	226.67	28.42	41.1	71640
Architecture and engineering	1.65	78.08	3.82	44.3	68576
Life physical and social science	-4.34	74.42	-7.06	42.8	55998
Community and social service	7.20	57.27	-5.65	43.9	37550
Legal occupations	15.19	132.26	7.69	46.0	70021
Education, training and library	8.76	112.08	5.08	43.3	42454
Arts, design and entertainment	6.90	79.41	14.50	40.5	41046
Healthcare practitioner and technical	19.67	126.40	21.43	43.7	56237
Health care support	19.14	51.19	27.16	39.1	23997
Protective service	9.48	47.47	6.03	40.8	33833
Food preparation and service	12.02	35.58	16.56	29.4	17501
Building and grounds cleaning	8.78	47.92	-13.28	44.1	21135
Personal care and service	18.35	71.50	28.67	40.0	19332
Sales and related	-2.79	34.48	6.03	39.9	23150
Office and administrative support	-9.01	44.01	-20.10	43.1	29453
Farming Fishing and Forestry	-2.76	10.00	9.52	37.2	17832
Construction and extraction	-17.83	36.54	-43.77	41.6	37421
Installation maintenance and repair	-2.09	83.50	-8.71	43.0	38130
Production	-13.40	26.36	-8.58	43.0	28754
Transportation and Moving	2.54	56.44	2.28	43.1	26775

Table 1: Changes are percent change from 2004 to 2013. Source: BLS Employed persons by detailed occupation.

2.2 Education

Proportion of 18-24 attending college has been steadily increasing last 50 years (Figure 7). As these students graduate and become older, they replace the cohorts before them, who have lower percentage of college graduates. Figure 8 shows the mechanical process of cohorts becoming more educated over time. Even if college attendance stagnates (it has actually decreased last few years), we will see an increase in the percentage of college graduates over time as the current old are relatively low in college graduates compared to the current young.

To predict future education rates, we regress each cohort's college proportion at year t to fixed effects for ages (21-74)⁴, its interaction to cohorts one year younger at time $t - 1$ and life expectancy⁵. The model essentially predicts college proportion of cohort at time t , by the college proportion of the one age younger cohort one year earlier, taking into account longevity. Equation 1 is a reduced form representation of the quantity of college educated cohorts, which is the market clearing quantity. Our goal here is not inferential and we ignore the usual covariates involved in supply and demand estimation. Our time series model accurately fits the data and the projections seem stationary, which is one of the criteria we required for the projection methods in this section.

Futures estimates of college educated cohorts are used in order to construct a time series of future employment percentage and cohort production efficiency. In transition analysis of OLG, steady state of the economy is assumed to occur many years ahead i.e. 100+ years. Therefore any estimation of future variables in a linear model can not have a time trend if we assume the variable needs to be at a steady state at some point in the future⁶. However the variables of interest do have strong time trend. We find by replacing time trend with life expectancy which we assume converges to the farthest projections by the Social Security Trust Administration, we can project variables which converges at the date of assumed steady state.

⁴Dummy Variable is for age - 1.

⁵This is defined as the probability of living up to age 85 given one reaches the age of 75. The regression results were robust to different end points of ages used in the definition.

⁶Non-linear estimation techniques can incorporate a time trend and still converge.

$$C_{i,t} = \alpha + \beta_1 \mathbb{1}_{(i-1=21)} + \beta_2 \mathbb{1}_{(i-1=22)} + \dots + \beta_{54} \mathbb{1}_{(i-1=74)} + \beta_{55} \mathbb{1}_{(i-1=20)} C_{20,t-1} \quad (1)$$

$$+ \beta_{56} \mathbb{1}_{(i-1=21)} C_{21,t-1} + \dots + \beta_{109} \mathbb{1}_{(i-1=74)} C_{74,t-1} + e_{i,t}$$

for $i = 21, 22, \dots, 75$ and $t = 1977$ to 2013 ⁷.

Dependent: ratio of college graduates to cohort size of age i at time t							
Variable	Estimate	SE	tStat	Variable	Estimate	SE	tStat
constant	0.0030	0.0029	1.0214				
$1(age - 1 = 21)$	0.0654	0.0046	14.0849	$1(age - 1 = 20)C_{20,t-1}$	3.0480	0.0038	1.7813
$1(age - 1 = 22)$	-0.0579	0.0124	-4.6650	$1(age - 1 = 21)C_{21,t-1}$	1.8160	0.2239	8.1121
$1(age - 1 = 23)$	-0.0360	0.0066	-5.4302	$1(age - 1 = 22)C_{22,t-1}$	2.4122	0.1244	19.3970
$1(age - 1 = 24)$	0.0037	0.0059	0.6236	$1(age - 1 = 23)C_{23,t-1}$	1.4131	0.0335	42.2029
$1(age - 1 = 25)$	-0.0125	0.0061	-2.0531	$1(age - 1 = 24)C_{24,t-1}$	1.0546	0.0236	44.7422
$1(age - 1 = 26)$	-0.0189	0.0061	-3.1124	$1(age - 1 = 25)C_{25,t-1}$	1.1169	0.0230	48.5146
$1(age - 1 = 27)$	-0.0074	0.0060	-1.2424	$1(age - 1 = 26)C_{26,t-1}$	1.1176	0.0216	51.7081
$1(age - 1 = 28)$	-0.0234	0.0059	-3.9742	$1(age - 1 = 27)C_{27,t-1}$	1.0336	0.0203	50.8596
$1(age - 1 = 29)$	0.0246	0.0056	4.3460	$1(age - 1 = 28)C_{28,t-1}$	1.1391	0.0199	57.2824
$1(age - 1 = 30)$	-0.0005	0.0062	-0.0797	$1(age - 1 = 29)C_{29,t-1}$	0.9317	0.0179	52.0824
$1(age - 1 = 31)$	-0.0195	0.0063	-3.1167	$1(age - 1 = 30)C_{30,t-1}$	0.9971	0.0197	50.7104
$1(age - 1 = 32)$	-0.0097	0.0061	-1.6045	$1(age - 1 = 31)C_{31,t-1}$	1.0798	0.0202	53.5313
$1(age - 1 = 33)$	0.0072	0.0060	1.1999	$1(age - 1 = 32)C_{32,t-1}$	1.0266	0.0192	53.5601
$1(age - 1 = 34)$	-0.0254	0.0061	-4.1796	$1(age - 1 = 33)C_{33,t-1}$	0.9849	0.0191	51.4700
$1(age - 1 = 35)$	-0.0027	0.0057	-0.4770	$1(age - 1 = 34)C_{34,t-1}$	1.1101	0.0193	57.4387
$1(age - 1 = 36)$	-0.0175	0.0056	-3.1399	$1(age - 1 = 35)C_{35,t-1}$	1.0003	0.0174	57.3642
$1(age - 1 = 37)$	0.0073	0.0053	1.3659	$1(age - 1 = 36)C_{36,t-1}$	1.0631	0.0173	61.5003
$1(age - 1 = 38)$	-0.0192	0.0053	-3.5828	$1(age - 1 = 37)C_{37,t-1}$	0.9713	0.0163	59.6754
$1(age - 1 = 39)$	0.0141	0.0051	2.7928	$1(age - 1 = 38)C_{38,t-1}$	1.0679	0.0166	64.3028
$1(age - 1 = 40)$	0.0170	0.0051	3.3376	$1(age - 1 = 39)C_{39,t-1}$	0.9780	0.0155	63.2523
$1(age - 1 = 41)$	0.0124	0.0052	2.3686	$1(age - 1 = 40)C_{40,t-1}$	0.9150	0.0153	59.6605
$1(age - 1 = 42)$	-0.0140	0.0053	-2.6564	$1(age - 1 = 41)C_{41,t-1}$	0.9429	0.0164	57.3363
$1(age - 1 = 43)$	0.0080	0.0050	1.6156	$1(age - 1 = 42)C_{42,t-1}$	1.0521	0.0170	61.7688
$1(age - 1 = 44)$	0.0074	0.0049	1.5122	$1(age - 1 = 43)C_{43,t-1}$	0.9587	0.0156	61.3315
$1(age - 1 = 45)$	-0.0187	0.0049	-3.8024	$1(age - 1 = 44)C_{44,t-1}$	0.9890	0.0157	62.9442
$1(age - 1 = 46)$	0.0012	0.0045	0.2566	$1(age - 1 = 45)C_{45,t-1}$	1.0493	0.0155	67.7918
$1(age - 1 = 47)$	0.0136	0.0044	3.0613	$1(age - 1 = 46)C_{46,t-1}$	0.9891	0.0140	70.4169
$1(age - 1 = 48)$	-0.0043	0.0045	-0.9722	$1(age - 1 = 47)C_{47,t-1}$	0.9326	0.0136	68.5402
$1(age - 1 = 49)$	0.0003	0.0043	0.0610	$1(age - 1 = 48)C_{48,t-1}$	1.0123	0.0141	71.8614
$1(age - 1 = 50)$	-0.0005	0.0042	-0.1099	$1(age - 1 = 49)C_{49,t-1}$	1.0223	0.0135	75.9646
$1(age - 1 = 51)$	0.0057	0.0042	1.3738	$1(age - 1 = 50)C_{50,t-1}$	0.9698	0.0127	76.3320
$1(age - 1 = 52)$	-0.0054	0.0041	-1.3096	$1(age - 1 = 51)C_{51,t-1}$	0.9485	0.0127	74.6761
$1(age - 1 = 53)$	0.0133	0.0040	3.3262	$1(age - 1 = 52)C_{52,t-1}$	1.0217	0.0129	79.2462
$1(age - 1 = 54)$	-0.0073	0.0041	-1.8028	$1(age - 1 = 53)C_{53,t-1}$	0.9495	0.0123	77.3463
$1(age - 1 = 55)$	-0.0044	0.0039	-1.1096	$1(age - 1 = 54)C_{54,t-1}$	1.0301	0.0126	82.0132
$1(age - 1 = 56)$	-0.0133	0.0039	-3.4358	$1(age - 1 = 55)C_{55,t-1}$	0.9875	0.0119	82.9685
$1(age - 1 = 57)$	0.0032	0.0037	0.8449	$1(age - 1 = 56)C_{56,t-1}$	1.0609	0.0118	89.7314
$1(age - 1 = 58)$	0.0001	0.0037	0.0265	$1(age - 1 = 57)C_{57,t-1}$	0.9646	0.0110	87.4295
$1(age - 1 = 59)$	-0.0144	0.0037	-3.8532	$1(age - 1 = 58)C_{58,t-1}$	0.9806	0.0115	85.2953
$1(age - 1 = 60)$	-0.0082	0.0036	-2.2572	$1(age - 1 = 59)C_{59,t-1}$	1.1046	0.0118	93.5721
$1(age - 1 = 61)$	0.0076	0.0036	2.1185	$1(age - 1 = 60)C_{60,t-1}$	1.0269	0.0108	95.2062
$1(age - 1 = 62)$	-0.0083	0.0036	-2.2685	$1(age - 1 = 61)C_{61,t-1}$	0.9309	0.0107	86.6512
$1(age - 1 = 63)$	-0.0035	0.0036	-0.9692	$1(age - 1 = 62)C_{62,t-1}$	1.0369	0.0117	88.2525
$1(age - 1 = 64)$	0.0053	0.0036	1.5293	$1(age - 1 = 63)C_{63,t-1}$	1.0089	0.0116	86.6551
$1(age - 1 = 65)$	-0.0088	0.0036	-2.4081	$1(age - 1 = 64)C_{64,t-1}$	0.9880	0.0119	83.1922
$1(age - 1 = 66)$	-0.0035	0.0036	-0.9689	$1(age - 1 = 65)C_{65,t-1}$	0.9909	0.0124	79.8082
$1(age - 1 = 67)$	0.0039	0.0036	1.0744	$1(age - 1 = 66)C_{66,t-1}$	1.0082	0.0132	76.5209
$1(age - 1 = 68)$	-0.0010	0.0037	-0.2547	$1(age - 1 = 67)C_{67,t-1}$	0.9579	0.0139	68.7877
$1(age - 1 = 69)$	-0.0053	0.0038	-1.4110	$1(age - 1 = 68)C_{68,t-1}$	0.9957	0.0154	64.5789
$1(age - 1 = 70)$	-0.0083	0.0038	-2.2081	$1(age - 1 = 69)C_{69,t-1}$	1.0261	0.0162	63.1752
$1(age - 1 = 71)$	-0.0059	0.0037	-1.5967	$1(age - 1 = 70)C_{70,t-1}$	1.0442	0.0164	63.7795
$1(age - 1 = 72)$	0.0008	0.0037	0.2145	$1(age - 1 = 71)C_{71,t-1}$	1.0093	0.0163	62.0104
$1(age - 1 = 73)$	0.0021	0.0037	0.5648	$1(age - 1 = 72)C_{72,t-1}$	0.9587	0.0167	57.3361
$1(age - 1 = 74)$	0.0067	0.0038	1.7813	$1(age - 1 = 73)C_{73,t-1}$	0.9706	0.0182	53.2632
				$1(age - 1 = 74)C_{74,t-1}$	0.9564	0.0191	50.1636

Table 2: Regression results for eqn 1. $R^2=0.997$. Fstat=5.13e3. p-value=0.

⁷The model regression results assume homoscedasticity, where regression errors $e_{i,t}$ are independent on i and t .

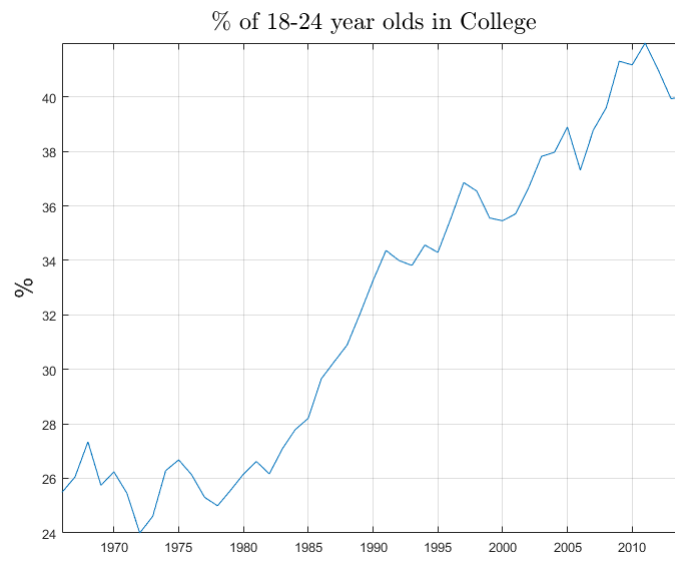


Figure 7: Source: <http://www.census.gov/hhes/school/data/cps/historical/>

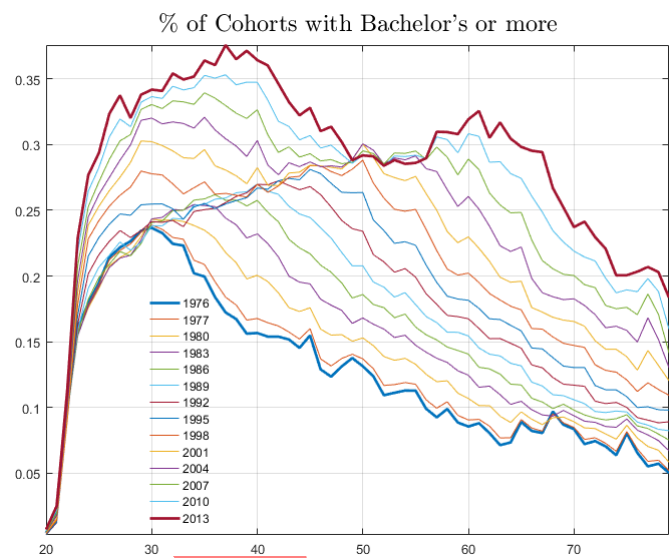


Figure 8: Historical. Source: ASEC CPS

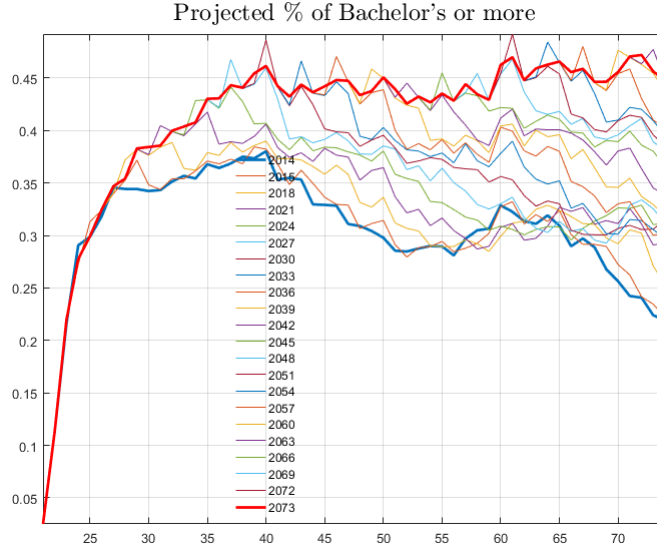


Figure 9: Projected

Figure 9 shows the predictions of the model up to year 2073. The model predicts almost a homogeneous ratio of college graduates after the age of 40 as would be the case if college graduate ratio stabilizes and the lower percentage age groups are replaced over time. The oldest age group will have the largest increase in college graduate proportions. We will see what this implies in employment rates in the following section.

2.3 Employment Percentage

Extensive margins account for most of the fluctuations in labor movement (cite Heckman 1984). Our model is not able to capture both intensive and extensive margin of labor. We take the view, the labor at the extensive is important and needs to be taken into account. However, in our model paradigm, agents' labor choices do not generate interesting extensive labor choices⁸. We will assume employment participation is exogenous process that is a function of time and education levels of the cohorts. Our regression model for employed percentage is similar to the model for education (equation 1). Again we emphasize the model's objective is projection rather than inference. We find the time series model accurately fits the data.

⁸Agents will choose to work when they are most productive, implying they will all work when they are old.

$$\begin{aligned}
E_{i,t} = & \alpha + \beta_1 \mathbb{1}_{(i=21)} + \beta_2 \mathbb{1}_{(i=22)} + \dots + \beta_{55} \mathbb{1}_{(i=75)} + \beta_{56} \mathbb{1}_{(i=20)} C_{20,t} \\
& + \beta_{57} \mathbb{1}_{(i=21)} C_{21,t} + \dots + \beta_{111} \mathbb{1}_{(i=75)} C_{75,t} + \beta_{112} L E_t + e_{i,t}
\end{aligned} \tag{2}$$

for $i = 20, 22, \dots, 75$ and $t = 1976$ to 2013.

The regression results are displayed in Table 3. Life expectancy has a negative coefficient. Although this may be counter intuitive as longer life necessitates more saving therefore longer employment, note there is no time trend in this regression. Life expectancy is pseudo time trend in this regression, as it is a monotonically increasing. The economic interpretation can be that technological progress has created jobs that require specific set of skills, education, or experience and has replaced some labor. For the youngest cohorts, the coefficients to college proportion is strongly negative. This can also be interpreted as longer training and education needed for technology driven jobs. The linear model predicts most of the age groups will be employed at a higher percentage as the proportion of college educated rises (Figure 11).

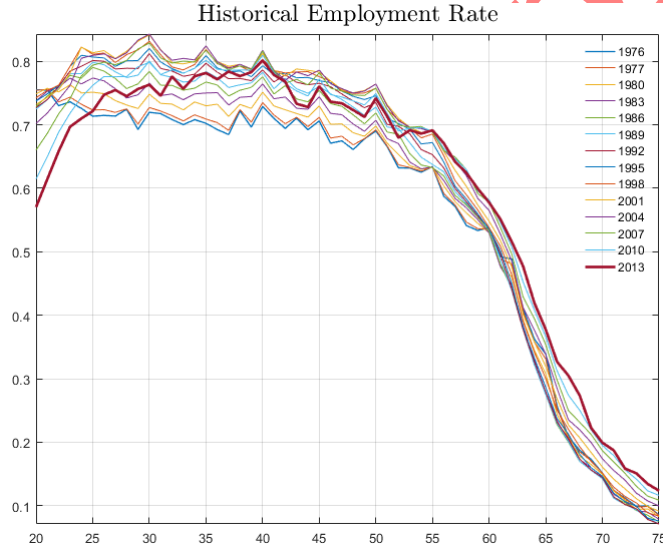


Figure 10: Historical Source: ASEC CPS

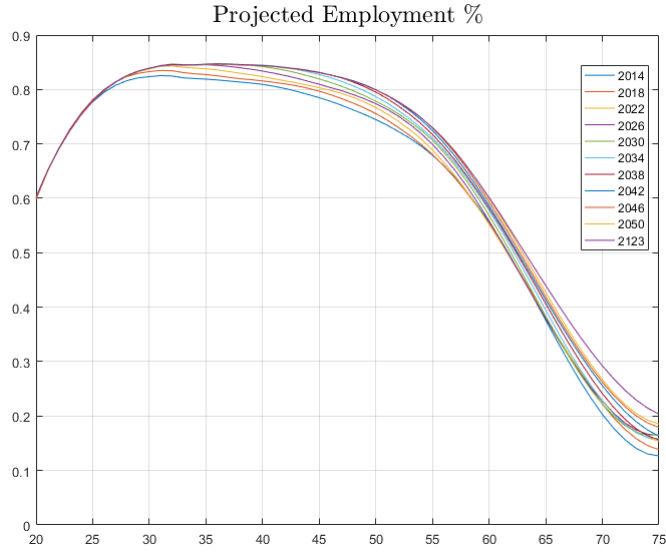


Figure 11: Projected

Dependent: Employment percentage of cohort i at time t							
Variable	Estimate	SE	tStat	Variable	Estimate	SE	tStat
constant	0.9332	0.0243	38.4439				
Life Expectancy	-0.1940	0.0435	-4.4635				
				$1(age = 20)C_{20,t}$	-27.7719	0.0224	-7.0068
$1(age = 21)$	0.9332	0.0243	38.4439	$1(age = 21)C_{21,t}$	-7.5795	1.3164	-5.7578
$1(age = 22)$	0.0178	0.0270	0.6607	$1(age = 22)C_{22,t}$	-3.5825	0.7315	-4.8977
$1(age = 23)$	0.2592	0.0723	3.5848	$1(age = 23)C_{23,t}$	-0.4391	0.2084	-2.1066
$1(age = 24)$	0.0112	0.0396	0.2834	$1(age = 24)C_{24,t}$	0.3370	0.1494	2.2564
$1(age = 25)$	-0.1261	0.0354	-3.5659	$1(age = 25)C_{25,t}$	0.3527	0.1461	2.4146
$1(age = 26)$	-0.1295	0.0367	-3.5258	$1(age = 26)C_{26,t}$	0.3972	0.1349	2.9432
$1(age = 27)$	-0.1437	0.0362	-3.9676	$1(age = 27)C_{27,t}$	0.4762	0.1271	3.7476
$1(age = 28)$	-0.1766	0.0356	-4.9586	$1(age = 28)C_{28,t}$	0.6319	0.1270	4.9767
$1(age = 29)$	-0.2168	0.0358	-6.0616	$1(age = 29)C_{29,t}$	0.7516	0.1143	6.5765
$1(age = 30)$	-0.2513	0.0342	-7.3463	$1(age = 30)C_{30,t}$	0.6297	0.1252	5.0300
$1(age = 31)$	-0.2062	0.0374	-5.5133	$1(age = 31)C_{31,t}$	0.5945	0.1283	4.6356
$1(age = 32)$	-0.2156	0.0380	-5.6757	$1(age = 32)C_{32,t}$	0.5813	0.1208	4.8107
$1(age = 33)$	-0.2181	0.0364	-5.9887	$1(age = 33)C_{33,t}$	0.6154	0.1207	5.1001
$1(age = 34)$	-0.2317	0.0361	-6.4176	$1(age = 34)C_{34,t}$	0.6538	0.1221	5.3563
$1(age = 35)$	-0.2375	0.0366	-6.4892	$1(age = 35)C_{35,t}$	0.6990	0.1101	6.3478
$1(age = 36)$	-0.2402	0.0341	-7.0402	$1(age = 36)C_{36,t}$	0.6040	0.1090	5.5429
$1(age = 37)$	-0.2239	0.0335	-6.6925	$1(age = 37)C_{37,t}$	0.6617	0.1013	6.5308
$1(age = 38)$	-0.2483	0.0315	-7.8809	$1(age = 38)C_{38,t}$	0.5426	0.1039	5.2206
$1(age = 39)$	-0.2068	0.0318	-6.4939	$1(age = 39)C_{39,t}$	0.5824	0.0964	6.0436
$1(age = 40)$	-0.2198	0.0299	-7.3425	$1(age = 40)C_{40,t}$	0.5923	0.0969	6.1155
$1(age = 41)$	-0.2033	0.0305	-6.6761	$1(age = 41)C_{41,t}$	0.5651	0.1025	5.5115
$1(age = 42)$	-0.2150	0.0310	-6.9290	$1(age = 42)C_{42,t}$	0.5759	0.1065	5.4085
$1(age = 43)$	-0.2186	0.0315	-6.9452	$1(age = 43)C_{43,t}$	0.5199	0.0995	5.2231
$1(age = 44)$	-0.2083	0.0298	-6.9894	$1(age = 44)C_{44,t}$	0.5694	0.1003	5.6778
$1(age = 45)$	-0.2235	0.0296	-7.5431	$1(age = 45)C_{45,t}$	0.5237	0.0987	5.3062
$1(age = 46)$	-0.2024	0.0296	-6.8492	$1(age = 46)C_{46,t}$	0.5872	0.0902	6.5091
$1(age = 47)$	-0.2321	0.0274	-8.4777	$1(age = 47)C_{47,t}$	0.4950	0.0874	5.6608
$1(age = 48)$	-0.2145	0.0266	-8.0576	$1(age = 48)C_{48,t}$	0.5698	0.0908	6.2749
$1(age = 49)$	-0.2396	0.0269	-8.9042	$1(age = 49)C_{49,t}$	0.5705	0.0871	6.5471
$1(age = 50)$	-0.2426	0.0261	-9.2812	$1(age = 50)C_{50,t}$	0.5206	0.0824	6.3194
$1(age = 51)$	-0.2197	0.0256	-8.5846	$1(age = 51)C_{51,t}$	0.4851	0.0824	5.8864
$1(age = 52)$	-0.2382	0.0250	-9.5151	$1(age = 52)C_{52,t}$	0.4269	0.0837	5.1034
$1(age = 53)$	-0.2381	0.0248	-9.6072	$1(age = 53)C_{53,t}$	0.4894	0.0798	6.1352
$1(age = 54)$	-0.2629	0.0241	-10.9261	$1(age = 54)C_{54,t}$	0.5600	0.0817	6.8547
$1(age = 55)$	-0.2895	0.0244	-11.8798	$1(age = 55)C_{55,t}$	0.5062	0.0774	6.5395
$1(age = 56)$	-0.2778	0.0236	-11.7515	$1(age = 56)C_{56,t}$	0.4870	0.0766	6.3539
$1(age = 57)$	-0.2957	0.0231	-12.8072	$1(age = 57)C_{57,t}$	0.4945	0.0711	6.9526
$1(age = 58)$	-0.3222	0.0223	-14.4774	$1(age = 58)C_{58,t}$	0.5119	0.0737	6.9431
$1(age = 59)$	-0.3425	0.0223	-15.3804	$1(age = 59)C_{59,t}$	0.4093	0.0753	5.4348
$1(age = 60)$	-0.3436	0.0222	-15.4986	$1(age = 60)C_{60,t}$	0.3609	0.0689	5.2373
$1(age = 61)$	-0.3537	0.0216	-16.3918	$1(age = 61)C_{61,t}$	0.4400	0.0681	6.4593
$1(age = 62)$	-0.4072	0.0213	-19.1347	$1(age = 62)C_{62,t}$	0.3962	0.0745	5.3178
$1(age = 63)$	-0.4334	0.0216	-20.0920	$1(age = 63)C_{63,t}$	0.4417	0.0729	6.0548
$1(age = 64)$	-0.5053	0.0212	-23.7929	$1(age = 64)C_{64,t}$	0.4966	0.0747	6.6511
$1(age = 65)$	-0.5602	0.0212	-26.4423	$1(age = 65)C_{65,t}$	0.5296	0.0781	6.7787
$1(age = 66)$	-0.6074	0.0215	-28.2350	$1(age = 66)C_{66,t}$	0.5931	0.0815	7.2745
$1(age = 67)$	-0.6644	0.0213	-31.2320	$1(age = 67)C_{67,t}$	0.5971	0.0851	7.0193
$1(age = 68)$	-0.6970	0.0213	-32.6471	$1(age = 68)C_{68,t}$	0.7160	0.0952	7.5220
$1(age = 69)$	-0.7383	0.0219	-33.6521	$1(age = 69)C_{69,t}$	0.6067	0.1009	6.0104
$1(age = 70)$	-0.7400	0.0222	-33.3013	$1(age = 70)C_{70,t}$	0.5341	0.1031	5.1811
$1(age = 71)$	-0.7464	0.0222	-33.6077	$1(age = 71)C_{71,t}$	0.6258	0.1017	6.1553
$1(age = 72)$	-0.7808	0.0219	-35.6630	$1(age = 72)C_{72,t}$	0.5768	0.1048	5.5054
$1(age = 73)$	-0.7853	0.0218	-36.0474	$1(age = 73)C_{73,t}$	0.5864	0.1135	5.1670
$1(age = 74)$	-0.7961	0.0221	-36.0952	$1(age = 74)C_{74,t}$	0.4981	0.1208	4.1230
$1(age = 75)$	-0.7949	0.0224	-35.4982	$1(age = 75)C_{75,t}$	0.5927	0.1291	4.5909

Table 3: Regression results for eqn 2. $R^2=.986$. Fstat=1.27e3. p-value=0.

2.4 Efficiency

In an OLG model, a hump shaped age-productivity profile is used to better match the age-dependent labor profile. The age dependent productivity is multiplied to each cohort's wage, changing the labor choice to reflect rising/decreasing opportunity cost of leisure. Using the proportional relationship between productivity and wages in a Cobb Douglas production, Hansen (1993) calculates efficiency unit by normalizing wages per hour of various subgroups by average wages per hour of the population for the period 1979 to 1987. Using Hansen's methodology we calculate the age-dependent productivity profile from 1976 to 2013⁹. Figure 12 shows the efficiency weights for the period 1976 to 2013¹⁰.

We highlight three features of this time series. First the productivity of the 65 year and over have increased over 50% since 1976. Second the peak of the highest productivity has shifted to close 60 years old. Third the younger age group's productivity relative to the peak is smaller today than it was in 1976. We believe the age-dependent profile has sufficiently changed that Hansen's 1993 curve is no longer accurate.

The implications of the new profile matches the current labor patterns. Higher proportion of the older workforce delays full retirement as higher wages increase the opportunity cost of retirement. The lower productivity for the younger workforce makes schooling or pure unemployment more attractive. And as longevity increases, the workers are able to be productive at later ages, while their experiences continue to accumulate. Lemieux (2006) shows wages are increasing in experience controlling for level of education.

⁹We use a different data set. He uses BLS aggregation of the CPS. We use ASEC of the CPS, in order to create subgroups by education.

¹⁰Selected years for visual aid.

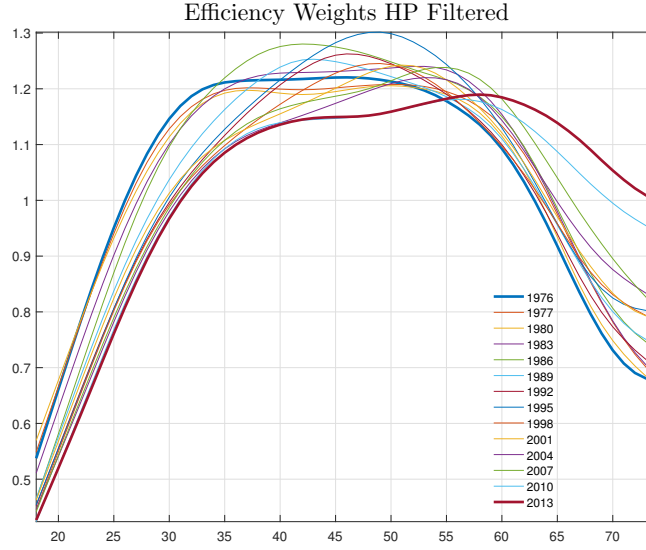


Figure 12: Efficiency Weights 1976-2013 Using Hansen's methodology. Data: ASEC CPS.

In order to project age dependent productivity in the future, we use a different specification than Hansen's efficiency weight. We project log hourly wages and normalize them. We do this for two reasons. First Mincer equation, which links log wages to education has a large literature supporting its form. Second when Hansen's efficiency weights are used as the dependent variable, the coefficient for Life Expectancy is negative. This implies the efficiency for all age groups will be declining, which is counter intuitive. The coefficient for life expectancy is positive for the regression specification in equation 3, which provides a productivity profile that is not declining over time.

$$\begin{aligned} \log w_{i,t} = & \alpha + \beta_1 \mathbb{1}_{(i=21)} + \beta_2 \mathbb{1}_{(i=22)} + \dots + \beta_{55} \mathbb{1}_{(i=75)} + \beta_{56} \mathbb{1}_{(i=20)} C_{20,t} \\ & + \beta_{57} \mathbb{1}_{(i=21)} C_{21,t} + \dots + \beta_{111} \mathbb{1}_{(i=75)} C_{75,t} + \beta_{112} LE_t + e_{i,t} \end{aligned} \quad (3)$$

for $i = 20, 22, \dots, 75$ and $t = 1976$ to 2013.

Dependent: log hourly wage of age i at time t							
Variable	Estimate	SE	tStat	Variable	Estimate	SE	tStat
constant	1.6692	0.0215	77.7918				
Life Expectancy	0.5789	0.0384	15.0697				
				$1(age = 20)C_{20,t}$	24.6504	0.0198	7.0358
$1(age = 21)$	1.6692	0.0215	77.7918	$1(age = 21)C_{21,t}$	7.5648	1.1636	6.5011
$1(age = 22)$	0.0735	0.0238	3.0853	$1(age = 22)C_{22,t}$	2.9787	0.6466	4.6068
$1(age = 23)$	-0.0138	0.0639	-0.2162	$1(age = 23)C_{23,t}$	1.1907	0.1842	6.4631
$1(age = 24)$	0.1379	0.0350	3.9352	$1(age = 24)C_{24,t}$	0.8869	0.1320	6.7171
$1(age = 25)$	0.2264	0.0313	7.2431	$1(age = 25)C_{25,t}$	0.9401	0.1291	7.2801
$1(age = 26)$	0.2615	0.0325	8.0578	$1(age = 26)C_{26,t}$	0.9307	0.1193	7.8028
$1(age = 27)$	0.3047	0.0320	9.5173	$1(age = 27)C_{27,t}$	0.9348	0.1123	8.3220
$1(age = 28)$	0.3430	0.0315	10.8942	$1(age = 28)C_{28,t}$	1.0639	0.1122	9.4793
$1(age = 29)$	0.3511	0.0316	11.1081	$1(age = 29)C_{29,t}$	1.0091	0.1010	9.9890
$1(age = 30)$	0.3895	0.0302	12.8823	$1(age = 30)C_{30,t}$	1.1407	0.1107	10.3080
$1(age = 31)$	0.3785	0.0331	11.4495	$1(age = 31)C_{31,t}$	1.2084	0.1134	10.6591
$1(age = 32)$	0.3884	0.0336	11.5679	$1(age = 32)C_{32,t}$	1.1502	0.1068	10.7690
$1(age = 33)$	0.4252	0.0322	13.2090	$1(age = 33)C_{33,t}$	1.1812	0.1067	11.0739
$1(age = 34)$	0.4397	0.0319	13.7797	$1(age = 34)C_{34,t}$	1.2221	0.1079	11.3275
$1(age = 35)$	0.4428	0.0324	13.6849	$1(age = 35)C_{35,t}$	1.1540	0.0973	11.8555
$1(age = 36)$	0.4719	0.0302	15.6488	$1(age = 36)C_{36,t}$	1.1494	0.0963	11.9319
$1(age = 37)$	0.4903	0.0296	16.5787	$1(age = 37)C_{37,t}$	1.1004	0.0896	12.2872
$1(age = 38)$	0.5165	0.0279	18.5427	$1(age = 38)C_{38,t}$	1.1738	0.0919	12.7765
$1(age = 39)$	0.5085	0.0282	18.0622	$1(age = 39)C_{39,t}$	1.1176	0.0852	13.1211
$1(age = 40)$	0.5346	0.0265	20.2018	$1(age = 40)C_{40,t}$	1.1971	0.0856	13.9833
$1(age = 41)$	0.5129	0.0269	19.0526	$1(age = 41)C_{41,t}$	1.2817	0.0906	14.1428
$1(age = 42)$	0.5055	0.0274	18.4348	$1(age = 42)C_{42,t}$	1.4102	0.0941	14.9831
$1(age = 43)$	0.4830	0.0278	17.3561	$1(age = 43)C_{43,t}$	1.4268	0.0880	16.2146
$1(age = 44)$	0.4866	0.0263	18.4714	$1(age = 44)C_{44,t}$	1.4958	0.0886	16.8731
$1(age = 45)$	0.4789	0.0262	18.2828	$1(age = 45)C_{45,t}$	1.5120	0.0872	17.3301
$1(age = 46)$	0.4730	0.0261	18.1067	$1(age = 46)C_{46,t}$	1.4376	0.0797	18.0275
$1(age = 47)$	0.5065	0.0242	20.9293	$1(age = 47)C_{47,t}$	1.4235	0.0773	18.4157
$1(age = 48)$	0.5169	0.0235	21.9670	$1(age = 48)C_{48,t}$	1.5247	0.0803	18.9942
$1(age = 49)$	0.5007	0.0238	21.0507	$1(age = 49)C_{49,t}$	1.5132	0.0770	19.6471
$1(age = 50)$	0.5088	0.0231	22.0242	$1(age = 50)C_{50,t}$	1.4642	0.0728	20.1080
$1(age = 51)$	0.5147	0.0226	22.7572	$1(age = 51)C_{51,t}$	1.4829	0.0729	20.3550
$1(age = 52)$	0.5232	0.0221	23.6434	$1(age = 52)C_{52,t}$	1.5284	0.0739	20.6690
$1(age = 53)$	0.5234	0.0219	23.8943	$1(age = 53)C_{53,t}$	1.4727	0.0705	20.8864
$1(age = 54)$	0.5368	0.0213	25.2417	$1(age = 54)C_{54,t}$	1.5303	0.0722	21.1909
$1(age = 55)$	0.5197	0.0215	24.1287	$1(age = 55)C_{55,t}$	1.4709	0.0684	21.4962
$1(age = 56)$	0.5311	0.0209	25.4126	$1(age = 56)C_{56,t}$	1.4729	0.0678	21.7398
$1(age = 57)$	0.5368	0.0204	26.3021	$1(age = 57)C_{57,t}$	1.3766	0.0629	21.8955
$1(age = 58)$	0.5520	0.0197	28.0636	$1(age = 58)C_{58,t}$	1.4458	0.0652	22.1860
$1(age = 59)$	0.5360	0.0197	27.2299	$1(age = 59)C_{59,t}$	1.5187	0.0666	22.8119
$1(age = 60)$	0.5176	0.0196	26.4157	$1(age = 60)C_{60,t}$	1.4361	0.0609	23.5771
$1(age = 61)$	0.5135	0.0191	26.9192	$1(age = 61)C_{61,t}$	1.4710	0.0602	24.4286
$1(age = 62)$	0.4959	0.0188	26.3625	$1(age = 62)C_{62,t}$	1.6881	0.0659	25.6344
$1(age = 63)$	0.4460	0.0191	23.3916	$1(age = 63)C_{63,t}$	1.7310	0.0645	26.8438
$1(age = 64)$	0.4201	0.0188	22.3774	$1(age = 64)C_{64,t}$	1.8850	0.0660	28.5596
$1(age = 65)$	0.3733	0.0187	19.9365	$1(age = 65)C_{65,t}$	2.1040	0.0691	30.4674
$1(age = 66)$	0.3020	0.0190	15.8836	$1(age = 66)C_{66,t}$	2.3263	0.0721	32.2769
$1(age = 67)$	0.2591	0.0188	13.7786	$1(age = 67)C_{67,t}$	2.5799	0.0752	34.3078
$1(age = 68)$	0.1961	0.0189	10.3919	$1(age = 68)C_{68,t}$	3.0615	0.0841	36.3867
$1(age = 69)$	0.1008	0.0194	5.1994	$1(age = 69)C_{69,t}$	3.4420	0.0892	38.5728
$1(age = 70)$	0.0210	0.0196	1.0709	$1(age = 70)C_{70,t}$	3.7144	0.0911	40.7635
$1(age = 71)$	-0.0406	0.0196	-2.0665	$1(age = 71)C_{71,t}$	3.8462	0.0899	42.7952
$1(age = 72)$	-0.0782	0.0194	-4.0391	$1(age = 72)C_{72,t}$	4.0717	0.0926	43.9644
$1(age = 73)$	-0.1163	0.0193	-6.0411	$1(age = 73)C_{73,t}$	4.6079	0.1003	45.9347
$1(age = 74)$	-0.1921	0.0195	-9.8549	$1(age = 74)C_{74,t}$	5.1939	0.1068	48.6342
$1(age = 75)$	-0.2837	0.0198	-14.3322	$1(age = 75)C_{75,t}$	5.5554	0.1141	48.6800

Table 4: Regression results for eqn 3. $R^2=0.993$ Fstat=2.39e3. p-value=0.

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