Introduction

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What is this course for?

- In this course, we will learn the *modern* macroeconomics.
- What is this?

Keynesian macroeconomics

- In undergraduate courses, you may have learned so-called Keynesian macroeconomics such as the IS-LM and AD-AS models.
- The IS-LM-AS model is static.
- Keynesian macroeconomics is at odd with classical theories such as the quantity theory of money and neutrality of money.

Lucas critique

- In 1970s, performances of Keynesian large-scale macroeconomic models were deteriorated.
- It is partly due to changes in expectations of agents (e.g., households and firms) in the economy and policy responses.
  - "... it is naive to try to predict the effects of a change in economic policy entirely on the basis of relationships observed in historical data, especially highly aggregated historical data."
- Since then, macroeconomists have tried to build models with solid *microfundations*, i.e., optimizing behavior of agents.

RBC as a point of departure for modern macro

- Real Business Cycle (RBC) theory explains that changes in the total factor productivity (TFP) are driving forces of economic fluctuations.
    - They characterize economic fluctuations as "optimal responses to uncertainty in the rate of technological change," and offers the policy advice that "costly efforts at stabilization are likely to be counterproductive." (Prescott, 1996)
    - "They assert that monetary policies have no effect on real activity, [...,] and that economic fluctuations are caused entirely by supply rather than demand shocks." (Summers, 1996)

New Keynesian macro

- RBC suggests no role of monetary policy for stabilizing the economy.
- New Keynesians correspond to the Lucas critique in 1990s and 2000s.
  - The theory they have developed is based on RBC. However, they assume market imperfection and price stickiness in the short run.
- New Keynesian models are now widely used in central banks and other institutions for forecasting and policy simulations.

What is DSGE?

- The modern macroeconomics = DSGE.
- DSGE = <u>D</u>ynamic <u>S</u>tochastic <u>G</u>eneral <u>E</u>quilibrium
  - Dynamic: Intertemporal behavior of agents, e.g., Euler equation, capital accumulation, etc.
  - Stochastic: Exogenous shocks drive short-run economic fluctuations. Agents' expectations are usually rational.
  - General Equilibrium: All markets clear simultaneously as a result of agents' optimizations.
- Both RBC and NK models are variants of DSGE model.

Two examples

- Asset price equation
- Cagan's (1958) model of hyperinflation

Asset price equation

- The process of asset prices follows:

$$p_t = d_t + \beta p_{t+1},$$

for $t = 0, 1, ...,$ where p_t is the asset price and d_t is dividend which is exogenously given.

- How to "solve" for the price?

Solving for the price

- Substitute the next period's equation...

$$
\begin{aligned}
p_t &= d_t + \beta p_{t+1}, \\
p_{t+1} &= d_{t+1} + \beta p_{t+2}, \\
p_{t+2} &= d_{t+2} + \beta p_{t+3}, \\
&\vdots
\end{aligned}
$$

Solving for the price, cont'd

- Substitute the next period's equation...

$$
\begin{aligned}
p_t &= d_t + \beta\left(d_{t+1} + \beta p_{t+2}\right), \\
p_t &= d_t + \beta d_{t+1} + \beta^2 p_{t+2}, \\
p_t &= d_t + \beta d_{t+1} + \beta^2\left(d_{t+2} + \beta p_{t+3}\right), \\
&\vdots
\end{aligned}
$$

Solving for the price, cont'd

- Substitute the next period's equation...

$$p_t = d_t + \beta d_{t+1} + \beta^2 d_{t+2} + \cdots \beta^s d_{t+s} + \beta^{s+1} p_{t+s+1},$$
$$p_t = \sum_{i=0}^{s} \beta^i d_{t+i} + \beta^{s+1} p_{t+s+1}.$$

- Finally, let $s \to \infty$:

$$p_t = \lim_{s\to\infty} \sum_{i=0}^{s} \beta^i d_{t+i}.$$

Stochastic case

- Assume d_t is i.i.d. with zero mean and variance σ^2. Then we have

$$p_t = d_t + \beta E_t p_{t+1},$$

  where E_t is called expectational operator.

- We have a solution

$$p_t = \lim_{s \to \infty} \sum_{i=0}^{s} \beta^i E_t d_{t+i} = d_t.$$

Money demand and supply

- Money demand function is given by

$$\ln \frac{M_t^d}{P_t} = \gamma - \alpha_1 i_t + \alpha_2 \log Y_t.$$

This is based on liquidity preference.

- Money supply rule is

$$M_t^s = (1 + \mu_t) M_{t-1}^s,$$

where μ_t is money growth rate.

- In equilibrium, $M_t^d = M_t^s$ holds.

Cagan's model

- Cagan's (1958) model:

$$m_t - p_t = \gamma - \alpha_1 E_t \pi_{t+1},$$
$$\Delta m_t = \mu + \varepsilon_t,$$

  where $\alpha_1 > 0$ and γ are parameters, and
  - m_t is logged quantity of money,
  - p_t is logged price level,
  - $\pi_t = p_t - p_{t-1}$ is the inflation rate, and
  - $\varepsilon_t \sim N(0, \sigma^2)$ is a shock to money growth.

- Note that $i_t = r_t + E_t \pi_{t+1}$ (Fisher equation). We assume r_t is very small so ignore it. We also assume $\alpha_2 = 0$.

Adaptive vs. rational expectations

- Adaptive expectation: Agents in the model form expectations based on observations in the past.
- Rational expectation: Agents in the model form expectations based on all the information available for the modeler.

Adaptive expectation

- Assuming adaptive expectation $E_t\pi_{t+1} = \pi_t$, we have

$$
\begin{aligned}
& \begin{cases} m_t - p_t = \gamma - \alpha\pi_t, \\ m_{t-1} - p_{t-1} = \gamma - \alpha\pi_{t-1}, \end{cases} \\
\Leftrightarrow \quad & m_t - m_{t-1} - (p_t - p_{t-1}) = -\alpha(\pi_t - \pi_{t-1}), \\
\Leftrightarrow \quad & \mu + \varepsilon_t - \pi_t = -\alpha(\pi_t - \pi_{t-1}), \\
\Leftrightarrow \quad & (1-\alpha)\pi_t = -\alpha\pi_{t-1} + \mu + \varepsilon_t, \\
\therefore \quad & \pi_t = \frac{-\alpha}{1-\alpha}\pi_{t-1} + \frac{1}{1-\alpha}(\mu + \varepsilon_t).
\end{aligned}
$$

Steady state

- Assume $\varepsilon_t = 0$ and $\pi_t = \pi_{t-1} = \pi$, which is called *the steady state*. Then we have

$$
\begin{aligned}
& \pi = \frac{-\alpha}{1-\alpha}\pi + \frac{1}{1-\alpha}\mu, \\
\Leftrightarrow \quad & (1-\alpha)\pi = -\alpha\pi + \mu, \\
\therefore \quad & \pi = \mu.
\end{aligned}
$$

- The steady state condition holds only in the long run.

A diagram

- [Show on the board. How the sequence of inflation converges to the steady state?]

Rational expectation

- What is a solution with rational expectation?

- Conjecture $\pi_t = a_0 + a_1\varepsilon_t$. Then we have

$$
\begin{aligned}
& \begin{cases} m_t - p_t = \gamma - \alpha E_t \pi_{t+1}, \\ m_{t-1} - p_{t-1} = \gamma - \alpha E_{t-1}\pi_t, \end{cases} \\
\Leftrightarrow \quad & \mu + \varepsilon_t - \pi_t = -\alpha(E_t\pi_{t+1} - E_{t-1}\pi_t), \\
\therefore \quad & \mu + \varepsilon_t - a_0 - a_1\varepsilon_t = 0,
\end{aligned}
$$

  Note that $E_t\pi_{t+1} = E_{t-1}\pi_t = a_0$. Then $a_0 = \mu$ and $a_1 = 1$ hold.

- $E_t\pi_{t+1} = \mu$ holds even in short-run fluctuations.

Cagan's model: Simulation results