

Monetary Policy Tradeoffs: Discretion vs. Commitment

Takeki Sunakawa

Quantitative Methods for Monetary Economics

University of Mannheim

May 2018

- The efficient allocation is obtained under fully flexible prices. The optimal policy fully stabilizes the price level.
- In practice, central banks face short-run tradeoffs: inflation vs. real variables such as output and employment.
- We need a monetary policy design in environment in which the central bank faces a nontrivial tradeoff.

The case of an efficient steady state

- Consider a situation in which the flexible price equilibrium allocation is inefficient. The natural level of output y_t^n deviates from its efficient counterpart y_t^e in the short run.
- Some real imperfections generates a time-varying gap $u_t \equiv y_t^n - y_t^e$ even in the absence of price rigidities.

The CB's problem

- The central bank (CB hereafter) will minimize the welfare loss function:

$$E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2),$$

where $x_t \equiv y_t - y_t^e$ is the welfare-relevant output gap with y_t^e denoting the efficient level of output. $\vartheta = \kappa/\epsilon$ is the weight of output gap fluctuations.

- Minimization is subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t,$$

where $u_t \equiv \kappa(y_t^e - y_t^n)$. [Note that $y_t - y_t^n = \underbrace{(y_t - y_t^e)}_{x_t} + \underbrace{(y_t^e - y_t^n)}_{u_t}$.]

Cost-push shock

- u_t follows the exogenous AR(1) process

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u,$$

where $\rho_u \in [0, 1)$ and ε_t^u is white noise with variance σ_u^2 .

- u_t generates a tradeoff between stabilizing π_t and stabilizing x_t .

Optimal Discretionary Policy

- Each period the CB minimizes the period losses

$$\pi_t^2 + \vartheta x_t^2,$$

subject to the constraint

$$\pi_t = \kappa x_t + \nu_t,$$

where $\nu_t \equiv \beta E_t \pi_{t+1} + u_t$ is taken as given by the central bank, as there are no endogenous state variables.

Optimal Discretionary Policy, cont'd

- The optimality condition is given by

$$x_t = -\frac{\kappa}{\vartheta} \pi_t,$$

for $t = 0, 1, 2, \dots$

- Substituting it into the NKPC and after some manipulation (e.g., undetermined coefficient method), we have

$$\begin{aligned}\pi_t &= \frac{\vartheta}{\kappa^2 + \vartheta(1 - \beta\rho)} u_t, \\ \pi_t &= -\frac{\kappa}{\kappa^2 + \vartheta(1 - \beta\rho)} u_t.\end{aligned}$$

Optimal Commitment Policy

- Now we assume that the CB is able to commit to future policies. The CB will choose a state-contingent sequence $\{x_t, \pi_t\}_{t=0}^{\infty}$ so as to minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2),$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t,$$

and u_t follows the AR(1) process:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u.$$

Optimal Commitment Policy, cont'd

- It is useful to set up the Lagrangian as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} (\pi_t^2 + \vartheta x_t^2) + \xi_t (\pi_t - \beta E_t \pi_{t+1} - \kappa x_t) \right],$$

where $\{\xi_t\}$ is a sequence of Lagrange multipliers.

- The FONCs are

$$\vartheta x_t - \kappa \xi_t = 0,$$

$$\pi_t + \xi_t - \xi_{t-1} = 0,$$

for $t = 0, 1, 2, \dots$, given $\xi_{-1} = 0$, because there is no commitment in period 0.

Optimal Commitment Policy, cont'd

- Combining the two, we have

$$x_0 = -\frac{\kappa}{\vartheta}\pi_0,$$

and

$$x_t - x_{t-1} = -\frac{\kappa}{\vartheta}\pi_t$$

for $t = 1, 2, 3, \dots$

Solving for the policy function under commitment

- We assume: $x_t = a_x u_t + b_x x_{t-1}$ and $\pi_t = a_\pi u_t + b_\pi x_{t-1}$ with the initial condition $x_{-1} = 0$.
- Substitute them into the NKPC and the tradeoff equation,

$$\begin{aligned} a_\pi u_t + b_\pi x_{t-1} &= \beta E_t(a_\pi u_{t+1} + b_\pi x_t) + \kappa x_t + u_t, \\ &= (1 + \beta \rho_u a_\pi) u_t + (\beta b_\pi + \kappa)(a_x u_t + b_x x_{t-1}), \\ &= (1 + (\beta b_\pi + \kappa) a_x + \beta \rho_u a_\pi) u_t + (\beta b_\pi + \kappa) b_x x_{t-1}. \end{aligned}$$

$$\begin{aligned} a_\pi u_t + b_\pi x_{t-1} &= -(\vartheta/\kappa)(x_t - x_{t-1}), \\ &= -(\vartheta/\kappa)(a_x u_t + (b_x - 1)x_{t-1}). \end{aligned}$$

Solving for the policy function under commitment, cont'd

- Then we have

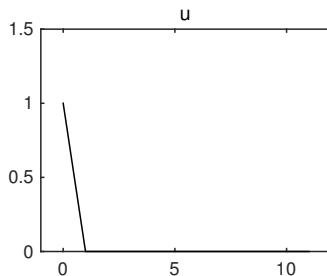
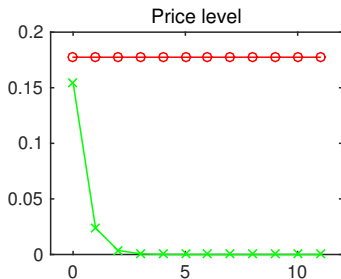
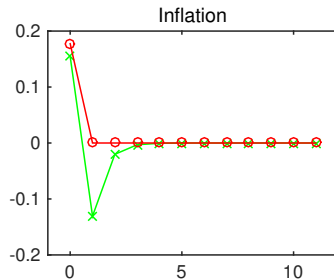
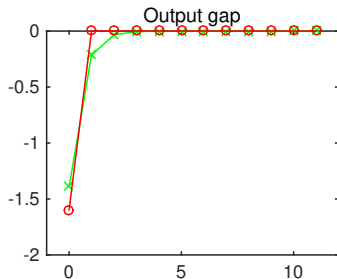
$$\begin{aligned}a_{\pi} &= -(\vartheta/\kappa)a_x \\b_{\pi} &= (\vartheta/\kappa)(1 - b_x) \\a_{\pi} &= 1 + (\beta b_{\pi} + \kappa)a_x + \beta \rho_u a_{\pi}, \\b_{\pi} &= (\beta b_{\pi} + \kappa)b_x\end{aligned}$$

- These can be solved for

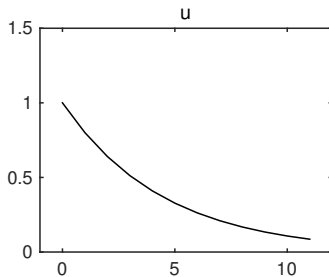
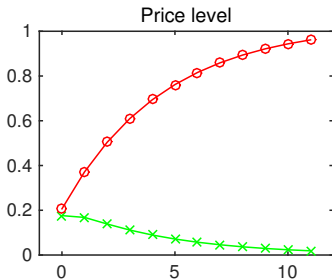
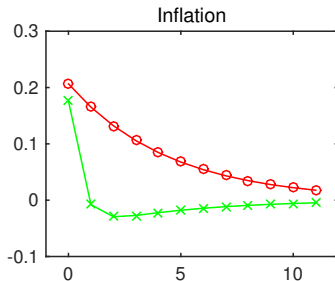
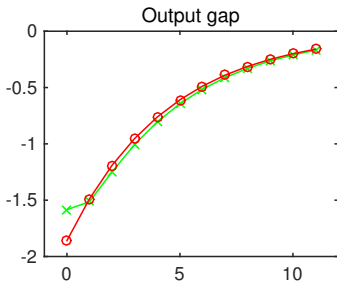
$$\begin{aligned}a_x &= -(\kappa/\vartheta)/[\beta(\delta^+ - \rho_u)], & a_{\pi} &= 1/[\beta(\delta^+ - \rho_u)], \\b_x &= \delta^- \in (0, 1), & b_{\pi} &= (\vartheta/\kappa)(1 - \delta^-).\end{aligned}$$

where $\delta^{\pm} = \left(1 \pm \sqrt{1 - 4\beta\gamma^2}\right) / (2\beta\gamma)$ is the solution of a quadratic equation $\beta\gamma\delta^2 - \delta + \gamma = 0$ where $\gamma = (1 + \beta + \kappa^2/\vartheta)^{-1}$.

Discretion vs. Commitment: $\rho_u = 0.0$ (when $\alpha = 0$)



Discretion vs. Commitment: $\rho_u = 0.8$ (when $\alpha = 0$)



The case of a distorted steady state

- Consider the case in which there is a permanent gap $x \equiv y^n - y^e$. Specifically,

$$-\frac{U_n}{U_c} = (1 - \Phi)MPN,$$

where $\Phi \geq 0$ measures the wedge between the marginal product of labor and marginal rate of substitution.

- For example, monopolistic competition and associated markup is a source of the distortion, $\Phi \equiv 1 - [(1 - \tau)\mathcal{M}]^{-1} \geq 0$.

Second-order approximation to the household utility

- Note that $-U_n/U_c = (1 - \Phi)MPN = (1 - \Phi)C/N$ implies $(1 - \Phi)U_c C = -U_n N$. Then we have

$$\begin{aligned}\frac{U_t - U}{U_c C} &\simeq \left(\hat{y}_t(1 + z_t) + \frac{1 - \sigma}{2} \hat{y}_t^2 \right) \\ &\quad - (1 - \Phi) \left(\hat{y}_t(1 + z_t) + d_t + \frac{1 + \varphi}{2} (\hat{y}_t - a_t)^2 \right) + t.i.p., \\ &= \Phi \left(\hat{y}_t(1 + z_t) + d_t + \frac{1 + \varphi}{2} (\hat{y}_t - a_t)^2 \right) \\ &\quad - \left(d_t + \frac{\sigma + \varphi}{2} \hat{y}_t^2 + (\sigma + \varphi) \hat{y}_t a_t \right) + t.i.p., \\ &= \Phi \hat{y}_t - \left(d_t + \frac{\sigma + \varphi}{2} \hat{y}_t^2 + (\sigma + \varphi) \hat{y}_t a_t \right) + t.i.p.,\end{aligned}$$

Under the small distortion assumption, the product of Φ with second-order terms is negligible. Also, $\Phi \hat{y}_t$ can be considered as a second-order term.

The CB's problem: The case of small SS distortions

- Under the small distortion assumption, the welfare loss function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} (\pi_t^2 + \vartheta \hat{x}_t^2) - \Lambda \hat{x}_t \right],$$

where $\Lambda \equiv \Phi \lambda / \epsilon > 0$ and $\hat{x}_t \equiv x_t - x$ with $x \equiv y^n - y^e$.

- Similarly, the NKPC can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + u_t.$$

- Each period the CB minimizes the period losses

$$\frac{1}{2} (\pi_t^2 + \vartheta \hat{x}_t^2) - \Lambda \hat{x}_t,$$

subject to the constraint

$$\pi_t = \kappa \hat{x}_t + \nu_t,$$

where $\nu_t \equiv \beta E_t \pi_{t+1} + u_t$ is taken as given by the central bank.

Optimal Discretionary Policy, cont'd

- The optimality condition is given by

$$\hat{x}_t = \frac{\Lambda}{\vartheta} - \frac{\kappa}{\vartheta} \pi_t,$$

for $t = 0, 1, 2, \dots$

- Substituting it into the NKPC and we have

$$\begin{aligned}\pi_t &= \frac{\Lambda\kappa}{\kappa^2 + \vartheta(1 - \beta)} + \frac{\vartheta}{\kappa^2 + \vartheta(1 - \beta\rho)} u_t, \\ \hat{x}_t &= \frac{\Lambda(1 - \beta)}{\kappa^2 + \vartheta(1 - \beta)} - \frac{\kappa}{\kappa^2 + \vartheta(1 - \beta\rho)} u_t.\end{aligned}$$

The constant terms are known as [the inflation bias](#).

Optimal Commitment Policy

- The Lagrangian is set up as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} (\pi_t^2 + \vartheta \hat{x}_t^2) - \Lambda \hat{x}_t + \xi_t (\pi_t - \beta E_t \pi_{t+1} - \kappa \hat{x}_t) \right],$$

where $\{\xi_t\}$ is a sequence of Lagrange multipliers.

- The FONCs are

$$\vartheta x_t - \kappa \xi_t - \Lambda = 0,$$

$$\pi_t + \xi_t - \xi_{t-1} = 0,$$

for $t = 0, 1, 2, \dots$ and $\xi_{-1} = 0$.

Optimal Commitment Policy, cont'd

- Combining the two, we have

$$\hat{x}_0 = -\frac{\kappa}{\vartheta}\pi_0 + \frac{\Lambda}{\vartheta},$$

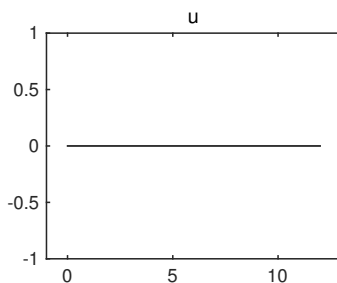
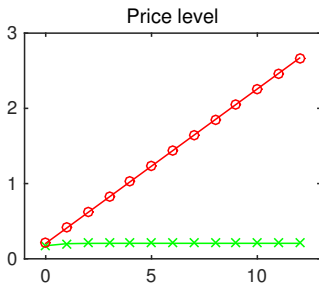
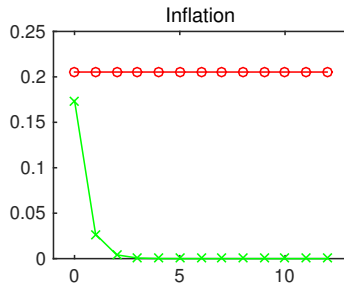
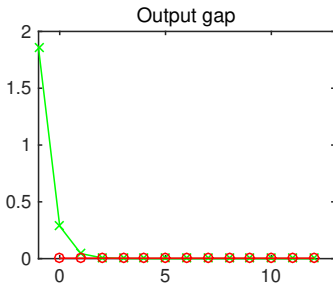
and

$$\hat{x}_t - \hat{x}_{t-1} = -\frac{\kappa}{\vartheta}\pi_t$$

for $t = 1, 2, 3, \dots$

- Note that the equilibrium conditions has the same form in the case of the efficient steady state except for $t = 0$.
- $\hat{x}_{-1} = \Lambda/\vartheta$ is given as the initial condition.

Initial dynamics



The zero lower bound on nominal interest rates

- In our cashless economy, money has no nominal payoffs but otherwise is identical to short-term nominal debt. This fact may impose the zero lower bound (ZLB) on the nominal return of such debt:

$$i_t \geq 0.$$

The zero lower bound on nominal interest rates

- Then the CB minimizes

$$E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2),$$

subject to

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t, \\ x_t &\leq x_{t+1} + \sigma^{-1} (\pi_{t+1} + r_t^n).\end{aligned}$$

The last equation is an **occasionally binding constraint**. The equality holds when $i_t = 0$.

The natural rate and the ZLB

- We assume the natural rate r_t^n follows an exogenous deterministic path:
 - In the steady state, $r_t^n = \rho$.
 - For $t = 0, 1, \dots, t_Z$, it **unexpectedly** drops to and remains at $r_t^n = -\epsilon < 0$.
 - From period $t_Z + 1$ onward, it reverts to $r_t^n = \rho$ again.
- We assume **the perfect foresight** (and get rid of expectational operators hereafter). Agents know the subsequent path of the natural rate in period 0.

The natural rate and the ZLB

- Whenever $r_t^n < 0$, the efficient allocation, implied by $i_t = r_t^n < 0$, is no longer attainable, as the nominal rate is stacked at $i_t = 0$.
- Note that the ZLB and the dynamic IS curve imply

$$\sigma^{-1}i_t = -x_t + x_{t+1} + \sigma^{-1}(\pi_{t+1} + r_t^n) \geq 0.$$

Optimal Discretionary Policy

- Each period the CB minimizes the period losses

$$\pi_t^2 + \vartheta x_t^2,$$

subject to the constraint

$$\pi_t = \kappa x_t + \nu_{0,t},$$

$$x_t \leq \nu_{1,t},$$

where $\nu_{0,t} \equiv \beta E_t \pi_{t+1}$ and $\nu_{1,t} \equiv x_{t+1} + \sigma^{-1}(\pi_{t+1} + r_t^n)$ are taken as given.

Optimal Discretionary Policy, cont'd

- The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2} (\pi_t^2 + \vartheta x_t^2) + \xi_{1,t} (\pi_t - \kappa x_t - \nu_{1,t}) + \xi_{2,t} (x_t - \nu_{2,t}).$$

- The FONCs are

$$\begin{aligned} x_t + \xi_{1,t} &= 0, \\ \vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} &= 0, \end{aligned}$$

and the slackness conditions

$$\xi_{2,t} \geq 0, \quad i_t \geq 0, \quad \xi_{2,t} i_t = 0.$$

for $t = 0, 1, 2, \dots$

The solution

- From $t = t_Z + 1$ onward, $i_t = \rho > 0$, $\xi_{2,t} = 0$, and $x_t = \pi_t = 0$ hold.
- For $t = 0, 1, \dots, t_Z$, $i_t = 0$, $\xi_{2,t} > 0$, and $\vartheta x_t = -\kappa\pi_t - \xi_{2,t}$ hold.
- For π_t and x_t , we can solve the two equations backward

$$\begin{aligned}\pi_t &= \beta\pi_{t+1} + \kappa x_t, \\ x_t &= x_{t+1} + \sigma^{-1}(\pi_{t+1} + r_t^n),\end{aligned}$$

from $t = t_Z, t_Z - 1, \dots, 0$, given $x_{t_Z+1} = \pi_{t_Z+1} = 0$.

Optimal Commitment Policy

- The CB will choose a state-contingent sequence $\{x_t, \pi_t\}_{t=0}^{\infty}$ so as to minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2),$$

subject to

$$\begin{aligned}\pi_t &= \beta \pi_{t+1} + \kappa x_t, \\ x_t &\leq x_{t+1} + \sigma^{-1} (\pi_{t+1} + r_t^n).\end{aligned}$$

Optimal Commitment Policy, cont'd

- It is useful to set up the Lagrangian as

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t & \left[\frac{1}{2} (\pi_t^2 + \vartheta x_t^2) + \xi_{1,t} (\pi_t - \beta \pi_{t+1} - \kappa x_t) \right. \\ & \left. + \xi_{2,t} (x_t - x_{t+1} - \sigma^{-1} (\pi_{t+1} + r_t^n)) \right].\end{aligned}$$

- The FONCs are

$$\partial \pi_t : \pi_t + \xi_{1,t} - \xi_{1,t-1} - (\beta \sigma)^{-1} \xi_{2,t-1} = 0,$$

$$\partial x_t : \vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} - \beta^{-1} \xi_{2,t-1} = 0,$$

and the slackness conditions

$$\xi_{2,t} \geq 0, \quad i_t \geq 0, \quad \xi_{2,t} i_t = 0.$$

for $t = 0, 1, 2, \dots$, given the initial conditions $\xi_{1,-1} = \xi_{2,-1} = 0$.

A conjectured solution

- Under the commitment policy, the CB wants to overshoot inflation and output by remaining at the ZLB for a longer period $t_C \geq t_Z$ than under the discretion policy.
- The solution is conjectured and verified as follows;
 - 1 From $t = 0$ to $t_C \geq t_Z$: $i_t = 0$ and $\xi_{2,t} > 0$.
 - 2 For $t = t_C + 1$: $i_t > 0$: $\xi_{2,t} = \xi_{2,t_C+1} = 0$ and $\xi_{2,t-1} = \xi_{2,t_C} > 0$.
 - 3 From $t = t_C + 2$ and onward: $i_t > 0$ and $\xi_{2,t} = \xi_{2,t-1} = 0$,

$t = t_C + 2$ and onward

- Note that $i_t > 0$ and $\xi_{2,t} = \xi_{2,t-1} = 0$, then from the FONCs,

$$\pi_t + \xi_{1,t} - \xi_{1,t-1} = 0,$$

$$\vartheta x_t - \kappa \xi_{1,t} = 0,$$

$$\pi_t = \beta \pi_{t+1} + \kappa x_t,$$

hold, given the initial condition ξ_{1,t_C+1} . The dynamic IS equation is irrelevant as $i_t > 0$.

- These equations can be solved for

$$x_{t_C+2+k} = -\frac{\kappa \delta^{k+1}}{\vartheta} \xi_{1,t_C+1},$$

$$\pi_{t_C+2+k} = (1 - \delta) \delta^k \xi_{1,t_C+1}.$$

for $k = 0, 1, 2, \dots$ $\{x_{t_C+2+k}, \pi_{t_C+2+k}\}$ can be obtained once we find ξ_{1,t_C+1} .

$$t = t_C + 1$$

- Note that $i_{t_C+1} > 0$ and $\xi_{2,t_C+1} = 0$, then from the FONCs,

$$\begin{aligned}\pi_{t_C+1} + \xi_{1,t_C+1} - \xi_{1,t_C} - (\beta\sigma)^{-1}\xi_{2,t_C} &= 0, \\ \vartheta x_{t_C+1} - \kappa\xi_{1,t_C+1} - \beta^{-1}\xi_{2,t_C} &= 0, \\ \pi_{t_C+1} &= \underbrace{\beta(1-\delta)\xi_{1,t_C+1}}_{=\pi_{t_C+2}} + \kappa x_{t_C+1},\end{aligned}$$

hold, given $(\xi_{1,t_C}, \xi_{2,t_C})$.

- By substituting out $\xi_{1,t_C+1} = [\beta(1-\delta)]^{-1}(\pi_{t_C+1} - \kappa x_{t_C+1})$ from the last equation, we solve

$$\begin{aligned}[1 + \beta(1-\delta)]\pi_{t_C+1} - \kappa x_{t_C+1} - \beta(1-\delta)\xi_{1,t_C} - [(1-\delta)/\sigma]\xi_{2,t_C} &= 0, \\ [\beta(1-\delta)\vartheta + \kappa^2]x_{t_C+1} - \kappa\pi_{t_C+1} - (1-\delta)\xi_{2,t_C} &= 0.\end{aligned}$$

for (x_{t_C+1}, π_{t_C+1}) .

$$t = 0, 1, \dots, t_Z, \dots, t_C$$

- Note that $i_t = 0$, then

$$\begin{aligned}\pi_t &= \beta\pi_{t+1} + \kappa x_t, \\ x_t &= x_{t+1} + \sigma^{-1}(\pi_{t+1} + r_t^n),\end{aligned}$$

hold, where

$$r_t^n = \begin{cases} -\rho, & \text{when } t = 0, 1, \dots, t_Z, \\ \rho, & \text{when } t = t_Z + 1, \dots, t_C. \end{cases}$$

- In addition, the original FONCs

$$\begin{aligned}\pi_t + \xi_{1,t} - \xi_{1,t-1} - (\beta\sigma)^{-1}\xi_{2,t-1} &= 0, \\ \vartheta x_t - \kappa\xi_{1,t} + \xi_{2,t} - \beta^{-1}\xi_{2,t-1} &= 0,\end{aligned}$$

hold for $t = 0, \dots, t_C + 1$, given the initial conditions $\xi_{1,-1} = \xi_{2,-1} = 0$.

The algorithm for the solution

- The algorithm is as follows:
 - 1 Fix $t_C \geq t_Z$. Solve $4 \times (t_C + 2)$ equations for $4 \times (t_C + 2)$ variables $\{x_t, \pi_t, \xi_{1,t}, \xi_{2,t}\}_{t=0}^{t_C+1}$ for $t = 0, 1, \dots, t_C + 1$.
 - 2 Obtain $x_{t_C+2+k} = -\frac{\kappa\delta^{k+1}}{\vartheta}\xi_{1,t_C+1}$ and $\pi_{t_C+2+k} = (1 - \delta)\delta^k\xi_{1,t_C+1}$ for $k = 0, 1, \dots$, given ξ_{1,t_C+1} at hand.
 - 3 Check $i_t = r_t^n + \pi_{t+1} + \sigma(x_{t+1} - x_t) = 0$ for $t = 0, 1, \dots, t_C$ and $i_t > 0$ $t = t_C + 1, \dots$. If not, increase t_C and do 1-3 again.

The relevant equations

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} r_t^n,$$

for $t = 0, 1, \dots, t_C$, and

$$\begin{bmatrix} x_{t_C+1} \\ \pi_{t_C+1} \end{bmatrix} = \underbrace{\begin{bmatrix} -\kappa & 1 + \beta(1 - \delta) \\ \beta(1 - \delta) + \frac{\kappa^2}{\vartheta} & -\frac{\kappa}{\vartheta} \end{bmatrix}}_{\mathbf{M}}^{-1} \begin{bmatrix} \beta(1 - \delta) & \frac{1 - \delta}{\frac{\sigma}{\vartheta}} \\ 0 & \frac{1 - \delta}{\frac{\sigma}{\vartheta}} \end{bmatrix} \times \begin{bmatrix} \xi_{1,t_C} \\ \xi_{2,t_C} \end{bmatrix}.$$

$$\begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & (\beta\sigma)^{-1} \\ \kappa & \beta^{-1}(1 + \kappa\sigma^{-1}) \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} \xi_{1,t-1} \\ \xi_{2,t-1} \end{bmatrix} - \underbrace{\begin{bmatrix} 0 & 1 \\ \vartheta & \kappa \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix},$$

for $t = 0, 1, \dots, t_C + 1$.

The relevant equations

- Or, equivalently,

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} - \mathbf{A} \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{r_t^n}{\sigma} \\ \frac{\kappa T_t^n}{\sigma} \end{bmatrix},$$

for $t = 0, 1, \dots, t_C$, and

$$-\mathbf{M} \begin{bmatrix} \xi_{1,t_C} \\ \xi_{2,t_C} \end{bmatrix} + \begin{bmatrix} x_{t_C+1} \\ \pi_{t_C+1} \end{bmatrix} = 0,$$

for $t = t_C + 1$, and

$$-\mathbf{H} \begin{bmatrix} \xi_{1,t-1} \\ \xi_{2,t-1} \end{bmatrix} + \mathbf{J} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix} = 0,$$

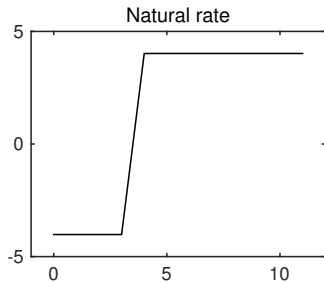
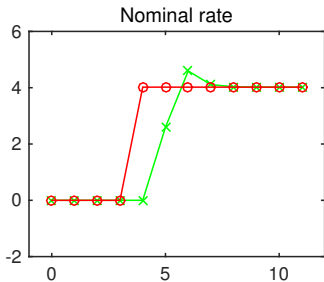
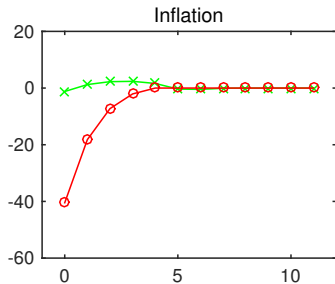
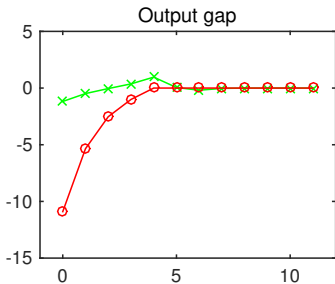
for $t = 0, 1, \dots, t_C + 1$. Note that $\xi_{1,-1} = \xi_{2,-1} = 0$.

The relevant equations, cont'd

- These equations can be stacked into

$$\begin{bmatrix} I_2 & & -\mathbf{A} \\ \mathbf{J} & I_2 & \\ & -\mathbf{H} & I_2 \\ & & \ddots \\ & & & \ddots \\ & & & & I_2 \\ & & & & -\mathbf{A} \\ & & & & & \ddots \\ & & & & & & \ddots \\ & & & & & & & I_2 \\ & & & & & & & -\mathbf{M} \\ & & & & & & & -\mathbf{H} \\ & & & & & & & & I_2 \\ & & & & & & & & \mathbf{J} \\ & & & & & & & & & I_2 \end{bmatrix} \begin{bmatrix} x_0 \\ \pi_0 \\ \xi_{1,0} \\ \xi_{2,0} \\ x_1 \\ \pi_1 \\ \xi_{1,1} \\ \xi_{2,1} \\ \vdots \\ \vdots \\ x_{t_C} \\ \pi_{t_C} \\ \xi_{1,t_C} \\ \xi_{2,t_C} \\ x_{t_C+1} \\ \pi_{t_C+1} \\ \xi_{1,t_C+1} \\ \textcolor{red}{0} \end{bmatrix} = \begin{bmatrix} \frac{r_0^n}{\sigma} \\ \frac{\kappa r_0^n}{\sigma} \\ 0 \\ 0 \\ \frac{r_1^n}{\sigma} \\ \frac{\kappa r_1^n}{\sigma} \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \frac{r_{t_C}^n}{\sigma} \\ \frac{\kappa r_{t_C}^n}{\sigma} \\ 0 \\ 0 \\ 0 \\ 0 \\ \textcolor{red}{0} \\ 0 \\ 0 \end{bmatrix}$$

Discretion vs. Commitment with the ZLB (when $\alpha = 0$)



- We have assumed the perfect foresight.
- **Uncertainty** about the natural rate matters. We need to solve the model with stochastic settings (e.g., Adam and Billi, 2006; 2007, Nakov, 2008. Also see Nakata, 2017).

Final Project

- Choose *either of* the following papers and replicate their results.
 - Write down the model, the equilibrium conditions, and (if necessary) other results showing that your replication is reasonable.
- 1 [Medium] Christiano and Eichenbaum (1992, AER): “Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuation.” Replicate Table 3 in p. 443.
 - 2 [Hard] Steinsson (2003, JME; also see Erratum): “Optimal monetary policy in an economy with inflation persistence.” Replicate **Figure 1-9 in Erratum**.
 - 3 [Very Hard] Adam and Billi (2007, JME): “Discretionary monetary policy and the zero lower bound on nominal interest rates.” Replicate Figure 5 and 9.