Monetary Policy Tradeoffs: Discretion vs. Commitment

Takeki Sunakawa

Quantitative Methods for Monetary Economics
University of Mannheim
May 2018

Introduction

- The efficient allocation is obtained under fully flexible prices. The optimal policy fully stabilizes the price level.
- In practice, central banks face short-run tradeoffs: inflation vs. real variables such as output and employment.
- We need a monetary policy design in environment in which the central bank faces a nontrivial tradeoff.

The case of an efficient steady state

- Consider a situation in which the flexible price equilibrium allocation is inefficient. The natural level of output y_t^n deviates from its efficient counterpart y_t^e in the short run.
- Some real imperfections generates a time-varying gap $u_t \equiv y_t^n y_t^e$ even in the absence of price rigidities.

The CB's problem

The central bank (CB hereafter) will minimize the welfare loss function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2 \right),\,$$

where $x_t \equiv y_t - y_t^e$ is the welfare-relevant output gap with y_t^e denoting the efficient level of output. $\vartheta = \kappa/\epsilon$ is the weight of output gap fluctuations.

Minimization is subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t,$$

where $u_t \equiv \kappa(y_t^e - y_t^n)$. [Note that $y_t - y_t^n = \underbrace{(y_t - y_t^e)}_{x_t} + \underbrace{(y_t^e - y_t^n)}_{u_t}$.]

Cost-push shock

• u_t follows the exogenous AR(1) process

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u,$$

where $\rho_u \in [0,1)$ and ε_t^u is white noise with variance σ_u^2 .

• u_t generates a tradeoff between stabilizing π_t and stabilizing x_t .

Optimal Discretionary Policy

Each period the CB minimizes the period losses

$$\pi_t^2 + \vartheta x_t^2,$$

subject to the constraint

$$\pi_t = \kappa x_t + \nu_t,$$

where $\nu_t \equiv \beta E_t \pi_{t+1} + u_t$ is taken as given by the central bank, as there are no endogenous state variables.

Optimal Discretionary Policy, cont'd

The optimality condition is given by

$$x_t = -\frac{\kappa}{\vartheta}\pi_t,$$

for t = 0, 1, 2,

• Substituting it into the NKPC and after some manipulation (e.g., undetermined coefficient method), we have

$$\pi_t = \frac{\vartheta}{\kappa^2 + \vartheta(1 - \beta \rho)} u_t,$$

$$\pi_t = -\frac{\kappa}{\kappa^2 + \vartheta(1 - \beta \rho)} u_t.$$

Optimal Commitment Policy

• Now we assume that the CB is able to commit to future policies. The CB will choose a state-contingent sequence $\{x_t, \pi_t\}_{t=0}^{\infty}$ so as to minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2 \right),\,$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t,$$

and u_t follows the AR(1) process:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u.$$

Optimal Commitment Policy, cont'd

• It is useful to set up the Lagrangian as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \vartheta x_t^2 \right) + \xi_t \left(\pi_t - \beta E_t \pi_{t+1} - \kappa x_t \right) \right],$$

where $\{\xi_t\}$ is a sequence of Lagrange multipliers.

The FONCs are

$$\vartheta x_t - \kappa \xi_t = 0,$$

$$\pi_t + \xi_t - \xi_{t-1} = 0,$$

for t=0,1,2,..., given $\xi_{-1}=0$, because there is no commitment in period 0.

Optimal Commitment Policy, cont'd

• Combining the two, we have

$$x_0 = -\frac{\kappa}{\vartheta}\pi_0,$$

and

$$x_t - x_{t-1} = -\frac{\kappa}{\vartheta} \pi_t$$

for t = 1, 2, 3,

Solving for the policy function under commitment

- We assume: $x_t = a_x u_t + b_x x_{t-1}$ and $\pi_t = a_\pi u_t + b_\pi x_{t-1}$ with the initial condition $x_{-1} = 0$.
- Substitute them into the NKPC and the tradeoff equation,

$$\begin{array}{rcl} a_{\pi}u_{t} + b_{\pi}x_{t-1} & = & \beta E_{t}(a_{\pi}u_{t+1} + b_{\pi}x_{t}) + \kappa x_{t} + u_{t}, \\ & = & (1 + \beta \rho_{u}a_{\pi})u_{t} + (\beta b_{\pi} + \kappa)(a_{x}u_{t} + b_{x}x_{t-1}), \\ & = & (1 + (\beta b_{\pi} + \kappa)a_{x} + \beta \rho_{u}a_{\pi})u_{t} + (\beta b_{\pi} + \kappa)b_{x}x_{t-1}. \end{array}$$

$$a_{\pi}u_{t} + b_{\pi}x_{t-1} = -(\vartheta/\kappa)(x_{t} - x_{t-1}),$$

= $-(\vartheta/\kappa)(a_{x}u_{t} + (b_{x} - 1)x_{t-1}).$



Solving for the policy function under commitment, cont'd

• Then we have

$$a_{\pi} = -(\vartheta/\kappa)a_{x}$$

$$b_{\pi} = (\vartheta/\kappa)(1 - b_{x})$$

$$a_{\pi} = 1 + (\beta b_{\pi} + \kappa)a_{x} + \beta \rho_{u}a_{\pi},$$

$$b_{\pi} = (\beta b_{\pi} + \kappa)b_{x}$$

These can be solved for

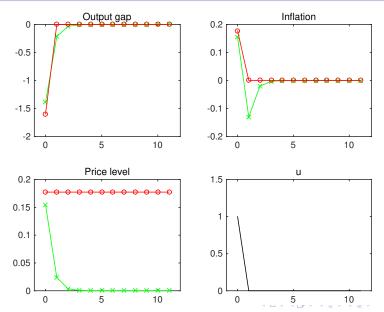
$$a_x = -(\kappa/\vartheta)/[\beta(\delta^+ - \rho_u)], \qquad a_\pi = 1/[\beta(\delta^+ - \rho_u)],$$

$$b_x = \delta^- \in (0, 1), \qquad b_\pi = (\vartheta/\kappa)(1 - \delta^-).$$

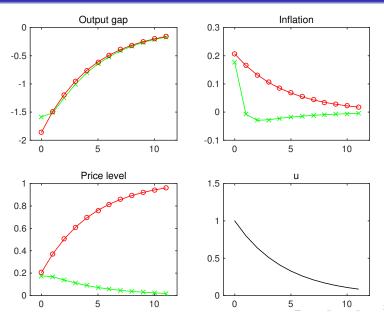
where $\delta^{\pm} = \left(1 \pm \sqrt{1 - 4\beta\gamma^2}\right)/(2\beta\gamma)$ is the solution of a quadratic equation $\beta\gamma\delta^2 - \delta + \gamma = 0$ where $\gamma = (1 + \beta + \kappa^2/\vartheta)^{-1}$.



Discretion vs. Commitment: $\rho_u = 0.0$ (when $\alpha = 0$)



Discretion vs. Commitment: $\rho_u = 0.8$ (when $\alpha = 0$)



The case of a distorted steady state

 \bullet Consider the case in which there is a permanent gap $x\equiv y^n-y^e.$ Specifically,

$$-\frac{U_n}{U_c} = (1 - \Phi)MPN,$$

where $\Phi \geq 0$ measures the wedge between the marginal product of labor and marginal rate of substitution.

• For example, monopolistic competition and associated markup is a source of the distortion, $\Phi \equiv 1 - [(1-\tau)\mathcal{M}]^{-1} \geq 0$.

Second-order approximation to the household utility

• Note that $-U_n/U_c=(1-\Phi)MPN=(1-\Phi)C/N$ implies $(1-\Phi)U_cC=-U_nN.$ Then we have

$$\begin{split} \frac{U_t - U}{U_c C} & \simeq & \left(\hat{y}_t (1 + z_t) + \frac{1 - \sigma}{2} \hat{y}_t^2 \right) \\ & - (1 - \Phi) \left(\hat{y}_t (1 + z_t) + d_t + \frac{1 + \varphi}{2} (\hat{y}_t - a_t)^2 \right) + t.i.p., \\ & = & \Phi \left(\hat{y}_t (1 + z_t) + d_t + \frac{1 + \varphi}{2} (\hat{y}_t - a_t)^2 \right) \\ & - \left(d_t + \frac{\sigma + \varphi}{2} \hat{y}_t^2 + (\sigma + \varphi) \hat{y}_t a_t \right) + t.i.p., \\ & = & \Phi \hat{y}_t - \left(d_t + \frac{\sigma + \varphi}{2} \hat{y}_t^2 + (\sigma + \varphi) \hat{y}_t a_t \right) + t.i.p., \end{split}$$

Under the small distortion assumption, the product of Φ with second-order terms is negligible. Also, $\Phi \hat{y}_t$ can be considered as a second-order term.

Quant Money Econ

The CB's problem: The case of small SS distortions

• Under the small distortion assumption, the welfare loss function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \vartheta \hat{x}_t^2 \right) - \Lambda \hat{x}_t \right],$$

where $\Lambda \equiv \Phi \lambda / \epsilon > 0$ and $\hat{x}_t \equiv x_t - x$ with $x \equiv y^n - y^e$.

• Similarly, the NKPC can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + u_t.$$

Optimal Discretionary Policy

• Each period the CB minimizes the period losses

$$\frac{1}{2} \left(\pi_t^2 + \vartheta \hat{x}_t^2 \right) - \Lambda \hat{x}_t,$$

subject to the constraint

$$\pi_t = \kappa \hat{x}_t + \nu_t,$$

where $\nu_t \equiv \beta E_t \pi_{t+1} + u_t$ is taken as given by the central bank.

Optimal Discretionary Policy, cont'd

The optimality condition is given by

$$\hat{x}_t = \frac{\Lambda}{\vartheta} - \frac{\kappa}{\vartheta} \pi_t,$$

for t = 0, 1, 2,

• Substituting it into the NKPC and we have

$$\begin{array}{rcl} \pi_t & = & \frac{\Lambda \kappa}{\kappa^2 + \vartheta(1-\beta)} + \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho)} u_t, \\ \hat{x}_t & = & \frac{\Lambda(1-\beta)}{\kappa^2 + \vartheta(1-\beta)} - \frac{\kappa}{\kappa^2 + \vartheta(1-\beta\rho)} u_t. \end{array}$$

The constant terms are known as the inflation bias.



Optimal Commitment Policy

The Lagrangian is set up as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \vartheta \hat{x}_t^2 \right) - \Lambda \hat{x}_t + \xi_t \left(\pi_t - \beta E_t \pi_{t+1} - \kappa \hat{x}_t \right) \right],$$

where $\{\xi_t\}$ is a sequence of Lagrange multipliers.

• The FONCs are

$$\vartheta x_t - \kappa \xi_t - \Lambda = 0,$$

$$\pi_t + \xi_t - \xi_{t-1} = 0,$$

for $t = 0, 1, 2, \dots$ and $\xi_{-1} = 0$.

Optimal Commitment Policy, cont'd

Combining the two, we have

$$\hat{x}_0 = -\frac{\kappa}{\vartheta}\pi_0 + \frac{\Lambda}{\vartheta},$$

and

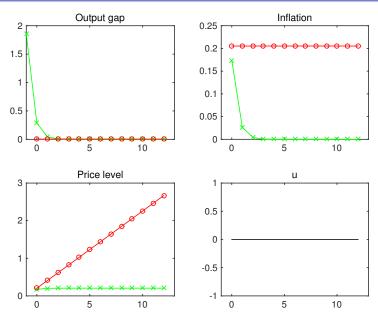
$$\hat{x}_t - \hat{x}_{t-1} = -\frac{\kappa}{\vartheta} \pi_t$$

for t = 1, 2, 3,

- Note that the equilibrium conditions has the same form in the case of the efficient steady state except for t=0.
- $\hat{x}_{-1} = \Lambda/\vartheta$ is given as the initial condition.



Initial dynamics



The zero lower bound on nominal interest rates

 Money has no nominal payoffs but otherwise is identical to short-term nominal debt. This fact may impose the zero lower bound (ZLB) on the nominal return of such debt:

$$i_t \geq 0$$
.

Then the CB minimizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2 \right),\,$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,$$

$$x_t \leq x_{t+1} + \sigma^{-1} (\pi_{t+1} + r_t^n).$$

The last equation is an occasionally binding constraint. The equality holds when $i_t=0$.

The natural rate and the ZLB

- We assume the natural rate r^n_t follows an exogenous deterministic path: In the steady state, $r^n_t = \rho$. It unexpectedly drops to and remains at $r^n_t = -\epsilon < 0$ for $t = 0, 1, ..., t_Z$. From period $t_Z + 1$ onward, it reverts to $r^n_t = \rho$ again.
- We assume the perfect foresight (and get rid of expectational operators hereafter). Agents know the subsequent path of the natural rate in period 0.
- Whenever $r_t^n < 0$, the efficient allocation, implied by $i_t = r_t^n$, is no longer attainable.

Optimal Discretionary Policy

Each period the CB minimizes the period losses

$$\pi_t^2 + \vartheta x_t^2,$$

subject to the constraint

$$\pi_t = \kappa x_t + \nu_{0,t},
x_t \leq \nu_{1,t},$$

where $\nu_{0,t} \equiv \beta E_t \pi_{t+1}$ and $\nu_{1,t} \equiv x_{t+1} + \sigma^{-1}(\pi_{t+1} + r_t^n)$ are taken as given.

Optimal Discretionary Policy, cont'd

• The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2} \left(\pi_t^2 + \vartheta x_t^2 \right) + \xi_{1,t} \left(\pi_t - \kappa x_t - \nu_{1,t} \right) + \xi_{2,t} \left(x_t - \nu_{2,t} \right).$$

The FONCs are

$$x_t + \xi_{1,t} = 0,$$

$$\vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} = 0,$$

and the slackness conditions

$$\xi_{2,t} \ge 0, \quad i_t \ge 0, \quad \xi_{2,t} i_t = 0.$$

for t = 0, 1, 2,



The solution

- From $t=t_Z+1$ onward, $i_t=\rho>0$, $\xi_{2,t}=0$, and $x_t=\pi_t=0$ hold.
- For $t = 0, 1, ..., t_Z$, $i_t = 0$, $\xi_{2,t} > 0$, and $\vartheta x_t = -\kappa \pi_t \xi_{2,t}$ hold.
- Then we can solve the two equations backward

$$\pi_t = \beta \pi_{t+1} + \kappa x_t,
x_t = x_{t+1} + \sigma^{-1} (\pi_{t+1} + r_t^n),$$

from $t = t_Z, t_Z - 1, ..., 0$, given $x_{t_{Z+1}} = \pi_{t_{Z+1}} = 0$.

Optimal Commitment Policy

 \bullet The CB will choose a state-contingent sequence $\{x_t,\pi_t\}_{t=0}^{\infty}$ so as to minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2 \right),\,$$

subject to

$$\pi_t = \beta \pi_{t+1} + \kappa x_t,$$

$$x_t \leq x_{t+1} + \sigma^{-1} (\pi_{t+1} + r_t^n).$$

Optimal Commitment Policy, cont'd

It is useful to set up the Lagrangian as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \vartheta x_t^2 \right) + \xi_{1,t} \left(\pi_t - \beta \pi_{t+1} - \kappa x_t \right) + \xi_{2,t} \left(x_t - x_{t+1} - \sigma^{-1} \left(\pi_{t+1} + r_t^n \right) \right) \right].$$

The FONCs are

$$\pi_t + \xi_{1,t} - \xi_{1,t-1} - (\beta \sigma)^{-1} \xi_{2,t-1} = 0,$$

$$\vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} - \beta^{-1} \xi_{2,t-1} = 0,$$

and the slackness conditions

$$\xi_{2,t} \ge 0, \quad i_t \ge 0, \quad \xi_{2,t} i_t = 0.$$

for t = 0, 1, 2, ..., given the initial conditions $\xi_{1,-1} = \xi_{2,-1} = 0$.



A conjectured solution

- The solution is conjectured and verified as follows;
- $\bullet \ \, \text{From} \,\, t=0 \,\, \text{to} \,\, t_C \geq t_Z \,\, \text{,} \,\, i_t=0 \,\, \text{and} \,\, \xi_{2,t}>0.$
- ② For $t = t_C + 1$, $i_t = 0$, $\xi_{2,t} = 0$ and $\xi_{2,t} > 0$.
- $\textbf{ § From } t=t_C+2 \text{ and onward, } i_t>0 \text{ and } \xi_{2,t}=\xi_{2,t-1}=0,$

$t = t_C + 2$ and onward

• Note that $\xi_{2,t}=\xi_{2,t-1}=0$, then

$$\begin{split} \pi_t + \xi_{1,t} - \xi_{1,t-1} &= 0, \\ \vartheta x_t - \kappa \xi_{1,t} &= 0, \\ \pi_t &= \beta \pi_{t+1} + \kappa x_t, \end{split}$$

hold, given the initial condition ξ_{1,t_C+1} .

• These equations can be solved for

$$\begin{split} x_{t_C+2+k} &= -\frac{\kappa \delta^{k+1}}{\vartheta} \xi_{1,t_C+1}, \\ \pi_{t_C+2+k} &= (1-\delta) \delta^k \xi_{1,t_C+1}. \end{split}$$

for $k = 0, 1, 2, \dots$ [check by yourself]



$t = t_C + 1$

• Note that $i_{t_C+1} > 0$ and $\xi_{2,t_C+1} = 0$, then

$$\pi_{t_C+1} + \xi_{1,t_C+1} - \xi_{1,t_C} - (\beta \sigma)^{-1} \xi_{2,t_C} = 0,$$

$$\vartheta x_{t_C+1} - \kappa \xi_{1,t_C+1} - \beta^{-1} \xi_{2,t_C} = 0,$$

$$\pi_{t_C+1} = \beta \underbrace{(1 - \delta) \xi_{1,t_C+1}}_{=\pi_{t_C+2}} + \kappa x_{t_C+1},$$

hold.

• By substituting out $\xi_{1,t_C+1}=[\beta(1-\delta)]^{-1}(\pi_{t_C+1}-\kappa x_{t_C+1})$, we have

$$[1 + \beta(1 - \delta)]\pi_{t_C + 1} - \kappa x_{t_C + 1} - \beta(1 - \delta)\xi_{1, t_C} - [(1 - \delta)/\sigma]\xi_{2, t_C} = 0,$$

$$[\beta(1 - \delta)\vartheta + \kappa^2]x_{t_C + 1} - \kappa \pi_{t_C + 1} - (1 - \delta)\xi_{2, t_C} = 0.$$

$t=0,1,...,t_C$ and so on

• Note that $i_t = 0$, then

$$\pi_t = \beta \pi_{t+1} + \kappa x_t,
x_t = x_{t+1} + \sigma^{-1} (\pi_{t+1} + r_t^n),$$

hold, where

$$r_t^n = \begin{cases} -\rho, & \text{when } t = 0, 1, \dots, t_Z, \\ \rho, & \text{when } t = t_Z + 1, \dots, t_C. \end{cases}$$

• In addition, the original FONCs

$$\pi_t + \xi_{1,t} - \xi_{1,t-1} - (\beta \sigma)^{-1} \xi_{2,t-1} = 0,$$

$$\vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} - \beta^{-1} \xi_{2,t-1} = 0,$$

hold for $t=0,...,t_C+1$, given the initial conditions $\xi_{1,-1}=\xi_{2,-1}=0$.



The algorithm for the solution

- The algorithm is as follows:
- Fix $t_C \ge t_Z$. Solve $4 \times (t_C + 2)$ equations for $4 \times (t_C + 2)$ variables $\{x_t, \pi_t, \xi_{1,t} \xi_{2,t}\}_{t=0}^{t_C + 1}$.
- ② Obtain $x_{t_C+2+k} = -\frac{\kappa \delta^{k+1}}{\vartheta} \xi_{1,t_C+1}$ and $\pi_{t_C+2+k} = (1-\delta) \delta^k \xi_{1,t_C+1}$ for k=0,1,..., given ξ_{1,t_C+1} at hand.
- $\begin{array}{l} \bullet \quad \text{Check } i_t=r_t^n+\pi_{t+1}+\sigma(x_{t+1}-x_t)=0 \text{ for } t=0,1,...t_C \text{ and } i_t>0 \\ t=t_C+1,.... \text{ If not, increase } t_C \text{ and do 1-3 again.} \end{array}$

The relevant equations

$$\left[\begin{array}{c} x_t \\ \pi_t \end{array}\right] \quad = \quad \underbrace{\left[\begin{array}{cc} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{array}\right]}_{\mathbf{A}} \left[\begin{array}{c} x_{t+1} \\ \pi_{t+1} \end{array}\right] + \left[\begin{array}{c} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{array}\right] r_t^n,$$

for $t = 0, 1, ..., t_C$,

$$\begin{bmatrix} x_{t_C+1} \\ \pi_{t_C+1} \end{bmatrix} = \underbrace{\begin{bmatrix} -\kappa & 1 + \beta(1-\delta) \\ \beta(1-\delta) + \frac{\kappa^2}{\vartheta} & -\frac{\kappa}{\vartheta} \end{bmatrix}^{-1} \begin{bmatrix} \beta(1-\delta) & \frac{1-\delta}{\sigma} \\ 0 & \frac{1-\delta}{\vartheta} \end{bmatrix}}_{\mathbf{M}} \times \begin{bmatrix} \xi_{1,t_C} \\ \xi_{2,t_C} \end{bmatrix},$$

$$\begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & (\beta\sigma)^{-1} \\ \kappa & \beta^{-1}(1+\kappa\sigma^{-1}) \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} \xi_{1,t-1} \\ \xi_{2,t-1} \end{bmatrix} - \underbrace{\begin{bmatrix} 0 & 1 \\ \vartheta & \kappa \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix},$$

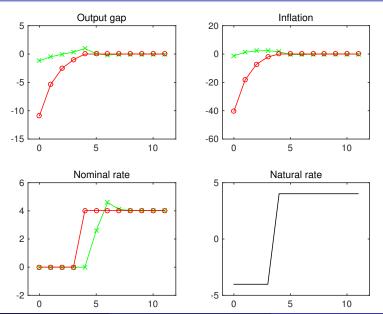
for $t = 0, 1, ..., t_C + 1$.



The relevant equations, cont'd

• These equations can be stacked into

Discretion vs. Commitment with the ZLB (when $\alpha = 0$)



Caveats

- We have assumed the perfect foresight.
- Uncertainty about the natural rate matters. We need to solve the model with stochastic settings (e.g., Adam and Billi, 2006; 2007, Nakov, 2008).

Final Project

- Choose either of the following papers and replicate their results.
- Write down the model, the equilibrium conditions, and (if necessary) other results showing that your replication is reasonable.
- Christiano and Eichenbaum (1992, AER): "Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuation." Replicate Table 3 in p. 443.
- ② Steinsson (2003, JME; also see Erratum): "Optimal monetary policy in an economy with inflation persistence." Replicate Figure 1-9 in Erratum.