Monetary Policy Tradeoffs: Discretion vs. Commitment

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Introduction

- The efficient allocation is obtained under fully flexible prices. The optimal policy fully stabilizes the price level.
- In practice, central banks face short-run tradeoffs: inflation vs. real variables such as output and employment.
- We need a monetary policy design in environment in which the central bank faces a nontrivial tradeoff.

The case of an efficient steady state

- Consider a situation in which the flexible price equilibrium allocation is inefficient. The natural level of output y_t^n deviates from its efficient counterpart y_t^e in the short run.

- Some real imperfections generates a time-varying gap $u_t \equiv y_t^n - y_t^e$ even in the absence of price rigidities.

The CB's problem

- The central bank (CB hereafter) will minimize the welfare loss function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2 \right),$$

where $x_t \equiv y_t - y_t^e$ is the welfare-relevant output gap with y_t^e denoting the efficient level of output. $\vartheta = \kappa/\epsilon$ is the weight of output gap fluctuations.

- Minimization is subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t,$$

where $u_t \equiv \kappa(y_t^e - y_t^n)$. [Note that $y_t - y_t^n = \underbrace{(y_t - y_t^e)}_{x_t} + \underbrace{(y_t^e - y_t^n)}_{u_t}$.]

Cost-push shock

- u_t follows the exogenous AR(1) process

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u,$$

where $\rho_u \in [0, 1)$ and ε_t^u is white noise with variance σ_u^2.

- u_t generates a tradeoff between stabilizing π_t and stabilizing x_t.

Optimal Discretionary Policy

- Each period the CB minimizes the period losses

$$\pi_t^2 + \vartheta x_t^2,$$

subject to the constraint

$$\pi_t = \kappa x_t + \nu_t,$$

where $\nu_t \equiv \beta E_t \pi_{t+1} + u_t$ is taken as given by the central bank, as there are no endogenous state variables.

Optimal Discretionary Policy, cont'd

- The optimality condition is given by

$$x_t = -\frac{\kappa}{\vartheta}\pi_t,$$

for $t = 0, 1, 2, ...$

- Substituting it into the NKPC and after some manipulation (e.g., undetermined coefficient method), we have

$$\begin{aligned} \pi_t &= \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho)} u_t, \\ \pi_t &= -\frac{\kappa}{\kappa^2 + \vartheta(1-\beta\rho)} u_t. \end{aligned}$$

Optimal Commitment Policy

- Now we assume that the CB is able to commit to future policies. The CB will choose a state-contingent sequence $\{x_t, \pi_t\}_{t=0}^{\infty}$ so as to minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right),$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t,$$

and u_t follows the AR(1) process:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u.$$

Optimal Commitment Policy, cont'd

- It is useful to set up the Lagrangian as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \vartheta x_t^2 \right) + \xi_t \left(\pi_t - \beta E_t \pi_{t+1} - \kappa x_t \right) \right],$$

where $\{\xi_t\}$ is a sequence of Lagrange multipliers.

- The FONCs are

$$\begin{aligned} \vartheta x_t - \kappa \xi_t &= 0, \\ \pi_t + \xi_t - \xi_{t-1} &= 0, \end{aligned}$$

for $t = 0, 1, 2, ...$, given $\xi_{-1} = 0$, because there is no commitment in period 0.

Optimal Commitment Policy, cont'd

- Combining the two, we have

$$x_0 = -\frac{\kappa}{\vartheta}\pi_0,$$

and

$$x_t - x_{t-1} = -\frac{\kappa}{\vartheta}\pi_t$$

for $t = 1, 2, 3, ...$

Solving for the policy function under commitment

- We assume: $x_t = a_x u_t + b_x x_{t-1}$ and $\pi_t = a_\pi u_t + b_\pi x_{t-1}$ with the initial condition $x_{-1} = 0$.
- Substitute them into the NKPC and the tradeoff equation,

$$\begin{aligned} a_\pi u_t + b_\pi x_{t-1} &= \beta E_t(a_\pi u_{t+1} + b_\pi x_t) + \kappa x_t + u_t, \\ &= (1 + \beta\rho_u a_\pi)u_t + (\beta b_\pi + \kappa)(a_x u_t + b_x x_{t-1}), \\ &= (1 + (\beta b_\pi + \kappa)a_x + \beta\rho_u a_\pi)u_t + (\beta b_\pi + \kappa)b_x x_{t-1}. \end{aligned}$$

$$\begin{aligned} a_\pi u_t + b_\pi x_{t-1} &= -(\vartheta/\kappa)\left(x_t - x_{t-1}\right), \\ &= -(\vartheta/\kappa)\left(a_x u_t + (b_x - 1)x_{t-1}\right). \end{aligned}$$

Solving for the policy function under commitment, cont'd

- Then we have

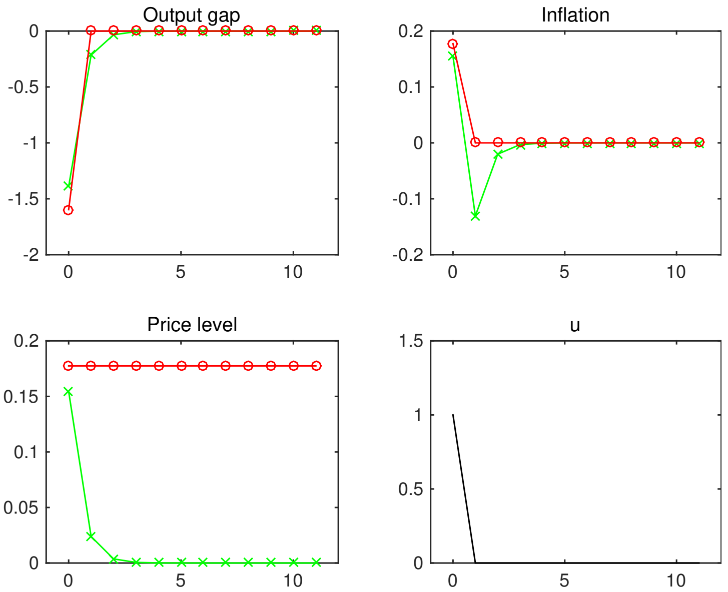
$$
\begin{aligned}
a_\pi &= -(\vartheta/\kappa)a_x \\
b_\pi &= (\vartheta/\kappa)(1-b_x) \\
a_\pi &= 1+(\beta b_\pi+\kappa)a_x+\beta\rho_u a_\pi, \\
b_\pi &= (\beta b_\pi+\kappa)b_x
\end{aligned}
$$

- These can be solved for

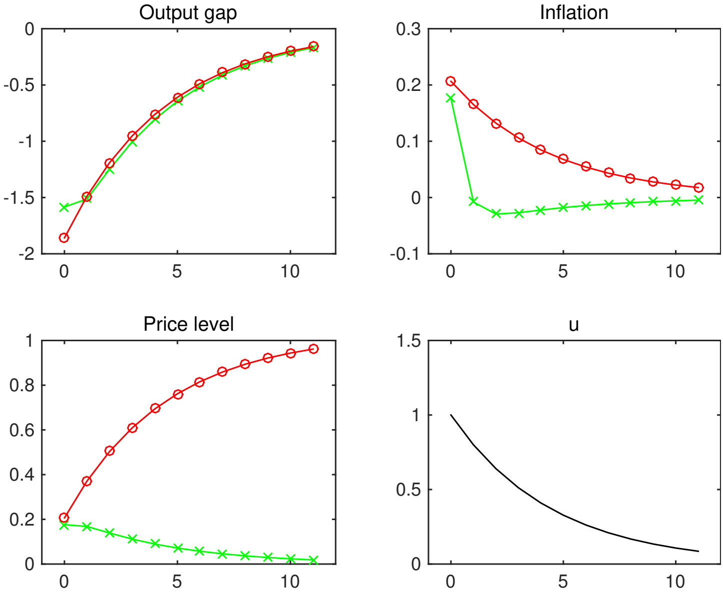
$$
\begin{aligned}
a_x = -(\kappa/\vartheta)/[\beta(\delta^+-\rho_u)], &\qquad a_\pi = 1/[\beta(\delta^+-\rho_u)], \\
b_x = \delta^- \in (0,1), &\qquad b_\pi = (\vartheta/\kappa)(1-\delta^-).
\end{aligned}
$$

where $\delta^\pm = \left(1 \pm \sqrt{1-4\beta\gamma^2}\right) / (2\beta\gamma)$ is the solution of a quadratic equation $\beta\gamma\delta^2-\delta+\gamma=0$ where $\gamma=(1+\beta+\kappa^2/\vartheta)^{-1}$.

Discretion vs. Commitment: $\rho_u = 0.0$ (when $\alpha = 0$)



Discretion vs. Commitment: $\rho_u = 0.8$ (when $\alpha = 0$)



The case of a distorted steady state

- Consider the case in which there is a permanent gap $x \equiv y^n - y^e$. Specifically,

$$-\frac{U_n}{U_c} = (1-\Phi)MPN,$$

  where $\Phi \geq 0$ measures the wedge between the marginal product of labor and marginal rate of substitution.

- For example, monopolistic competition and associated markup is a source of the distortion, $\Phi \equiv 1 - [(1-\tau)\mathcal{M}]^{-1} \geq 0$.

Second-order approximation to the household utility

- Note that $-U_n/U_c = (1-\Phi)MPN = (1-\Phi)C/N$ implies $(1-\Phi)U_cC = -U_nN$. Then we have

$$
\begin{aligned}
\frac{U_t - U}{U_cC} &\simeq \left(\hat{y}_t(1+z_t) + \frac{1-\sigma}{2}\hat{y}_t^2\right) \\
&\quad -(1-\Phi)\left(\hat{y}_t(1+z_t) + d_t + \frac{1+\varphi}{2}(\hat{y}_t - a_t)^2\right) + t.i.p., \\
&= \Phi\left(\hat{y}_t(1+z_t) + d_t + \frac{1+\varphi}{2}(\hat{y}_t - a_t)^2\right) \\
&\quad - \left(d_t + \frac{\sigma+\varphi}{2}\hat{y}_t^2 + (\sigma+\varphi)\hat{y}_t a_t\right) + t.i.p., \\
&= \Phi\hat{y}_t - \left(d_t + \frac{\sigma+\varphi}{2}\hat{y}_t^2 + (\sigma+\varphi)\hat{y}_t a_t\right) + t.i.p.,
\end{aligned}
$$

Under the small distortion assumption, the product of Φ with second-order terms is negligible. Also, $\Phi\hat{y}_t$ can be considered as a second-order term.

The CB's problem: The case of small SS distortions

- Under the small distortion assumption, the welfare loss function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \vartheta \hat{x}_t^2 \right) - \Lambda \hat{x}_t \right],$$

where $\Lambda \equiv \Phi\lambda/\epsilon > 0$ and $\hat{x}_t \equiv x_t - x$ with $x \equiv y^n - y^e$.

- Similarly, the NKPC can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + u_t.$$

Optimal Discretionary Policy

- Each period the CB minimizes the period losses

$$\frac{1}{2}\left(\pi_t^2 + \vartheta \hat{x}_t^2\right) - \Lambda \hat{x}_t,$$

subject to the constraint

$$\pi_t = \kappa \hat{x}_t + \nu_t,$$

where $\nu_t \equiv \beta E_t \pi_{t+1} + u_t$ is taken as given by the central bank.

Optimal Discretionary Policy, cont'd

- The optimality condition is given by

$$\hat{x}_t = \frac{\Lambda}{\vartheta} - \frac{\kappa}{\vartheta}\pi_t,$$

for $t = 0, 1, 2, ...$

- Substituting it into the NKPC and we have

$$\begin{aligned}
\pi_t &= \frac{\Lambda\kappa}{\kappa^2 + \vartheta(1-\beta)} + \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho)}u_t, \\
\hat{x}_t &= \frac{\Lambda(1-\beta)}{\kappa^2 + \vartheta(1-\beta)} - \frac{\kappa}{\kappa^2 + \vartheta(1-\beta\rho)}u_t.
\end{aligned}$$

The constant terms are known as the inflation bias.

Optimal Commitment Policy

- The Lagrangian is set up as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \vartheta \hat{x}_t^2 \right) - \Lambda \hat{x}_t + \xi_t \left(\pi_t - \beta E_t \pi_{t+1} - \kappa \hat{x}_t \right) \right],$$

where $\{\xi_t\}$ is a sequence of Lagrange multipliers.

- The FONCs are

$$\vartheta x_t - \kappa \xi_t - \Lambda = 0,$$
$$\pi_t + \xi_t - \xi_{t-1} = 0,$$

for $t = 0, 1, 2, ...$ and $\xi_{-1} = 0$.

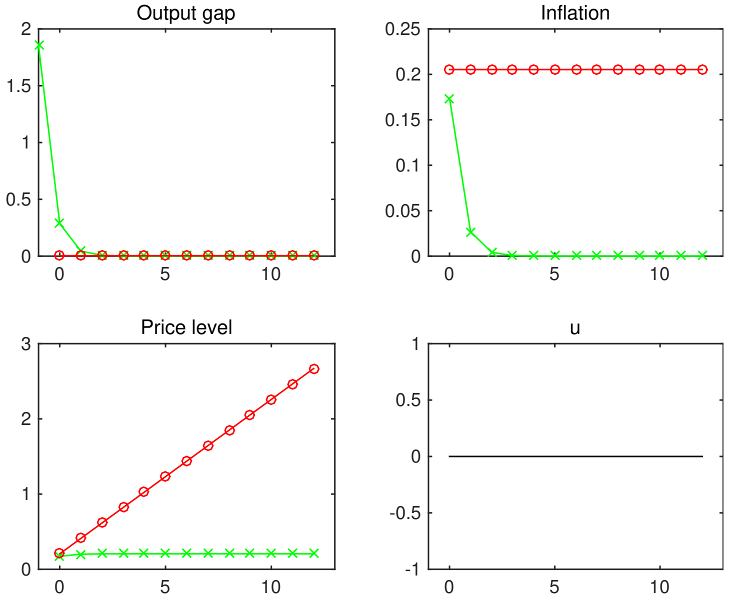
Optimal Commitment Policy, cont'd

- Combining the two, we have
$$\hat{x}_0 = -\frac{\kappa}{\vartheta}\pi_0 + \frac{\Lambda}{\vartheta},$$
and
$$\hat{x}_t - \hat{x}_{t-1} = -\frac{\kappa}{\vartheta}\pi_t$$
for $t = 1, 2, 3, ...$

- Note that the equilibrium conditions has the same form in the case of the efficient steady state except for $t = 0$.

- $\hat{x}_{-1} = \Lambda/\vartheta$ is given as the initial condition.

Initial dynamics



The zero lower bound on nominal interest rates

- Money has no nominal payoffs but otherwise is identical to short-term nominal debt. This fact may impose the zero lower bound (ZLB) on the nominal return of such debt:

$$i_t \geq 0.$$

- Then the CB minimizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2 \right),$$

subject to

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t, \\ x_t &\leq x_{t+1} + \sigma^{-1} \left(\pi_{t+1} + r_t^n \right). \end{aligned}$$

The last equation is an occasionally binding constraint. The equality holds when $i_t = 0$.

The natural rate and the ZLB

- We assume the natural rate r_t^n follows an exogenous deterministic path: In the steady state, $r_t^n = \rho$. It unexpectedly drops to and remains at $r_t^n = -\epsilon < 0$ for $t = 0, 1, ..., t_Z$. From period $t_Z + 1$ onward, it reverts to $r_t^n = \rho$ again.

- We assume the perfect foresight (and get rid of expectational operators hereafter). Agents know the subsequent path of the natural rate in period 0.

- Whenever $r_t^n < 0$, the efficient allocation, implied by $i_t = r_t^n$, is no longer attainable.

Optimal Discretionary Policy

- Each period the CB minimizes the period losses

$$\pi_t^2 + \vartheta x_t^2,$$

subject to the constraint

$$\begin{aligned} \pi_t &= \kappa x_t + \nu_{0,t}, \\ x_t &\leq \nu_{1,t}, \end{aligned}$$

where $\nu_{0,t} \equiv \beta E_t \pi_{t+1}$ and $\nu_{1,t} \equiv x_{t+1} + \sigma^{-1}(\pi_{t+1} + r_t^n)$ are taken as given.

Optimal Discretionary Policy, cont'd

- The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}\left(\pi_t^2 + \vartheta x_t^2\right) + \xi_{1,t}\left(\pi_t - \kappa x_t - \nu_{1,t}\right) + \xi_{2,t}\left(x_t - \nu_{2,t}\right).$$

- The FONCs are

$$x_t + \xi_{1,t} = 0,$$
$$\vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} = 0,$$

and the slackness conditions

$$\xi_{2,t} \geq 0, \quad i_t \geq 0, \quad \xi_{2,t} i_t = 0.$$

for $t = 0, 1, 2, \ldots$.

The solution

- From $t = t_Z + 1$ onward, $i_t = \rho > 0$, $\xi_{2,t} = 0$, and $x_t = \pi_t = 0$ hold.
- For $t = 0, 1, ..., t_Z$, $i_t = 0$, $\xi_{2,t} > 0$, and $\vartheta x_t = -\kappa\pi_t - \xi_{2,t}$ hold.
- Then we can solve the two equations backward

$$\begin{aligned} \pi_t &= \beta\pi_{t+1} + \kappa x_t, \\ x_t &= x_{t+1} + \sigma^{-1}\left(\pi_{t+1} + r_t^n\right), \end{aligned}$$

from $t = t_Z, t_Z - 1, ..., 0$, given $x_{t_{Z+1}} = \pi_{t_{Z+1}} = 0$.

Optimal Commitment Policy

- The CB will choose a state-contingent sequence $\{x_t, \pi_t\}_{t=0}^{\infty}$ so as to minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right),$$

subject to

$$\begin{aligned} \pi_t &= \beta\pi_{t+1} + \kappa x_t, \\ x_t &\leq x_{t+1} + \sigma^{-1}\left(\pi_{t+1} + r_t^n\right). \end{aligned}$$

Optimal Commitment Policy, cont'd

- It is useful to set up the Lagrangian as

$$\begin{aligned}\mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \vartheta x_t^2 \right) + \xi_{1,t} \left(\pi_t - \beta \pi_{t+1} - \kappa x_t \right) \right. \\ & \left. + \xi_{2,t} \left(x_t - x_{t+1} - \sigma^{-1} \left(\pi_{t+1} + r_t^n \right) \right) \right].\end{aligned}$$

- The FONCs are

$$\pi_t + \xi_{1,t} - \xi_{1,t-1} - (\beta\sigma)^{-1} \xi_{2,t-1} = 0,$$
$$\vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} - \beta^{-1} \xi_{2,t-1} = 0,$$

and the slackness conditions

$$\xi_{2,t} \geq 0, \quad i_t \geq 0, \quad \xi_{2,t} i_t = 0.$$

for $t = 0, 1, 2, \ldots$, given the initial conditions $\xi_{1,-1} = \xi_{2,-1} = 0$.

A conjectured solution

- The solution is conjectured and verified as follows;

1. From $t = 0$ to $t_C \geq t_Z$, $i_t = 0$ and $\xi_{2,t} > 0$.
2. For $t = t_C + 1$, $i_t = 0$, $\xi_{2,t} = 0$ and $\xi_{2,t} > 0$.
3. From $t = t_C + 2$ and onward, $i_t > 0$ and $\xi_{2,t} = \xi_{2,t-1} = 0$,

$t = t_C + 2$ and onward

- Note that $\xi_{2,t} = \xi_{2,t-1} = 0$, then

$$\pi_t + \xi_{1,t} - \xi_{1,t-1} = 0,$$
$$\vartheta x_t - \kappa \xi_{1,t} = 0,$$
$$\pi_t = \beta \pi_{t+1} + \kappa x_t,$$

  hold, given the initial condition ξ_{1,t_C+1}.

- These equations can be solved for

$$x_{t_C+2+k} = -\frac{\kappa \delta^{k+1}}{\vartheta} \xi_{1,t_C+1},$$
$$\pi_{t_C+2+k} = (1-\delta)\delta^k \xi_{1,t_C+1}.$$

  for $k = 0, 1, 2, ...$ [check by yourself]

$t = t_C + 1$

- Note that $i_{t_C+1} > 0$ and $\xi_{2,t_C+1} = 0$, then

$$\begin{aligned}
&\pi_{t_C+1} + \xi_{1,t_C+1} - \xi_{1,t_C} - (\beta\sigma)^{-1}\xi_{2,t_C} = 0,\\
&\vartheta x_{t_C+1} - \kappa\xi_{1,t_C+1} - \beta^{-1}\xi_{2,t_C} = 0,\\
&\pi_{t_C+1} = \beta \underbrace{(1-\delta)\xi_{1,t_C+1}}_{=\pi_{t_C+2}} + \kappa x_{t_C+1},
\end{aligned}$$

  hold.

- By substituting out $\xi_{1,t_C+1} = [\beta(1-\delta)]^{-1}(\pi_{t_C+1} - \kappa x_{t_C+1})$, we have

$$\begin{aligned}
&[1+\beta(1-\delta)]\pi_{t_C+1} - \kappa x_{t_C+1} - \beta(1-\delta)\xi_{1,t_C} - [(1-\delta)/\sigma]\xi_{2,t_C} = 0,\\
&[\beta(1-\delta)\vartheta + \kappa^2]x_{t_C+1} - \kappa\pi_{t_C+1} - (1-\delta)\xi_{2,t_C} = 0.
\end{aligned}$$

$t = 0, 1, ..., t_C$ and so on

- Note that $i_t = 0$, then

$$\begin{aligned} \pi_t &= \beta \pi_{t+1} + \kappa x_t, \\ x_t &= x_{t+1} + \sigma^{-1} \left(\pi_{t+1} + r_t^n \right), \end{aligned}$$

hold, where

$$r_t^n = \begin{cases} -\rho, & \text{when } t = 0, 1, ..., t_Z, \\ \rho, & \text{when } t = t_Z + 1, ..., t_C. \end{cases}$$

- In addition, the original FONCs

$$\begin{aligned} &\pi_t + \xi_{1,t} - \xi_{1,t-1} - (\beta\sigma)^{-1} \xi_{2,t-1} = 0, \\ &\vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} - \beta^{-1} \xi_{2,t-1} = 0, \end{aligned}$$

hold for $t = 0, ..., t_C + 1$, given the initial conditions $\xi_{1,-1} = \xi_{2,-1} = 0$.

The algorithm for the solution

- The algorithm is as follows:

1. Fix $t_C \geq t_Z$. Solve $4 \times (t_C + 2)$ equations for $4 \times (t_C + 2)$ variables $\{x_t, \pi_t, \xi_{1,t}\xi_{2,t}\}_{t=0}^{t_C+1}$.
2. Obtain $x_{t_C+2+k} = -\frac{\kappa\delta^{k+1}}{\vartheta}\xi_{1,t_C+1}$ and $\pi_{t_C+2+k} = (1-\delta)\delta^k\xi_{1,t_C+1}$ for $k = 0, 1, ...$, given ξ_{1,t_C+1} at hand.
3. Check $i_t = r_t^n + \pi_{t+1} + \sigma(x_{t+1} - x_t) = 0$ for $t = 0, 1, ...t_C$ and $i_t > 0$ $t = t_C + 1, ...$ If not, increase t_C and do 1-3 again.

The relevant equations

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} r_t^n,$$

for $t = 0, 1, ..., t_C$,

$$\begin{bmatrix} x_{t_C+1} \\ \pi_{t_C+1} \end{bmatrix} = \underbrace{\begin{bmatrix} -\kappa & 1 + \beta(1-\delta) \\ \beta(1-\delta) + \frac{\kappa^2}{\vartheta} & -\frac{\kappa}{\vartheta} \end{bmatrix}^{-1} \begin{bmatrix} \beta(1-\delta) & \frac{1-\delta}{\sigma} \\ 0 & \frac{1-\delta}{\vartheta} \end{bmatrix}}_{\mathbf{M}}$$

$$\times \begin{bmatrix} \xi_{1,t_C} \\ \xi_{2,t_C} \end{bmatrix},$$

$$\begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & (\beta\sigma)^{-1} \\ \kappa & \beta^{-1}(1 + \kappa\sigma^{-1}) \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} \xi_{1,t-1} \\ \xi_{2,t-1} \end{bmatrix} - \underbrace{\begin{bmatrix} 0 & 1 \\ \vartheta & \kappa \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix},$$

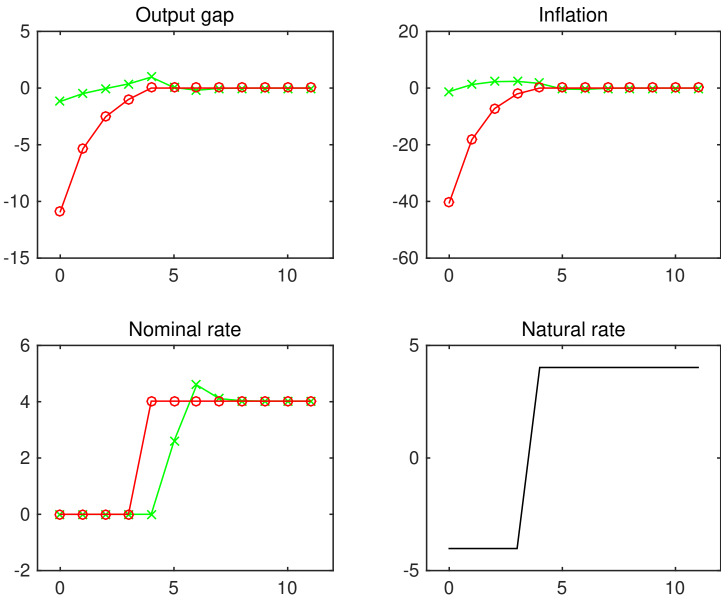
for $t = 0, 1, ..., t_C + 1$.

The relevant equations, cont'd

- These equations can be stacked into

$$\begin{bmatrix} I_2 & & -\mathbf{A} & & & & & \\ \mathbf{J} & I_2 & & & & & & \\ & & I_2 & & -\mathbf{A} & & & \\ & -\mathbf{H} & \mathbf{J} & I_2 & & & & \\ & & & & \ddots & & & \\ & & & & & \ddots & & \\ & & & & & -\mathbf{M} & I_2 & \\ & & & & & -\mathbf{H} & \mathbf{J} & I_2 \end{bmatrix} \begin{bmatrix} x_0 \\ \pi_0 \\ \xi_{1,0} \\ \xi_{2,0} \\ x_1 \\ \pi_1 \\ \xi_{1,1} \\ \xi_{2,1} \\ \vdots \\ x_{t_C+1} \\ \pi_{t_C+1} \\ \xi_{1,t_C+1} \\ {\color{red}0} \end{bmatrix} = \begin{bmatrix} \frac{r_0^n}{\sigma} \\ \frac{\kappa r_0^n}{\sigma} \\ 0 \\ 0 \\ \frac{r_1^n}{\sigma} \\ \frac{\kappa r_1^n}{\sigma} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Discretion vs. Commitment with the ZLB (when $\alpha = 0$)



Caveats

- We have assumed the perfect foresight.
- Uncertainty about the natural rate matters. We need to solve the model with stochastic settings (e.g., Adam and Billi, 2006; 2007, Nakov, 2008).