Proposed Answer to Assignment #2

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(1)

Note that the following holds:

$$\begin{aligned} T_t &= \tau(r_t - \delta)k_{t-1}, \\ \pi_t &= y_t - r_t k_{t-1}. \end{aligned}$$

By substituting them into the household's budget constraint, we have

$$c_t + k_t \leq (1-\delta)k_{t-1} + y_t.$$

(2)

The social planner's economy is the same as the one without tax and transfer. In the competitive (decentralized) economy, the household's Lagrangean is given by

$$L_0 \equiv \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t - \lambda_t \left(c_t + k_t - (1-\delta)k_{t-1} - r_t k_{t-1} + \tau(r_t - \delta)k_{t-1} - \pi_t - T_t \right) \right\}.$$

FOCs are

$$\begin{aligned} \partial c_t : &\quad \lambda_t = 1/c_t, \\ \partial k_t : &\quad \lambda_t = \beta \lambda_{t+1} \left(1 + (1-\tau)(r_{t+1} - \delta) \right), \\ \partial \lambda_t : &\quad c_t + k_t - (1-\delta)k_{t-1} = r_t k_{t-1} - \tau(r_t - \delta)k_{t-1} + \pi_t. \end{aligned}$$

The firm's profit maximization yields

$$\begin{aligned} y_t &= A_t k_{t-1}^{\alpha}, \\ r_t &= \alpha y_t / k_{t-1}, \end{aligned}$$

The resouce constraint holds with equality. In other words, the good market clears as

$$c_t + k_t - (1-\delta)k_{t-1} = y_t.$$

In sum, we have

$$
\begin{aligned}
1 &= \beta \frac{c_t}{c_{t+1}} \left(1 + (1-\tau)(r_{t+1} - \delta)\right), \\
y_t &= A_t k_{t-1}^{\alpha} \\
r_t &= \alpha y_t / k_{t-1}, \\
c_t + k_t - (1-\delta) k_{t-1} &= y_t.
\end{aligned}
$$

There are four equations and four variables $\{c_t, k_t, y_t, r_t\}$. This can be further summarized as

$$
\begin{aligned}
1 &= \beta \frac{c_t}{c_{t+1}} \left(1 + (1-\tau)(\alpha A_t k_{t-1}^{\alpha-1} - \delta)\right), \\
c_t + k_t - (1-\delta) k_{t-1} &= A_t k_{t-1}^{\alpha}.
\end{aligned}
$$