Steady state and log-linearized equations in Cooper and Hansen

- In equilibrium, all households are the same: $C_t = c_t^i$, $H_t = h_t^i$, $K_{t+1} = k_{t+1}^i$ and $\hat{M}_t = \hat{m}_t^i = 1$. Then we have

$$
\begin{aligned}
1 &= \beta E_t \left\{ \tfrac{w_t}{w_{t+1}} \left(1 + r_{t+1} - \delta\right) \right\}, \\
\tfrac{B}{w_t \hat{p}_t} &= -\beta E_t \left\{ \tfrac{1}{\hat{p}_{t+1} C_{t+1} g_{t+1}} \right\}, \\
\hat{p}_t C_t g_t &= \hat{m}_{t-1}^i + g_t - 1, \\
K_{t+1} + \tfrac{\hat{m}_t^i}{\hat{p}_t} &= w_t H_t + r_t K_t + (1-\delta) K_t, \\
w_t &= (1-\theta) \lambda_t K_t^{\theta} H_t^{-\theta}, \\
r_t &= \theta \lambda_t K_t^{\theta - 1} H_t^{1-\theta}.
\end{aligned}
$$

  There are six unknowns and six equations (note that $\hat{m}_t^i = 1$), so we can solve the model.

- In the steady state, the six equations of the model become

$$
\begin{aligned}
1 &= \beta \left(1 + r - \delta\right), \\
B w^{-1} &= -\beta / (gC), \\
\hat{p} C &= 1, \\
\hat{p}^{-1} &= (r - \delta) K + wH, \\
w &= (1-\theta)(K/H)^{\theta}, \\
r &= \theta (K/H)^{\theta - 1}.
\end{aligned}
$$

  The steady-state conditions can be solved for $(r, w, C, \hat{p}, H, K)$.

- After log-linearization, we have

$$
\begin{aligned}
&-\hat{w}_t = \beta r E_t \hat{r}_{t+1} - E_t \hat{w}_{t+1}, \\
&-\left(\hat{p}_t + \hat{w}_t\right) = -\pi \hat{g}_t, \\
&k \hat{k}_{t+1} + \frac{m}{p} \left(\hat{m}_t - \hat{p}_t\right) = wh \left(\hat{w}_t + \hat{h}_t\right) + rk \left(\hat{r}_t + \hat{k}_t\right) + (1-\delta) k \hat{k}_t, \\
&\hat{r}_t = \hat{\lambda}_t + (\theta - 1) \left(\hat{k}_t - \hat{h}_t\right), \\
&\hat{w}_t = \hat{\lambda}_t + \theta \left(\hat{k}_t - \hat{h}_t\right),
\end{aligned}
$$

and two stochastic processes

$$\hat{\lambda}_{t+1} = \gamma \hat{\lambda}_t + \varepsilon^{\lambda}_{t+1},$$
$$\hat{g}_{t+1} = \pi \hat{g}_t + \varepsilon^{g}_{t+1}.$$