Classical Monetary Model, Pt. I

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Introduction

- In classical economics, money is neutral so that real variables are determined independently of monetary policy.
- We will cover Cooley and Hansen's (1989) model, in which money has a very limited role in the normal time.

Cooley and Hansen (1989)

- Cooley and Hansen (1989) introduce money into Hansen's model with indivisible labor.
- Households hold cash in advance to purchase consumption goods.
- The second welfare theorem fails to hold. Labor and capital markets need to be considered explicitly.

Households

- A household (family) $i \in [0,1]$ want to maximize the discounted expected utility
$$\sum_{t=0}^{\infty} \beta^t u(c_t^i, h_t^i).$$
- Following Cooley and Hansen, the utility function is written by
$$u(c_t^i, h_t^i) = \ln c_t^i + \left[A \frac{\ln(1-h_0)}{h_0} \right] h_t^i.$$

That is, labor is indivisible.

Firms

- The representative firm can access to the Cobb-Douglas production function technology:
$$y_t = \lambda_t K_t^{\theta} H_t^{1-\theta},$$
and λ_t follows a stochastic process,
$$\ln \lambda_{t+1} = \gamma \ln \lambda_t + \varepsilon_{t+1},$$
where $\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$.

Firms, cont'd

- As a result of profit maximization, the wage rate at time t equals

$$w_t = (1-\theta)\lambda_t K_t^{\theta} H_t^{-\theta},$$

  and the rental rate is

$$r_t = \theta\lambda_t K_t^{\theta-1} H_t^{1-\theta}.$$

- Thanks to the constant-returns-to-scale (CRS) production function, there are no excess profits.

Capital and labor markets

- The aggregate amount of labor available at time t is equal to

$$H_t = \int_0^1 h_t^i di,$$

and the aggregate amount of capital available at time t is

$$K_t = \int_0^1 k_t^i di.$$

Monetary policy rule

- Monetary policy rule is given by

$$M_t = g_t M_{t-1}$$

  where g_t is the gross growth rate of money and M_{t-1} is the per capita money stock.

- Seigniorage is given by

$$M_t - M_{t-1} = (g_t - 1)M_{t-1}.$$

Cash in advance

- Household i carries over m_{t-1}^{i} from the previous period and receives a transfer $M_t - M_{t-1} = (g_t - 1)M_{t-1}$.
- The cash-in-advance (CIA) constraint is given by

$$p_t c_t^i \leq m_{t-1}^i + (g_t - 1)M_t,$$

where p_t is the price level in period t.

- For simplicity, we assume that the CIA constraint always holds.

Households' budget constraint

- Household i holds capital k_t^i and money m_{t-1}^i. In addition to the CIA constraint, household i faces the budget constraint:

$$c_t^i + k_{t+1}^i + \frac{m_t^i}{p_t} = w_t h_t^i + r_t k_t^i + (1-\delta)k_t^i + \frac{m_{t-1}^i}{p_t} + \frac{(g_t - 1)M_{t-1}}{p_t}.$$

Note that the variables are measured in real terms.

A normalization

- Cooley and Hansen normalize nominal variables as $\hat{p}_t = p_t/M_t$, $\hat{m}_t^i = m_t^i/M_t$ and $\hat{M}_t = M_t/M_t = 1$.
- Using these definitions, The CIA and budget constraints become

$$\hat{p}_t c_t^i \leq \frac{\hat{m}_{t-1}^i + (g_t - 1)}{g_t},$$

$$c_t^i + k_{t+1}^i + \frac{\hat{m}_t^i}{\hat{p}_t} = w_t h_t^i + r_t k_t^i + (1-\delta)k_t^i + \frac{m_{t-1}^i + (g_t - 1)}{g_t \hat{p}_t}.$$

Lagrangean

- We set up the Lagrangean as

$$L_0^i \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t^i + Bh_t^i + \chi_t^1 \left(\hat{p}_t c_t^i - \frac{\hat{m}_{t-1}^i + (g_t - 1)}{g_t} \right) \right.$$
$$\left. + \chi_t^2 \left(k_{t+1}^i + \frac{\hat{m}_t^i}{\hat{p}_t} - w_t h_t^i - r_t k_t^i - (1-\delta) k_t^i \right) \right].$$

- Taking the derivatives of the Lagrangean and set them to zero,

$$\begin{aligned}
\partial c_t^i : &\quad 1/c_t^i + \chi_t^1 \hat{p}_t = 0, \\
\partial h_t^i : &\quad B - \chi_t^2 w_t = 0, \\
\partial k_{t+1}^i : &\quad \chi_t^2 = \beta E_t \chi_{t+1}^2 \left(1 + r_{t+1} - \delta\right), \\
\partial \hat{m}_t^i : &\quad \chi_t^2 \hat{p}_t^{-1} = \beta E_t \chi_{t+1}^1 g_{t+1}^{-1}.
\end{aligned}$$

Equilibrium

- Note that $\chi_t^1 = -(\hat{p}_t c_t^i)^{-1}$ and $\chi_t^2 = B w_t^{-1}$. Now we have the following equilibrium conditions:

$$1 = \beta E_t \left\{ \frac{w_t}{w_{t+1}} \left(1 + r_{t+1} - \delta\right) \right\},$$

$$\frac{B}{w_t \hat{p}_t} = -\beta E_t \left\{ \frac{1}{\hat{p}_{t+1} c_{t+1}^i g_{t+1}} \right\},$$

$$\hat{p}_t c_t^i = \frac{\hat{m}_{t-1}^i + (g_t - 1)}{g_t},$$

$$k_{t+1}^i + \frac{\hat{m}_t^i}{\hat{p}_t} = w_t h_t^i + r_t k_t^i + (1 - \delta) k_t^i,$$

where $w_t = (1 - \theta)\lambda_t K_t^\theta H_t^{-\theta}$ and $r_t = \theta \lambda_t K_t^{\theta - 1} H_t^{1-\theta}$.

Equilibrium, cont'd

- In equilibrium, all households are the same: $C_t = c_t^i$, $H_t = h_t^i$, $K_{t+1} = k_{t+1}^i$ and $\hat{M}_t = \hat{m}_t^i = 1$. Then we have

$$
\begin{aligned}
1 &= \beta E_t \left\{ \frac{w_t}{w_{t+1}} \left(1 + r_{t+1} - \delta\right) \right\}, \\
\frac{B}{w_t \hat{p}_t} &= -\beta E_t \left\{ \frac{1}{\hat{p}_{t+1} C_{t+1} g_{t+1}} \right\}, \\
\hat{p}_t C_t g_t &= \hat{m}_{t-1}^i + g_t - 1, \\
K_{t+1} + \frac{\hat{m}_t^i}{\hat{p}_t} &= w_t H_t + r_t K_t + (1-\delta) K_t, \\
w_t &= (1-\theta)\lambda_t K_t^{\theta} H_t^{-\theta}, \\
r_t &= \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta}.
\end{aligned}
$$

- There are six unknowns and six equations (note that $\hat{m}_t^i = 1$), so we can solve the model.

Output and welfare in the steady state

- Output is from the household budget constraint

$$Y = C + \delta K.$$

- Welfare is

$$W = \sum_{t=0}^{\infty} \beta^t \left(\ln C + BH \right) = (1-\beta)^{-1} \left(\ln C + BH \right).$$

The steady state values

- We use the parameter values: $\theta = 0.36$, $\delta = 0.025$, $\beta = 0.99$, $A = 1.72$, and $h_0 = 0.583$, so $B = -2.5805$.

- By varying g, we obtain the corresponding steady state values.

Annual Inflation $\tilde{\pi}$	-4%	0%	10%	100%	400%
$g = (1 + \tilde{\pi})^{1/4}$	0.9898	1.0000	1.0241	1.1892	1.4142
Output	1.2356	1.2231	1.1943	1.0285	0.8648
Consumption	0.9188	0.9095	0.8881	0.7648	0.6431
Investment	0.3168	0.3136	0.3062	0.2637	0.2217
Capital stock	12.6726	12.5440	12.2486	10.5482	8.8699
Hours worked	0.3336	0.3302	0.3224	0.2777	0.2335
Welfare loss, %	0.00	0.16	0.55	4.14	10.41

- Note that Friedman rule holds so that $i = r + \pi = 0$ is optimal.

Log-linearization

- Recap: Useful formulae

$$
\begin{aligned}
x_t y_t &= xy \exp(\hat{x}_t + \hat{y}_t) \\
&\approx xy(1 + \hat{x}_t + \hat{y}_t), \\
x_t / y_t &= (x/y) \exp(\hat{x}_t - \hat{y}_t) \\
&\approx (x/y)(1 + \hat{x}_t - \hat{y}_t), \\
y_t^a &= y^a \exp(a\hat{y}_t) \\
&\approx y^a(1 + a\hat{y}_t).
\end{aligned}
$$

$$
\hat{x}_t^n = 0 \text{ for } n > 1, \qquad \hat{x}_t \hat{y}_t = 0,
$$

$$
\begin{aligned}
E_t y_{t+1}^a &= E_t y^a \exp(a\hat{y}_{t+1}). \\
&\approx y^a(1 + aE_t\hat{y}_{t+1}).
\end{aligned}
$$

Stochastic process of money growth

- Cooley and Hansen estimate the stochastic process using the U.S. data

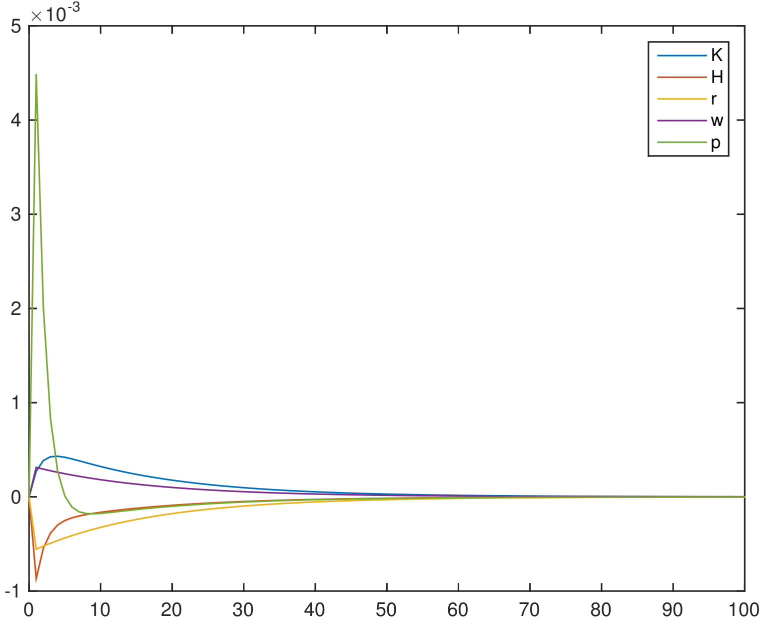
$$\Delta \log(M1)_t = 0.00798 + 0.481 \Delta \log(M1)_{t-1}, \qquad \hat{\sigma} = 0.0086.$$

- In the model, we use

$$\log g_{t+1} = 0.481 \log g_t + \varepsilon_{g,t+1}, \quad \varepsilon_{g,t+1} \sim N(0, \hat{\sigma}^2)$$

- The effect of erratic money growth on real variables is very limited. Money is almost neutral.

Impulse responses to money growth shock



Impulse responses to technology shock