Classical Monetary Model, Pt. II

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Gali's classical monetary model

- This is a simple model of a classical monetary economy with perfect competition and fully flexible prices in all markets.
- The classical economy provides a reference benchmark that will be useful later, when imperfect competition and sticky prices are introduced.
- The resulting framework is referred to as the basic New Keynesian model, which will be discussed in the next week.

Households

- The economy is inhabited by a large number of identical households. The representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t),$$

where

  - C_t is the quantity of consumption,
  - N_t is hours of work or employment, and
  - Z_t is an exogenous preference shifter.

Household's budget constraint

- Maximization is subject to a sequence of budget constraints:

$$P_tC_t + Q_tB_t \leq B_{t-1} + W_tN_t + D_t,$$

where

  - P_t is the price of consumption goods,
  - W_t is nominal wage,
  - B_t is one-period riskless discount bonds with its price Q_t, and
  - D_t represents dividends.

Lagrangean

- We set up the Lagrangean as

$$
\begin{aligned}
L_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t & \left[U(C_t, N_t; Z_t) \right. \\
& \left. + \lambda_t \left(B_{t-1} + W_t N_t + D_t - P_t C_t - Q_t B_t \right) \right].
\end{aligned}
$$

- Taking the derivatives of the Lagrangean and set them to zero,

$$
\begin{aligned}
\partial C_t : &\quad U_{c,t} = P_t \lambda_t, \\
\partial N_t : &\quad U_{n,t} = -W_t \lambda_t, \\
\partial B_t : &\quad Q_t \lambda_t = \beta E_t \lambda_{t+1},
\end{aligned}
$$

where $U_{c,t} = \partial U_t / \partial C_t$ and $U_{n,t} = \partial U_t / \partial N_t$.

Household's optimality conditions

- Eliminating the Lagrange multiplier λ_t, we have

$$\begin{aligned} \frac{W_t}{P_t} &= -\frac{U_{n,t}}{U_{c,t}}, \\ Q_t &= \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}. \end{aligned}$$

- Also, the transversality condition is given by

$$\lim_{T \to \infty} E_t \left\{ \Lambda_{t,T} \frac{B_T}{P_T} \right\} = 0,$$

where $\Lambda_{t,T} = \beta^{T-t} U_{c,T} / U_{c,t}$ is called the stochastic discount factor.

Utility function

- The utility function takes the form

$$U(C_t, N_t; Z_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t, & \text{for } \sigma \neq 1, \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t, & \text{for } \sigma = 1, \end{cases}$$

  where $\sigma \geq 0$ and $\varphi \geq 0$ are the curvature of the utility of consumption and the disutility of labor.

- $z_t \equiv \log Z_t$ follows an exogenous AR(1) process:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z.$$

Household's optimality conditions, cont'd

- After having the utility function, the optimality conditions become

$$
\begin{aligned}
\frac{W_t}{P_t} &= C_t^{\sigma} N_t^{\varphi}, \\
Q_t &= \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right\}.
\end{aligned}
$$

- The log-linearized version of the optimality conditions are

$$
\begin{aligned}
w_t - p_t &= \sigma c_t + \varphi n_t, \\
c_t &= E_t c_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1} - \rho) + \sigma^{-1} (1 - \rho_z) z_t,
\end{aligned}
$$

where

- $i_t \equiv -\log Q_t$ is the nominal interest rate,
- $\rho = -\log \beta$ is the household's discount rate, and
- $\pi_{t+1} \equiv p_{t+1} - p_t$ is the inflation rate.

Firms

- A large number of identical firms operate in the economy. The representative firm's production function is

$$Y_t = A_t N_t^{1-\alpha},$$

where A_t is the level of technology and $a_t \equiv \log A_t$ follows

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a.$$

Firm's optimality conditions

- Each period the firm maximizes profit

$$P_t Y_t - W_t N_t$$

subject to the production function. This maximization yields

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}.$$

- Its log-linearized version is

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha).$$

Summary

- Now we have the following log-linearized equilibrium conditions

$$
\begin{aligned}
w_t - p_t &= \sigma c_t + \varphi n_t, \\
c_t &= E_t c_{t+1} - \sigma^{-1}\left(i_t - E_t\left(p_{t+1} - p_t\right) - \rho\right) + \sigma^{-1}(1-\rho_z) z_t, \\
w_t - p_t &= a_t - \alpha n_t + \log(1-\alpha), \\
y_t &= a_t + (1-\alpha) n_t, \\
y_t &= c_t
\end{aligned}
$$

Given the policy rate i_t and exogeneous variables (a_t, z_t), we have 5 equations and 5 variables.

Solving for real variables

- From the equilibrium conditions, we have

$$\sigma y_t + \varphi n_t = a_t - \alpha n_t + \log(1-\alpha),$$
$$y_t = a_t + (1-\alpha) n_t.$$

Then one can determine the equilibrium levels of employment and output

$$\begin{aligned} n_t &= \psi_{na} a_t + \psi_n, \\ y_t &= \psi_{ya} a_t + \psi_y, \end{aligned}$$

where $\psi_{na} = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$, $\psi_n = \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$, $\psi_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$, and $\psi_y = (1-\alpha)\psi_n$.

Solving for real variables, cont'd

- Further, the real interest rate $r_t \equiv i_t - E_t\pi_{t+1}$ is given by

$$\begin{aligned} r_t &= \rho + (1-\rho_z)z_t + \sigma E_t(y_{t+1} - y_t), \\ &= \rho + (1-\rho_z)z_t + \sigma\psi_{ya}a_t. \end{aligned}$$

- Note that the equilibrium levels of employment, output, and the real interest rate are determined independently of monetary policy. In other words, monetary policy is neutral.

Monetary policy and price level determination

- In contrast with real variables, nominal variables cannot be determined independently of monetary policy.
- We will see that
  - Inflation and price level is undetermined under a fixed interest rate, and
  - Inflation is pinned down with the Taylor rule.

Fisherian equation

- The Fisherian equation is given by

$$i_t = E_t \pi_{t+1} + r_t.$$

- In the steady state (i.e., $z_t = a_t = 0$), $r = \rho$ and

$$i = \rho + \pi.$$

Exogenous nominal interest rate

- A monetary policy rule is given by

$$i_t = i + \nu_t,$$

where ν_t follows

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu.$$

ν_t is called a monetary policy shock. It should be interpreted as a random and transitory deviation from the "usual" conduct of monetary policy.

Expected inflation determined

- Combining the Fisherian equation and monetary policy rule, we have

$$\begin{aligned} E_t \pi_{t+1} &= i_t - r_t, \\ &= \pi + \nu_t - \underbrace{(r_t - \rho)}_{\hat{r}_t}. \end{aligned}$$

  the expected inflation is pinned down uniquely, as it is a function of exogenous variables.

Price level indeterminacy

- However, actual inflation is not. Any inflation path that satisfies

$$\pi_t = \pi + \nu_{t-1} - \hat{r}_{t-1} + \xi_t,$$

is consistent with equilibrium. ξ_t is called sunspot shocks.

- An equilibrium in which nonfundamental factors may cause fluctuations is referred to as an indeterminate equilibrium.

A simple interest rate rule

- Suppose that the central bank (CB) adjusts the nominal interest rate in response to deviations of inflation from a target π, according to the interest rate rule

$$i_t = \rho + \pi + \phi_\pi \underbrace{(\pi_t - \pi)}_{\hat{\pi}_t} + \nu_t,$$

where $\phi_\pi \geq 0$ is a degree of the endogenous response of monetary policy.

- Combining the Fisherian equation and this rule, we have

$$\phi_\pi \hat{\pi}_t = E_t \hat{\pi}_{t+1} + \hat{r}_t - \nu_t.$$

The Taylor principle

- If $\phi_\pi > 1$, the previous difference equation has only one nonexplosive solution:

$$\hat{\pi}_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t(r_{t+k} - \rho - \nu_{t+k}).$$

- In particular, using the previous solution for r_{t+k}, we have [demonstrated on the white board]

$$\pi_t = \pi - \frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi - \rho_a} a_t + \frac{1-\rho_z}{\phi_\pi - \rho_z} z_t - \frac{1}{\phi_\pi - \rho_\nu} \nu_t.$$

  Through the choice of ϕ_π, the CB can influence the degree of inflation volatility.

- The condition for determinacy, $\phi_\pi > 1$, is known as the Taylor principle.

Beyond the cashless economy

- In the previous model, money plays only the role of numeraire. This is called cashless economy.
- It is unclear why agents would want to hold an asset that is dominated in return by bonds.
- There are two (somewhat incomplete) frameworks which can explain why.
  - Cash-in-advance (CIA) constraint: Cooley and Hansen
  - Money-in-the-utiliy (MIU) function

Money in the utility

- The representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_t}{P_t}, N_t; Z_t),$$

subject to

$$P_t C_t + Q_t A_t + (1 - Q_t) M_t \leq A_{t-1} + W_t N_t + D_t,$$

where $A_t = B_t + M_t$. Real money holdings, M_t/P_t, enter the utility function. Money provides a "transaction service" that households value.

- The additional optimality condition is given by

$$\frac{U_{m,t}}{U_{c,t}} = 1 - Q_t = 1 - \exp(-i_t).$$

An example with separable utility

- Assume that the household's utility function takes the form

$$U(C_t, N_t; Z_t) = \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu} - 1}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t.$$

- Then the optimality condition becomes

$$\frac{M_t}{P_t} = C_t^{\sigma/\nu} (1 - \exp(-i_t))^{-1/\nu},$$

and its log-linearized version is

$$m_t - p_t = \frac{\sigma}{\nu} c_t - \eta i_t,$$

where $\eta \equiv [\nu(\exp(i) - 1)]^{-1}$. This equation can be interpreted as a demand for real balances.