Hayashi and Prescott (2002): "The 1990s in Japan: A Lost Decade"

Takeki Sunakawa

Quantitative Methods for Monetary Economics

University of Mannheim

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Notes

- Hayashi, Fumio and Edward C. Prescott (2002), "The 1990s in Japan: A lost decade." Review of Economic Dynamics, Vol 5(1), 206-235.
- The paper is published in the special issue of Great Depressions of the 20th Century. (Other papers are also found at https://www.sciencedirect.com/journal/review-of-economic-dynamics/vol/5/issue/1)

Introduction

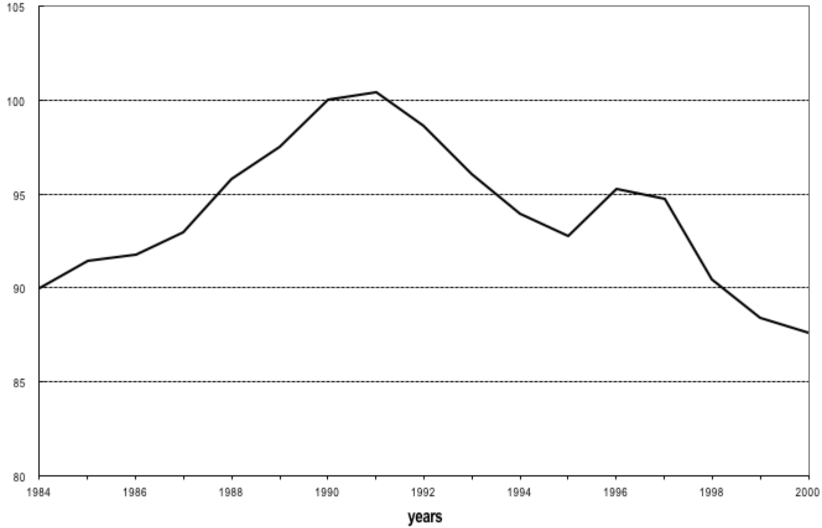
- The Japanese economy has been stuck in depression since the 1990s.
  - The average growth of real GDP per capita is 0.5% for 1991-2000.
- What is the problem?
  - Inadequate fiscal policy, over investment in the 1980s, problems with financial intermediation, etc.

Introduction, cont'd

- The problem is a low growth rate of total factor productivity (TFP).
  - A breakdown of financial system in 1997-98 is *not* the cause of the problem.
- The reduction of the workweek length also contributes to the low growth rates.

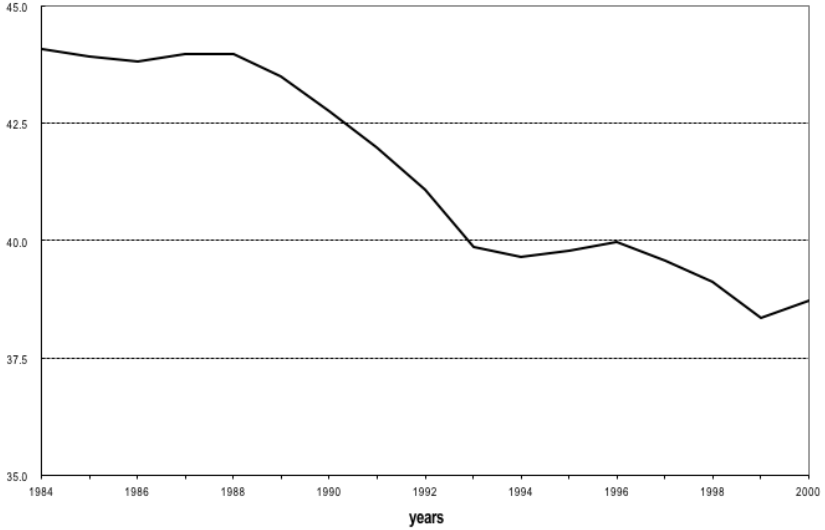
Poor performance in the 1990s

FIG. 1. Detrended real GNP per working-age person (1990=100)



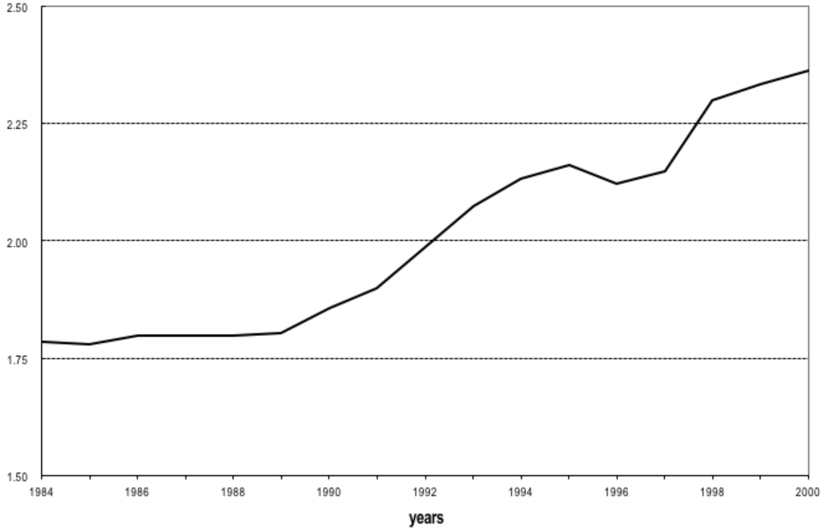
Workweek falls in the 1998-93 period

FIG. 2. Length of workweek



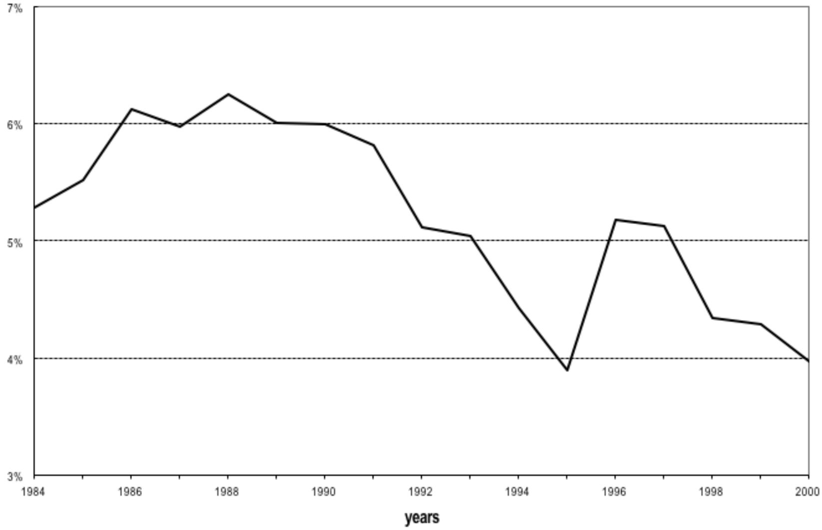
Capital deepens...

FIG. 3. Capital-output ratio



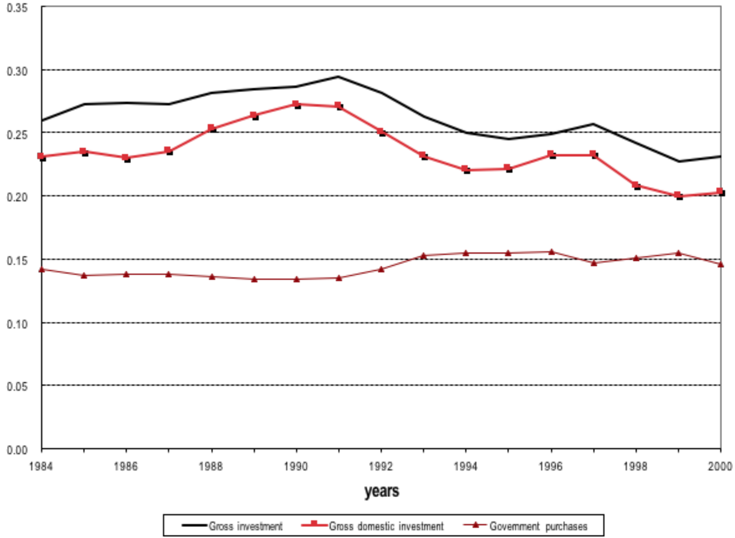
...as the rate of return declines

FIG. 4. After-tax rate of return



Government expenditure and investment

FIG. 5. Government purchases and investment as a share of output



Production function

- The representative firm's production function is given by:

$$Y_t = A_t K_t^{\theta} (h_t E_t)^{1-\theta},$$

where Y_t is aggregate output, A_t is the TFP, K_t is aggregate capital, h_t is hours per worker (workweek), and E_t is employment.

Growth accounting

- Let N_t be the working-age population and define $y = Y/N$, $e = E/N$, and $x = K/Y$. Then we have

$$\begin{aligned} & Y_t = A_t K_t^{\theta} (h_t E_t)^{1-\theta}, \\ \Leftrightarrow \quad & Y_t^{1/(1-\theta)} = A_t^{1/(1-\theta)} K_t^{\theta/(1-\theta)} h_t E_t, \\ \Leftrightarrow \quad & Y_t = A_t^{1/(1-\theta)} x_t^{\theta/(1-\theta)} h_t E_t, \\ \therefore \quad & y_t = A_t^{1/(1-\theta)} x_t^{\theta/(1-\theta)} h_t e_t. \end{aligned}$$

Growth accounting, cont'd

- By taking the difference of logarithm, growth of output per capita is decomposed as

$$g_{yt} = \frac{1}{1-\theta} g_{At} + \frac{\theta}{1-\theta} g_{xt} + g_{ht} + g_{et},$$

where $\theta = 0.362$.

Table I: Accounting for Japanese Growth per Person Aged 20-69

Period	Growth rate	Factors			
		TFP factor	Capital intensity	Workweek length	Employment rate
1960-1973	7.2%	6.5%	2.3%	-0.8%	-0.7%
1973-1983	2.2%	0.8%	2.1%	-0.4%	-0.3%
1983-1991	3.6%	3.7%	0.2%	-0.5%	0.1%
1991-2000	0.5%	0.3%	1.4%	-0.9%	-0.4%

Household's utility

- There are N_t households in working age. The utility function is

$$\sum_{t=0}^{\infty} N_t \left(\log c_t - g(h_t; z_t) e_t \right),$$

  where $e_t = E_t / N_t$ is the fraction of working households and $c_t = C_t / N_t$ is consumption per capita.

- Labor is indivisible so that a person eigher works h_t hours or does not work at all.

- Households also choose h_t if they work. z_t is the number of days worked and exogenously given.

Household's budget constraint

- The household's budget constraint is given by

$$\begin{aligned} C_t + I_t &\leq w_t h_t E_t + r_t K_t - \tau(r_t - \delta)K_t - \pi_t, \\ I_t &= K_{t+1} - (1-\delta)K_t, \end{aligned}$$

  where C_t is aggregate consumption, I_t is aggregate investment, w_t is wage rate and r_t is real interest rate.

- A distortionary tax is imposed at rate τ on capital income after depreciation. π_t is a lumpsum tax.

- Second welfare theorem does not hold.

Closing the model

- Resource constraint is given by

$$C_t + I_t + G_t = Y_t.$$

- The government's budget constraint is implicitly obtained as

$$G_t = \tau(r_t - \delta)K_t + \pi_t.$$

Solving the model: Firm

- The representative firm maximizes its profit: (marginal cost) = (marginal product)

$$w_t = (1-\theta)y_t/(h_t e_t),$$
$$r_t = \theta y_t/k_t.$$

where y_t and k_t are output and capital per capita.

Solving the model: Household

- The household's Lagrangean is

$$L_t \equiv \sum_{t=0}^{\infty} \beta^t \left\{ N_t \left(\log c_t - g(h_t; z_t) e_t \right) \right\} .$$
$$+ \lambda_t N_t \left(w_t h_t e_t + r_t k_t - \tau (r_t - \delta) k_t - \tilde{\pi}_t - c_t - n_t k_{t+1} + (1 - \delta) k_t \right) ,$$

where $n_t \equiv N_{t+1}/N_t$ and $\tilde{\pi}_t \equiv \pi_t / N_t$.

- The FOCs are

$$\begin{aligned} \partial c_t : &\quad c_t^{-1} = \lambda_t, \\ \partial e_t : &\quad g(h_t; z_t) = \lambda_t w_t h_t, \\ \partial h_t : &\quad g'(h_t; z_t) = \lambda_t w_t, \\ \partial k_{t+1} : &\quad 1 = \beta \frac{\lambda_{t+1}}{\lambda_t} \left(1 + (1 - \tau)(r_{t+1} - \delta) \right) . \end{aligned}$$

Solving the model: Aggregation

- The resource constraint (per capita) is

$$c_t + n_t k_{t+1} - (1-\delta)k_t + g_t = y_t.$$

- The production function is

$$y_t = A_t k_t^{\theta}(h_t e_t)^{1-\theta}.$$

- We have $\{c_t, k_t, y_t, h_t, e_t, \lambda_t, w_t, r_t\}$ and 8 equations, so we can solve the model.

Equilibrium conditions

- The equlibrium conditions are summerized as

$$
\begin{aligned}
&c_t g(h_t; z_t) = (1-\theta) y_t / e_t, \\
&c_t g'(h_t; z_t) = (1-\theta) y_t / (h_t e_t), \\
&1 = \beta \frac{c_t}{c_{t+1}} \left(1 + (1-\tau)(r_{t+1} - \delta)\right), \\
&c_t + n_t k_{t+1} - (1-\delta) k_t + g_t = y_t, \\
&y_t = A_t k_t^{\theta} (h_t e_t)^{1-\theta}, \\
&w_t = (1-\theta) y_t / (h_t e_t), \\
&r_t = \theta y_t / k_t.
\end{aligned}
$$

- Be aware of time subscripts.

Labor disutility and hours worked

- Combining the first two equations

$$hg'(h;z) = g(h;z),$$

where $g(h;z)$ is convex in h and $g(0;z) > 0$ because of the fixed cost of going work. $h(z)$ is determined separately from the rest of the model.

- Further, $g(h)$ is approximated around $\bar{h} = 40$ as

$$g(h(z)) \approx \frac{\alpha h(z)}{\bar{h}}.$$

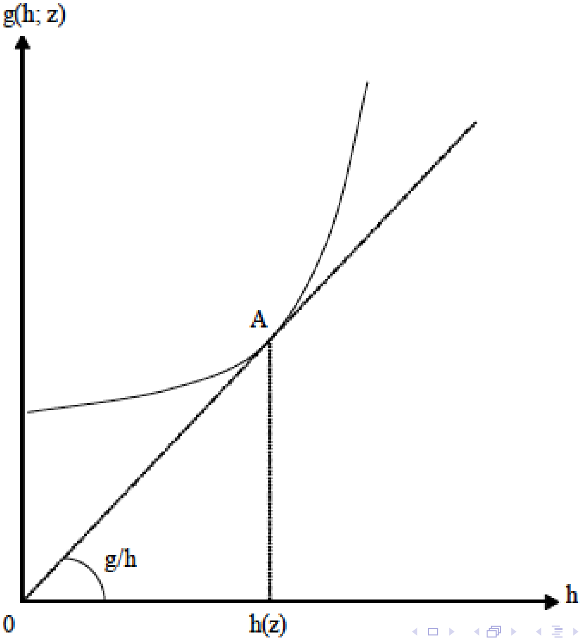
Indivisible labor

- Why $g(h)$ is convex and $g(0) > 0$? e_t is also the probability of employed, and the household's utility function is

$$\log(c_t) + \phi(h_t)e_t + \phi_0(1 - e_t),$$

where $\phi'(h) < 0$ and $\phi''(h) < 0$ (e.g., $\phi(h) = A\log(1-h)$). Ignoring a constant term, $g(h) \equiv \phi_0 - \phi(h)$.

How hours worked are determined



Detrending

- Let $Z_t \equiv A_t^{\frac{1}{1-\theta}}$ and define

$$\tilde{c}_t \equiv c_t/Z_t, \quad \tilde{k}_t \equiv k_t/Z_t, \quad \tilde{y}_t \equiv y_t/Z_t,$$
$$\gamma_t \equiv Z_{t+1}/Z_t, \quad \psi_t \equiv g_t/y_t, \quad n_t \equiv N_{t+1}/N_t.$$

- Finally, we have

$$(\alpha h_t/\bar{h})\tilde{c}_t = (1-\theta)\tilde{y}_t/e_t,$$
$$1 = \frac{\beta}{\gamma_t}\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\left(1+(1-\tau)(r_{t+1}-\delta)\right),$$
$$\tilde{c}_t + \gamma_t n_t \tilde{k}_{t+1} - (1-\delta)\tilde{k}_t = (1-\psi_t)\tilde{y}_t,$$
$$\tilde{y}_t = \tilde{k}_t^{\theta}(h_t e_t)^{1-\theta},$$
$$r_t = \theta\tilde{y}_t/\tilde{k}_t.$$

Calibration

- The model is calibrated using the data during 1984-89 (which is found in columns FX-GB in FED_data.xls).
  - $\theta = 0.362$: average of the capital income share in GNP.
  - $\delta = 0.089$: ratio of depreciation to the beginning-of-the-year capital stock.
  - $\tau = 0.480$: average of the capital tax.
  - $\beta = 0.976$: obtained from the Euler equation

$$\frac{c_{t+1}}{c_t} = \beta\left(1 + (1-\tau)(\theta y_{t+1}/k_{t+1} - \delta)\right).$$

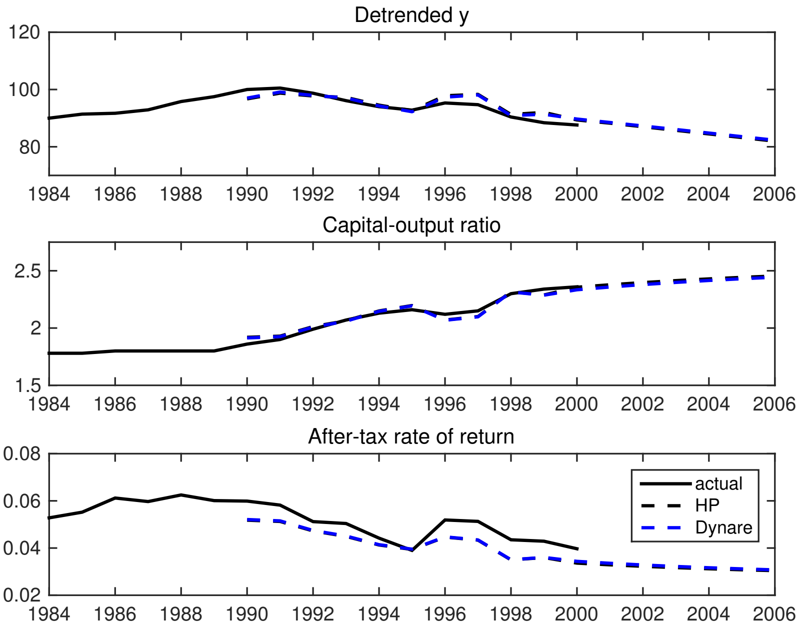
  - $\alpha = 1.373$: obtained from the wage equation so that $h(z_t) = \bar{h}$

$$\alpha c_t = (1-\theta) y_t / e_t = w_t h_t.$$

Simulation

- The exogenous variables are (A_t, N_t, ψ_t) where $\psi_t = G_t/Y_t$.
- For the 1990s ($t = 1990, 1991, ..., 2000$), actual values are used.
- For $t = 2001, 2002, ...,$
  - The growth rate of $A_t^{1/(1-\theta)}$is set to its 1991-2000 average of 0.29%.
  - N_t is 2000 value; no population growth.
  - ψ_t is set to 1999-2000 value of 15%.
- Deterministic simulation is done. The decline of TFP growth in the 1990s is forecasted in 1990.
- Also, in our Dynare simulation, we set $h(z_t) = \bar{h} = 40$ for all t.

Figure 6-8 in the paper



Some comments

- Detrended output is close to the model predictions.
- Capital-output ratio rises and the rate of return falls as output (consumption) growth falls, which is seen in the Euler equation.
- The model's prediction is sensitive to the exogenous variables in the 1990s. The most important variable is TFP.
- Dynare simulation yields almost the same result as in HP.