Basic New Keynesian Model

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Introduction

- The classical monetary economy with perfect competition and fully flexible prices in all markets provides a reference benchmark.
- The model to be considered has two departures from the assumptions of the classical monetary model:
  - Imperfect competition in the goods market: Each firm produces a differentiated good for which it set the price.
  - Price stikiness: Only a fraction of firms can reset their prices in any given period (Calvo, 1983).
- The resulting framework is referred to as the basic New Keynesian model.

Roadmap

- We will look at:

1. Household's problem: They consume differentiated goods so that the demand schedule for each good is derived.
2. Firm's problem: Given the demand schedule, each firm set the price in a monopolistic competition. As prices are sticky, firms maximize the discounted sum of their future profits.
3. In equilibrium, the two key dynamic equations will be derived: New Keynesian Phillips curve (NKPC) and Dynamic IS curve.

Households

- The representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t),$$

where C_t is now a consumption index given by

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

and $C_t(i)$ denotes the quantity of good $i \in [0,1]$ consumed. This is called Dixit-Stiglitz aggregator.

Household's budget constraint

- Maximization is subject to a sequence of budget constraints:

$$\int_0^1 P_t(i)C_t(i)di + Q_tB_t \leq B_{t-1} + W_tN_t + D_t,$$

where

- $P_t(i)$ is the price of good i,
- W_t is nominal wage,
- B_t is one-period riskless discount bonds with its price Q_t,
- and D_t represents dividends.

Household's cost minimization

- The household minimizes the cost of purchasing goods,

$$\int_0^1 P_t(i)C_t(i)di,$$

subject to

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

for any given level of C_t.

Aggregate price index and consumption

- After some algebra, we have a downward-sloping demand schedule

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t, \tag{1}$$

for all $i \in [0,1]$.

- Furthermore, we have

$$\int_0^1 P_t(i)C_t(i)di = P_tC_t.$$

Pluging this into the budget constraint yields

$$P_tC_t + Q_tB_t \leq B_{t-1} + W_tN_t + D_t$$

which is the exactly same form as we have in the classical monetary model.

Lagrangean

- Taking the choice of $C_t(i)$ as given, we have the same budget constraint and utility function. Then we set up the Lagrangean as before:

$$L_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t, N_t; Z_t) \right.$$
$$\left. + \lambda_t \left(B_{t-1} + W_t N_t + D_t - P_t C_t - Q_t B_t \right) \right].$$

- Taking the derivatives of the Lagrangean and set them to zero,

$$\begin{aligned} \partial C_t : &\quad U_{c,t} = P_t \lambda_t, \\ \partial N_t : &\quad U_{n,t} = -W_t \lambda_t, \\ \partial B_t : &\quad Q_t \lambda_t = \beta E_t \lambda_{t+1}, \end{aligned}$$

where $U_{c,t} = \partial U_t / \partial C_t$ and $U_{n,t} = \partial U_t / \partial N_t$.

Household's optimality conditions

- Recap: Given the form of the utility function, the optimality conditions become

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi},$$
$$Q_t = \beta E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{-\sigma} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right\}.$$

- The log-linearized version of the optimality conditions are

$$p_t - w_t = \sigma c_t + \varphi n_t,$$
$$c_t = E_t c_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1} - \rho) + \sigma^{-1} (1 - \rho_z) z_t.$$

- Also, an ad-hoc log-linear money demand equation is given by

$$m_t - p_t = \frac{\sigma}{\nu} c_t - \eta i_t.$$

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Firms

- A continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good, but access to the same technology of production

$$Y_t(i) = A_t N_t(i),$$

where A_t is the level of technology and $a_t \equiv \log A_t$ follows

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a.$$

- [For simplicity, we consider the special case of $\alpha = 1$ in Gali's textbook.]

Firms, cont'd

- Each firm faces the demand schedule (1) by the household:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t,$$

and takes P_t and Y_t as given. Note that $Y_t(i) = C_t(i)$ and $Y_t = C_t$ holds in equilibrium.

Firms' problem

- We will consider the firm i's problem in the following two stages:

1. The firm minimizes the production cost in each period by choosing the amount of labor $N_t(i)$, and
2. The firm maximizes the discounted sum of their future profits by setting the price $P_t(i)$ in the current period. Note that $P_t(i)$ will affect the future profits as the prices will be unchanged for some time.

Firm's cost minimization

- The firm minimizes the cost of production. Lagrangean is

$$L_t = W_t N_t(i) - \Psi_t(i) \left(A_t N_t(i) - Y_t(i) \right),$$

which yields the real marginal cost $\tilde{\Psi}_t(i) = \Psi_t(i)/P_t$ as

$$\tilde{\Psi}_t(i) A_t = \frac{W_t}{P_t}.$$

Real marginal cost multiplied by the marginal product of labor = real wage.

- Thanks to the CRS technology, the marginal cost is common across firms, i.e., $\tilde{\Psi}_t(i) = \tilde{\Psi}_t$.

Firm's profit maximization

- Each firm may adjust its price only with probability $(1-\theta)$ in any given period, independent of the time elapsed from the last adjustment.

- The firm's expected future profit is

$$\Pi_t(P_t^*) + \theta\Lambda_{t,t+1}\Pi_{t+1}(P_t^*) + \theta^2\Lambda_{t,t+2}\Pi_{t+2}(P_t^*) + \cdots$$

  where θ is the probability of the price being fixed at P_t^* and $\Lambda_{t,t+k}$ is a stochastic discount factor.

Firm's profit maximization, cont'd

- A reoptimizing firm chooses the price P_t^* so as to maximize (in real terms)

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \left(\frac{P_t^*}{P_{t+k}} - \tilde{\Psi}_{t+k} \right) Y_{t+k|t} \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}.$$

$Y_{t+k|t}$ is the demand for the goods produced by the firm given that the price is unchanged.

Optimality condition

- The optimality condition takes the form

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t+k}} - \underbrace{\frac{\varepsilon}{\varepsilon - 1}}_{\mathcal{M}} \tilde{\Psi}_{t+k} \right) \right\} = 0. \tag{2}$$

- Note that when $\theta = 0$, the above equation collapses to

$$P_t^* = \mathcal{M} \Psi_t.$$

That is, the optimal price is the nomial marginal cost multiplied by the optimal markup.

Firm's optimality condition, cont'd

- The log-linearized version of (2) is given by

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t \psi_{t+k}, \tag{3}$$

where $\mu \equiv \log \mathcal{M}$ is the logged optimal markup and $\psi_{t+k} \equiv \log \Psi_{t+k}$ is the logged nominal marginal cost.

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Aggregate price dynamics

- A measure of $(1-\theta)$ producers reset their prices, whereas θ producers keep their price unchenged. Then we have the aggregate price dynamics

$$
\begin{aligned}
P_t^{1-\varepsilon} &= \int_0^1 P_t(i)^{1-\varepsilon} di, \\
&= \int_{S(t)} P_{t-1}(i)^{1-\varepsilon} di + (1-\theta)(P_t^*)^{1-\varepsilon}, \\
&= \theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon},
\end{aligned}
$$

  where $S(t)$ is the set of firms not adjusting their price in period t.

- The log-linearized version of the aggregate price dynamics is given by

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}).$$

Inflation dynamics

- Eq. (3) can be rewritten recursively

$$p_t^* = \beta\theta E_t p_{t+1}^* + (1-\beta\theta)\left(\mu + \psi_t\right).$$

- Combinining it with the aggregate price dynamics, we have

$$\pi_t = \beta E_t \pi_{t+1} - \underbrace{\frac{(1-\beta\theta)(1-\theta)}{\theta}}_{=:\lambda} \hat{\mu}_t,$$

where $\hat{\mu}_t \equiv \mu_t - \mu$ is the deviation between the average and desired markups and $\mu_t \equiv p_t - \psi_t$.

Inflation dynamics, cont'd

- Solving this forward, inflation is expressed as the discounted sum of current and expected future deviations of average markups from their desired levels:

$$\pi_t = -\lambda \sum_{k=0}^{\infty} \beta^k E_t \{\hat{\mu}_{t+k}\}$$

  Thus, inflation will be positive when firms expect average markups to be below (or real marginal cost to be above) their desired level.

- Or, using the real marginal cost $\tilde{\psi}_t = \psi_t - p_t = -\mu_t$,

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k E_t \left\{\tilde{\psi}_{t+k}\right\}$$

Aggregate employment

- Aggregate employment is given by

$$\begin{aligned} N_t &= \int_0^1 N_t(i)di, \\ &= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right) di, \\ &= \left(\frac{Y_t}{A_t}\right) \underbrace{\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di}_{=:D_t \geq 1}. \end{aligned}$$

Note that $Y_t D_t = A_t N_t$ holds. Staggered price setting takes real resources.

Solving for the output gap

- Taking logs yields
$$n_t = y_t - a_t + d_t.$$
d_t is a measure of price dispersion, which is equal to zero up to a first-order approximation. [See Appendix 3.4 in the text.]

- The average price markup is given by
$$\begin{aligned}\mu_t &= p_t - \psi_t \\ &= p_t - w_t + a_t, \\ &= -(\sigma y_t + \varphi n_t) + a_t, \\ &= -(\sigma + \varphi) y_t + (1 + \varphi) a_t.\end{aligned}$$
Note that we have also used $\psi_t = w_t - a_t$ and $p_t - w_t = -(\sigma y_t + \varphi n_t)$.

Solving for the output gap, cont'd

- Define the natural level of output y_t^n under flexible prices,

$$\mu = -(\sigma + \varphi)y_t^n + (1 + \varphi)a_t,$$

which implies

$$y_t^n = \psi_{ya} a_t + \psi_y$$

where $\psi_{ya} = \frac{1+\varphi}{\sigma+\varphi}$ and $\psi_y = \frac{-\mu}{\sigma+\varphi}$.

- Then we have

$$\hat{\mu}_t = -(\sigma + \varphi)(y_t - y_t^n),$$

that is, the markup gap is proportional to the output gap, $\tilde{y}_t = y_t - y_t^n$.

Output dynamics

- By substituting the output gap into the Euler equation

$$y_t = E_t y_{t+1} - \sigma^{-1}\left(i_t - E_t \pi_{t+1} - \rho\right) + \sigma^{-1}(1-\rho_z) z_t$$

we have

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma^{-1}\left(i_t - E_t \pi_{t+1} - r_t^n\right)$$

where the natural rate of interest is given by

$$r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t.$$

Output dynamics, cont'd

- Solving this forward, inflation is expressed as the discounted sum of current and expected future deviations of real interest rate from the natural rate:

$$\tilde{y}_t = -\sigma^{-1} \sum_{k=0}^{\infty} E_t \left\{ r_{t+k} - r_{t+k}^n \right\} .$$

Key equations

- Now we have two key equations:
- New Keynesian Phillips curve (NKPC)

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t,$$

where $\kappa = \frac{(1-\beta\theta)(1-\theta)(\sigma+\varphi)}{\theta}$.

- Dynamic IS curve

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^n \right).$$

- In order to close the model, one or more equations determining the nominal interest rate i_t are needed.

Simple interest rate rule

- Consider a simple interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y (y_t - y) + \nu_t,$$

where ϕ_π and ϕ_y are chosen by the monetary authority.

- Note that a zero inflation steady state implies $i = \rho$. This is known as the "Taylor rule."

Equilibrium under a simple interest rate rule

- Combining the Taylor rule with NKPC and IS curve, we have

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \underbrace{\Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa+\beta(\sigma+\phi_y) \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} E_t\tilde{y}_{t+1} \\ E_t\pi_{t+1} \end{bmatrix} + \underbrace{\Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}}_{\mathbf{B}} u_t,$$

where $\Omega = (\sigma + \phi_y + \kappa\phi_\pi)^{-1}$ and

$$\begin{aligned} u_t &= (r_t^n - \rho) - \phi_y(y_t^n - \psi_y) - \nu_t, \\ &= -\psi_{ya}(\phi_y + \sigma(1-\rho_a))a_t + (1-\rho_z)z_t - \nu_t. \end{aligned}$$

We will consider each shock $\{a_t, z_t, \nu_t\}$ at a time.

Stability condition

- When $\mathbf{A}$ ($\mathbf{A}^{-1}$) has both eigenvalues inside (outside) the unit circle, the solution is locally unique. A necessary and sufficient condition for uniqueness is given by
$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$
which is assumed to hold.

The effects of a monetary policy shock

- Assume that $a_t = z_t = 0$ and $\nu_t = -u_t$ follows an AR(1) process:

$$\nu_{t+1} = \rho_\nu \nu_t + \varepsilon^\nu_{t+1}.$$

Also conjecture the solution is of the form:

$$\tilde{y}_t = \psi_x \nu_t, \qquad \pi_t = \psi_\pi \nu_t.$$

- Then we obtain

$$\begin{aligned} \psi_x &= \Omega[\sigma\psi_x + (1-\beta\phi_\pi)\psi_\pi]\rho_u + \Omega, \\ \psi_\pi &= \Omega[\sigma\kappa\psi_x + (\kappa + \beta(\sigma + \phi_y))\psi_\pi]\rho_u + \Omega\kappa. \end{aligned}$$

The effects of a monetary policy shock, cont'd

- After some algebra, we have the decision rules

$$\begin{aligned}\tilde{y}_t &= -(1-\beta\rho_\nu)\Lambda_\nu \nu_t, \\ \pi_t &= -\kappa\Lambda_\nu \nu_t,\end{aligned}$$

  where $\Lambda_\nu = \frac{1}{(1-\beta\rho_\nu)[\sigma(1-\rho_\nu)+\phi_y]+\kappa(\phi_\pi-\rho_\nu)}$. We immediately know that $\tilde{y}_t$ and π_t respond negatively to a positive shock to ν_t.

- Also,

$$\begin{aligned}r_t &= \rho + \sigma(1-\rho_\nu)(1-\beta\rho_\nu)\Lambda_\nu \nu_t, \\ i_t &= r_t + E_t\pi_{t+1}, \\ & \rho + [\rho + \sigma(1-\rho_\nu)(1-\beta\rho_\nu) - \rho_\nu\kappa]\Lambda_\nu \nu_t.\end{aligned}$$

The effects of a technology shock

- Similarly we have

$$\begin{aligned}\tilde{y}_t &= -\psi_{ya}(\phi_y + \sigma(1-\rho_a))(1-\beta\rho_a)\Lambda_a a_t,\\ \pi_t &= -\psi_{ya}(\phi_y + \sigma(1-\rho_a))\kappa\Lambda_a a_t,\end{aligned}$$

where $\Lambda_a = \frac{1}{(1-\beta\rho_a)[\sigma(1-\rho_a)+\phi_y]+\kappa(\phi_\pi-\rho_a)}$.

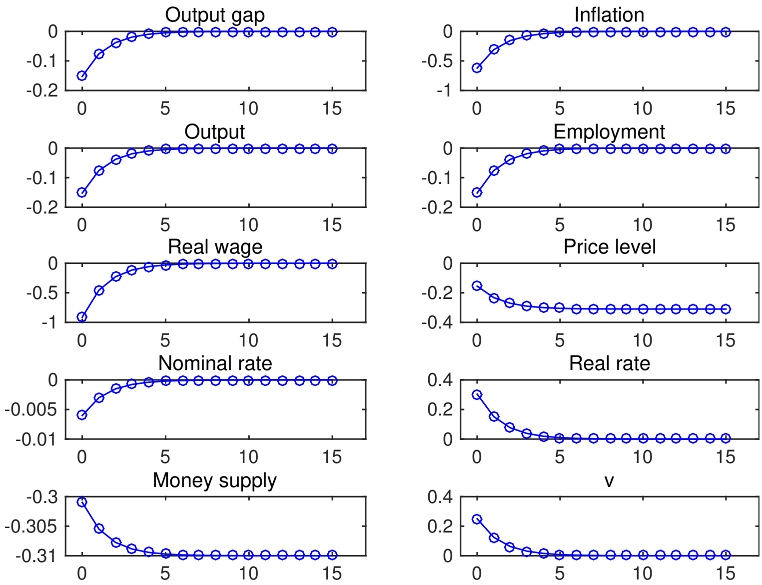
- Also,

$$\begin{aligned}y_t &= \psi_{ya}\kappa(\phi_\pi - \rho_a)\Lambda_a a_t,\\ n_t &= y_t - a_t\\ &= \left[\frac{(1-\sigma)\kappa(\phi_\pi-\rho_a)}{\sigma+\varphi} - (\phi_y + \sigma(1-\rho_a))(1-\beta\rho_a)\right]\Lambda_a a_t.\end{aligned}$$

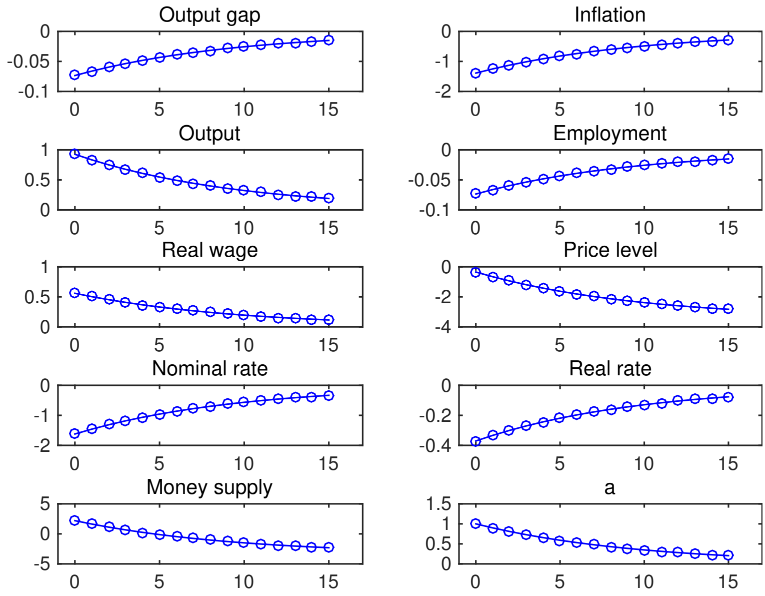
Numerical exercises

- We set: $\beta = 0.99$, $\sigma = 1$, $\varphi = 5$, $\epsilon = 9$, $\theta = 3/4$, $\phi_\pi = 1.5$, $\phi_y = 0.5/4$, and $\eta = 4$.
- We consider:
  - An increase of 25 basis points in ε_t^ν with $\rho_\nu = 0.5$.
  - An increase of 100 basis points in ε_t^a with $\rho_a = 0.9$.

Impulse responses to monetary policy shock



Impulse responses to technology shock



Assignment #4

1. Derive the downward-sloping demand curve (1) by solving the household's cost minimization problem.
2. Derive the firm's optimality condition (2) and its log-linear counterpart (3).
3. Calculate the impulse responses to a decrease of 50 basis points in discount rate shock z_t with $\rho_z = 0.5$. Discuss the difference from the impulse responses to a monetary policy shock shown in this slide.