# Proposed Answers to Assignment #3 and #5

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## Assignment #3 (1)

The equilibrium conditions are

$$
\begin{aligned}
&1 = \beta\left(1 + \theta\frac{y}{k} - \delta\right), \\
&(1-\theta)\frac{y}{h} = Bc, \\
&y = k^{\theta}h^{1-\theta}, \\
&c + \delta k = y.
\end{aligned}
$$

These are solved for the ratio values

$$
\begin{aligned}
&\frac{y}{k} = \theta^{-1}(\beta^{-1} - 1 + \delta), \\
&\frac{k}{h} = \left(\frac{y}{k}\right)^{\frac{1}{\theta-1}}, \\
&\frac{c}{k} = \frac{y}{k} - \delta.
\end{aligned}
$$

Then we have

$$
\begin{aligned}
h &= \frac{(1-\theta)\frac{y}{k}}{B\frac{c}{k}}, \\
&= \frac{(1-\theta)\frac{y}{k}}{B(\frac{y}{k} - \delta)},
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1-\theta)(\beta^{-1}-1+\delta)}{B(\beta^{-1}-1+(1-\theta)\delta)}, \\
&= \frac{(1-\theta)[1-\beta(1-\delta)]}{B(1-\beta[1-(1-\theta)\delta])}.
\end{aligned}
$$

Also, $k = (k/h)h$, $y = (y/k)k$ and $c = (c/k)k$ are obtained.

As for the log-linearization, only two equations are new.

$$
\begin{aligned}
& (1-\theta)\frac{y_t}{h_t} = Bc_t, \\
\Rightarrow \quad & (1-\theta)(1+\hat{y}_t - \hat{h}_t) = Bc(1+\hat{c}_t), \\
\therefore \quad & \hat{y}_t - \hat{h}_t = \hat{c}_t.
\end{aligned}
$$

Note that we have used $(1-\theta) = Bc$.

$$
\begin{aligned}
& \lambda_t = (1-\gamma) + \gamma\lambda_{t-1} + \varepsilon_t, \\
\Rightarrow \quad & \lambda(1+\hat{\lambda}_t) = (1-\gamma) + \gamma\lambda(1+\hat{\lambda}_{t-1}) + \varepsilon_t, \\
\therefore \quad & \hat{\lambda}_t = \gamma\hat{\lambda}_{t-1} + \varepsilon_t.
\end{aligned}
$$

Note that we have used $\lambda = (1-\gamma)+\gamma\lambda$ therefore $\lambda = 1$. Also, net investment is given by

$$
\begin{aligned}
& x_t = k_t - (1-\delta)k_{t-1} \\
\Rightarrow \quad & x(1+\hat{x}_t) = k(1+\hat{k}_t) - (1-\delta)k(1+\hat{k}_{t-1}), \\
\Rightarrow \quad & x\hat{x}_t = k\hat{k}_t - (1-\delta)k\hat{k}_{t-1}, \\
\therefore \quad & \delta\hat{x}_t = k\hat{k}_t - (1-\delta)k\hat{k}_{t-1}.
\end{aligned}
$$

Note that we have used $x = \delta k$.

## Assignment #5 (1)

The planner's problem is

$$
\max_{\{c_t, k_t\}} \sum_{t=0}^{\infty} \beta^t \ln c_t
$$

subject to

$$c_t + k_t \le k_{t-1}^{\theta},$$

given $k_{-1}$.

[Note that $A$ in the production function is set as $A = 1$.] The value function is defined as

$$V(k_{t-1}) \equiv \max_{k_t} \sum_{t=0}^{\infty} \beta^t \ln(k_{t-1}^{\theta} - k_t).$$

The Bellman equation is written as

$$V(k_{t-1}) = \max_{k_t} \left\{ \ln(k_{t-1}^{\theta} - k_t) + \beta V(k_t) \right\}.$$

## Assignment #5 (2)

Conjecture $V(k) = A + B \ln k$. Then we have

$$V(k) = \max_{k'} \left\{ \ln(k^{\theta} - k') + \beta \left( A + B \ln k' \right) \right\}.$$

The FOC is

$$\frac{1}{k^{\theta} - k'} = \frac{\beta B}{k'}.$$

It can be solved for

$$k' = \frac{\beta B}{1 + \beta B} k^{\theta}.$$

Pluging it into the Bellman equation, we have

$$\begin{aligned}
& \ln(k^{\theta} - k') + \beta \left( A + B \ln k' \right), \\
= \; & \ln \left( k^{\theta} - \frac{\beta B}{1 + \beta B} k^{\theta} \right) + \beta A + \beta B \ln \left( \frac{\beta B}{1 + \beta B} k^{\theta} \right), \\
= \; & \ln \left( \frac{1}{1 + \beta B} k^{\theta} \right) + \beta A + \beta B \ln \left( \frac{\beta B}{1 + \beta B} k^{\theta} \right), \\
= \; & \beta A + \ln \left( \frac{1}{1 + \beta B} \right) + \beta B \ln \left( \frac{\beta B}{1 + \beta B} \right) + \theta \ln k + \beta B \theta \ln k.
\end{aligned}$$

Thus we have

$$\begin{aligned}
A &= \beta A + \ln\left(\frac{1}{1+\beta B}\right) + \beta B \ln\left(\frac{\beta B}{1+\beta B}\right), \\
B &= \theta(1+\beta B).
\end{aligned}$$

Finally, we have

$$B = \theta/(1-\beta\theta),$$

and

$$A = (1-\beta)^{-1} \ln\left(\frac{1}{1+\beta B}\right)\left(\frac{\beta B}{1+\beta B}\right)^{\beta B}.$$