Monetary Policy Design in the Basic New Keynesian Model

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Introduction

- How monetary policy should be conducted?
- Monopolistic competition and sticky prices are the source of inefficiency in the basic NK model.
- Fiscal subsidy eliminates the inefficiency stemming from the monopolistic competition. Monetary policy can achieve the efficient allocation by fully stabilizing the price level.

The efficient allocation

- The benevolent social planner maximizes the representative household's utility:

$$U(C_t, N_t; Z_t)$$

subject to

$$\begin{aligned} C_t &\equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \\ C_t(i) &= A_t N_t, \end{aligned}$$

where $N_t = \int_0^1 N_t(i) di$, for each period.

The conditions for the efficient allocation

- The optimiality condition should satisfy

$$
\begin{aligned}
C_t(i) &= C_t, \\
N_t(i) &= N_t,
\end{aligned}
$$

for all $i \in [0, 1]$. Given this, we have

$$
\underbrace{-\frac{U_{n,t}}{U_{c,t}}}_{MRS_t} = MPN_t.
$$

$MPN_t = A_t$ denotes the economy's average product of labor (=the marginal product of each firm).

Distortions from monopolistic competition

- Suppose that prices are fully flexible. In this case, the firm i maximizes

$$P_t(i)Y_t(i) - W_t N_t(i),$$

subject to

$$\begin{aligned} Y_t(i) &= A_t N_t(i), \\ Y_t(i) &= \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t. \end{aligned}$$

Distortions from monopolistic competition, cont'd

- The optimal price setting rule is given by

$$P_t = \mathcal{M}\frac{W_t}{MPN_t}.$$

where $\mathcal{M} = \varepsilon/(\varepsilon - 1)$. Note that firms are symmetric without Calvo-type price stickiness: $P_t(i) = P_t(j)$, $Y_t(i) = Y_t(j)$ and $N_t(i) = N_t(j)$ for $i \neq j$ hold.

- Accordingly,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t.$$

The presence of a markup leads to an inefficiently low level of employment and output. [graph]

Subsidy to eliminate the distortions

- The above inefficiency can be eliminated by an employment subsidy (financed by means of lump-sum tax):

$$P_t(i)Y_t(i) - (1-\tau)W_t N_t(i).$$

- Then, under flexible prices, $P_t = \mathcal{M}\frac{(1-\tau)W_t}{MPN_t}$ holds. Accordingly,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}.$$

  The optimal allocation can be attained if $\mathcal{M}(1-\tau) = 1$.

- By construction, the equilibrium under flexible prices is efficient.

Distortions related to sticky prices

- Under sticky prices, the economy's average markup is defined as

$$\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t\mathcal{M}}{(W_t/MPN_t)},$$

where we have used $\mathcal{M}(1-\tau)=1$. In this case of the efficient steady state,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t\frac{\mathcal{M}}{\mathcal{M}_t}.$$

The optimal allocation can be attained if $\mathcal{M}_t = \mathcal{M}$.

- Yet another source of distortion: When prices are sticky, $P_t(i) \neq P_t(j)$ holds. Then the efficiency conditions $C_t(i) = C_t$ and $N_t(i) = N_t$ are also violated.

The case of efficient natural allocation

- Optimal policy (which achieves the efficient allocation) requires that

$$y_t = y_t^n,$$

or equivalently, the output gap $x_t = y_t - y_t^n = 0$ for all t. Then the NKPC implies $\pi_t = 0$ and the dynamic IS curve implies $i_t = r_t^n$ for all t.

- Two features of the optimal policy:

1. Output stability is not optimal; output should vary with the natural level of output.
2. Price stability implies an efficient level of output, and vice versa. This is called the divine coincidence (Blanchard and Gali, 2007).

The basic NK model

- Recall: New Keynesian Phillips curve (NKPC)

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,$$

where $x_t = y_t - y_t^n$, $y_t^n = \psi_{ya} a_t + \psi_y$, and $\kappa = \frac{(1-\beta\theta)(1-\theta)(\sigma+\varphi)}{\theta}$.

- Dynamic IS curve

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^n \right),$$

where the narutal rate of interest is given by

$$r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t.$$

- Question: What is the optimal interest rate rule for i_t?

Optimal monetary policy rules

- If we consider the following rule

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y x_t,$$

where $\phi_\pi > 0$ and $\phi_y > 0$, It implements the efficient allocation iff the equilibrium is determinate, i.e.,

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

holds.

- However, it is unlikely to observe the narural rate of interest, r_t^n, in reality.

Simple monetary policy rules

- Instead, we consider

$$i_t = \rho + \phi_\pi \pi_t + \phi_y (y_t - y),$$

where $\rho = -\log \beta$. This is a function of observable variables only, and called "simple rule."

- How to evaluate the simple rules?

Welfare loss function

- Following Rotemberg and Woodford (1999), a welfare-based criterion from a second-order approximation to the representative household's utility is used.

- Recall: the representative household's utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t).$$

- It can be approximated by

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\epsilon}{\lambda} \pi_t^2 + (\sigma + \varphi) x_t^2 \right).$$

This is called welfare loss function.

Welfare loss function, cont'd

- The average welfare loss per period is given by a linear combination of the variance of the output gap and inflation

$$\mathbb{L} = \frac{1}{2}\left[\frac{\epsilon}{\lambda}\text{var}(\pi_t) + (\sigma + \varphi)\text{var}(x_t)\right].$$

- We can obtain the variance and the avarage welfare loss once we solve the model.

- The analysis is conducted on technology and demand shocks separately.

The effects of a technology shock

- Recall: After some algebra, we have the decision rules

$$
\begin{aligned}
x_t &= -\psi_{ya}(\phi_y + \sigma(1-\rho_a))(1-\beta\rho_a)\Lambda_a a_t, \\
\pi_t &= -\psi_{ya}(\phi_y + \sigma(1-\rho_a))\kappa\Lambda_a a_t,
\end{aligned}
$$

where $\Lambda_a = \frac{1}{(1-\beta\rho_a)[\sigma(1-\rho_a)+\phi_y]+\kappa(\phi_\pi-\rho_a)}$.

- Also,

$$
y_t = \psi_{ya}\kappa(\phi_\pi - \rho_a)\Lambda_a a_t.
$$

The effects of a demand shock

- We have

$$\begin{aligned} x_t = y_t &= -(1-\beta\rho_\nu)\Lambda_z z_t, \\ \pi_t &= -\kappa\Lambda_z z_t, \end{aligned}$$

where $\Lambda_z = \frac{1}{(1-\beta\rho_z)[\sigma(1-\rho_z)+\phi_y]+\kappa(\phi_\pi-\rho_z)}$.

Numerical examples (when $\alpha = 0$)

- The standard deviation of innovations in both shocks is set to one percent. What if $\phi_\pi \to \infty$ or $\phi_y \to \infty$?

	Technology				Demand			
ϕ_π	1.5	1.5	5	1.5	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1	0.125	0	0	1
$\sigma(y)$	2.126	2.216	2.282	1.653	0.351	0.380	0.113	0.229
$\sigma(x)$	0.169	0.078	0.012	0.641	0.351	0.380	0.113	0.229
$\sigma(\pi)$	0.797	0.369	0.056	3.030	0.358	0.387	0.116	0.234
$(1-\beta)\mathbb{L}$	0.334	0.072	0.002	4.826	0.071	0.083	0.007	0.030

Appendices

- A model with Rotemberg (1982) adjustment cost
- Second-order approximation of the utility function

Firm's price setting: Price adjustment cost

- Rotemberg (1982) introduced price stickiness in a form of price adjustment cost. Firms incur a cost when they set price different from yesterday.
- The firm $i \in [0,1]$ chooses the price $P_t(i)$ so as to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t}{C_0} \right)^{-\sigma} \left[\left(\frac{P_t(i)}{P_t} - (1-\tau)\tilde{\Psi}_t \right) Y_t(i) - \frac{\phi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t \right],$$

subject to the sequence of demand constraints

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t.$$

Firm's price setting, cont'd

- The optimality condition takes the form

$$
\begin{aligned}
&(1-\varepsilon)\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}\frac{Y_t}{P_t}+\varepsilon\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1}\frac{(1-\tau)\tilde{\Psi}_t Y_t}{P_t}\\
&-\phi\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)\frac{Y_t}{P_{t-1}(i)}\\
&+\beta\phi E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_{t+1}(i)}{P_t(i)}-1\right)\frac{P_{t+1}(i)Y_{t+1}}{P_t(i)^2}=0.
\end{aligned}
$$

- Note that firms are symmetric in the equilibrium; $P_t(i)=P_t$. Then we have

$$
\begin{aligned}
&(1-\varepsilon)+\varepsilon(1-\tau)\tilde{\Psi}_t\\
&-\phi\left(\Pi_t-1\right)\Pi_t+\beta\phi E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\Pi_{t+1}-1\right)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}=0.
\end{aligned}
$$

Firm's price setting, cont'd

- In the steady state, we have

$$\mathcal{M}(1-\tau)\tilde{\Psi} = 1.$$

- The log-linearized version is given by

$$\begin{aligned}
&(1-\epsilon) + \epsilon(1-\tau)\tilde{\Psi}(1+\hat{\tilde{\psi}}_t) \\
&-\phi\Pi^2(1+2\pi_t) + \beta\phi\Pi^2 E_t\left\{1+2\pi_{t+1}+(1-\sigma)(\hat{c}_{t+1}-\hat{c}_t)\right\} \\
&+\phi\Pi(1+\pi_t) - \beta\phi\Pi E_t\left\{1+\pi_{t+1}+(1-\sigma)(\hat{c}_{t+1}-\hat{c}_t)\right\} = 0,
\end{aligned}$$

$$\Leftrightarrow \pi_t = \beta E_t\pi_{t+1} + \frac{\varepsilon-1}{\phi}\hat{\tilde{\psi}}_t.$$

Given that $\Pi = 1$, $\pi_t = \log \Pi_t$ and $\tilde{\psi}_t = \log \tilde{\Psi}_t$.

Inflation dynamics coincide

- Under staggered prices by following Calvo (1983), we have

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta}\hat{\tilde{\psi}}_t,$$

- If $\frac{\varepsilon-1}{\phi} = \frac{(1-\beta\theta)(1-\theta)}{\theta}$, the Calvo and Rotemberg price stickiness has the same first-order inflation dynamics (Roberts, 1995).

Nonlinear equilibrium conditions

- The good market clearing implies

$$Y_t = C_t + \frac{\phi}{2}\left(\Pi_t - 1\right)^2 Y_t.$$

- Then we have the following equilibrium conditions:

$$\begin{aligned}
\frac{W_t}{P_t} &= C_t^{\sigma} N_t^{\varphi}, \\
1 &= \beta R_t E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right\}, \\
C_t &= \left[1 - \frac{\phi}{2}\left(\Pi_t - 1\right)^2 \right] Y_t, \\
Y_t &= A_t N_t, \\
\tilde{\Psi}_t &= \frac{W_t}{P_t} \frac{1}{A_t},
\end{aligned}$$

Nonlinear equilibrium conditions, cont'd

- and

$$
\begin{aligned}
&(1-\epsilon)+\epsilon(1-\tau)\tilde{\Psi}_t \\
&-\phi\left(\Pi_t-1\right)\Pi_t+\beta\phi E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\Pi_{t+1}-1\right)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}=0.
\end{aligned}
$$

  There are 7 variables $\{C_t, Y_t, N_t, \tilde{\Psi}_t, W_t, P_t, R_t\}$. Note that $\Pi_t = P_t/P_{t-1}$.

- Also, a nonlinear version of the Taylor rule is given by

$$
R_t = R\Pi_t^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y}\exp(\nu_t).
$$

  Then we have 7 equations and can solve the model.

Flexible-price equilibrium and steady state

- In flexible-price equilibrium (when $\phi = 0$), $C_t^n = Y_t^n = A_t N_t^n$ and

$$\begin{aligned}\tilde{\Psi} &= [(1-\tau)\mathcal{M}]^{-1} \\ &= (C_t^n)^{\sigma}(N_t^n)^{\varphi}A_t^{-1} \\ &= (Y_t^n)^{\sigma+\varphi}A_t^{-(1+\varphi)}.\end{aligned}$$

- In steady state,

$$\begin{aligned}&\Pi = 1, \\ &R = \beta^{-1}, \\ &C = N = \tilde{\Psi}^{\frac{1}{\sigma+\varphi}}, \\ &\tilde{\Psi} = [(1-\tau)\mathcal{M}]^{-1}.\end{aligned}$$

Note that when $(1-\tau)\mathcal{M} = 1$, the steady state is efficient.

Log-linearization

- We have

$$
\begin{aligned}
& i_t - E_t\pi_{t+1} - \rho - \sigma(E_t c_{t+1} - c_t) + E_t z_{t+1} - z_t = 0, \\
& c_t = y_t = a_t + n_t, \\
& \tilde{\psi}_t = \sigma c_t + \varphi n_t - a_t, \\
& \pi_t = \beta E_t \pi_{t+1} + \frac{\varepsilon - 1}{\phi}(\tilde{\psi}_t - \tilde{\psi}), \\
& i_t = \rho + \phi_\pi \pi_t + \phi_y (y_t - y) + \nu_t.
\end{aligned}
$$

There are 5 variables, $\{c_t, \pi_t, i_t, n_t, \tilde{\psi}_t\}$, and 5 equations.

Log-linearization, cont'd

- In flexible-price equilibrium,

$$\tilde{\psi} = (\sigma + \varphi) y_t^n - (1 + \varphi) a_t.$$

- Then we have

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^n), \\ \pi_t &= \beta E_t \pi_{t+1} + \frac{(\varepsilon - 1)(\sigma + \varphi)}{\phi} x_t, \\ i_t &= \rho + \phi_\pi \pi_t + \phi_y (y_t - y) + \nu_t, \end{aligned}$$

where

$$\begin{aligned} x_t &= y_t - y_t^n, \\ y_t^n &= \frac{1 + \varphi}{\sigma + \varphi} a_t + \frac{1}{\sigma + \varphi} \tilde{\psi}, \\ r_t^n &= \rho + \sigma (E_t y_{t+1}^n - y_t^n) - E_t z_{t+1} + z_t. \end{aligned}$$

Second-order approximation

- The second order approximation of relative deviation:

$$\frac{X_t - X}{X} \simeq \hat{x}_t + \frac{1}{2}\hat{x}_t,$$

where $\hat{x}_t \equiv \log(X_t/X)$.

- This can be obtained by the second-order Taylor expansion of:

$$\begin{aligned} X_t/X = \exp(\hat{x}_t) &\simeq \exp(\hat{x}) + \exp(\hat{x})(\hat{x}_t - \hat{x}) + \frac{1}{2}\exp(\hat{x})(\hat{x}_t - \hat{x})^2, \\ &= 1 + \hat{x}_t + \frac{1}{2}\hat{x}_t^2. \end{aligned}$$

Second-order apploximation to the household utility

- The second order Taylor expansion of $U_t = U(C_t, N_t; Z_t)$ around a steady state (C, N, Z) yields

$$
\begin{aligned}
U_t - U \quad \simeq \quad & U_c C \left(\frac{C_t - C}{C} \right) + U_n N \left(\frac{N_t - N}{N} \right) + \frac{1}{2} U_{cc} C^2 \left(\frac{C_t - C}{C} \right)^2 \\
& + \frac{1}{2} U_{nn} N^2 \left(\frac{N_t - N}{N} \right) + U_c C \left(\frac{C_t - C}{C} \right) \left(\frac{Z_t - Z}{Z} \right) \\
& + U_n N \left(\frac{N_t - N}{N} \right) \left(\frac{Z_t - Z}{Z} \right) + t.i.p.
\end{aligned}
$$

where $t.i.p.$ stands for terms independent of policy. Note that we have used $U_{cn} = 0$.

Second-order apploximation to the household utility, cont'd

- In terms of log deviations,

$$
\begin{aligned}
U_t - U &\simeq U_c C\left(\hat{y}_t(1+z_t) + \frac{1-\sigma}{2}\hat{y}_t^2\right) \\
&\quad + U_n N\left(\hat{n}_t(1+z_t) + \frac{1+\varphi}{2}\hat{n}_t^2\right) + t.i.p.
\end{aligned}
$$

where we have used $\sigma \equiv -\frac{U_{cc}}{U_c}C$ and $\varphi \equiv \frac{U_{nn}}{U_n}N$ and $\hat{c}_t = \hat{y}_t$. Also, using $\hat{n}_t = \hat{y}_t - a_t + d_t$ and the fact that d_t is at second order:

$$
\begin{aligned}
U_t - U &\simeq U_c C\left(\hat{y}_t(1+z_t) + \frac{1-\sigma}{2}\hat{y}_t^2\right) \\
&\quad + U_n N\left(\hat{y}_t(1+z_t) + d_t + \frac{1+\varphi}{2}(\hat{y}_t - a_t)^2\right) \\
&\quad + t.i.p.
\end{aligned}
$$

Second-order apploximation to the household utility, cont'd

- The efficient steady state implies $U_c C = -U_n N$. Then we have

$$
\begin{aligned}
\frac{U_t - U}{U_c C} &\simeq \left(\hat{y}_t(1+z_t) + \frac{1-\sigma}{2}\hat{y}_t^2\right) \\
&\quad - \left(\hat{y}_t(1+z_t) + d_t + \frac{1+\varphi}{2}(\hat{y}_t - a_t)^2\right) + t.i.p., \\
&= -\left(d_t - \frac{1-\sigma}{2}\hat{y}_t^2 + \frac{1+\varphi}{2}(\hat{y}_t - a_t)^2\right) + t.i.p., \\
&= -\left(d_t + \frac{\sigma+\varphi}{2}\hat{y}_t^2 + (\sigma+\varphi)\hat{y}_t a_t\right) + t.i.p., \\
&= -\left(d_t + \frac{\sigma+\varphi}{2}x_t^2\right) + t.i.p.
\end{aligned}
$$

where we have used that $\hat{y}_t^n = \frac{1+\varphi}{\sigma+\varphi}a_t$ and $x_t = \hat{y}_t - \hat{y}_t^n$.

Welfare loss function derived

- We use the following lemmas:

1. In a neighborhood of a symmetric steady state and up to the second order approximation, $d_t = (\epsilon/2)var_i\{p_t(i)\}$. [See Appendix 3.4 in the text.]
2. $\sum_{t=0}^{\infty}\beta^t var_i\{p_t(i)\} = \lambda^{-1}\sum_{t=0}^{\infty}\beta^t\pi_t^2$. [See Woodford (2003, ch. 6).]

- Then we obtain

$$\begin{aligned} E_0\sum_{t=0}^{\infty}\beta^2\left(\frac{U_t-U}{U_cC}\right) &\simeq -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^2\left(\frac{\epsilon}{\lambda}\pi_t^2+(\sigma+\varphi)x_t^2\right)+t.i.p., \\ &= -\frac{\lambda(\sigma+\varphi)}{2\epsilon}E_0\sum_{t=0}^{\infty}\beta^2\left(\pi_t^2+\frac{\kappa}{\epsilon}x_t^2\right)+t.i.p. \end{aligned}$$