Macro III: Problem Set 2
Deadline: Wed, 29/8/2018

Tiago Cavalcanti

August 2018

1. **Stochastic Processes**

   (a) Explain the procedures to approximate an AR(1) process

   $$y_t = \mu(1-\rho) + \rho y_{t_1} + \epsilon_t,$$

   where $\epsilon_t \sim N(0, \sigma^2)$ with a Markov chain, based on the Tauchen method (with equal intervals).

   (b) Use the code sent to you to generate and plot $T = 1000$ realisations from a Markov chain approximation of the AR(1) process

   $$y_t = 0.8 y_{t-1} + \epsilon_t,$$

   where $\epsilon_t \sim N(0, 0.01)$. To generate the realisations, use as initial state of the chain the one that best approximates $y_0 = 0$, and use $r = 3$. Do the following experiments (remember to always use the same seed):

   i. Start by generating the series using $N = 3$ grid points for the approximation. What do you observe? Why?

   ii. Next, use $N = 7$ and $N = 15$ and compare how the results differ in terms of quality of approximation.

2. **RBC Model.** Consider the following RBC model:

   $$\max_{c_t, k_{t+1}, h_t} E_0 \left(\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t) \right),$$

   subject to

   $$c_t + k_{t+1} = z_t F(k_t, h_t) + (1-\delta) k_t,$$

   $$\ln(z_{t+1}) = \rho \ln(z_t) + \epsilon_{t+1}, \ \ \epsilon_{t+1} \sim N(0, \sigma^2)$$

The technology is given by

$$Y_t = z_t F(k_t, h_t) = z_t k_t^{\alpha} h_t^{1-\alpha}.$$

The first and second welfare theorems hold for this economy, therefore you can solve the social planner's problem. If needed you can recover prices using marginal productivities. Assume also that:

$$u(c, 1-h) = \frac{\left(c^{\gamma}(1-h)^{1-\gamma}\right)^{1-\mu}}{1-\mu}.$$

(a) Write down the Social Planner's problem in recursive formulation.

Calibration: We need to set the value of parameters. Use $\beta = 0.987$, $\mu = 2$. For the production function assume that $\alpha = 1/3$ and $\delta = 0.012$.

(b) Assume that the model period is a quarter, explain the intuition behind the value of each parameter above.

(c) For now, assume that there is no uncertainty (i.e., $\sigma = 0$). Derive the Euler Equation and the intra-temporal condition. Calibrate γ such that hours worked in the model is 1/3 of the time endowment in the steady-state, i.e., h=1/3.

Calibration (cont.): For the stochastic process, assume the values of Cooley and Prescott (1985): $\rho = 0.95$ and $\sigma = 0.007$.

(d) Now assume that there is uncertainty. Solve the model using the value function algorithm. Use the method of Tauchen (1986) with 7 grid points. For the capital grid you can use a linear grid with 101 points in the interval from $[0.75k_{ss}, 1.25k_{ss}]$. Report the number of value function iteration, the time it takes to find the optimal value function, plot figure of the policy function and calculate Euler Errors.

(e) Now solve the model using Howard's improvement algorithm. Iterate 20 times in the policy function before updating your value function. Report the number of value function iteration, the time it takes to find the optimal value function, plot the figure of the policy function and calculate Euler Errors.

(f) Calculate and report the first and second moments of consumption, hours worked, capital, investment and output.