Macro III: Problem Set 3
Deadline: Friday, 17/09/2018

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1. **Huggett Model.** Consider the Huggett (1993) economy. Use his benchmark parameter values: period length is two months, $\beta = 0.96$ (annual basis) coefficient of relative risk- aversion $\sigma = 1.5$, $e_h = 1$, $e_l = 0.1$, $\pi(e_h|e_h) = 0.925$, $\pi(e_h|e_l) = 0.5$. Consider a stationary equilibrium.

   (a) In this equilibrium what is average income, and what is the average duration of an unemployment spell? (You will need to calculate the unconditional stationary distribution of the associated Markov Process.).

   Set the borrowing constraint equal to one year's average income. Construct an equally-spaced grid on asset holdings, with a maximum value equal to $3\times$ average income. Let the number of grid points $N = 20$. Suppose the interest rate $r = 3.4\%$ (annual basis).

   (b) Solve for optimal decision rules across the grid.

   (c) Imagine one agent, who starts out with zero assets and the high endowment. Simulate the evolution of the agent's wealth, income and consumption for 10,000 periods, each period drawing an endowment according to the Markov process described above. Plot a histogram for asset holdings over this simulation.

   (d) Over the last 1,000 periods of this simulation: (a) What is the average value for asset holdings? (b) What is the average decline in consumption in response to entering unemployment? (c) What is the average value for consumption conditional on being (i) employed, (ii) unemployed, (iii) unemployed in the current period and in each of the previous 5 periods (ie unemployed for at least 12 months)? (d) What are the correlations between earnings, income, wealth and consumption?

   (e) Suppose the net supply of assets is zero (as in the original Huggett paper). What is the market-clearing interest rate?

(f) Suppose we were to increase the value for the risk aversion coefficient, σ, from 1.5 to 3. What would happen to (a) the equilibrium interest rate? (b) standard measures for wealth dispersion (eg. the Gini coefficient)?

2. **Aiyagari Model.** Time is discrete and indexed by $t = 0, 1, 2...$ Let $\beta \in (0, 1)$ be the subjective discount factor, $c_t \geq 0$ be consumption at period t and l_t be labor supply at t. Agents are *ex-ante* identical and have the following preferences:

Preferences:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma_c}}{1-\sigma_c} + \gamma \frac{(1-l_t)^{1-\sigma_l}}{1-\sigma_l} \right) \right],$$

where $\sigma_c,\ \sigma_l > 1, \gamma > 0$. Expectations are taken over an idiosyncratic shock, z_t, on labor productivity, where

$$\ln(z_{t+1}) = \rho \ln(z_t) + \epsilon_{t+1},\ \rho \in [0, 1].$$

Variable ϵ_{t+1} is an iid shock with zero mean and variance σ_ϵ^2. Markets are incomplete as in Huggett (1993) and Aiyagari (1994). There are no state contingent assets and agents trade a risk-free bond, a_{t+1}, which pays interest rate r_t at period t. In order to avoid a Ponzi game, we impose a natural borrowing limit.

Technology: There is no aggregate uncertainty and the technology is represented by $Y_t = K_t^\alpha N_t^{1-\alpha}$. Let I_t be investment at period t. Capital evolves according to:

$$K_{t+1} = (1-\delta)K_t + I_t.$$

Let $\delta = 0.08$, $\beta = 0.96$, $\alpha = 0.4$, $\gamma = 0.75$ and $\sigma_c = \sigma_l = 2$.

(a) Use a finite approximation for the autoregressive process

$$\ln(z') = \rho \ln(z) + \epsilon.$$

where ϵ' is normal iid with zero mean and variance σ_ϵ^2. Use a 7 state Markov process spanning 3 standard deviations of the log wage. Let ρ be equal to 0.98 and assume that $\sigma_z^2 = \frac{\sigma_\epsilon^2}{1-\rho^2} = 0.621$. Simulate this shock and report results.

(b) State the households' problem.

(c) State the representative firm's problem.

(d) Define the recursive competitive equilibrium for this economy.

(e) Write down a code to solve this problem. Find the policy functions for a', c, and l.

(f) Solve out for the equilibrium allocations and compute statistics for this economy. Report basic statistics about this economy, such as: the capital-to-output ratio, cumulative distribution of income (e.g., bottom 1%, 5%, 10%, 50%, top 1%, top 5%, top 10%), cumulative distribution of wealth (e.g., bottom 1%, 5%, 10%, 50%, top 1%, top 5%, top 10%).

(g) Introduce a government in this economy such that the government levies a payroll tax (firms have to pay $(1 + \tau^n)w_t N_t$) and a tax on revenue (revenues are $(1 - \tau^y)Y_t$). First, assume that $\tau^n = 0.25$ and $\tau^y = 0$. Assume that government spending are wasted (or alternatively (harder), the government makes equal lump-sum transfers to all households). Now, let $\tau^n = 0.20$, then find the value for τ^y such that in equilibrium total revenue is unchanged. Report statistics for both economies, including a measure of aggregate welfare and also distributional effects.