Macro III: Problem Set 4
Deadline: Monday, 08/10/2018

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1. **New Keynesian model.** *(Borrowed from Miao, Ch. 2, Question 2)* Consider a simple log-linearised New Keynesian model studied in the lectures:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1},$$

$$x_t = E_t x_{t+1} - \frac{1}{\gamma}(i_t - E_t \pi_{t+1}) + \epsilon_t^x,$$

where π_t is the log inflation rate, x_t is the output gap, i_t is the log gross nominal interest rate, and ϵ_t^x represents exogenous IID shocks. Parameters κ, β and γ are positive quantities.

   (a) Set the policy rules as:

$$i_t = \delta \pi_t + \epsilon_t.$$

   Prove that $\delta > 1$ is necessary and sufficient for the existence of a unique stable equilibrium. This condition is called the **Taylor Principle**. Explain the intuition behind this condition.

   (b) Suppose that the policy rule is given by the **Taylor Rule**

$$i_t = \delta_\pi \pi_t + \delta_x x_t + \epsilon_t.$$

   Derive the conditions for the existence of a unique stable equilibrium.

2. **Log-linearisation.** Consider the following model. Households maximize expected discounted lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t + A L_t\right)$$

over consumption C_t, leisure L_t, capital K_t and investment I_t, subject to the budget constraint

$$C_t + K_t - K_{t-1} = (1-\tau)(R_t - \delta)K_{t-1} + (1-\xi)W_t N_t$$

where $N_t \equiv 1 - L_t$ denotes hours worked and the evolution of capital is determined by $K_t = (1-\delta)K_{t-1} + I_t$, where R_t, W_t denote the price of capital rental and the wage rate respectively. The parameters β and δ denote the discount factor and the depreciation rate of capital respectively. Furthermore, τ, $\xi \in [0,1)$ denote tax rates on capital and labor income respectively. Taxes on capital are after depreciation allowances.

The representative firm produces output Y_t, rents capital, hires efficiency units of labor at a rate W_t, and maximizes profits

$$Y_t - W_t N_t - R_t K_{t-1}$$

in every period, subject to a Cobb-Douglas production function

$$Y_t = Z_t K_{t-1}^{\alpha} N_t^{1-\alpha}$$

with $0 < \alpha < 1$, and to a technological shock, which evolves according to

$$\log Z_t = \rho \log Z_{t-1} + \varepsilon_t$$

with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $0 < \rho < 1$. Finally, the government has expenditures G_t that are financed by levying taxes on capital and labor income, and maintains a balanced budget

$$G_t = \tau(R_t - \delta)K_{t-1} + \xi W_t N_t$$

This model is an extension of Hansen's (1985) model that includes government expenditures and taxation of capital and labor income.[1]

(a) Collect the equations that characterise the competitive equilibrium.

(b) Determine the steady-state equations.

(c) Log-linearise the equations, determine the endogenous and exogenous state variables and write the log-linearised equations in the form

$$\begin{aligned}
0 &= E_t\left[F x_{t+1} + G x_t + H x_{t-1} + J y_{t+1} + K y_t + L z_{t+1} + M z_t\right] \\
z_{t+1} &= N z_t + \varepsilon_{t+1} \\
0 &= A x_t + B x_{t-1} + C y_t + D z_t
\end{aligned}$$

i.e. determine the matrices $A, B, C, D, E, F, G, H, J, K, L, M$ and N.

From now own, you do this if you want. No required.

[1] IMPORTANT: The model is setup such that $R_t = \alpha Y_t / K_{t-1}$ rather than $(1-\delta) + \alpha Y_t/K_{t-1}$ as in Hansen's model. This does not make any quantitative difference: the only important thing is that now you cannot interpret R as interest rate, but only as capital rental price.

(d) Consider the basic parameterization

$$\alpha = 0.36, \delta = 0.025, \beta = 1/1.01, \rho = 0.95, \sigma = 0.712\% \text{ and } A = 2.5846.$$

Given this, solve the model, using the Uhlig's toolkit, for $\tau = 0.4$ and $\xi = 0.25$.

(e) Keeping the basic parameterisation fixed (i.e. do not change the values of α, δ, β, ρ, σ, A), show numerically that model is **regular** (i.e. there is always a unique stationary solution) for all possible combinations of $\tau, \xi \in [0, 1)$.

(f) Now keep the basic parameterisation fixed and solve the model first for $\tau = \xi = 0$ (i.e. no taxes) then for various combinations of τ and ξ. Given your findings, comment on the effects on the economic system of introducing taxes (for this part, it is up to you to determine which results to present and how to present them, and how you will make your arguments and comments).

3. **Ramsey model in continuous time.** Consider the decentralised Ramsey model in continuous time. Households solve

$$\max_{c,l} \int_0^\infty e^{-\rho t} u(c, l) dt,$$

subject to

$$\dot{a} = w(1 - l) + ra - c,$$

where c is consumption, a denotes assets and l is leisure. ρ is the subjective discount rate, r is the interest rate and w is the wage rate. Firms rent capital and labour from households to maximise

$$\max AK^\alpha N^{1-\alpha} - wN - (r + \delta)K,$$

where δ is the depreciation rate and A is a productivity factor.

(a) Write down the HJB associated with the households problem. Explain the steps to derive it.

From now on, assume that $u(c, l) = \log(c) + \eta \log(l)$. You can assume that $\rho = 0.04$ and $\eta = 0.75$

(b) For a given interest rate r and wage rate w, write down a code to solve the households problem.

(c) Write down the market clearing conditions.

Assume that $\delta = 0.06$, $A = 1$ and $\alpha = 0.33$.

(d) Write down the equations that describe the steady-state of the system and solve for the steady-state level of capital and labour supply.

(e) Write down a code to solve out the whole transition. Then simulate a permanent changein the TFP factor, such that A increases from $A = 1$ to $A = 1.2$. Plot the evolution of capital, labour and consumption.

4. **Aiyagari in continuous time.** For this problem set, you are asked to solve the Aiyagari model in continuous time step-by-step. To your help is an **m.file** called **household.m**, which solves the households problem given a wage and an interest rate. All needed parameters are in the code.

   (a) Start by familiarising yourself with the code. You can do so by setting Na to a small number and print the relevant matrices (no need to report this).

   (b) Calculate the steady state value of the employment rate and the unemployment rate, and derive the equilibrium tax rate described in the lectures.

   (c) Calculate the stationary endogenous distribution, and the associated value of capital supply.

   (d) Set $r_h = 1/\beta - 1$ and $r_l = 0$. Set $r = (r_h + r_l)/2$ and calculate the wage according to the firms optimisation problem.

   (e) Repeat step (iii) and find the interest rate implied by the associated capital supply, $\hat{r}$.

   (f) Update r_h or r_l according to the bisection method and return to step (vi) until $|r - \hat{r}| < 1e - 6$.

   (g) Report the stationary (graph) distribution of assets by employed and unemployed agents. Report (graph) the stationary policy functions (consumption and savings) by employed and unemployed agents for each level of asset.

   (h) Consider a permanent rise in the TFP. Say it increases by 10% (say from 1 to 1.2). Show the dynamics to the new steady-state.