Macroeconomics III
Lecture 6: Complete Markets

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Readings

Main reading:

Miao (2014). "*Economic Dynamics in Discrete Time,*" MIT Press. Chapter 13 (pages 319-328).

Other reading:

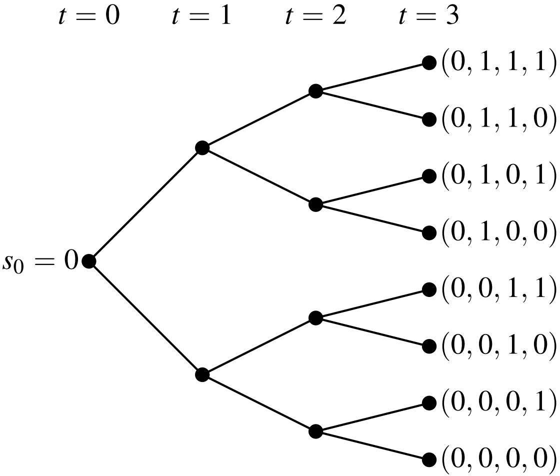
Ljungqvist and Sargent (2012). "*Recursive Macroeconomic Theory*", MIT Press. Chapter 8 (pages 251-265).

Environment I

- Time is discrete, $t = 0, 1, 2, ...$ and it can be finite or not.
- Uncertainty is captured by a finite state space $\mathbf{S} = \{1, 2, ..., S\}$.
- Debreu's approach: Goods are indexed by location, space, time and **states of nature**. People buy and sell claims to these goods:
  - *At the beginning of the period people buy (sell) a contract that promises so many units of a good to be delivered to them if the state of the world is s.*
- The history of events up to time t: $s^t = (s_0, s_1, ..., s_t)$.
- Commodity space: $L_\infty = \{x : \ sup_{t,s^t} |x_t(s^t)| < \infty\}$

Environment II

- For a two-event stochastic process $s_t \in S = \{0, 1\}$:



Environment III

- The unconditional probability of history s^t is $Pr(s^t)$.
- s_0 is given, so $Pr(s_0) = 1$.
- The conditional probability of observing history s^t given the realization of s^τ is $Pr(s^t/s^\tau)$.
- Consider now a **pure exchange economy.**
- Preferences of agent i
  - $U^i = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c_t^i(s^t)) Pr(s^t)$.
- An allocation is feasible if

$$\sum_i c^i(s^t) = \sum_i y^i(s^t) = \bar{y}(s^t).$$

Time-0 Trading: Competitive Equilibrium:

Definition: A Competitive Equilibrium for this economy is a series of prices $\{q_t^0(s^t)\}_{t=0}^{\infty}$, a series of allocations for each household i $\{c_t^i(s^t)\}_{t=0}^{\infty}$, such that:

1. Given $\{q_t^0(s^t)\}_{t=0}^{\infty}$, $\{c_t^i(s^t)\}_{t=0}^{\infty}$ solves:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c_t^i(s^t)) Pr(s^t) \text{ subject to}$$

$$\sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) c_t^i(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) y_t^i(s^t).$$

2. Market clearing:

$$\sum_i c^i(s^t) = \sum_i y^i(s^t) = \bar{y}(s^t), \ \forall t \text{ and } \forall s^t.$$

Equilibrium I:

$$L^i = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c_t^i(s^t)) Pr(s^t) + \mu^i \left[\sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t)[y_t^i(s^t) - c_t^i(s^t)] \right].$$

First order conditions: (recall that $s^0 = s_0$ is given, $Pr(s^0) = 1$ and $q_0^0(s^0) = 1$)

$$c_0^i(s^0) : \; u'(c_0^i(s^0)) - \mu^i = 0,$$

$$c_t^i(s^t) : \; \beta^t u'(c_t^i(s^t)) Pr(s^t) - q_t^0(s^t)\mu^i = 0, \forall t, \forall s^t$$

Arrow-Debrue state price:

$$q_t^0(s^t) = \frac{\beta^t u'(c_t^i(s^t)) Pr(s^t)}{u'(c_0^i(s^0))}.$$

Equilibrium II:

$$\frac{u'(c_t^i(s^t))}{u'(c_t^j(s^t))} = \frac{\mu^i}{\mu^j}.$$

Point: Ratios of marginal utility between pairs of agents are constant across all histories and dates! Solving for $c_t^i(s^t)$:

$$c_t^i(s^t) = (u')^{-1}\left[u'(c_t^1(s^t))\frac{\mu^i}{\mu^1}\right],\ \forall i \neq 1$$

Using the market clearing condition, we have:

$$\sum_i (u')^{-1}\left[u'(c_t^1(s^t))\frac{\mu^i}{\mu^1}\right] = \sum_i y^i(s^t) = \bar{y}(s^t),\ \forall t \text{ and } \forall s^t.$$

Proposition: The CE Allocation is a function of the realised aggregate endowment and does not depend on time, on the specific history, and on the cross section distribution of endowments, i.e., $c_t^i(s^t) = c_t^i(\tilde{s}^\tau)$ for all histories s^t and $\tilde{s}^\tau$ such that $\sum_i y_t^i(s^t) = \sum_i y_\tau^i(s^\tau)$.

Equilibrium Computation

1. Fix $\mu^1 > 0$. Guess the remaining $\mu^i > 0$. Then solve for $c_t^1(s^t)$ using:

$$\sum_i (u')^{-1}\left[u'(c_t^1(s^t))\frac{\mu^i}{\mu^1}\right] = \sum_i y^i(s^t) = \bar{y}(s^t), \ \forall t \text{ and } \forall s^t.$$

   and $c_t^i(s^t)$, $i = 2, ..., I$, with $c_t^i(s^t) = (u')^{-1}\left[u'(c_t^1(s^t))\frac{\mu^i}{\mu^1}\right]$.
2. Use $q_t^0(s^t) = \frac{\beta^t u'(c_t^i(s^t))Pr(s^t)}{u'(c_0^i(s^0))}$, to find $q_t^0(s^t)$.
3. For each i, check the budget constraint. For those i such that the cost of consumption exceeds the value of their endowment, then raise μ^i, while for the others decrease μ^i.
4. Iterate until convergence.

Example: $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}, \gamma > 0$

$$c_t^j(s^t) = c_t^i(s^t)\left(\frac{\mu^j}{\mu^i}\right)^{-\frac{1}{\gamma}}, \forall t, \text{ and } \forall s^t.$$

Risk sharing: Individual consumption is perfectly correlated.

$$c_t^i(s^t) = \frac{(\mu^i)^{-\frac{1}{\gamma}}}{\sum_j (\mu^j)^{-\frac{1}{\gamma}}}\sum_j y^j(s^t) = \frac{(\mu^i)^{-\frac{1}{\gamma}}}{\sum_j (\mu^j)^{-\frac{1}{\gamma}}}\bar{y}(s^t) = \alpha_i \bar{y}(s^t).$$

Individual consumption is a constant fraction of aggregate income!

$$FOC: \ \beta^t u'(c_t^i(s^t))Pr(s^t) - q_t^0(s^t)\mu^i = 0 \ \Rightarrow \ q_t^0(s^t) = (\mu^i)^{-1}\alpha_i^{-\gamma}\bar{y}(s^t)^{-\gamma}\beta^t Pr(s^t).$$

We are free to set $(\mu^i)^{-1}\alpha_i^{-\gamma}$ to an arbitrary positive number. So we can compute $q_t^0(s^t)$, and from the i's individual budget constraint:

$$\alpha_i = \frac{\sum_{t=0}^{\infty}\sum_{s^t} q_t^0(s^t) y_t^i(s^t)}{\sum_{t=0}^{\infty}\sum_{s^t} q_t^0(s^t)\bar{y}(s^t)}.$$

No Aggregate Uncertainty but Idiosyncratic Shocks: $c_t^i(s^t) = \bar{c}^i \; \forall t \; \forall s^t$

Consider an economy with two types of agents (A and B) with unit measure each.

Let $s_t \in [0,1]$ be an stochastic event ($s_t \sim U[0,1]$) such that $y_t^A(s^t) = s_t$ and $y_t^B = 1 - s_t$. This implies that $E[y_t^i(s^t)] = 0.5$. Feasibility: $c_t^A(s^t) + c_t^B(s^t) = 1 = \bar{y}(s^t) \forall t, \; \forall s^t$. Budget constraint:

$$\sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t)[\bar{c}^i - y_t^i(s^t)] = 0 \;\text{ since }\; q_t^0(s^t) = (\mu^i)^{-1} \beta^t u'(\bar{c}^i) Pr(s^t),$$

$$(\mu^i)^{-1} u'(\bar{c}^i) \sum_{t=0}^{\infty} \sum_{s^t} \beta^t Pr(s^t)[\bar{c}^i - y_t^i(s^t)] = 0 \;\Rightarrow\; \bar{c}^i = (1-\beta) \sum_{t=0}^{\infty} \beta^t \sum_{s^t} Pr(s^t) y_t^i(s^t).$$

Given s_0, we have

$$\bar{c}^A = (1-\beta)s_0 + \frac{\beta}{2}, \; \bar{c}^B = (1-\beta)(1-s_0) + \frac{\beta}{2}, \;\text{ and }\; q_t^0(s^t) = \beta^t Pr(s^t).$$

Bewley Model

The Bewley model with endowment process

- Type-A: $\omega^A = \{\omega_0^A, \omega_1^A, \omega_2^A, ...\} = \{1, 0, 1, 0, ...\}$.
- Type-B: $\omega^B = \{\omega_0^B, \omega_1^B, \omega_2^B, ...\} = \{0, 1, 0, 1, ...\}$.

is a particular case of the above model with $s_0 = 1$ and only one history $\tilde{s}^t = (1, 0, 1, 0, ...)$.

In this case: $q_t^0(\tilde{s}^t) = \beta^t$ and

$$\bar{c}^A = (1-\beta)\sum_{j=0}^{\infty}\beta^{2j} = \frac{1}{1+\beta}, \text{ and } \bar{c}^B = (1-\beta)\beta\sum_{j=0}^{\infty}\beta^{2j} = \frac{\beta}{1+\beta}.$$

Testing Complete Markets

With Isoelastic utility function:

$$c_t^i(s^t) = \frac{(\mu^i)^{-\frac{1}{\gamma}}}{\sum_j (\mu^j)^{-\frac{1}{\gamma}}} \sum_j y^j(s^t) = \frac{(\mu^i)^{-\frac{1}{\gamma}}}{\sum_j (\mu^j)^{-\frac{1}{\gamma}}} \bar{y}(s^t) = \alpha_i \bar{y}(s^t).$$

Complete markets can be tested using the following equation:

$$\Delta \ln(c_{it}) = \eta \Delta \ln(c_t) + \delta \Delta \ln(y_{it}) + u_{it}$$

Null hypothesis: $\eta = 1$ and $\delta = 0$

	Dependent variable: Growth of household consumption		
	Thailand	United States	Italy
Growth of agg cons	0.74^{**}	1.06^{**}	1.32^{**}
	(9.25)	(13.25)	(3.30)
Growth of hh income	0.34^{**}	0.04^{**}	0.53^{**}
	(2.00)	(4.00)	(5.30)

Note: Thailand (Townsend, 1989); United States (Mace, 1991); and Italy (Jappelli and Pistaferri, 2017). t-statistic in parentheses