Macro III: Problem Set 1
Deadline: Monday, 20/8/2018

August 2018

1. **Solving Nonlinear Equations - Bisection Method.** Consider the function:

$$b(x) = e^x - e^{2.2087}.$$

Starting from the interval $x \in [0, 4]$, find x^* such that $b(x^*) = 0$ using the bisection method.

2. **Solving Nonlinear Equations - Newton's Method.** Consider the function:

$$d(x) = x^{-5} - x^{-3} - c.$$

   (a) Set $c = 1$ and plot d on $x \in [0.6, 10]$. Find x^* such that $d(x^*) = 0$ using Newton's method.

   (b) Now construct an equidistant grid for c containing 10 nodes between 1 and 10, and for each value of c on the grid find x^*.

   (c) Construct a new equidistant grid for c containing 1000 nodes between 1 and 10, and plot your solution for each value on this grid using a spline.

   (d) Relabel your solution x^* from part (b) $x(c)$. Find the inverse function $c(x)$, and plot it on an equidistant grid of 1000 nodes on $x \in [0.6, 10]$ using a spline approximation.

   (e) Now find the solution to:

$$0 = c(x) + x.$$

3. **Approximation Methods: Finite Element Methods.** Consider the function $h(x)$ on the domain $x \in [-2, 2]$:

$$h(x) = \begin{cases} (x - 0.5)^2 & \text{for } 0 \leq x \leq 2, \\ (x + 0.5)^2 & \text{for } -2 \leq x < 0, \end{cases}$$

(a) Approximate $h(x)$ with a cubic spline, using $n = 5$ equally spaced nodes. Plot the function along with your approximation and calculate the root mean squared error of your approximation over the fine grid, with interval 0.0001.

(b) Approximate $h(x)$ with a cubic spline, using $n = 10$ equally spaced nodes. Plot this new approximation along with both the actual function and your approximation from (a) and calculate the root mean squared error of your approximation over the fine grid, with interval 0.0001. Explain the reasons for the differences in your answers.

4. **Growth Model**. Consider the following growth model with taxes.

Households. There is a continuum of identical households with measure one. Each household has N_t members, which grows at rate η. The representative household owns the initial capital stock, k_0, and each household member has a unit of productive time in each period. Let h_t and l_t be hours worked and leisure by each household member, respectively, such that $h_t + l_t = 1$. Preferences of the representative household is given by the following utility function:

$$U = \sum_{t=0}^{\infty} \beta^t N_t[\ln(c_t) + \theta \ln(l_t)].$$

The evolution of capital stock is:

$$k_{t+1} = (1 - \delta)k_t + x_t, \;\; \delta \in (0, 1)$$

where x_t is investment at t.

(Important: Note the difference between the household and a household member - You can think about this as a dynasty such that there is perfect altruism towards future generations)

Production Sector. There is a continuum of measure one of identical firms. Let A_t be a productivity factor which evolves according to $A_{t+1} = (1 + \gamma)A_t$. Let K_t be the capital stock, and H_t be the total hours employed in the production of good Y_t by the representative firm. The technology is given by

$$Y_t = K_t^{\alpha}(A_t H_t)^{1-\alpha}.$$

The representative firm rents capital and labor from households.

Government Sector. There is a government which finances G_t expenditures through taxes on consumption, τ^c, labor income, τ^h, and capital income, τ^k. Suppose that in every period the government balances its budget, such that

$$G_t = \tau^c N_t c_t + \tau^h w_t N_t h_t + \tau^k r_t k_t.$$

Resource Constraint:

$$Y_t = N_t c_t + x_t + G_t.$$

(Comment: Think about a continuous measure one of households. This implies that each household is very tiny. Since they are identical, this implies that you can solve the problem of a household who represents the others. Finally, the average of the economy is similar to the aggregate)

(a) Define a competitive equilibrium for this economy and write down the equations that describe the equilibrium of the system.

(This is the most important part of the problem. You can look at your macro text book)

(b) What are the variables growth rate along the balanced growth path equilibrium? (hint: Assume that taxes are constant and that G grows at the same rate of Y such that the "government size", G/Y, is also constant.)

(c) Write down the equivalent stationary system. (You have to transform your variable such that they are stationary in the long-run)

(d) Now suppose that $\beta = 0.98$, $\delta = 0.08$, $\gamma = 0.015$, $\alpha = 0.4$, $\theta = 2$, $\tau^c = 0.15$, $\tau^h = 0.25$, $\tau^k = 0.15$. Write down a program that solves the transitional dynamics of the equilibrium system by using one of the methods of solving a system of nonlinear equations (e.g., Newton, Secant or fsolve). Assume that $K_0 = 0.8 K_{SS}$.

(e) Plot the dynamics of capita, consumption, investment, labor and output per capita.

(f) Now let's study the following tax reform: Suppose that the government reduces the tax on capital income from 0.15 to 0.10 and the government increases the tax on labor income in order to finance the same level of spending (i.e. in the long run the tax reform is revenue neutral). Write a program to implement this tax reform. Assume that prior the reform the initial capital stock is in the steady-state. Show the dynamics and analyze this reform. Use economic arguments to interpret your results.

(g) Calculate the long run welfare implications of this tax reform in terms of consumption:

$$[\ln((1+\omega)c^{new}) + \gamma \ln(l^{new})] - [\ln(c^{old}) + \gamma \ln(l^{old})].$$

ω is the percentage of consumption that the household must be compensated (or pay) to accept this tax reform. How about the transitional dynamics?

(Related to this exercise see Cavalcanti (2008). “Tributos sobre a Folha ou sobre o Faturamento? Efeitos Quantitativos Para o Brasil” RBE.)

5. Write down a code by using one of the methods of solving a system of nonlinear equations (e.g., Newton, Secant or fsolve) to reproduce Figure 1 of Gourinchas and Jeanne (2006, *Review of Economic Studies*), “The Elusive Gains from International Financial Integration.”