

STATISTISCHE METHODEN DER DATENANALYSE

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## **Excercise Sheets**

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Physik

UNI WIEN

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# 1 Sheet

## 1.1 Introduction

### 1.1.1 Minimum Value of $P(A \cap B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (1.1)$$

$$\Leftrightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B) \quad (1.2)$$

To minimize  $P(A \cup B)$  maximize  $P(A \cup B)$ ! Since  $P(A) + P(B) \geq 1$  we can assume that  $A \cup B = \Omega$  in effort to maximize  $P(A \cup B)$ . Therefore set  $P(A \cup B) = 1$ . Now

$$\min_{A,B} P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{1}{21} \quad (1.3)$$

### 1.1.2 A and B independent

For A and B independent

$$P(A \cap B) = P(A)P(B) = \frac{5}{21} \quad (1.4)$$

### 1.1.3 C:= None of the Events occur

The Negations of Events are independent for independent Events. So,

$$P(A' \cap B') = P(A')P(B') = \frac{2}{3} \cdot \frac{2}{7} = \frac{4}{21} \quad (1.5)$$

### 1.1.4 D:= Exactly one of the two events occurs

Lets define  $AB := A \cap B$ .

$$P(A'B \cup AB') = P(A'B) + P(AB') - P(\underbrace{A'B \cap AB'}_{=\emptyset}) \quad (1.6)$$

$$= P(A'B) + P(AB') \quad (1.7)$$

$$= P(A')P(B) + P(A)P(B') \quad (1.8)$$

$$= \frac{2}{3} \cdot \frac{5}{7} + \frac{1}{3} \cdot \frac{2}{7} = \frac{4}{7} \quad (1.9)$$

### 1.1.5 E:= Both Events occur

For A and B independent

$$P(A \cap B) = P(A)P(B) = \frac{1}{3} \cdot \frac{5}{7} = \frac{5}{21} \quad (1.10)$$

**Sanity Check** Since the already calculated Events  $C := \text{No Event occurring}$ ,  $D := \text{One Event occurring}$  and  $E := \text{Both events occurring}$  span  $\Omega$ , we can check that

$$P(C) + P(D) + P(E) = \frac{4}{21} + \frac{4}{7} + \frac{5}{21} \stackrel{!}{=} 1 \quad (1.11)$$

and indeed it is.

### 1.1.6 $F := \text{At least one of the two Events occurs}$

**Fast Route** Since  $F = D \cup E$

$$P(F) = P(D) + P(E) = \frac{17}{21} \quad (1.12)$$

**Long, but instructive Route** Again, using  $AB := A \cap B$ ,

$$P(D \cup E) = P(A'B \cup AB' \cup AB) \quad (1.13)$$

$$= P(A'B) + P(AB' \cup AB) - P\left(\underbrace{A'B \cap (AB' \cup AB)}_{=\emptyset}\right) \quad (1.14)$$

$$= P(A'B) + P(AB') + P(AB) - P\left(\underbrace{AB' \cap AB}_{=\emptyset}\right) \quad (1.15)$$

$$= P(A')P(B) + P(A)P(B') + P(A)P(B) \quad (1.16)$$

$$= \frac{2}{3} \cdot \frac{5}{7} + \frac{1}{3} \cdot \frac{2}{7} + \frac{1}{3} \cdot \frac{5}{7} \quad (1.17)$$

$$= \frac{17}{21} \quad (1.18)$$

### 1.1.7 $G := \text{At most one of the two Events occurs}$

**Fast Route** Since  $G := C \cup D$

$$P(G) = P(C) + P(D) = \frac{16}{21} \quad (1.19)$$

**Long (shortened), but instructive Route** Using the same empty Intsection argument as in 1.6, 1.14 and 1.15

$$P(A'B' \cup A'B \cup AB') = P(A'B') + P(A'B) + P(AB') \quad (1.20)$$

$$= \frac{2}{3} \cdot \frac{2}{7} + \frac{2}{3} \cdot \frac{5}{7} + \frac{1}{3} \cdot \frac{2}{7} = \frac{16}{21} \quad (1.21)$$

## 1.2 Novel Detector Failure

There are two remoTES, each fully functional ( $c$ ) with a probability

$$P(c) = P(p_1) P(w) P(p_2) = 0.405 \quad (1.22)$$

, since the success events of three components ( $p_1$ ,  $w$  and  $p_2$ ) are independent. Each one will fail with a probability

$$P(\bar{c}) = 1 - 0.405 = 0.595 \quad (1.23)$$

The probability for both to fail is

$$P(\bar{c}_1 \cap \bar{c}_2) = P^2(\bar{c}) = 0.354025 \quad (1.24)$$

Which means at least one will function with probability

$$P(c_1 \cup c_2) = 1 - 0.354025 = 0.645975 \quad (1.25)$$

## 1.3 graduation rate

### 1.3.1 graduation rate

With the Events  $w := \text{is Woman}$ ,  $g := \text{graduated}$  the probability of degree completion is

$$P(g) = P(g|w) P(w) + P(g|\bar{w}) P(\bar{w}) \quad (1.26)$$

$$= 0.372 \cdot 0.625 + 0.311 \cdot (1 - 0.625) = 0.349125 \quad (1.27)$$

### 1.3.2 percentage of male dropouts

Since we now know the unconditional dropout rate, we can calculate

$$P(m|\bar{g}) = \frac{P(m \cap \bar{g})}{P(\bar{g})} \quad (1.28)$$

$$= \frac{P(\bar{g}|m) P(m)}{P(\bar{g})} \quad (1.29)$$

$$= \frac{(1 - 0.311)(1 - 0.625)}{1 - 0.349125} \quad (1.30)$$

$$\approx 0.397 \quad (1.31)$$

## 2 Sheet

### 2.1 Bad Detector

It makes sense to assume, that if an event is only registered with probability  $p$ , this can be translated to a reduced rate  $\lambda_r = p\lambda$ . Therefore the Number of registered Events is poisson distributed with density

$$f(k; \lambda_r) = \frac{\lambda_r^k}{k!} e^{-\lambda_r} \quad (2.1)$$

yes, but rigorous?

### 2.2 Uneconomical Warranty for faulty computer monitors

#### 2.2.1 Average faultless running time for economical warranty

The percentage of failed monitors after  $t$  years is

$$p_f = \int_0^t f_{EX}(t'; \tau) dt' \quad (2.2)$$

$$= \int_0^t \frac{1}{\tau} e^{-\frac{t'}{\tau}} dt' \quad (2.3)$$

$$= 1 - e^{-\frac{t}{\tau}} \quad (2.4)$$

For  $t = 5$  and  $p_f = 0.2$ :

$$0.2 \geq 1 - e^{-\frac{5}{\tau}} \quad (2.5)$$

$$\Leftrightarrow 0.8 \leq e^{-\frac{5}{\tau}} \quad (2.6)$$

$$\Leftrightarrow -\frac{5}{\ln(0.8)} \leq \tau \quad (2.7)$$

$$\Leftrightarrow 22.4 \leq \tau \quad (2.8)$$

#### 2.2.2 Shortened Warranty

$$\tau \geq -\frac{3}{\ln(0.8)} = 13.44 \quad (2.9)$$

#### 2.2.3 Monitors running after 9 years

Assuming at least 90% run after 3 years:

$$\tau \geq -\frac{3}{\ln(0.9)} = 28.47 \quad (2.10)$$

The percentage failed monitors after 5 years is

$$p_f = 1 - e^{-\frac{t}{\tau}} \quad (2.11)$$

$$= 0.161 \quad (2.12)$$

After 5 years 0.83% are still running.

## 2.3 Vulcano eruption

Let the waiting times to eruption for the  $N$  identical vulcanos be realized within the random variables  $X_1, X_2, \dots, X_N$ . Each one distributed exponentially

### 2.3.1 Mean Time to eruption

### 2.3.2 Time until first eruption

For each  $X_i$ :

$$P(X_i \leq t) = \int_0^t \frac{1}{\tau} e^{-\frac{t'}{\tau}} dt' = 1 - e^{-\frac{t}{\tau}} \quad (2.13)$$

$$\Rightarrow P(X_i > t) = e^{-\frac{t}{\tau}} \quad (2.14)$$

The time until the first eruption  $Y$  is now distributed as

$$P(Y \leq t) = P(\min\{X_1, X_2, \dots, X_N\} \leq t) \quad (2.15)$$

$$= 1 - P(X_1 > t, X_2 > t, \dots, X_N > t) \quad (2.16)$$

$$= 1 - e^{-\frac{nt}{\tau}} \quad (2.17)$$

$Y$  is distributed exponentially with mean  $\frac{\tau}{n}$ .

### 2.3.3 Time until last eruption

## 2.4 Electric Current

With

$$\frac{dR}{dI} = -\frac{U}{I^2} \quad (2.18)$$

and the Variance of  $R$

$$\text{Var}[R(I)] \approx R'(E[I])^2|_{I,U} \cdot \text{Var}[I] \quad (2.19)$$

$$\Rightarrow \Delta R = \left| \frac{dR}{dI} \right| \Delta I \quad (2.20)$$

the variance of the resistance in linear approximation is given by

$$\Delta R = \frac{230\text{V}}{2.5^2\text{A}^2} 0.15\text{A} = 5.52\Omega \quad (2.21)$$

The relative error ist then

$$\frac{\Delta R}{R} = 0.06 \quad (2.22)$$

### 2.4.1 Different Resistor

Halving the Resistance means halving the voltage or doubling the current.

$$R' = \frac{1}{2} \frac{U}{I} \quad (2.23)$$

Both lead to different outcomes. Assuming the voltage halves:

$$\Delta R = \frac{115\text{V}}{2.5^2\text{A}^2} 0.15\text{A} = 2.76\Omega \quad (2.24)$$

$$\Rightarrow \frac{\Delta R}{R} = 0.06 \quad (2.25)$$

, because  $U$  is linear in the error, as is the resistance.