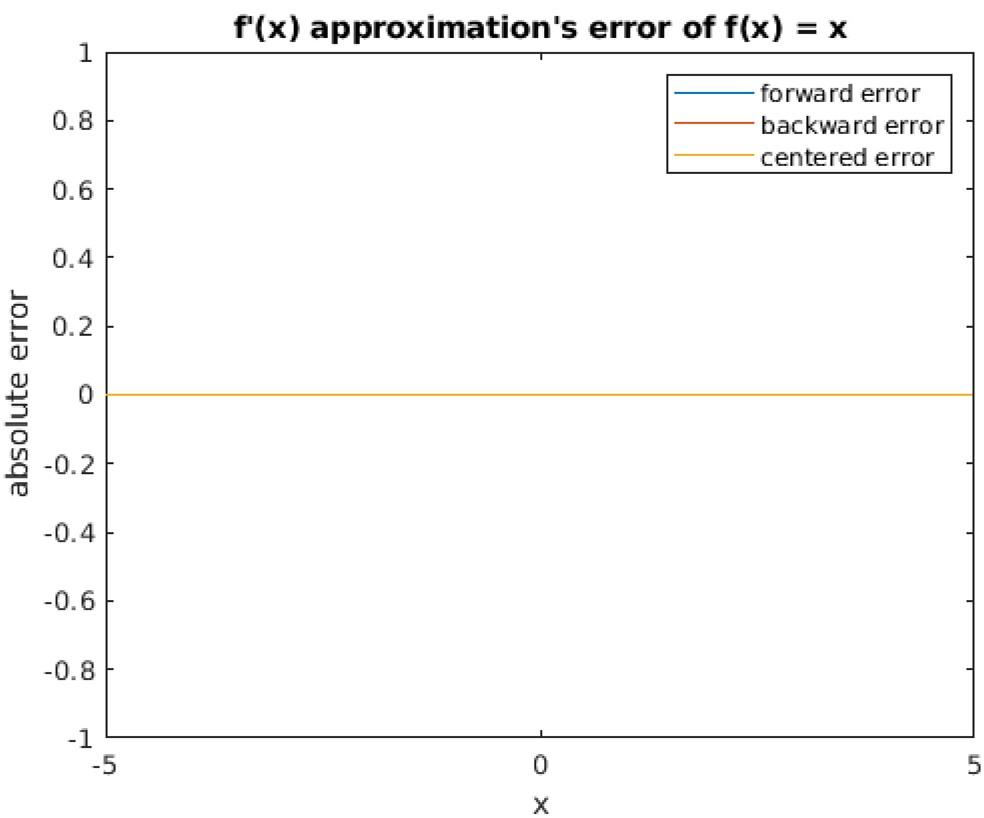
Question 1

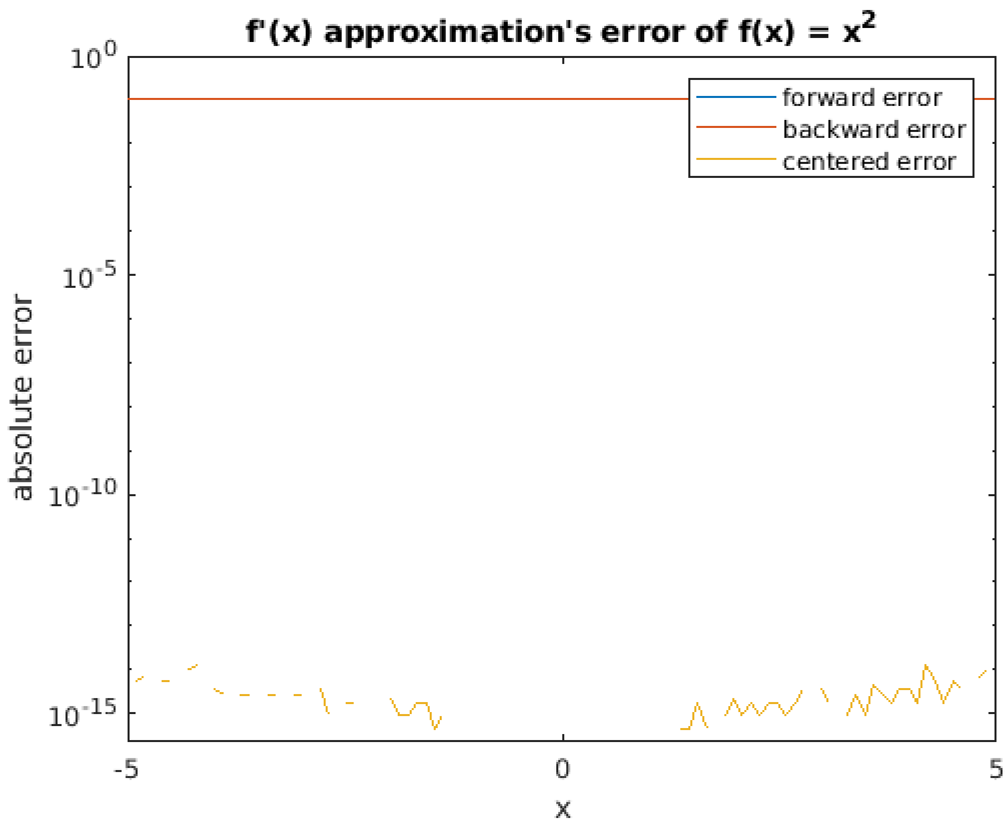
```
x = linspace(-5, 5, 100);

f = @(x) x;
fPrime = @(x) 1;
y = fPrime(x);
forwardFPrime = forward(f, x);
backwardFPrime = backward(f, x);
centeredFPrime = centered(f, x);
forwardError = abs(y - forwardFPrime);
backwardError = abs(y - backwardFPrime);
centeredError = abs(y - centeredFPrime);
plot(x, forwardError, x, backwardError, x, centeredError)
title("f'(x) approximation's error of f(x) = x")
legend("forward error", "backward error", "centered error")
ylabel("absolute error")
xlabel("x")
```



Accurate at all values because this graph is linear (polynomial power = 1)

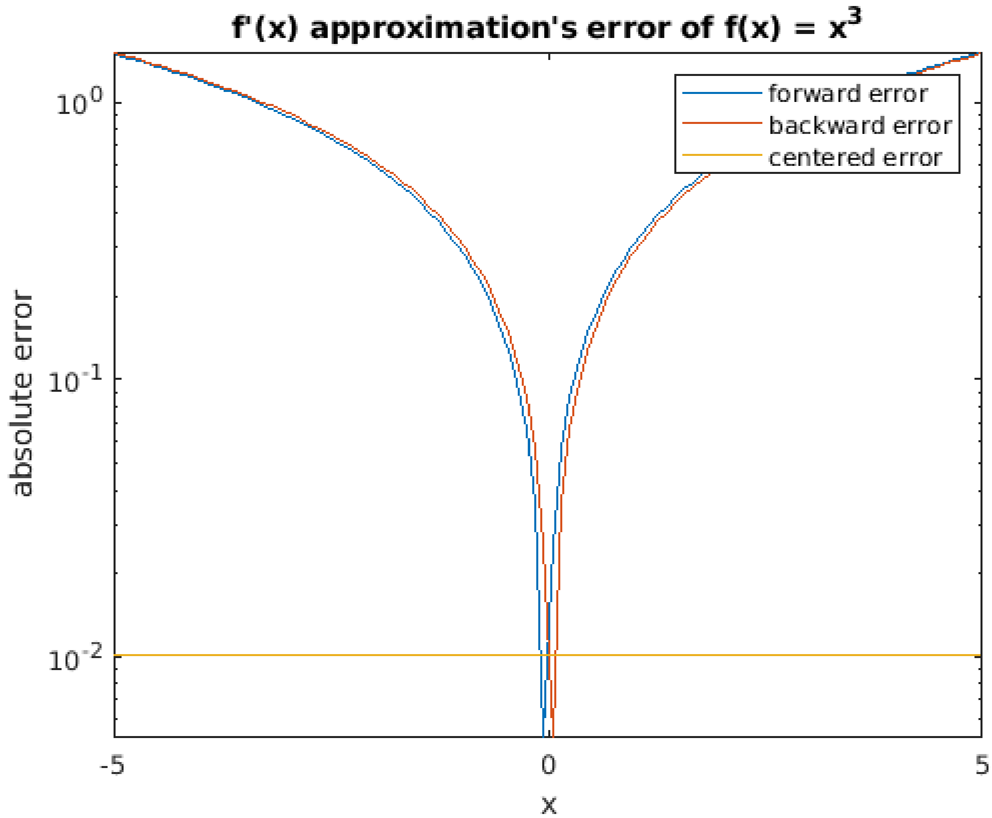
```
f = @(x) x.^2;
fPrime = @(x) 2.*x;
y = fPrime(x);
forwardFPrime = forward(f, x);
backwardFPrime = backward(f, x);
```

```
centeredFPrime = centered(f, x);
forwardError = abs(y - forwardFPrime);
backwardError = abs(y - backwardFPrime);
centeredError = abs(y - centeredFPrime);
semilogy(x, forwardError, x, backwardError, x, centeredError)
title("f'(x) approximation's error of f(x) = x^2")
legend("forward error", "backward error", "centered error")
ylabel("absolute error")
xlabel("x")
```



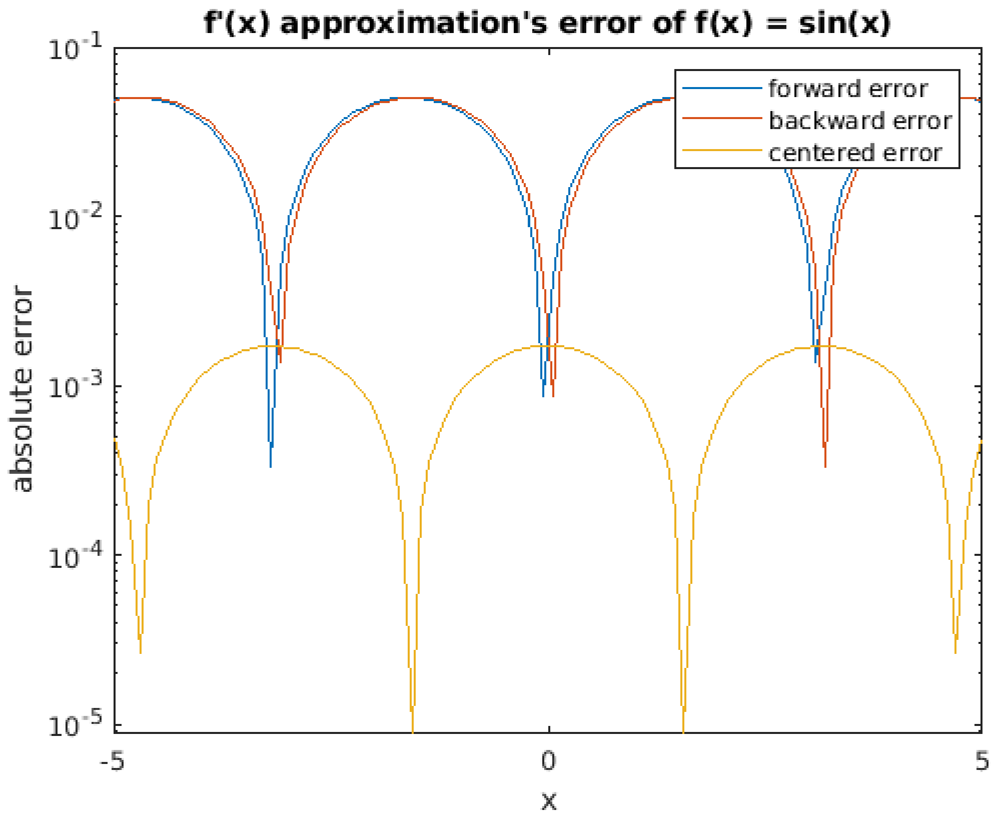
Centered seems to be VERY accurate for almost all values in this case, but the sparse lining of it are NONzero values where they are still very close to 0 (10^-15).

```
f = @(x) x.^3;
fPrime = @(x) 3.*x.^2;
y = fPrime(x);
forwardFPrime = forward(f, x);
backwardFPrime = backward(f, x);
centeredFPrime = centered(f, x);
forwardError = abs(y - forwardFPrime);
backwardError = abs(y - backwardFPrime);
centeredError = abs(y - centeredFPrime);
semilogy(x, forwardError, x, backwardError, x, centeredError)
title("f'(x) approximation's error of f(x) = x^3")
legend("forward error", "backward error", "centered error")
ylabel("absolute error")
xlabel("x")
```



For forward and backward, it is accurate at some value around -4 and 4 and centered is accurate at some value around -0.5 and 0.5.

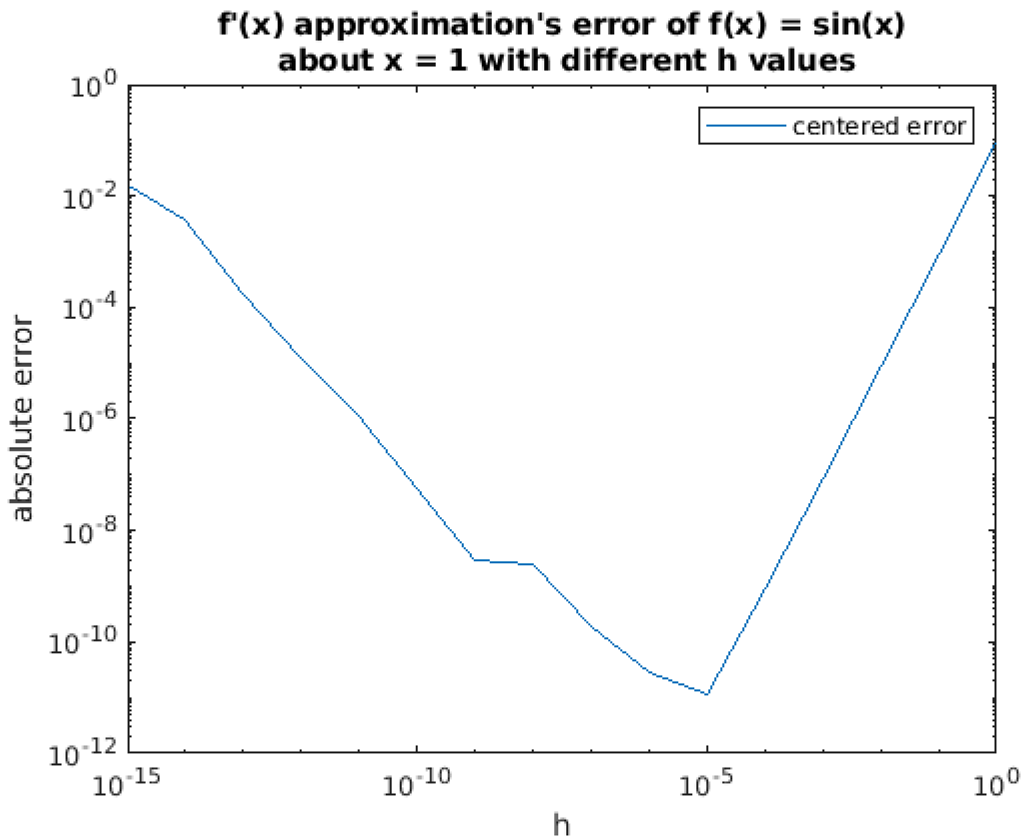
```
f = @(x) sin(x);
fPrime = @(x) cos(x);
y = fPrime(x);
forwardFPrime = forward(f, x);
backwardFPrime = backward(f, x);
centeredFPrime = centered(f, x);
forwardError = abs(y - forwardFPrime);
backwardError = abs(y - backwardFPrime);
centeredError = abs(y - centeredFPrime);
semilogy(x, forwardError, x, backwardError, x, centeredError)
title("f'(x) approximation's error of f(x) = sin(x)")
legend("forward error", "backward error", "centered error")
ylabel("absolute error")
xlabel("x")
```



Error tends to approach 0 at those troughs of each different function, so forward and backward will do it roughly around -3, 0, and 3. For centered, it does so as it approached -5, 5, 1.5, and -1.5.

Question 2

```
f = @(x) sin(x);
centeredApprox = @(h) (f(1 + h) - f(1 - h))/(2.*h);
x = [];
errors = [];
for i = 1:16;
    x(i) = 1/(10^(i - 1));
    errors(i) = abs(cos(1) - centeredApprox(x(i)));
end
loglog(x, errors);
title({"f'(x) approximation's error of f(x) = sin(x)", " about x = 1 with different h
legend("centered error")
xlabel("h")
ylabel("absolute error")
```



Error stops decreasing at about 10^-5, which is semi-reasonable since it starts to get bugged by machine roundoff.

Question 3

```
X = [1,2,3,4,5];
f = @(x) 5.*x.^3 + 2.*x.^2 + x + 0;
y = num_diff(X,f(X),4.5,3)
```

```
y = 30.0000
```

```
function fPrime = forward(f, x)
    h = x(2) - x(1);
    fPrime = [];
    for i = 1:numel(x)
        fPrime(i) = (f(x(i) + h) - f(x(i)))/h;
    end
end

function fPrime = backward(f, x)
    h = x(2) - x(1);
    fPrime = [];
    for i = 1:numel(x)
        fPrime(i) = (f(x(i)) - f(x(i) - h))/h;
```

```
    end
end

function fPrime = centered(f, x)
    h = x(2) - x(1);
    fPrime = [];
    for i = 1:numel(x)
        fPrime(i) = (f(x(i) + h) - f(x(i) - h))/(2*h);
    end
end

function a = linterp_poly(X, Y)
a = zeros(1,numel(X));
for i = 1:numel(X)
aa = poly(X([1:i-1 i+1:end]));
a = a + Y(i) * aa / polyval(aa, X(i));
end
end

function y = num_diff(X, Y, x, d)
    vals = linterp_poly(X, Y);
    for i = 1:d
        vals = polyder(vals);
    end
    y = polyval(vals, x);
end
```