Question 1

Implement the bisection method (textbook §2.1). Your function should have the following signature,

function r = bisection(f, a, b, tol, maxits)

```
f = @(x) sin(x);
a = 3;
b = 4;
tol = 1e-5;
maxits = 50;
fprintf('%.5f\n',bisection(f, a, b, tol, maxits, 'none'))
```

```
3.14159
```

```
fprintf('%.5f\n',bisectionrecursive(f, a, b, tol, maxits))
```

```
3.14159
```

a) tol 1e-5 and maxits 16 functions to get us accuracy to 5th decimal place.

b) replacing sinx with tanx causes a weird solution because there is a discontinuity between 1 and 2 and the bisection function does not know this.

c) done, named bisectionrecursive.

Question 2

```
fzero(@sin, [3 4])
```

```
ans =
   3.141592653589793
```

```
fzero(@tan, [1 2], optimset('Display','iter'))
```

Func-count	x	f(x)	Procedure
2	1	1.55741	initial
3	2	-2.18504	interpolation
4	1.85165	-3.46644	interpolation
5	1.41615	6.4146	bisection
6	1.47895	10.8572	interpolation
7	1.6339	-15.8261	bisection
8	1.61954	-20.4982	interpolation
9	1.58798	-58.1765	bisection
10	1.55642	69.5775	bisection
11	1.55783	77.1347	interpolation
12	1.56502	173.074	bisection
13	1.56861	457.679	bisection
14	1.5722	-710.251	bisection
15	1.57182	-980.91	interpolation
16	1.57041	2574.06	bisection

```
17      1.57111    -3169.73    interpolation
18      1.57104    -4124.11    interpolation
19      1.57088    -11801.6    bisection
20      1.57072     13697.2    bisection
21      1.57073     14893.3    interpolation
22      1.57077     32636.8    bisection
23      1.57078     80720.7    bisection
24       1.5708     -170547    bisection
25      1.57079      306518    interpolation
26       1.5708     -384460    interpolation
27       1.5708     -515558    interpolation
28       1.5708   1.51194e+06  bisection
29       1.5708  -1.56465e+06  interpolation
30       1.5708  -1.62117e+06  interpolation
31       1.5708  -3.36385e+06  bisection
32       1.5708  -7.27282e+06  bisection
33       1.5708  -1.73587e+07  bisection
34       1.5708   4.48791e+07  bisection
35       1.5708  -5.66155e+07  interpolation
36       1.5708  -7.66641e+07  interpolation
37       1.5708   2.16494e+08  bisection
38       1.5708  -2.37393e+08  interpolation
39       1.5708  -2.62758e+08  interpolation
40       1.5708  -5.88385e+08  bisection
41       1.5708  -1.54689e+09  bisection
42       1.5708   2.45914e+09  bisection
43       1.5708   3.48749e+09  interpolation
44       1.5708  -8.33979e+09  bisection
45       1.5708   1.19881e+10  interpolation
46       1.5708   2.13107e+10  interpolation
47       1.5708  -2.74039e+10  bisection
48       1.5708   -3.1975e+10  interpolation
49       1.5708  -7.67527e+10  bisection
50       1.5708   1.91689e+11  bisection
51       1.5708  -2.56007e+11  interpolation
52       1.5708  -3.85261e+11  interpolation
53       1.5708   7.63028e+11  bisection
54       1.5708   1.49707e+12  interpolation
55       1.5708  -1.55633e+12  bisection
56       1.5708  -1.58761e+12  interpolation
57       1.5708  -3.24064e+12  bisection
58       1.5708  -6.76496e+12  bisection
59       1.5708  -1.48279e+13  bisection
60       1.5708  -3.64003e+13  bisection
61       1.5708   7.86301e+13  bisection
62       1.5708  -1.33542e+14  interpolation
63       1.5708   1.93489e+14  interpolation
64       1.5708   3.66869e+14  interpolation
65       1.5708  -4.19946e+14  bisection
66       1.5708  -5.83048e+14  interpolation
67       1.5708  -1.20927e+15  bisection
```

Current point x may be near a singular point. The interval [1, 2]
reduced to the requested tolerance and the function changes sign in the interval,
but f(x) increased in magnitude as the interval reduced.

ans =

    1.570796326794897

```
fzero(@sin, [3 4], optimset('PlotFcns',@optimplotfval));
fprintf('%.5f\n',bisection(f, a, b, tol, maxits, 'iter'))
```

```
Iteration: 1 Value: 3.50000
Iteration: 2 Value: 3.25000
```

```
Iteration: 3 Value: 3.12500
Iteration: 4 Value: 3.18750
Iteration: 5 Value: 3.15625
Iteration: 6 Value: 3.14062
Iteration: 7 Value: 3.14844
Iteration: 8 Value: 3.14453
Iteration: 9 Value: 3.14258
Iteration: 10 Value: 3.14160
Iteration: 11 Value: 3.14111
Iteration: 12 Value: 3.14136
Iteration: 13 Value: 3.14148
Iteration: 14 Value: 3.14154
Iteration: 15 Value: 3.14157
Iteration: 16 Value: 3.14159
Iteration: 17 Value: 3.14159
3.14159
```

```
fprintf('%.5f\n',bisection(f, a, b, tol, maxits, 'plot'))
```
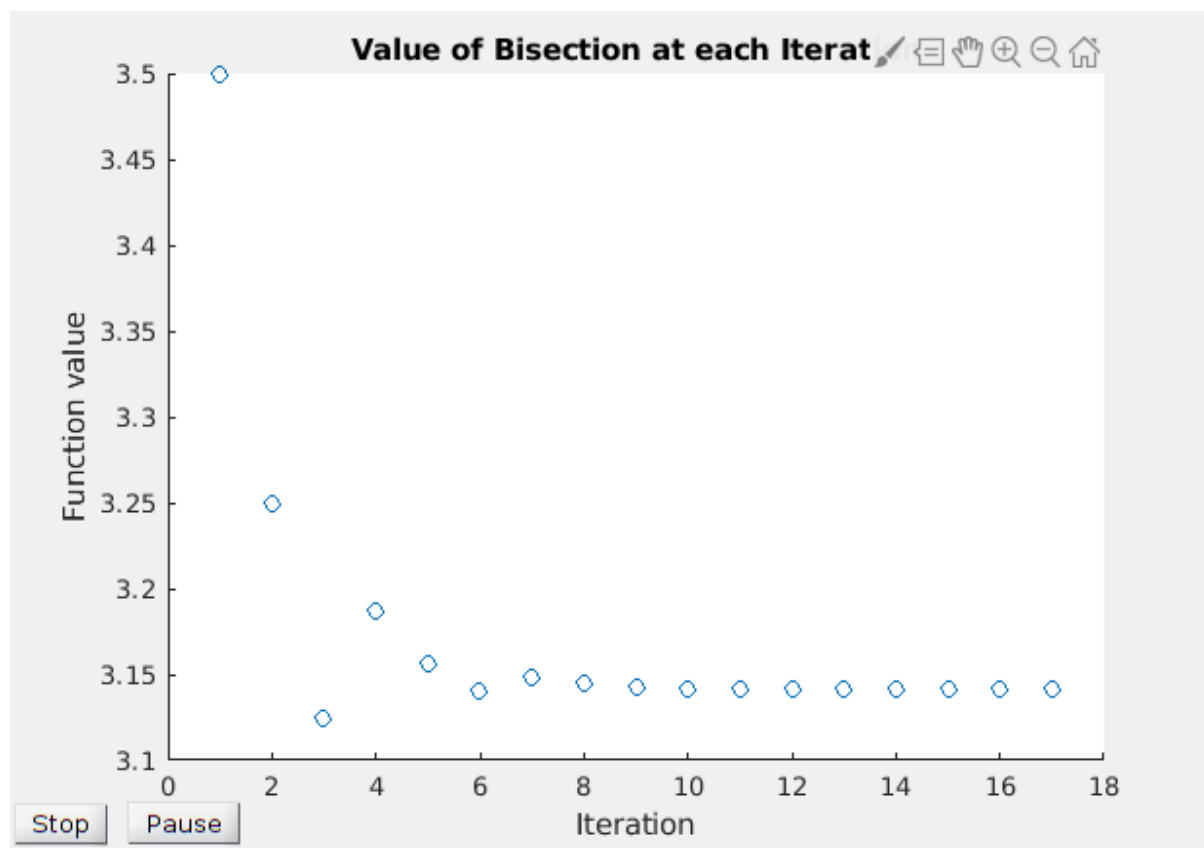


```
3.14159
```

a) Up to 5 digits are correct.

b) Done

c) Done

d) Done

```
function r = bisection(f, a, b, tol, maxits, mode)
    x = 0;
```

```matlab
    y = 0;
    format long
    for i = 1:maxits
        r = ( a + b ) / 2;
        if (mode == 'iter')
            fprintf('Iteration: %d Value: %.5f\n', i, r)
        end

        x(i) = i;
        y(i) = r;

        if (sign(f(r)) == sign(f(a)))
            a = r;
        else
            b = r;
        end

        if (abs(a - b)<tol)
            break
        end

    end

    if (mode == 'plot')
        title('Value of Bisection at each Iteration')
        scatter(x, y)
    end
end

function r = bisectionrecursive(f, a, b, tol, maxits)

    r = (a + b) / 2;
    % Base cases
    if (abs(a - b)<tol)
        return
    elseif (maxits == 0)
        return
    end

    if (sign(f(r)) == sign(f(a)))
        r = bisectionrecursive(f, r, b, tol, maxits - 1);
    else
        r = bisectionrecursive(f, a, r, tol, maxits - 1);
    end
end
```