# Consensus Problems in Networks of Agents with Switching Topology and Time-Delays

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分享暨个人总结

- Introduction
- Consensus Problems
- Consensus Protocols
- 4 Algebric Graph Theory: Properties of Laplacians
- 5 A Counterexample for Average-Consensus
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- Networks with Communication Time-Delays
- 8 Max-Consensus and Leader Determination
- Simulation Results

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# Some Basic Notations

- 邻接矩阵
- 邻居节点
- 决策值
- χ-一致性问题
- 平均/最大/最小一致性

- $\bullet \ \mathcal{A} = [a_{ij}]$
- $\bullet$   $N_i$
- α
- χ
- Ave(x)/Max(x)/Min(x)

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# Model Consensus Protocols

#### CT Model

$$\dot{x}_i = u_i(t)$$

#### DT Model

$$x_i(k+1) = x_i(k) + \epsilon u_i(k), \epsilon > 0$$

## A1 Zero Communication Time-Delay

$$u_i = \sum_{j \in N_i} a_{ij}(x_j - x_i)$$

### A2 Communication Time-Delay $\tau_{ii} > 0$

$$u_i(t) = \sum_{i \in N_i} a_{ij} [x_j(t - \tau_{ij}) - x_i(t - \tau_{ij})]$$

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# Laplacians

## Laplacians

$$I_{ij} = \begin{cases} \sum_{k=1, k \neq i}^{n} a_{ik}, & j = i \\ -a_{ij}, & j \neq i \end{cases}$$

#### A1 State Evolves

$$\dot{x}(t) = -Lx(t)$$

# A1 State Evolves with Switchin Topology

$$\dot{x}(t) = -L_k x(t), \ k = s(t)$$

#### A1 DT

$$x(k+1) = P_{\epsilon}x(k), P_{\epsilon} = I - \epsilon L$$



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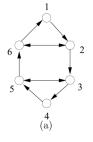
# Balanced Graph

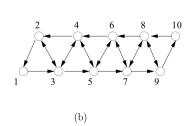
Balanced Node

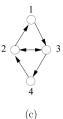
$$\deg_{out}(v_i) = \deg_{in}(v_i)$$

Balanced Graph

$$\deg_{in}(v_i) = \sum_{j=1}^n a_{ji}, \quad \deg_{out}(v_i) = \sum_{j=1}^n a_{ij}$$







# Laplacians

$$L = \mathcal{L}(G) = \Delta - \mathcal{A}$$

```
\mathcal{A} (adjacency matrix) \{0,1\} w_r w_l SC (Strongly Connected)
```

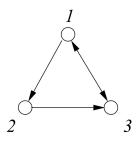
 $\Delta$  (degree matrix) diag(deg<sub>out</sub>( $v_i$ ))

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# Counterexample



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# 一致性协议

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