Consensus Problems in Networks of Agents with Switching Topology and Time-Delays

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分享暨个人总结

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Some Basic Notations

- adjacency matrix
- neighbors nodes
- decision value
- χ-Consensus Problesm
- Ave/Max/Min Consensus

- $\mathcal{A} = [a_{ij}]$
- N_i
- $\alpha(x^*)$
- χ
- Ave(x)/Max(x)/Min(x)

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Model Consensus Protocols

CT Model

$\dot{x}_i = u_i(t)$

DT Model

$x_i(k+1) = x_i(k) + \epsilon u_i(k), \epsilon > 0$

A1 Zero Communication Time-Delay

$u_i = \sum_{j \in N_i} a_{ij}(x_j - x_i)$

A2 Communication Time-Delay $\tau_{ij} > 0$

$u_i(t) = \sum_{j \in N_i} a_{ij}[x_j(t - \tau_{ij}) - x_i(t - \tau_{ij})]$

Remark

Consensus Problems:

1. Dynamic Networks
2. Consensus Protocols

Laplacians

Laplacians

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{n} a_{ik}, & j = i \\ -a_{ij}, & j \neq i \end{cases}$$

A1 State Evolves

$$\dot{x}(t) = -Lx(t)$$

A1 State Evolves with Switchin Topology

$$\dot{x}(t) = -L_k x(t), \;\; k = s(t)$$

A1 DT

$$x(k+1) = P_\epsilon x(k), \;\; P_\epsilon = I - \epsilon L$$

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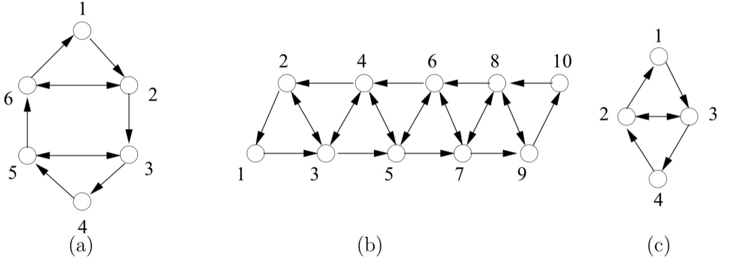
Balanced Graph

Balanced Node

$$\deg_{out}(v_i) = \deg_{in}(v_i)$$

Balanced Graph

$$\deg_{in}(v_i) = \sum_{j=1}^{n} a_{ji}, \quad \deg_{out}(v_i) = \sum_{j=1}^{n} a_{ij}$$



Laplacians

$$L = \mathcal{L}(G) = \Delta - \mathcal{A}$$

Δ (degree matrix) $\rightarrow$ diag$(\deg_{out}(v_i))$
$\mathcal{A}$ (adjacency matrix) $\in \{0,1\}$
$w_r : \quad Lw_r = \lambda w_r$
$w_l : \quad w_l L = w_l \lambda$
SC (Strongly Connected)

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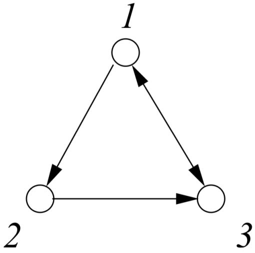
Counterexample



$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\mathcal{V} = \{1, 2, 3\}, \quad \mathcal{E} = \{12, 23, 31, 13\}.$$

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Fixed Topology

$$x^* = \lim_{t \to +\infty} x(t) = R x_0 = w_r (w_L^T x_0) = \frac{1}{\sqrt{n}} (w_l^T x_0) \mathbf{1}$$

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Time-Delays

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij}[x_j(t - \tau_{ij}) - x_i(t - \tau_{ij})].$$

$$\tau \leq \frac{\pi}{4 d_{max}(G)}$$

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Max-Consensus

$$x_i(k+1) = max(x_i(k), u_i(k))$$

$$x_i(k+1) = \frac{1}{2}(x_i(k) + u_i(k) + |x_i(k) - u_i(k)|) \quad u_i(k) = \max_{j \in N_i} x_j$$

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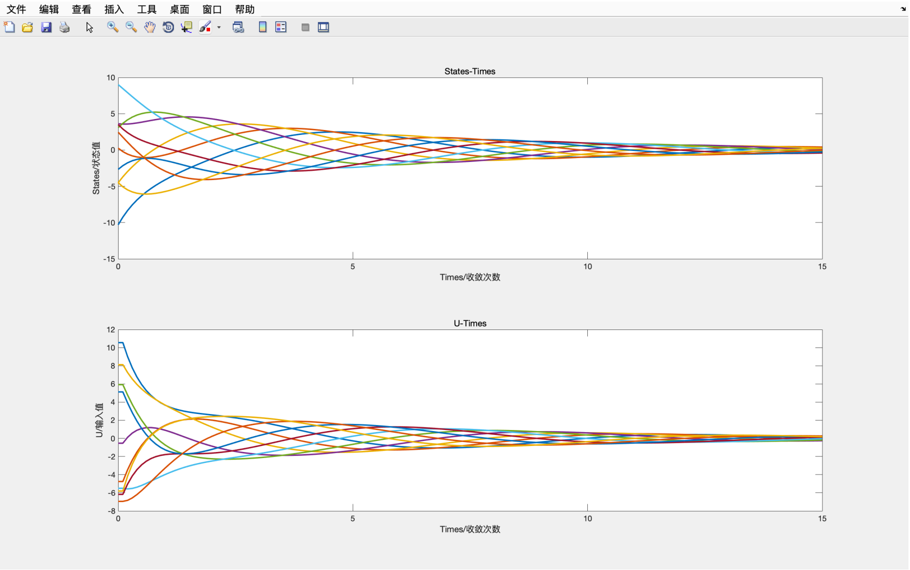
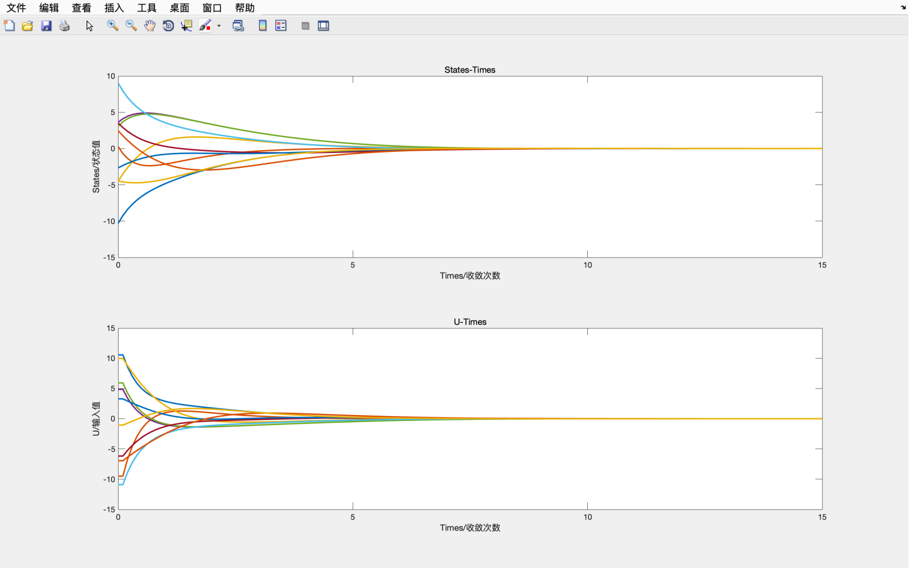
Simulation


文件 编辑 查看 插入 工具 桌面 窗口 帮助
States-Times
States/状态值
Times/收敛次数
U-Times
U/输入值
Times/收敛次数

Simulation



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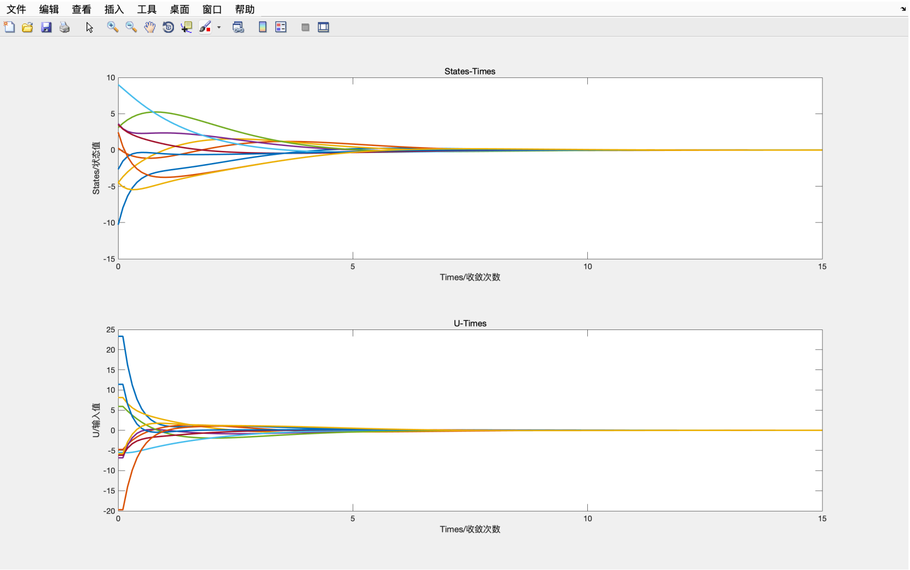
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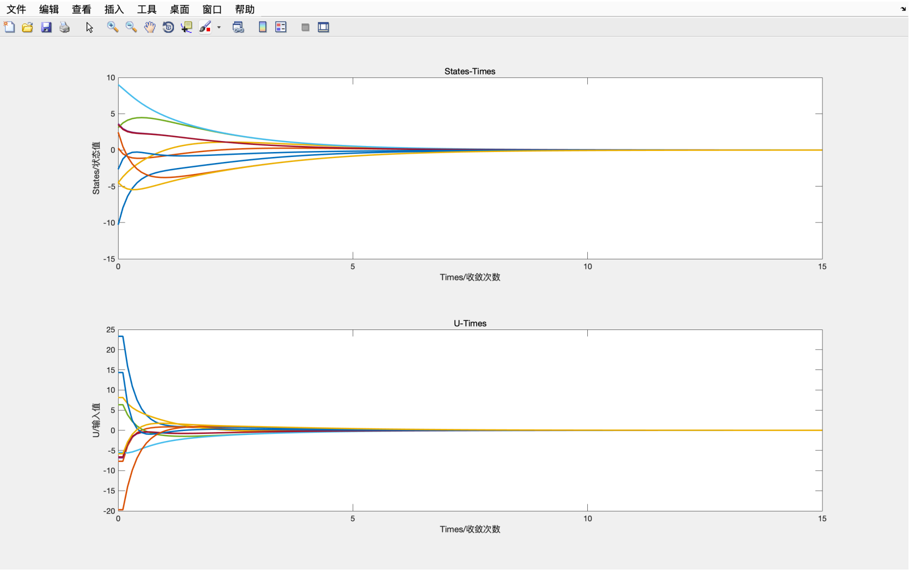

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