CS321 Introduction to Theory of Computation Assignment No. 1, Due: Friday January 20, 2023

Ivan Chan *933821369*

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1. Prove that $\overline{S_1 \cup S_2} = \overline{S_1} \cap \overline{S_2}$ where S_1 and S_2 are sets and $\bar{S}$ is the complement of the set S.

Proof. Let $x \in (\overline{S_1 \cup S_2})$ Thus,

$$\begin{aligned}
&x \in \overline{S_1} \text{ and } x \in \overline{S_2}\\
&x \notin S_1 \text{ and } x \notin S_2\\
&x \in \overline{S_1} \cap \overline{S_2}\\
&\text{Therefore, } \overline{S_1 \cup S_2} \subset \overline{S_1} \cap \overline{S_2}
\end{aligned}$$

Let another arbitrary variable $y \in (\overline{S_1} \cap \overline{S_2})$,

$$\begin{aligned}
&y \in (\overline{S_1} \cap \overline{S_2})\\
&y \in \overline{S_1} \text{ and } y \in \overline{S_2}\\
&y \notin S_1 \text{ and } y \notin S_2\\
&y \notin (S_1 \cup S_2)\\
&y \in (\overline{S_1} \cup \overline{S_2})\\
&\text{Therefore, } \overline{S_1} \cap \overline{S_2} \subset (\overline{S_1 \cup S_2})\\
&\text{Proof is also known as DeMorgan's Union Law}
\end{aligned}$$

□

2. A tree is a graph with no cycle. Show by induction that a tree with n nodes contains $n - 1$ edges.

Proof.

Base case: When n = 1, there are $(1) - 1 = 0$ edges.

The proof holds true that a tree with $n = 1$ nodes contain $n - 1 = 0$ edges

Induction Assumption:

k nodes $= k - 1$ edges for $k \geq 0$

Induction Step:

Let $n = (k + 1)$,

Then a tree containing $(k + 1)$ nodes $= (k + 1) - 1 = k$ edges

Thus, the $(k + 1)$th node will contain k edges

Hence, nodes containing n nodes contain $n - 1$ edges

□

3. A rational number is of the form m/n where m and n are integers. For example, $\frac{2}{3}, \frac{3}{4}, \frac{2}{5}, \frac{4}{7}, \frac{3}{8}, \frac{5}{9}, \frac{11}{18}, \frac{9}{25}$ are some rational numbers. Show by contradiction that $\sqrt{2}$ is not a rational number

Proof. Let $x \in \bar{(}S_1 \cup S_2)$ where $t = xy$ and $u = zw$. So,

$$Hellothere$$

□

4. Let the input symbols in a finite automata be $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Design a DFA that accepts all integers which are divisible by 3. (Hint: An integer is divisible by 3 if the sum of the digits is divisible by 3)

Proof. Let $x \in \bar{(}S_1 \cup S_2)$ where $t = xy$ and $u = zw$. So,

$$Hello$$

□

5. For this problem assume that the input symbols are $\{0, 1\}$. Design a DFA that accepts the binary string if it is divisible by 3.

Proof. Let $x \in \bar{(}S_1 \cup S_2)$ where $t = xy$ and $u = zw$. So,

$$Hello$$

□