JIDHUN PP | BSc(Hons) Computer Science| 20211419|Practical-6

SOLUTION OF CAUCHY PROBLEM FOR FIRST ORDER PDE

QUESTION 1:Obtain the solution of the linear equation u[(x,y),x]-u[(x,y),y]=1 with the Cauchy data u(x,0)=x*x

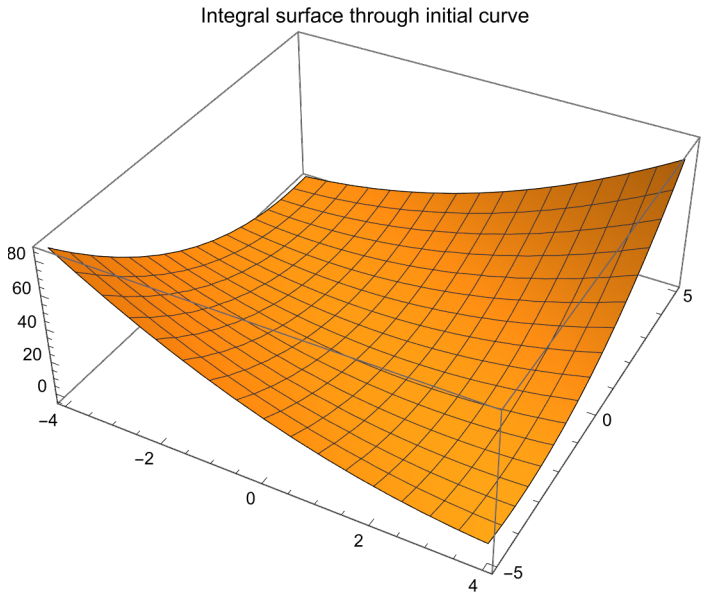
```
pde = D[u[x, y], x] - D[u[x, y], y] == 1
```

$$-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$$

```
sol = DSolve[{pde, u[x, 0] == x * x}, u[x, y], {x, y}]
```

$$\{\{u[x, y] \to x^2 - y + 2\,x\,y + y^2\}\}$$

```
Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -5, 5},
 PlotLabel → "Integral surface through initial curve"]
```



QUESTION 2:Obtain the solution of the linear equation y*u[(x,y),x]-2*x*u[(x,y)] with the Cauchy data u(0,y)=y*y*y

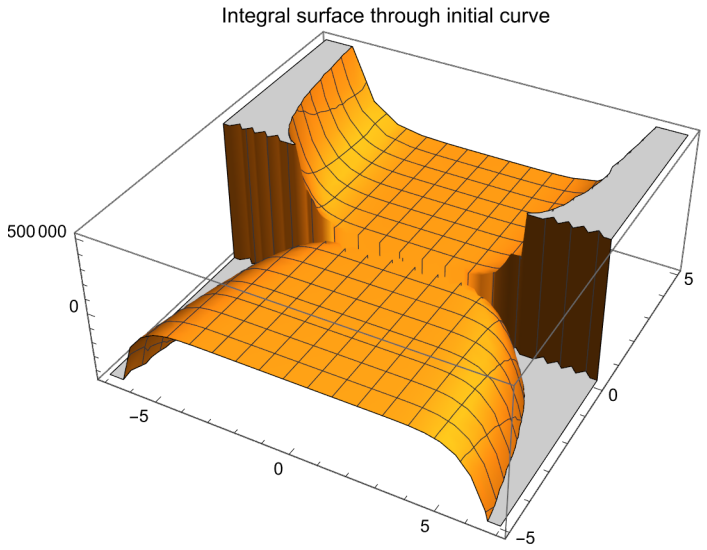
SOLUTION:

```
pde = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y] == 2 * x * u[x, y]
sol3 = DSolve[{pde, u[0, y] == y * y * y}, u[x, y], {x, y}]
```

$$-2\,x\,y\,u^{(0,1)}[x, y] + y\,u^{(1,0)}[x, y] == 2\,x\,u[x, y]$$

$$\left\{\left\{u[x, y] \to \frac{\left(x^2 + y\right)^4}{y}\right\}\right\}$$

```
Plot3D[u[x, y] /. sol3, {x, -7, 7}, {y, -5, 5},
 PlotLabel → "Integral surface through initial curve"]
```



QUESTION 3:Determine the integral surfaces of the equation u[(x,y),x]+u[(x,y),y] =u[x,y]*u[x,y], (a)with the data x+y=0,u=1. (b) with the data u(x,0)=tanh(x)

SOLUTION:

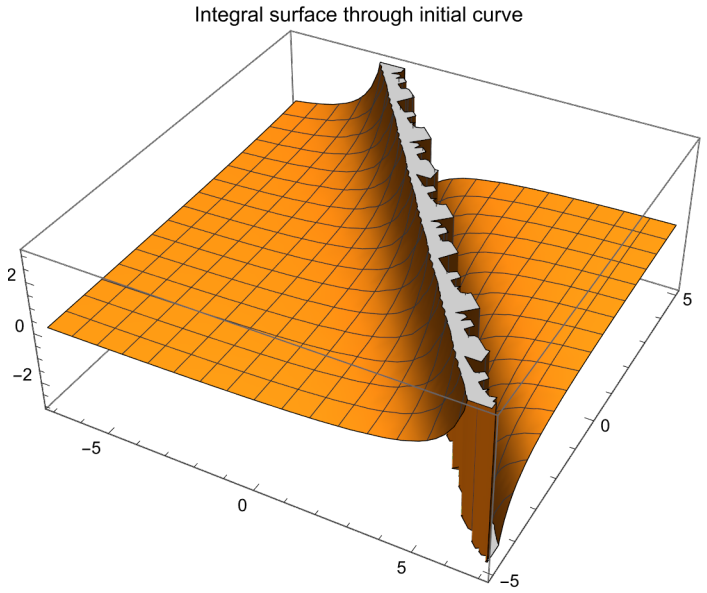
(a)

```
Eqn = D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]
sol4 =
 DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, -x] == 1}, u[x, y], {x, y}]
```

$$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2$$

$$\left\{\left\{u[x, y] \to -\frac{2}{-2 + x + y}\right\}\right\}$$

```
Plot3D[u[x, y] /. sol4, {x, -7, 7}, {y, -5, 5},
 PlotLabel → "Integral surface through initial curve"]
```



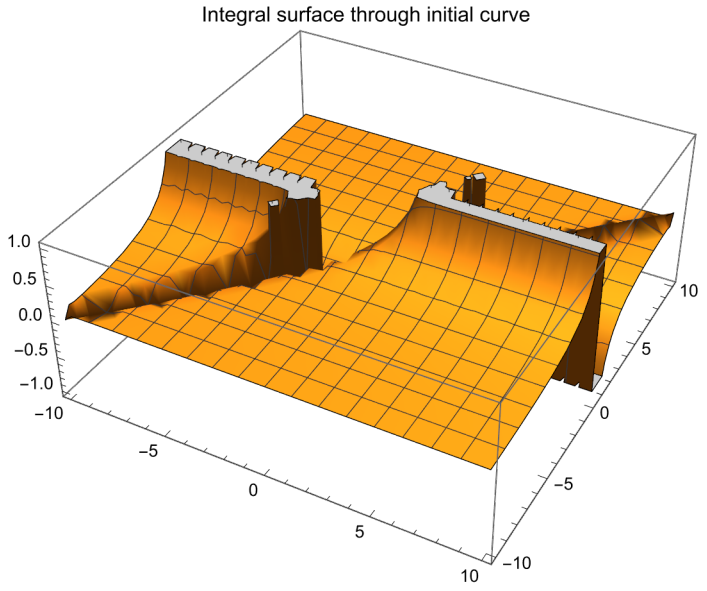
(b)

```
D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]
sol5 =
 DSolve[
  {D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, 0] == Tanh[x]}, u[x, y], {x, y}]
```

$$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2$$

$$\left\{\left\{u[x, y] \to \frac{1}{-y + \text{Coth}[x - y]}\right\}\right\}$$

```
Plot3D[u[x, y] /. sol5, {x, -10, 10}, {y, -10, 10},
 PlotLabel → "Integral surface through initial curve"]
```



QUESTION 4:Obtain the solution of the linear equation u[(x,y),x]+u[(x,y)]=1 with the Cauchy data u(x,2x)=x*x*x

SOLUTION:

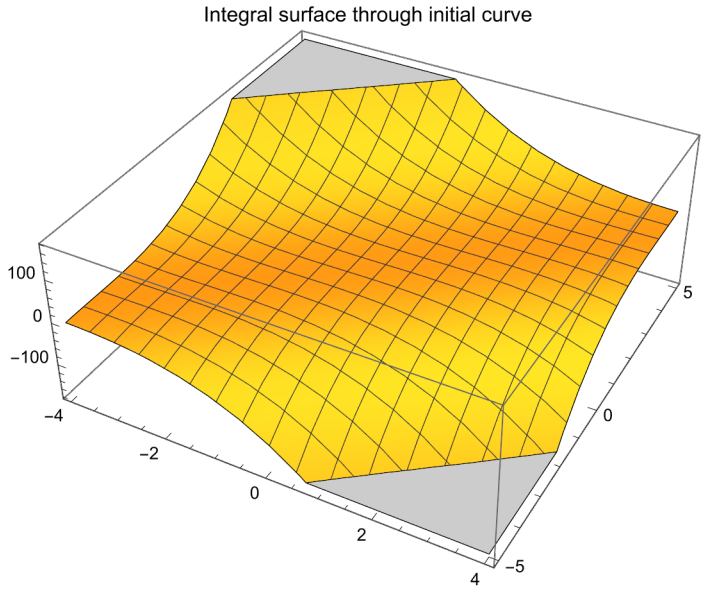
```
D[u[x, y], x] + D[u[x, y], y] == 1
```

$$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$$

```
sol6 =
 DSolve[{D[u[x, y], x] + D[u[x, y], y] == 1, u[x, 2 x] == x * x * x}, u[x, y], {x, y}]
```

$$\left\{\left\{u[x, y] \to 2\,x - x^3 - y + 3\,x^2\,y - 3\,x\,y^2 + y^3\right\}\right\}$$

```
Plot3D[u[x, y] /. sol6, {x, -4, 4}, {y, -5, 5},
 PlotLabel → "Integral surface through initial curve"]
```



QUESTION 5:Obtain the solution of the linear equation u(x+y)*u[(x,y),x]+u(x-y)* u[(x,y),y]=x*x+y*y with the Cauchy data u(x,2x)=0

SOLUTION:

```
u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == x * x + y * y
```

$$(x - y)\,u[x, y]\,u^{(0,1)}[x, y] + (x + y)\,u[x, y]\,u^{(1,0)}[x, y] == x^2 + y^2$$

```
DSolve[
 {u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == (x * x) + (y * y),
  u[x, 2 x] == 0}, u[x, y], {x, y}]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$\left\{\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\ \sqrt{2 x^2 + 3 x y - 2 y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\ \sqrt{2 x^2 + 3 x y - 2 y^2}\right\},\right.$$
$$\left.\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\ \sqrt{2 x^2 + 3 x y - 2 y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\ \sqrt{2 x^2 + 3 x y - 2 y^2}\right\}\right\}$$

```
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5},
 PlotLabel → "Integral surface through initial curve"]
```