JIDHUN PP | BSc(Hons) Computer Science| 20211419|Practical-4

Method of Variation of Parameters

QUESTION 1:Solve second order differential equation y''[x]+y[x]=tan[x] by method of variation of parameter

Solution:

Step 1:Find complementary function

```
eqn := y''[x] + y[x];
f[x_] := Tan[x];
P = DSolve[eqn == 0, y[x], x]
```

```
{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}
```

Step 2:Consider fundamental solution function u(x) and v(x)

```
u[x_] := Cos[x];
v[x_] := Sin[x];
```

Step 3:Find Wronskian W=({u[x],v[x],u'[x],v'[x]})

```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
1
```

Step 4:Find g[x]=(-v[x]f[x]/w and h[x]=(u[x]f[x]/w

```
g[x] := (-v[x] × f[x]) / w
h[x] := (u[x] × f[x]) / w
```

Step 5: Find G=Integrate[g[x],x] and H=Integrate[h[x],x]

```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

$$\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]-\mathrm{Sin}\left[\frac{x}{2}\right]\right]-\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right]+\mathrm{Sin}[x]$$

$$-\mathrm{Cos}[x]$$

Step 6:Find PI=u[x]G+v[x]H

```
PI = u[x] G + v[x] H
```

$$-\mathrm{Cos}[x]\,\mathrm{Sin}[x]+\mathrm{Cos}[x]\left(\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]-\mathrm{Sin}\left[\frac{x}{2}\right]\right]-\mathrm{Log}\left[\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right]+\mathrm{Sin}[x]\right)$$

QUESTION 2:Solve second order differential equation y’’[x]-2y’[x]=e^xSin[x] by method of variation of parameter

Step 1:Find complementary function

```
eqn := y''[x] - 2 y'[x];
f[x_] := e^x * Sin[x];
P = DSolve[eqn == 0, y[x], x]
```

$$\left\{\left\{y[x]\to\frac{1}{2}e^{2x}C[1]+C[2]\right\}\right\}$$

Step 2:Consider fundamental solution function u(x) and v(x)

```
u[x_] := 1 / 2 Exp[2 x]
v[x_] := 1
```

Step 3:Find Wronskian W=({u[x],v[x],u’[x],v’[x]})

```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```

$$-e^{2x}$$

Step 4:Find g[x]=(-v[x]f[x]/w and h[x]=(u[x]f[x]/w

```
g[x] := (-v[x] × f[x]) / w
h[x] := (u[x] × f[x]) / w
```

Step 5: Find G=Integrate[g[x],x] and H=Integrate[h[x],x]

```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

$$\frac{e^x\, e^{-2x}\, (-\text{Cos}[x] + (-2 + \text{Log}[e])\, \text{Sin}[x])}{5 - 4\, \text{Log}[e] + \text{Log}[e]^2}$$

$$\frac{e^x\, (\text{Cos}[x] - \text{Log}[e]\, \text{Sin}[x])}{2\, \left(1 + \text{Log}[e]^2\right)}$$

Step 6:Find PI=u[x]G+v[x]H

```
PI = u[x] G + v[x] H
```

$$\frac{e^x\, (-\text{Cos}[x] + (-2 + \text{Log}[e])\, \text{Sin}[x])}{2\, \left(5 - 4\, \text{Log}[e] + \text{Log}[e]^2\right)} + \frac{e^x\, (\text{Cos}[x] - \text{Log}[e]\, \text{Sin}[x])}{2\, \left(1 + \text{Log}[e]^2\right)}$$

QUESTION 3:Solve second order differential equation y’’[x]-2y’[x]+y[x]=e^xSin[x] by method of variation of parameter

Step 1:Find complementary function

```
eqn := y''[x] - 2 y'[x] + y[x];
f[x_] := e^x * Sin[x];
P = DSolve[eqn == 0, y[x], x]
```

$$\{\{y[x] \to e^x\, C[1] + e^x\, x\, C[2]\}\}$$

Step 2:Consider fundamental solution function u(x) and v(x)

```
u[x_] := Exp[x]
v[x_] := x * Exp[x]
```

Step 3:Find Wronskian W=({u[x],v[x],u’[x],v’[x]})

```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```

$$e^{2x}$$

Step 4:Find g[x]=(-v[x]f[x]/w and h[x]=(u[x]f[x]/w

```
g[x] := (-v[x] × f[x]) / w
h[x] := (u[x] × f[x]) / w
```

Step 5: Find G=Integrate[g[x],x] and H=Integrate[h[x],x]

```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

$$-\left(\left(e^{x}\, e^{-x}\left(-\mathrm{Cos}[x]\left(2\ (1+x)-2\ (1+x)\ \mathrm{Log}[e]+x\,\mathrm{Log}[e]^{2}\right)+\left(-2\,x+(2+4\,x)\ \mathrm{Log}[e]-(1+3\,x)\ \mathrm{Log}[e]^{2}+x\,\mathrm{Log}[e]^{3}\right)\mathrm{Sin}[x]\right)\right)\Big/\left(2-2\,\mathrm{Log}[e]+\mathrm{Log}[e]^{2}\right)^{2}\right)$$

$$-\frac{e^{x}\, e^{-x}\ (\mathrm{Cos}[x]+\mathrm{Sin}[x]-\mathrm{Log}[e]\ \mathrm{Sin}[x])}{2-2\,\mathrm{Log}[e]+\mathrm{Log}[e]^{2}}$$

Step 6:Find PI=u[x]G+v[x]H

```
PI = u[x] G + v[x] H
```

$$-e^{2\,x}\int\frac{e^{x}\, e^{-x}\,\mathrm{Sin}[x]}{-2+e^{2\,x}\ (1+x)}\,dx+\int\frac{e^{x}\, e^{x}\,\mathrm{Sin}[x]}{-2+e^{2\,x}\ (1+x)}\,dx$$

QUESTION 4:Solve second order differential equation y’’[x]-+y[x]=e^x by method of variation of parameter

Step 1:Find complementary function

```
eqn := y''[x] - y[x];
f[x_] := e^x * Sin[x];
P = DSolve[eqn == 0, y[x], x]
```

$$\{\{y[x]\to e^{x}\,C[1]+e^{-x}\,C[2]\}\}$$

Step 2: Consider fundamental solution function u(x) and v(x)

```
u[x_] := Exp[x]
v[x_] := Exp[-x]
```

Step 3:Find Wronskian W=({u[x],v[x],u’[x],v’[x]})

```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```

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Step 4:Find g[x]=(-v[x]f[x]/w and h[x]=(u[x]f[x]/w

```
g[x] := (-v[x] × f[x]) / w
h[x] := (u[x] × f[x]) / w
```

Step 5: Find G=Integrate[g[x],x] and H=Integrate[h[x],x]

```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

$$-\frac{e^{x}\, e^{-x}\, (\text{Cos}[x] + \text{Sin}[x] - \text{Log}[e]\, \text{Sin}[x])}{2\left(2 - 2\,\text{Log}[e] + \text{Log}[e]^{2}\right)}$$

$$\frac{e^{x}\, e^{x}\, (\text{Cos}[x] - (1 + \text{Log}[e])\, \text{Sin}[x])}{2\left(2 + 2\,\text{Log}[e] + \text{Log}[e]^{2}\right)}$$

Step 6:Find PI=u[x]G+v[x]H

```
PI = u[x] G + v[x] H
```

$$-\frac{e^{x}\, (\text{Cos}[x] + \text{Sin}[x] - \text{Log}[e]\, \text{Sin}[x])}{2\left(2 - 2\,\text{Log}[e] + \text{Log}[e]^{2}\right)} + \frac{e^{x}\, (\text{Cos}[x] - (1 + \text{Log}[e])\, \text{Sin}[x])}{2\left(2 + 2\,\text{Log}[e] + \text{Log}[e]^{2}\right)}$$