
JIDHUN PP | BSc(Hons) Computer Science|

20211419|Practical-7

Find the Characteristics for the first order PDE and Plotting them

Example 1: Find the characteristics of the equation $(u-y)u_x + yu_y = x+y$ and plot them

Solution:

The characteristics system is

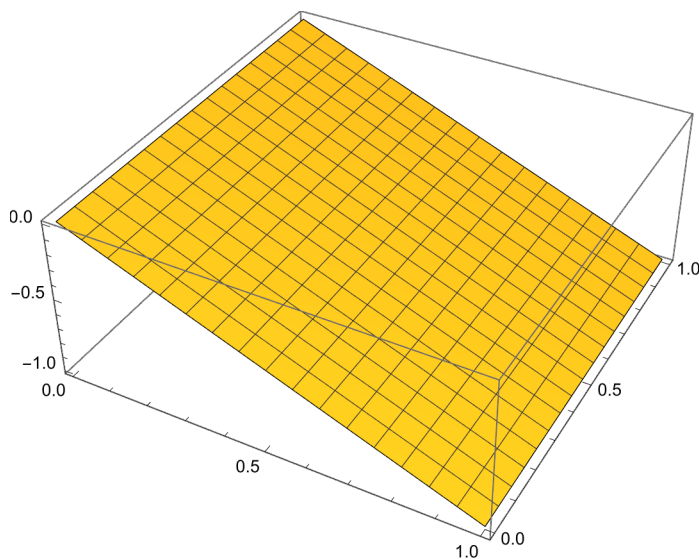
$$dx/(u-y) = dy/y = du/(x+y)$$

using (i)+(ii)+(iii), we have $v=(u+x)/y=c_1$, is a first integral

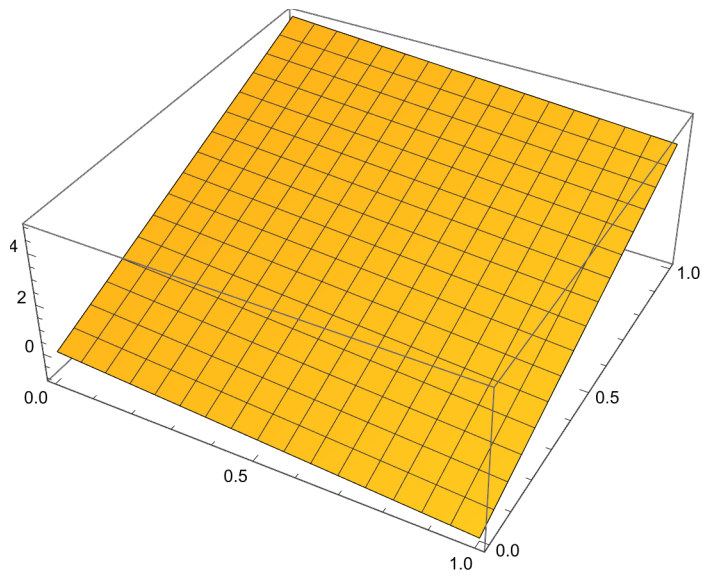
using (i)+(ii)=(iii), we have $w=(x+y)^2 - u^2 = c_2$

is a second integral

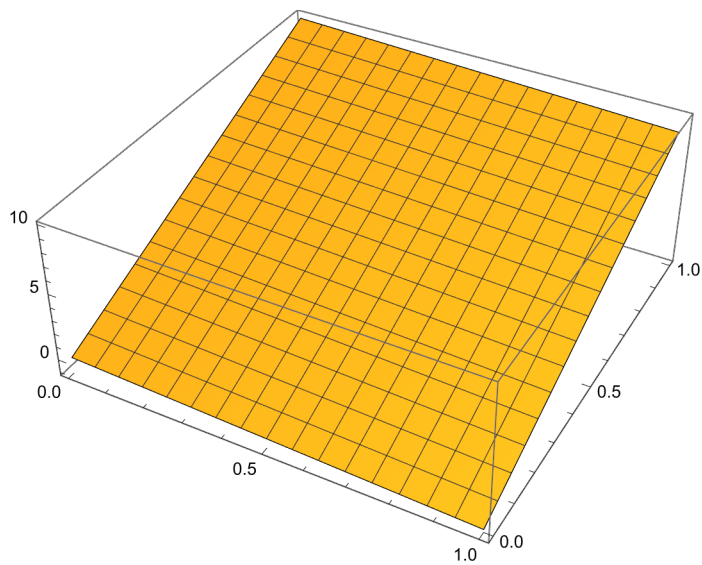
```
f0 = Plot3D[-x, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10]
```



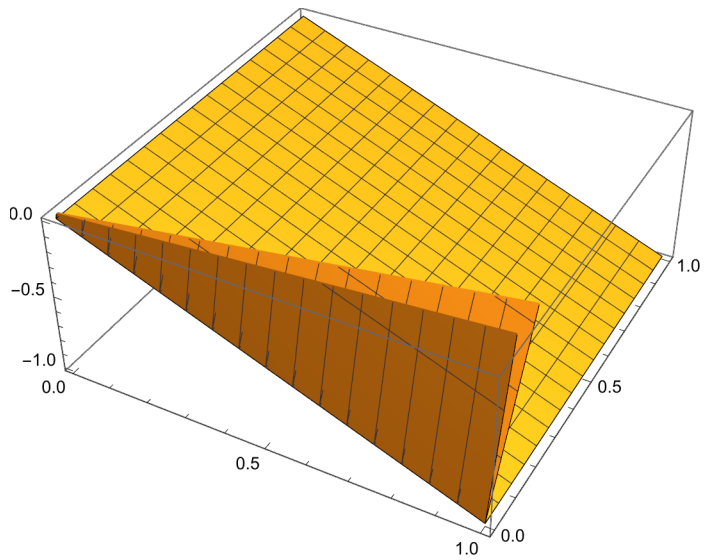
```
f1 = Plot3D[5 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10]
```



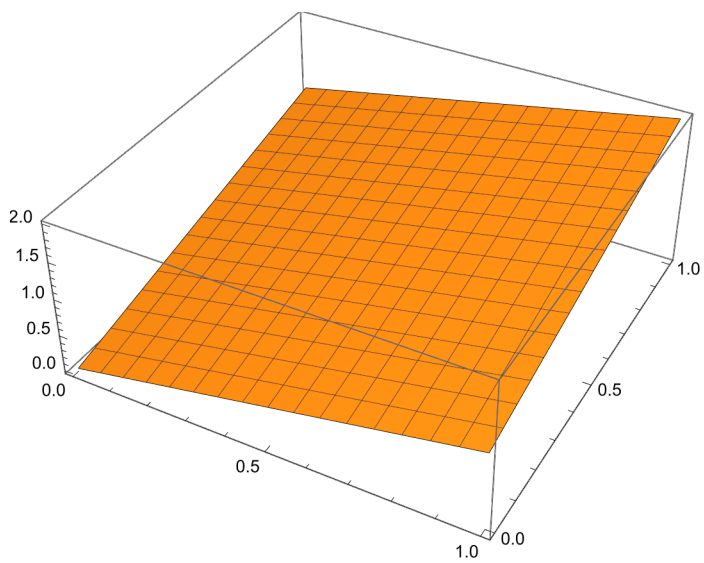
```
f2 = Plot3D[10 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10]
```



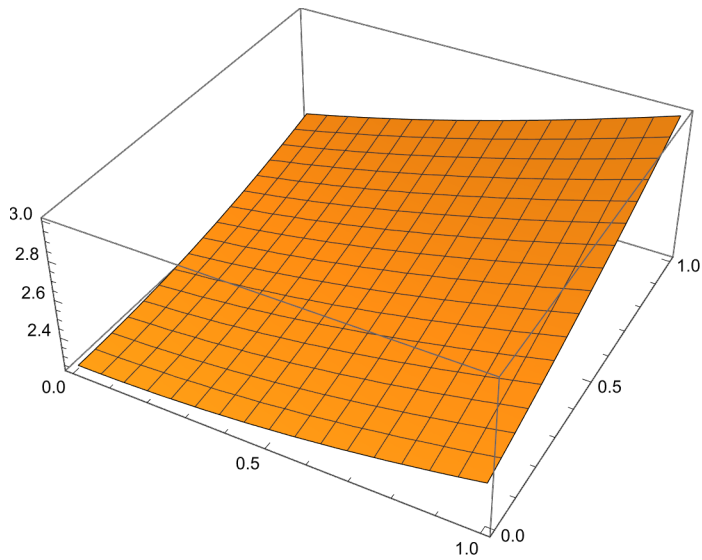
```
g1 = Show[f0, f1, f2]
```



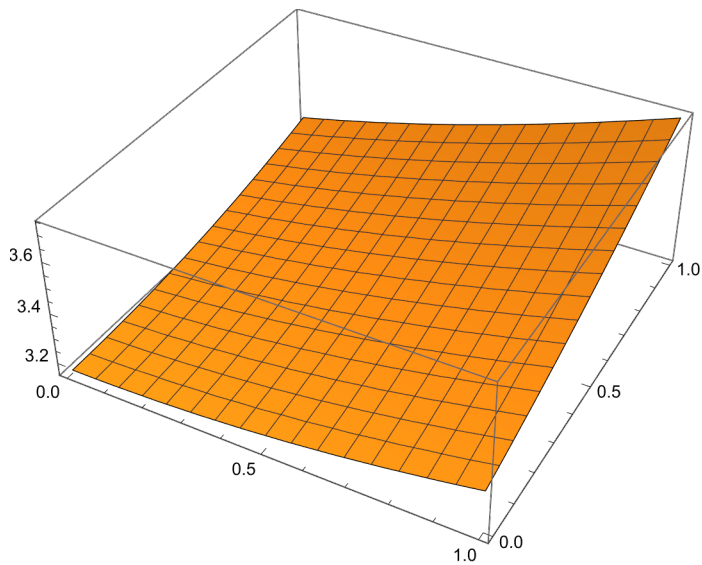
```
h0 = Plot3D[x + y, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10]
```



```
h1 = Plot3D[Sqrt[(x + y)^2 + 5], {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10]
```

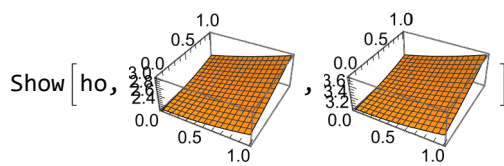
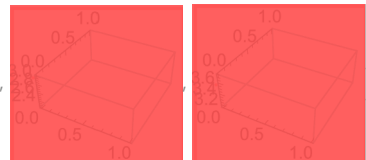


```
h2 = Plot3D[Sqrt[(x + y)^2 + 10], {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10]
```



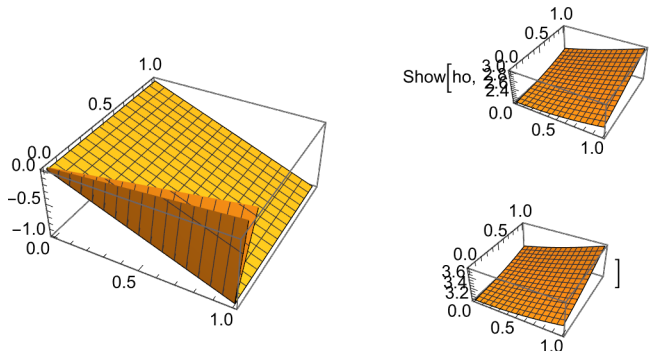
```
g2 = Show[ho, h1, h2]
```

 **Show:** Could not combine the graphics objects in Show[ho,



```
Show[GraphicsArray[{g1, g2}]]
```

⚠ **GraphicsArray**: GraphicsArray is obsolete. Switching to GraphicsGrid.



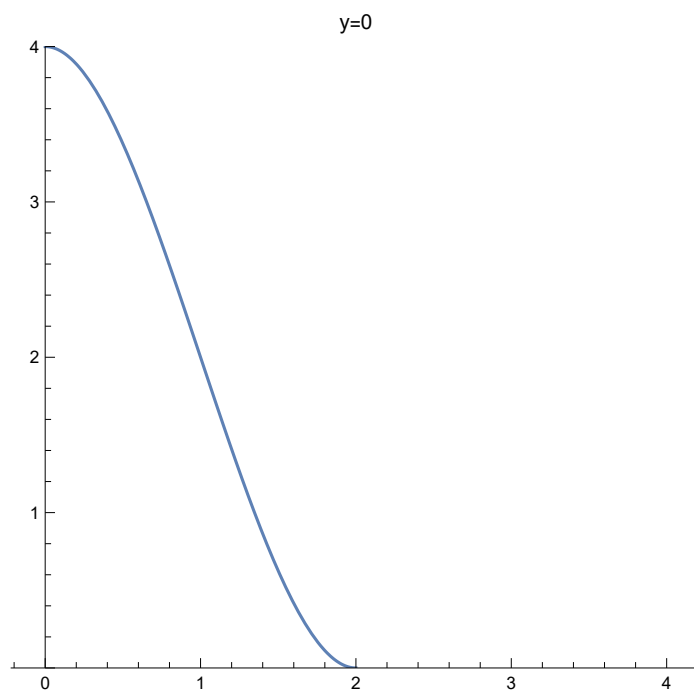
Example 2: The solution of the equation $u[(x,y),y] + u[x,y] \cdot u[(x,y),x] = 0$ can be interpreted as a vector field on the x axis varying with time y . Find the integral satisfying the initial condition $u(s,0) = h(s)$, where h is a given function
Solution:

We plot the curves

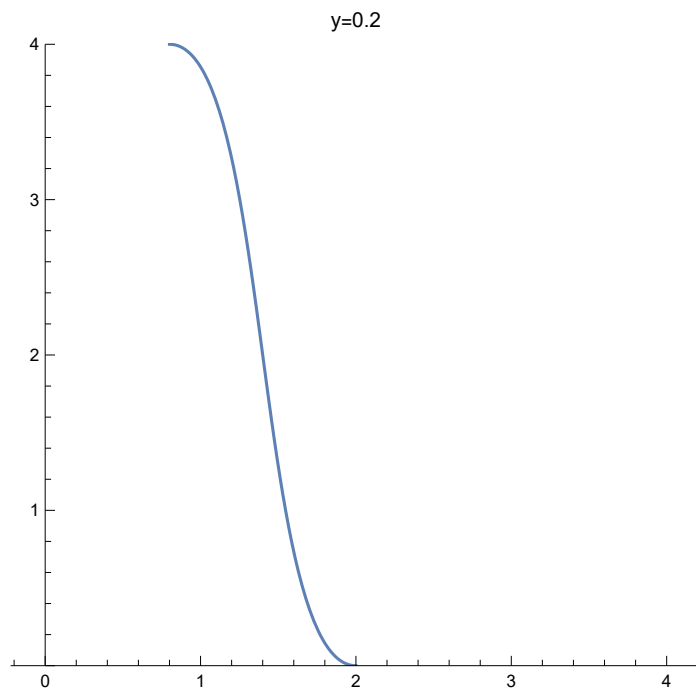
$$\{Ct: x = s + t(s^3 - 3s^2 + 4)\}$$

$$u = s^3 - 3s^2 + 4$$

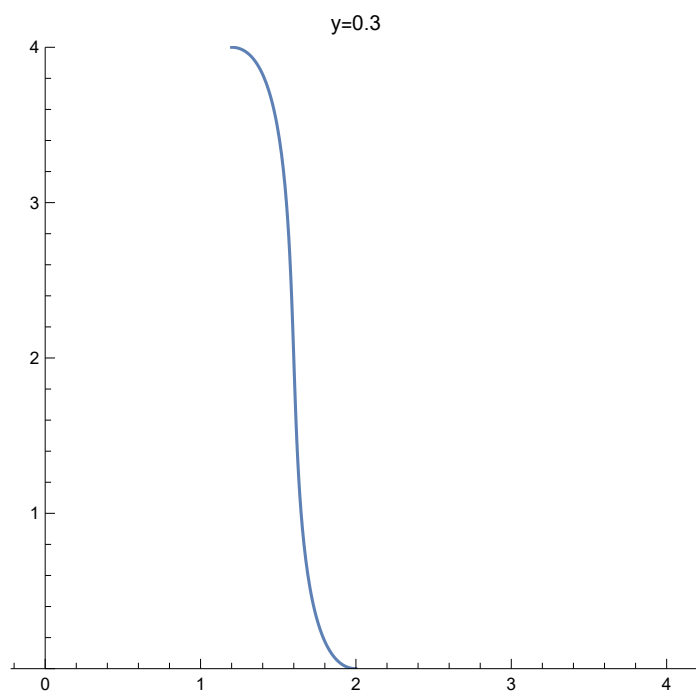
```
u[s_] := s^3 - 3 s^2 + 4;
x[s_, t_] := s + t * u[s];
ho = ParametricPlot[{x[s, 0], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0"]
```



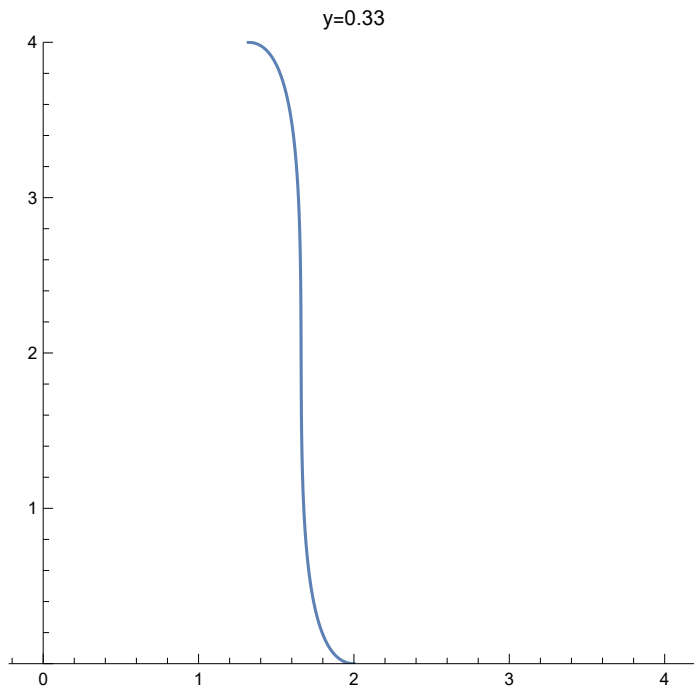
```
h1 = ParametricPlot[{x[s, 0.2], u[s]},  
  {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.2"]
```



```
h2 = ParametricPlot[{x[s, 0.3], u[s]},  
  {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.3"]
```



```
h3 = ParametricPlot[{x[s, 0.33], u[s]},
  {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.33"]
```



```
h4 = ParametricPlot[{x[s, 0.333], u[s]},
  {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.333"]
```

