Digital Signal Processing

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Submitted by

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1 Design of a Simple FIR Filter :-

1.1 Illustrate the use of zeros in filter design :-

- Aim :

  1. Design an FIR filter with two complex conjugate zeros on the unit circle at $z_1 = e^{j\theta}$ and $z_2 = e^{-j\theta}$. Understand how the placement of these zeros affects the filter's frequency response.

  2. Derive the transfer function for the filter is given by:

  $$H_f(z) = 1 - 2\cos(\theta)z^{-1} + z^{-2}$$

  3. Analyze and explain how the value of θ affects the magnitude of the filter's frequency response. Discuss the implications of zeros at $e^{\pm j\theta}$ and how they correspond to the frequencies that are filtered out.

- Difference Equation :

  To derive the difference equation, we can express the transfer function as:

  $$H_f(z) = \frac{Y(z)}{X(z)}$$

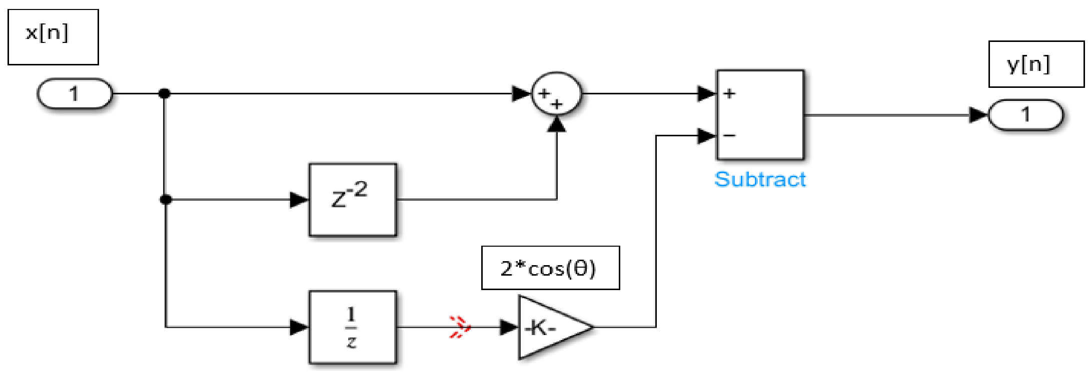
  where $Y(z)$ is the output and $X(z)$ is the input. Rearranging gives:

  $$Y(z) = H_f(z) \cdot X(z) = (1 - 2\cos\theta\, z^{-1} + z^{-2})X(z)$$

  In the time domain, this corresponds to the difference equation:

  $$y[n] = x[n] - 2\cos\theta\, x[n-1] + x[n-2]$$

- System Diagram :



- Impulse Response :

  The impulse response $h[n]$ can be computed from the transfer function as follows:

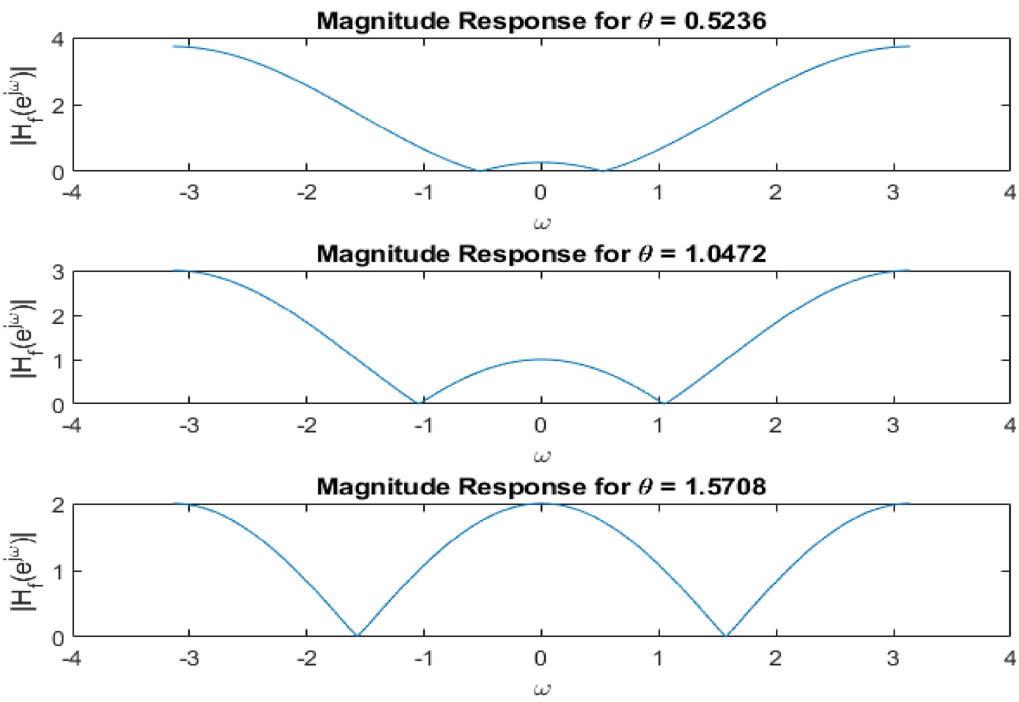
$$h[n] = \mathcal{Z}^{-1}\{H_f(z)\}$$

  Substituting $H_f(z)$:

$$h[n] = \delta[n] - 2\cos\theta\,\delta[n-1] + \delta[n-2]$$

  Thus, the impulse response is given by:

$$h[n] = \begin{cases} 1 & n = 0 \\ -2\cos\theta & n = 1 \\ 1 & n = 2 \\ 0 & \text{otherwise} \end{cases}$$



- Observation :

  The value of θ determines the frequency at which the filter's response becomes zero. For example, when $\theta = \frac{\pi}{6}$, the filter will not pass sine waves at $\omega = \frac{\pi}{6}$, effectively filtering out that frequency.

  As θ changes, the location of the zeros on the unit circle changes, shifting the notch (attenuation point) in the frequency response. The filter can be used to remove or attenuate specific frequencies in a signal, making it useful for applications such as noise reduction or interference removal.

1.2 Remove an undesirable sinusoidal interference from a speech signal :-

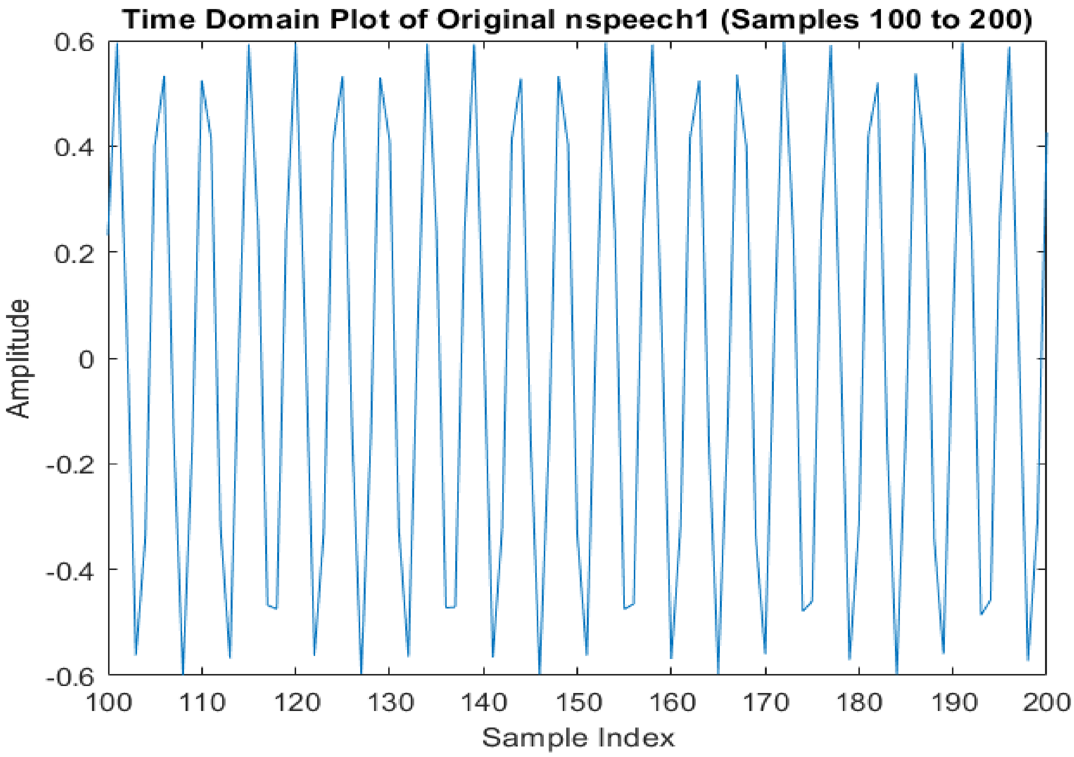
- Aim :-

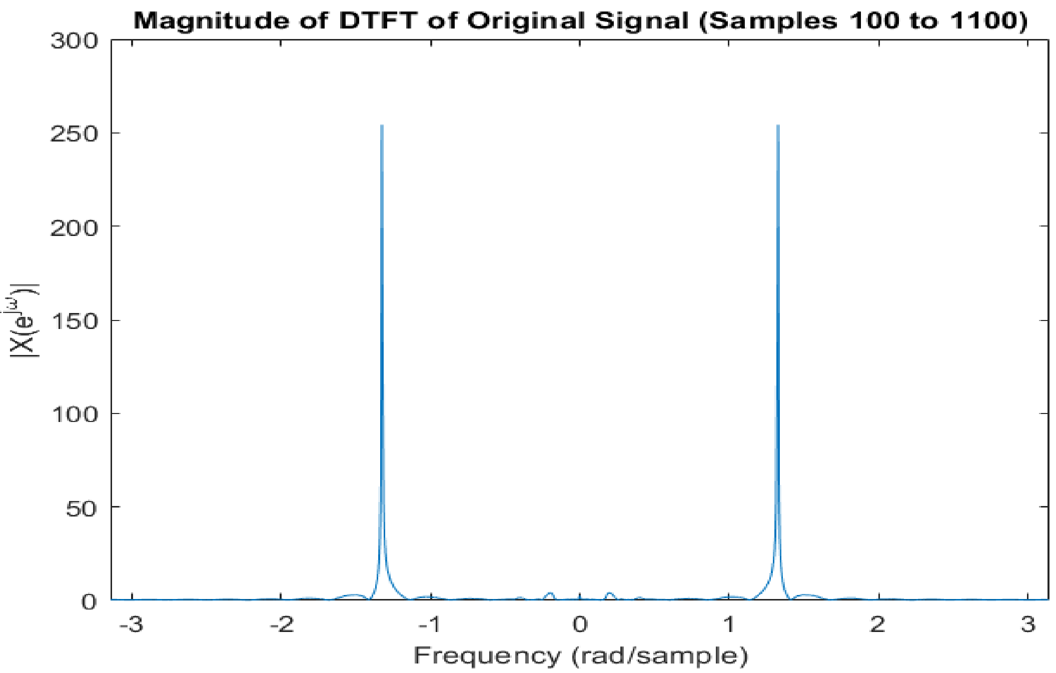
  1. To apply the filter to a speech signal (nspeech1) and analyze the effectiveness of the filtering process.

  2. Apply the FIRfilter function to the nspeech1 signal to attenuate the sinusoidal interference.

  3. Determine whether the FIR filter acts as a lowpass, highpass, bandpass, or bandstop filter based on its effect on the frequency spectrum.

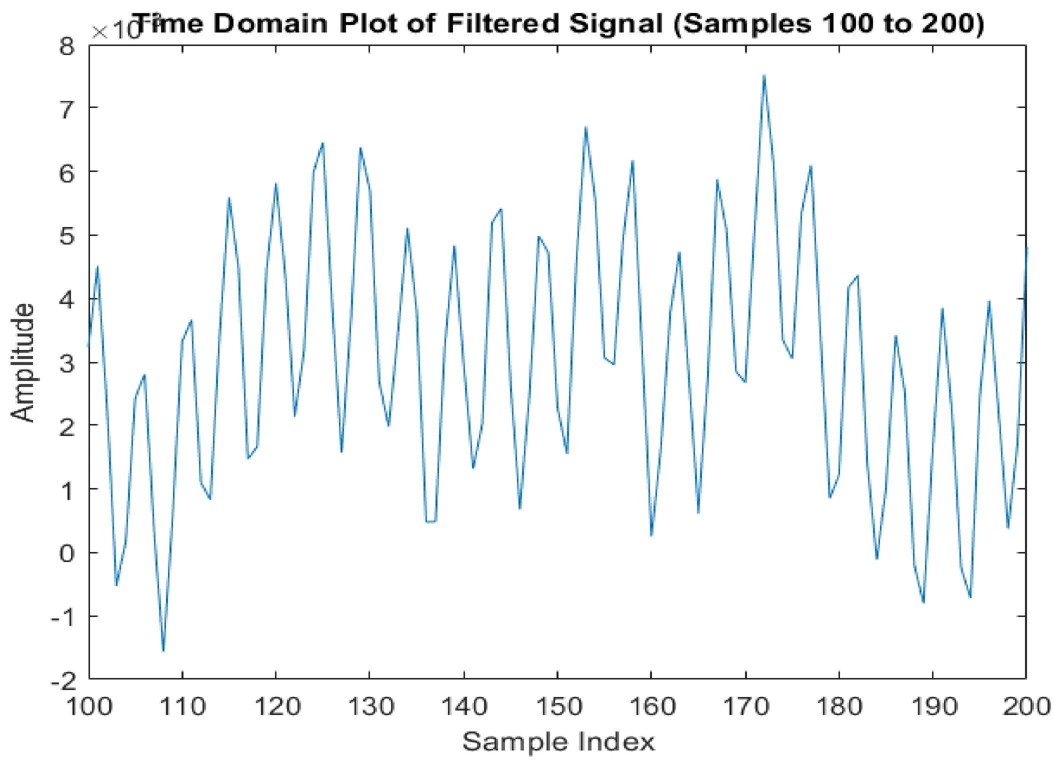
- Original Siganl Plots :

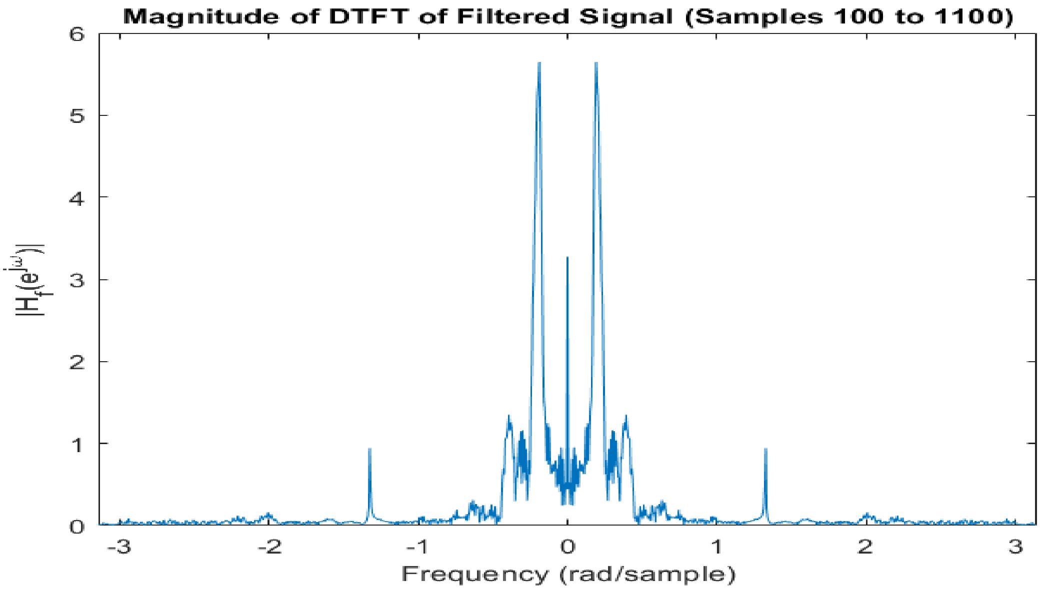




- We observe from the DTFT magnitude plot that the peaks occur at $\theta = \frac{\pi}{2.38}$. Hence we filter out component containing θ from the original signal.

- Filtered Signal Plots :





- Observation :

1. Frequency Content Change: After filtering, the peaks corresponding to the sinusoidal interference should be attenuated. The FIR filter designed here acts as a bandstop (notch) filter, removing or significantly reducing the amplitude at frequencies close to θ while allowing other frequencies to pass.

2. Filter Type: The FIR filter is a bandstop filter because it attenuates a specific frequency component the interference frequency while keeping other frequencies intact.

3. Quality of the Audio Signal: The filtered audio signal should have reduced interference, making the speech clearer. However, if r is too close to 1 (e.g., 0.9999999), the filter becomes extremely selective, which may lead to numerical instability and ringing effects in the time domain.

2 Design of a Simple IIR Filter :-

2.1 Illustrate use of poles to separate a narrowband signal from adjacent noise :-

- Aim :

1. To design and analyze a second-order IIR filter characterized by two complex-conjugate poles.

2. To study how the location of poles (defined by their distance r from the origin and angle affects the frequency response of the filter.

- Difference Equation :

The transfer function for the IIR filter is given by:

$$H_i(z) = \frac{1-r}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}$$

From this, we can derive the difference equation by multiplying through by the denominator:

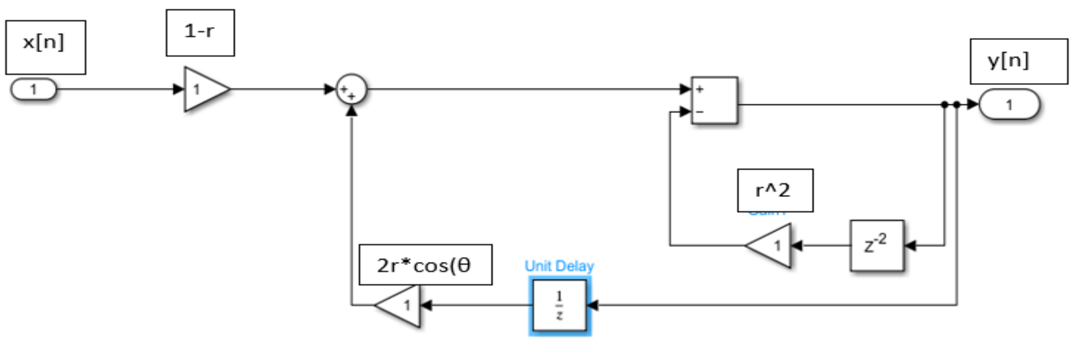
$$(1-2r\cos(\theta)z^{-1}+r^2z^{-2})Y(z) = (1-r)X(z)$$

Taking the inverse Z-transform, we obtain the time-domain difference equation:

$$y[n] = 2r\cos(\theta)y[n-1] - r^2y[n-2] + (1-r)x[n]$$

where $y[n]$ is the output and $x[n]$ is the input signal.

- System Diagram :



- Impulse Response :

Given the transfer function:

$$H_i(z) = \frac{1-r}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}$$

We need to determine the impulse response $h_i[n]$.

Step 1: Inverse Z-Transform To find $h_i[n]$, we take the inverse Z-transform of $H_i(z)$. The inverse Z-transform of a rational function can be found by expanding $H_i(z)$ in terms of its power series.

The recursive form of the difference equation from $H_i(z)$ is:

$$y[n] = 2r\cos(\theta)y[n-1] - r^2y[n-2] + (1-r)\delta[n]$$

Step 2: Initial Conditions Assume $y[n] = 0$ for $n < 0$ (causality).

Step 3: Solving for the Impulse Response To find $h_i[n]$, we can use the homogeneous and particular solutions approach:

$$h_i[n] = A_1\lambda_1^n + A_2\lambda_2^n$$

where λ_1 and λ_2 are the roots of the characteristic equation:

$$\lambda^2 - 2r\cos(\theta)\lambda + r^2 = 0$$

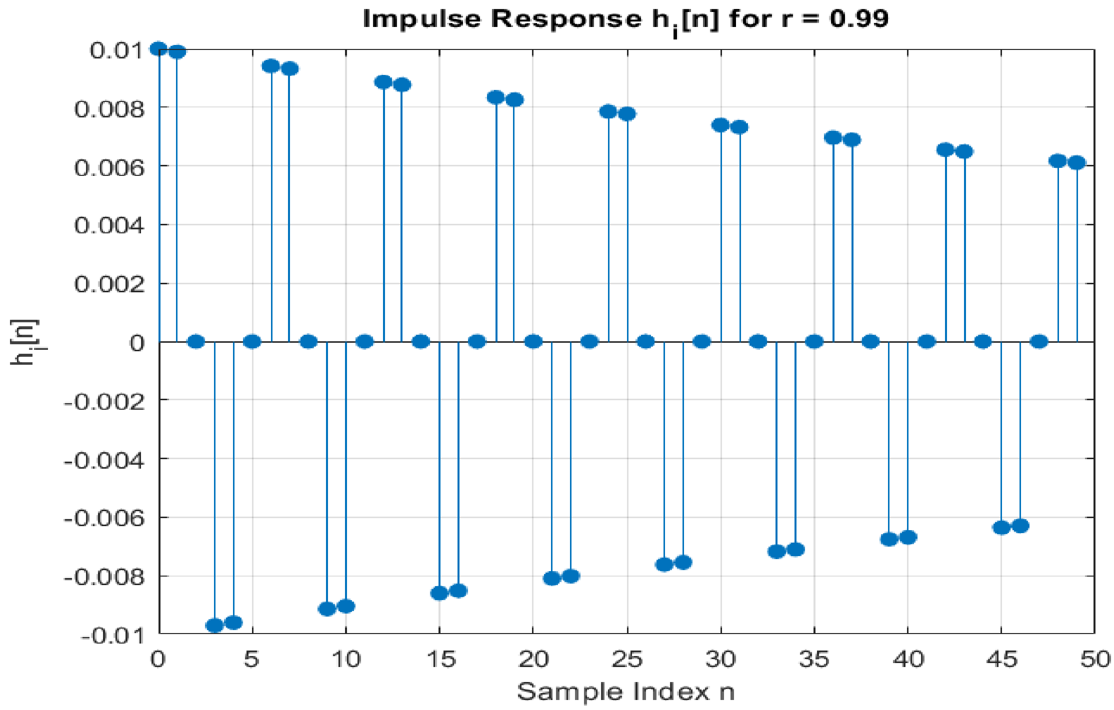
Solving for λ, we get:

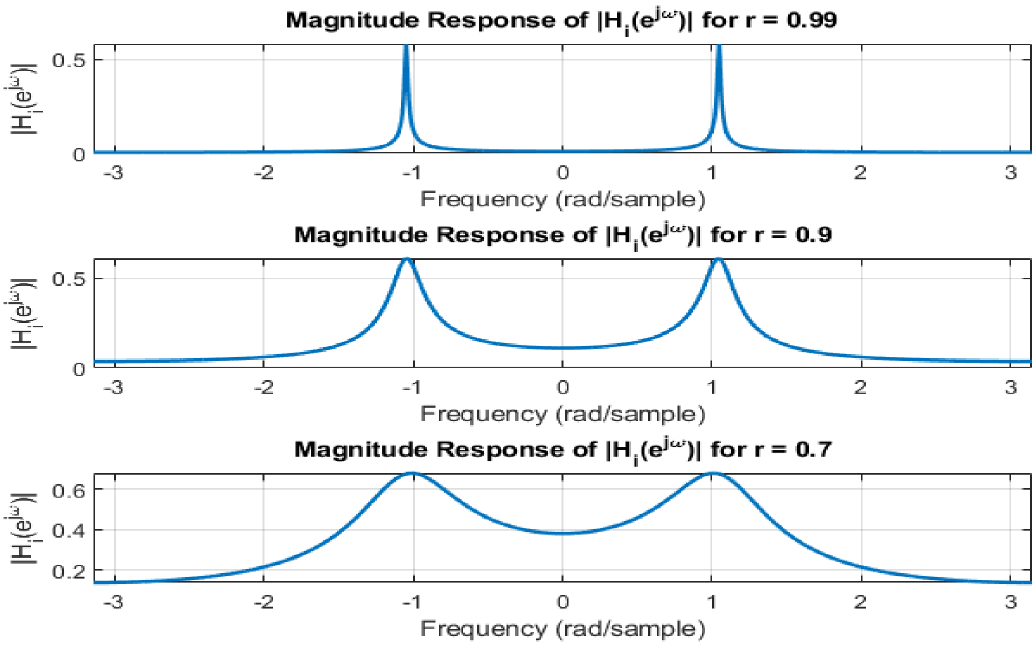
$$\lambda_{1,2} = re^{\pm j\theta}$$

Step 4: Final Impulse Response Expression Thus, the impulse response $h_i[n]$ can be expressed as:

$$h_i[n] = (1-r)r^n\cos(\theta n)u[n]$$

where $u[n]$ is the unit step function.





- Observation :

  1. Higher r (e.g., 0.99): Results in a narrower, sharper peak with a higher magnitude at the center frequency. The filter becomes more selective.

  2. Lower r (e.g., 0.7): Results in a wider, flatter peak with lower gain, making the filter less selective and covering a broader range of frequencies.

  3. r very close to 1 (e.g., 0.999): Can provide very sharp selectivity but risks numerical instability or excessive sensitivity to small perturbations in the signal.

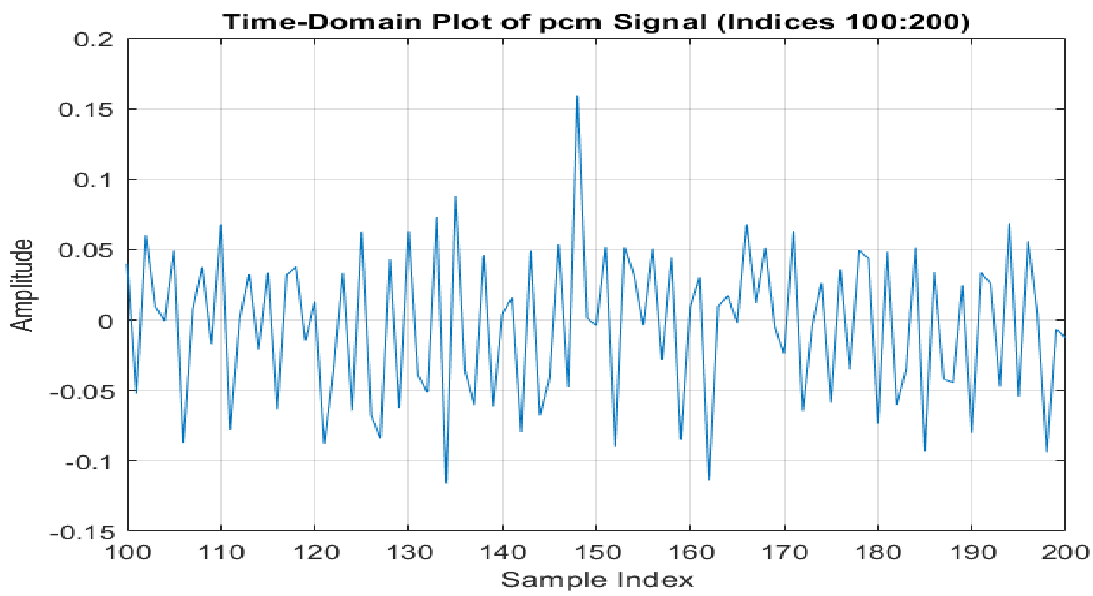
2.2 Separate a modulated sinusoid from background noise :-

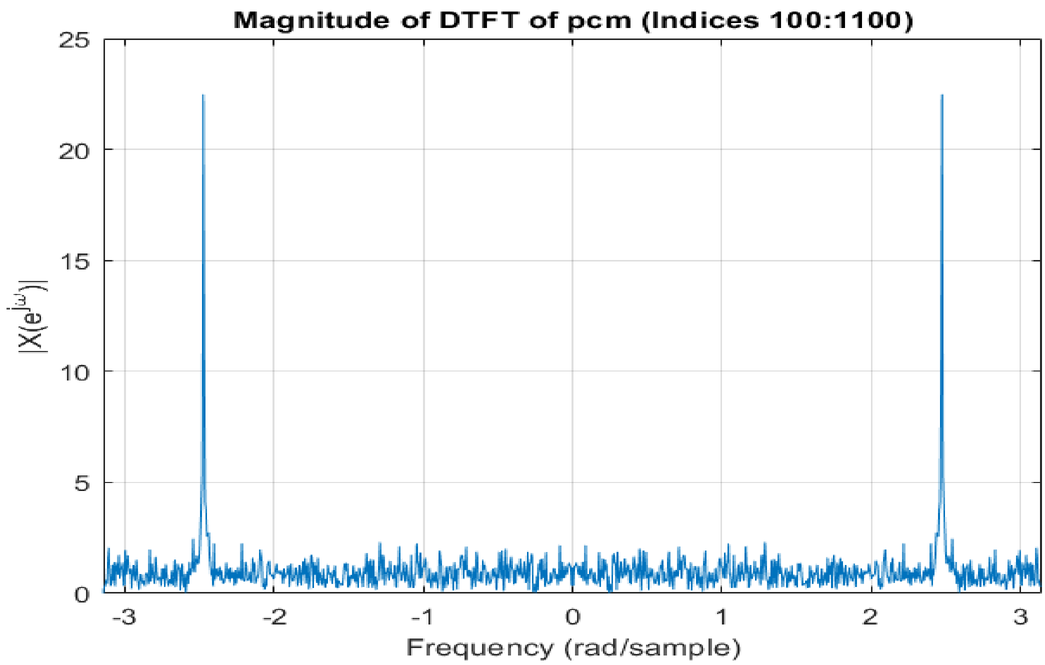
- Aim :

  1. To design and implement a second-order IIR filter using poles to amplify a modulated sinusoidal signal from background noise. The experiment will involve analyzing the frequency response of the filter, implementing it in MATLAB, and applying it to an audio signal.

  2. Apply the IIRfilter function to the pcm signal to enhance the desired frequency component relative to background noise.

- Original Signal Plots :





- Calculation of θ :

Modulation frequency: $f = 3146$ Hz Sampling frequency: $f_s = 8000$ Hz The normalized angular frequency ω can be calculated using:

$$\omega = 2\pi \frac{f}{f_s}$$

Substitute the values:

$$\omega = 2\pi \frac{3146}{8000}$$

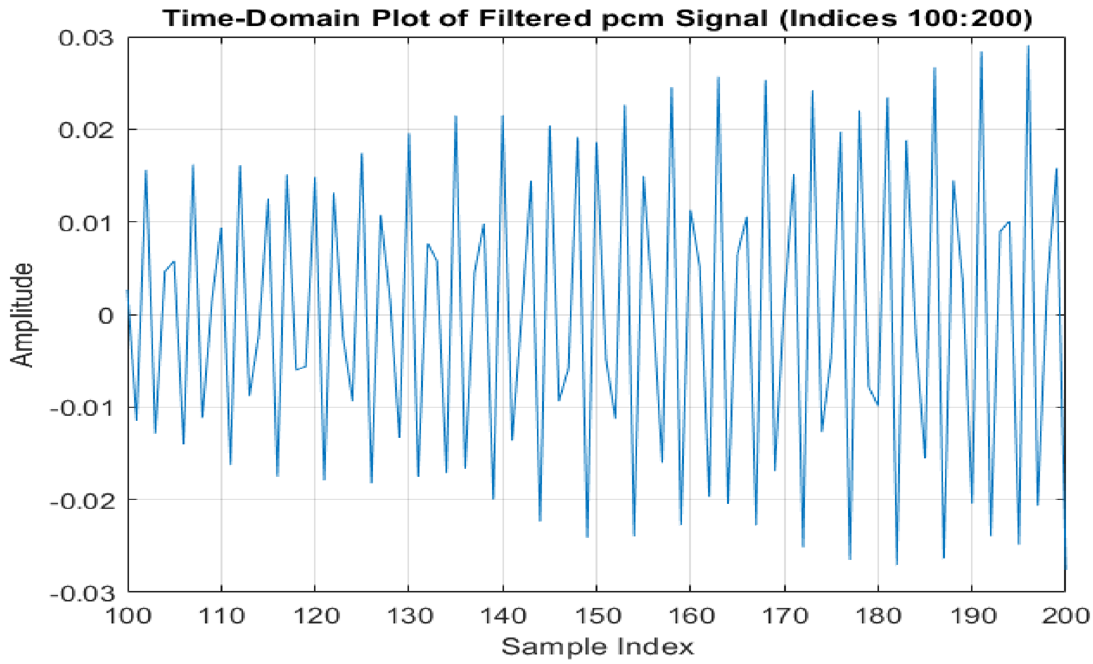
Simplify the expression:

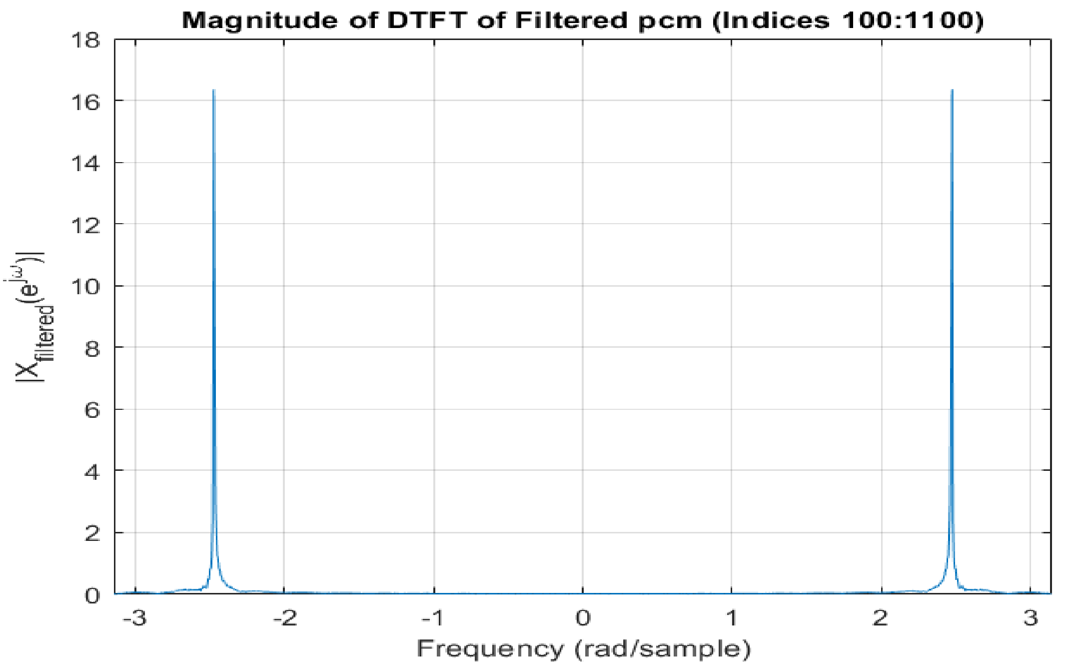
$$\omega = 2\pi \times 0.39325 \approx 2.47 \text{ radians}$$

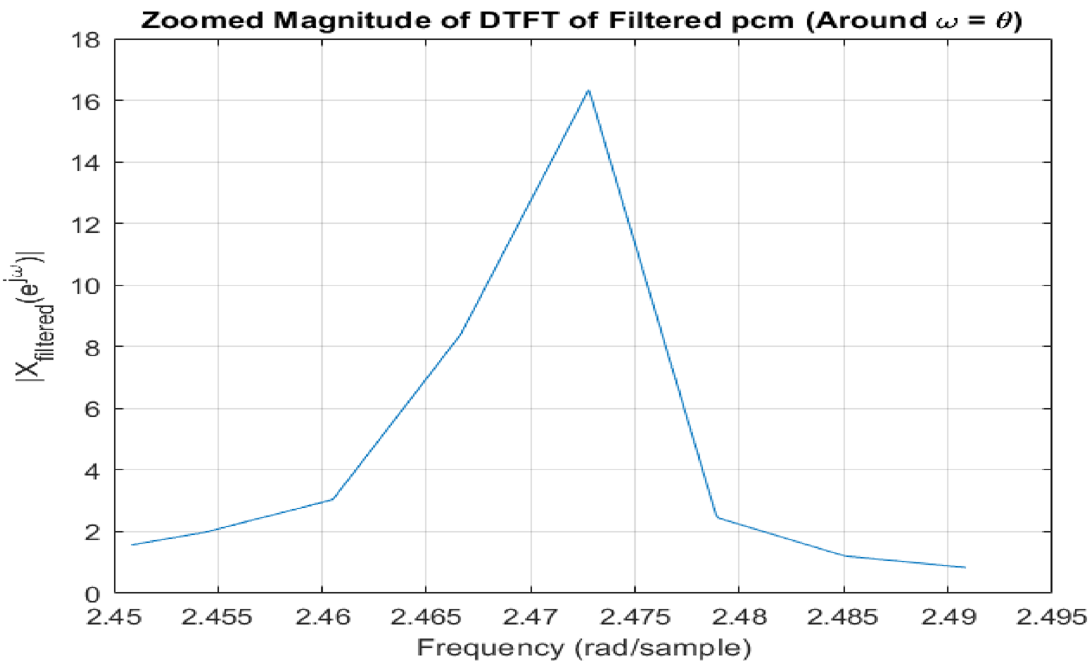
Since θ for the filter is equivalent to this angular frequency:

$$\theta = 2.47 \text{ radians}$$

- Filtered Signal Plots :







- Observation :

1. As r approaches 1, the poles of the filter get closer to the unit circle. This results in a narrower bandwidth around the center frequency θ. Consequently, the filter becomes more selective, allowing only frequencies very close to θ to pass through while attenuating frequencies further away. This can enhance the desired signal if it is centered around θ.

2. While r = 0.9999999 would theoretically provide a very narrow passband and significant amplification at the desired frequency, it risks introducing instability into the system. Such a high value could lead to an output that fluctuates wildly or diverges due to feedback, especially if the input signal has any noise components.

3 Lowpass Filter Design Parameters :-

3.1 Filter Design Using Truncation :-

- Aim :

1. To design a digital filter (low-pass, high-pass, or band-pass) that effectively approximates an ideal filter while adhering to specified performance criteria.

2. To analyze the passband and stopband characteristics of a real filter, identifying the cutoff frequency (w_c), passband (w_p), stopband (w_s), and transition band.

3. To implement a low-pass filter using the truncation method by deriving and truncating the impulse response to create a finite-duration, realizable filter.

4. To investigate how varying the filter size (N = 21 and N = 101) influences the stopband ripple and the overall filter performance.

- Function :

  The frequencies $|\omega| < \omega_p$ are known as the passband, and the frequencies $\omega_s < |\omega| \leq \pi$ are the stopband.

$$|H(e^{j\omega}) - 1| \leq \delta_p \quad \text{for } |\omega| < \omega_p$$
$$|H(e^{j\omega})| \leq \delta_s \quad \text{for } \omega_s < |\omega| \leq \pi$$

  where δ_p and δ_s are known as the passband and stopband ripple respectively.
  Ideally, a low-pass filter with cutoff frequency ω_c should have a frequency response of :

$$H_{\text{ideal}}(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \omega_c < |\omega| \leq \pi \end{cases}$$

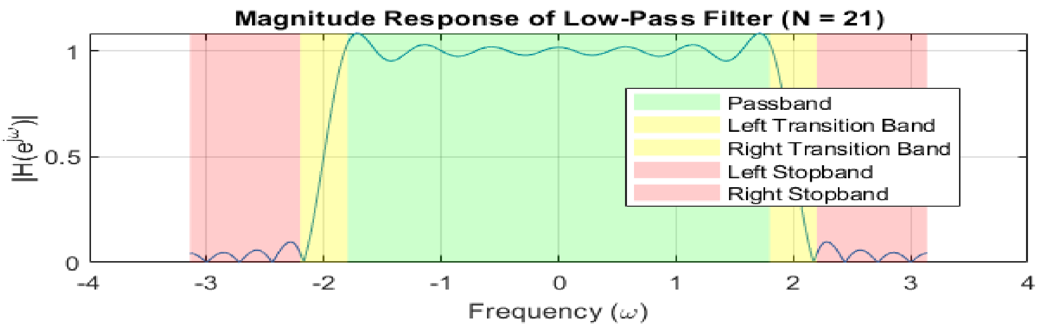
  and a corresponding impulse response of :

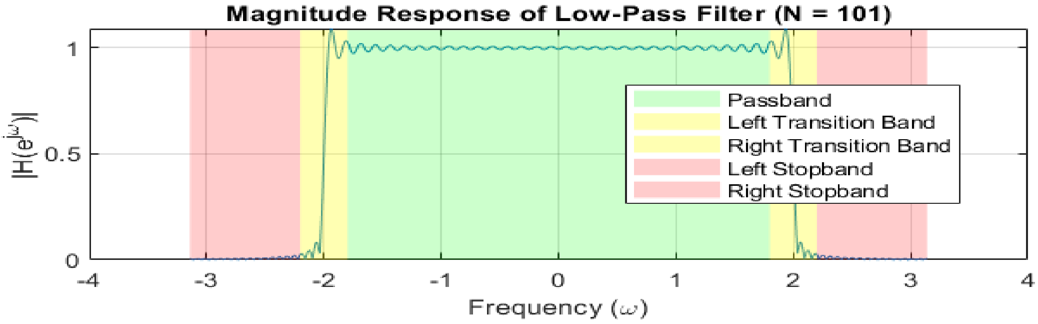
$$h_{\text{ideal}}(n) = \frac{\omega_c}{\pi}\text{sinc}\left(\frac{\omega_c n}{\pi}\right) \quad \text{for } -\infty < n < \infty$$

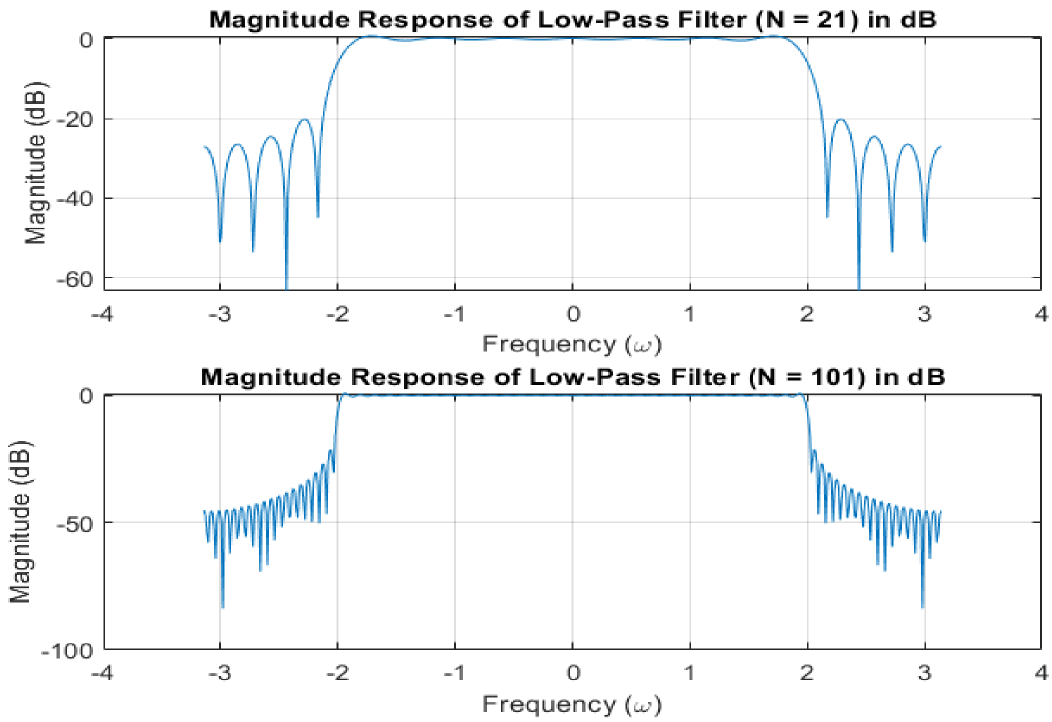
$$h_{\text{trunc}}(n) = \begin{cases} \frac{\omega_c}{\pi}\text{sinc}\left(\frac{\omega_c}{\pi}n\right) & n = -M, \ldots, -1, 0, 1, \ldots, M \\ 0 & \text{otherwise} \end{cases}$$

$$h(n) = \begin{cases} \frac{\omega_c}{\pi}\text{sinc}\left(\frac{\omega_c}{\pi}\left(n - \frac{N-1}{2}\right)\right) & n = 0, 1, \ldots, N-1 \\ 0 & \text{otherwise} \end{cases}$$

- Magnitude Plots of Filter :







- Observation :

1. As the filter size N increases, the stopband ripple decreases. This is because larger filter sizes lead to a more precise approximation of the ideal frequency response, resulting in reduced sidelobes in the DTFT.

2. A larger N allows for more coefficients in the impulse response, improving the filter's ability to attenuate unwanted frequencies while preserving the desired signal in the passband.

3. The filtered signals sounded clearer than the original signal, with a noticeable reduction in background noise.

4. The filter with N=101 provided a more substantial noise reduction compared to N=21. Listeners may perceive better clarity and quality in the speech signal filtered with the larger filter size.