Control and Electronic System Design

17th February 2025

Submitted by

Jidnyesh Chaudhari

3rd Year Undergraduate Student

Department of Electrical Engineering
Indian Institute of Technology Kharagpur

1 Magnetic Ball Suspension System Modeling and Controller Design :-

1.1 Schematic Diagram & Parameters :-

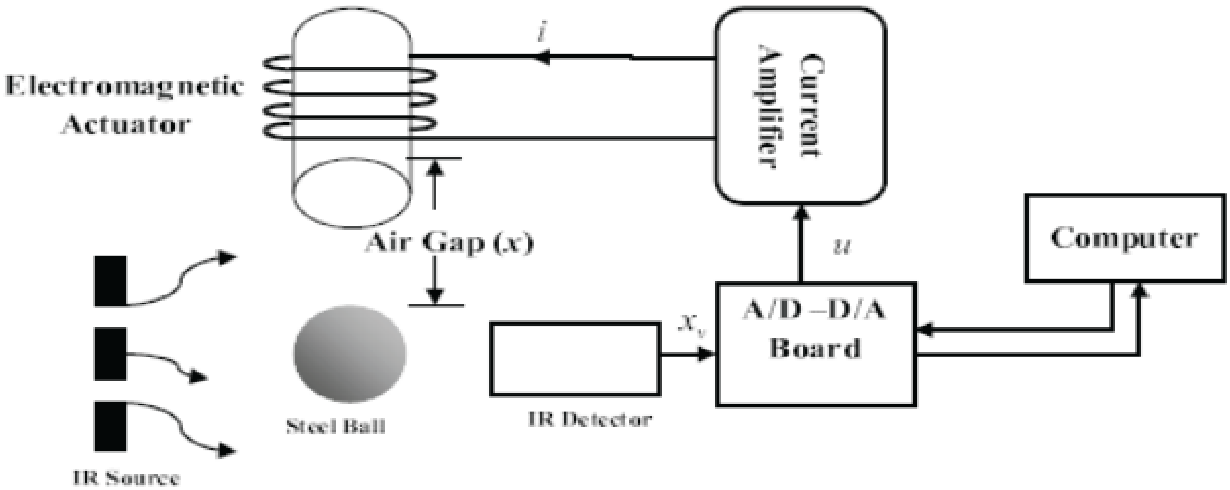


Fig. 1: Magnetic ball suspension system

Parameters	Value
m—Mass of the steel ball	0.02 kg
g—Acceleration due to gravity	9.81 m/s^2
i_0—Equilibrium value of current	0.8 A
x_0—Equilibrium value of position	0.009 m
k_1—Control voltage to coil current gain	1.05 A/V
k_2—Sensor gain, offset (η)	143.48 V/m, -2.8 V
Control input voltage level (u)	± 5 V
Sensor output voltage level (x_v)	$+1.25$ V to -3.75 V

1.2 Modeling :-

1.2.1 Objective:

- Derive the small-signal transfer function $\frac{\Delta x}{\Delta u}$ by linearizing the electromagnetic force $k\frac{i^2}{x^2}$ around the equilibrium point (x_0, i_0), incorporating the inner control loop $i = k_1 u$.
- Integrate the sensor gain k_2, mass m, and gravitational acceleration g into the model to establish the relationship between the control voltage and ball displacement, and validate the derived expression using the given parameters.

1.2.2 Derivation of the Linearized Transfer Function and Final Plant Transfer Function:

- **Nonlinear Model:** The simplest nonlinear model of MAGLEV is given by

$$m\ddot{x} = mg - \frac{k\,i^2}{x^2},$$

where k is a constant depending on the coil parameters and g is the acceleration due to gravity.

- **Control Voltage-to-Current Relation:** An inner control loop ensures

$$i = k_1\,u,$$

with k_1 being the voltage-to-current gain.

- **Equilibrium Condition:** At the equilibrium point (x_0, i_0) (with $\ddot{x} = 0$), the forces balance:

$$mg = \frac{k\,i_0^2}{x_0^2} \quad \Longrightarrow \quad k = \frac{mg\,x_0^2}{i_0^2}.$$

- **Perturbation and Linearization:** For small deviations, let

$$x = x_0 + \Delta x \quad \text{and} \quad i = i_0 + \Delta i.$$

Define the function

$$f(i, x) = \frac{k\,i^2}{m\,x^2}.$$

Linearize $f(i, x)$ using a first-order Taylor expansion about (x_0, i_0):

$$\Delta f \approx \left.\frac{\partial f}{\partial i}\right|_{(x_0,i_0)} \Delta i + \left.\frac{\partial f}{\partial x}\right|_{(x_0,i_0)} \Delta x.$$

The partial derivatives are:

$$\left.\frac{\partial f}{\partial i}\right|_{(x_0,i_0)} = \frac{2k\,i_0}{m\,x_0^2}, \quad \left.\frac{\partial f}{\partial x}\right|_{(x_0,i_0)} = -\frac{2k\,i_0^2}{m\,x_0^3}.$$

Define

$$k_i \frac{2k\,i_0}{m\,x_0^2} = \frac{2g}{i_0}, \quad k_x \frac{2k\,i_0^2}{m\,x_0^3} = \frac{2g}{x_0},$$

so that

$$\Delta f \approx k_i\,\Delta i - k_x\,\Delta x.$$

- **Linearized Dynamics:** The perturbed equation of motion becomes

$$\Delta\ddot{x} = -\left(k_i\,\Delta i - k_x\,\Delta x\right).$$

Taking the Laplace transform (assuming zero initial conditions) gives

$$\left(s^2 - k_x\right)\Delta X(s) = -\,k_i\,\Delta I(s).$$

- **Transfer Function Derivation:** The transfer function from the current deviation Δi to the position deviation Δx is

$$H(s) = \frac{\Delta X(s)}{\Delta I(s)} = -\frac{k_i}{s^2 - k_x}.$$

With the relation $\Delta i = k_1\,\Delta u$, the transfer function from control voltage Δu to position deviation Δx becomes

$$H_u(s) = \frac{\Delta X(s)}{\Delta u(s)} = -\frac{k_1\,k_i}{s^2 - k_x}.$$

- **Incorporating Sensor Gain:** If the measured position is amplified by a sensor with gain k_2, the overall plant transfer function is

$$P(s) = \frac{\Delta X_v(s)}{\Delta u(s)} = -\frac{k_1\,k_2\,k_i}{s^2 - k_x}.$$

- **Final Expression:** Given

$$k_1 = 1.05 \text{ A/V}, \quad k_2 = 143.38 \text{ V/m}, \quad x_0 = 0.009 \text{ m}, \quad i_0 = 0.8 \text{ A}, \quad g = 9.81 \text{ m/s}^2,$$

compute

$$k_i = \frac{2g}{i_0} = \frac{2 \times 9.81}{0.8} \approx 24.525, \quad k_x = \frac{2g}{x_0} = \frac{2 \times 9.81}{0.009} \approx 2180.$$

Thus, the plant transfer function is

$$P(s) = -\frac{1.05 \times 143.38 \times 24.525}{s^2 - 2180} \approx -\frac{3694.61}{s^2 - 2180}.$$

1.3 Controller Design :-

1.3.1 Objective :

- **2-DoF Controller Design:** Use pole-placement to design a minimal order, proper 2-DoF controller that achieves a settling time of less than 2 s and a peak overshoot of less than 5%. Ensure the non-dominant closed-loop poles satisfy the criteria $|GM| \geq 6dB$ and $|PM| \geq 60$.

- **Simulation Validation:** Validate the controller's tracking performance, as well as its gain margin (GM) and phase margin (PM), by simulating both the linear and nonlinear system models.

- **1-DoF vs. 2-DoF Comparison:** Implement a 1-DoF controller with identical closed-loop pole locations and compare its tracking accuracy and control effort to those of the 2-DoF controller to demonstrate the superiority of the latter.

1.3.2 Dominant and Non-Dominant Poles Calculation :-

- **Step 1: Determine the Dominant Poles Constraints**
  Given constraints:

$$T_s = \frac{4}{\zeta\omega_n} < 2 \quad \Rightarrow \quad \zeta\omega_n > 2.$$
$$M_p = e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)} < 0.05.$$

  Solving for ζ, taking the natural logarithm on both sides:

$$-\frac{\pi\zeta}{\sqrt{1-\zeta^2}} < \ln(0.05).$$

  Evaluating $\ln(0.05) \approx -3$:

$$\frac{\pi\zeta}{\sqrt{1-\zeta^2}} > 3.$$

  Rearranging and solving numerically, we get:

$$\zeta > 0.69.$$

  Choosing $\zeta = 0.7$ for good damping, we solve for ω_n:

$$0.69 * \omega_n > 2 \quad \Rightarrow \quad \omega_n > \frac{2}{0.69} \approx 3.0627.$$

  Selecting $\omega_n = 3$, the dominant poles are:

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -2.117 \pm j2.2211.$$

- **Step 2: Define Non-Dominant Poles**
  We take two real non-dominant poles with real parts equal to α times the dominant pole real part and imaginary part 0:

$$s_3, s_4 = -\alpha$$

- **Step 3: Form the Characteristic Equation**
  The characteristic equation is given by:

$$\delta(s) = (s - s_1)(s - s_2)(s - s_3)(s - s_4).$$

  Expanding the first two terms:

$$(s + 2.117 - j2.2211)(s + 2.117 + j2.2211) = (s + 2.1)^2 + 2.14^2.$$
$$= s^2 + 4.234s + 4.482.$$

  Multiplying both factors:

$$\delta(s) = (s^2 + 4.234s + 4.482)(s + \alpha)^2.$$

1.3.3 Controller Gain Calculation :

- **Controller :**

  The controller is chosen as

$$C(s) = \frac{h_2 s^2 + h_1 s + h_0}{s^2 + s k_1}.$$

- **MATLAB Code for Control Gain tuning :**

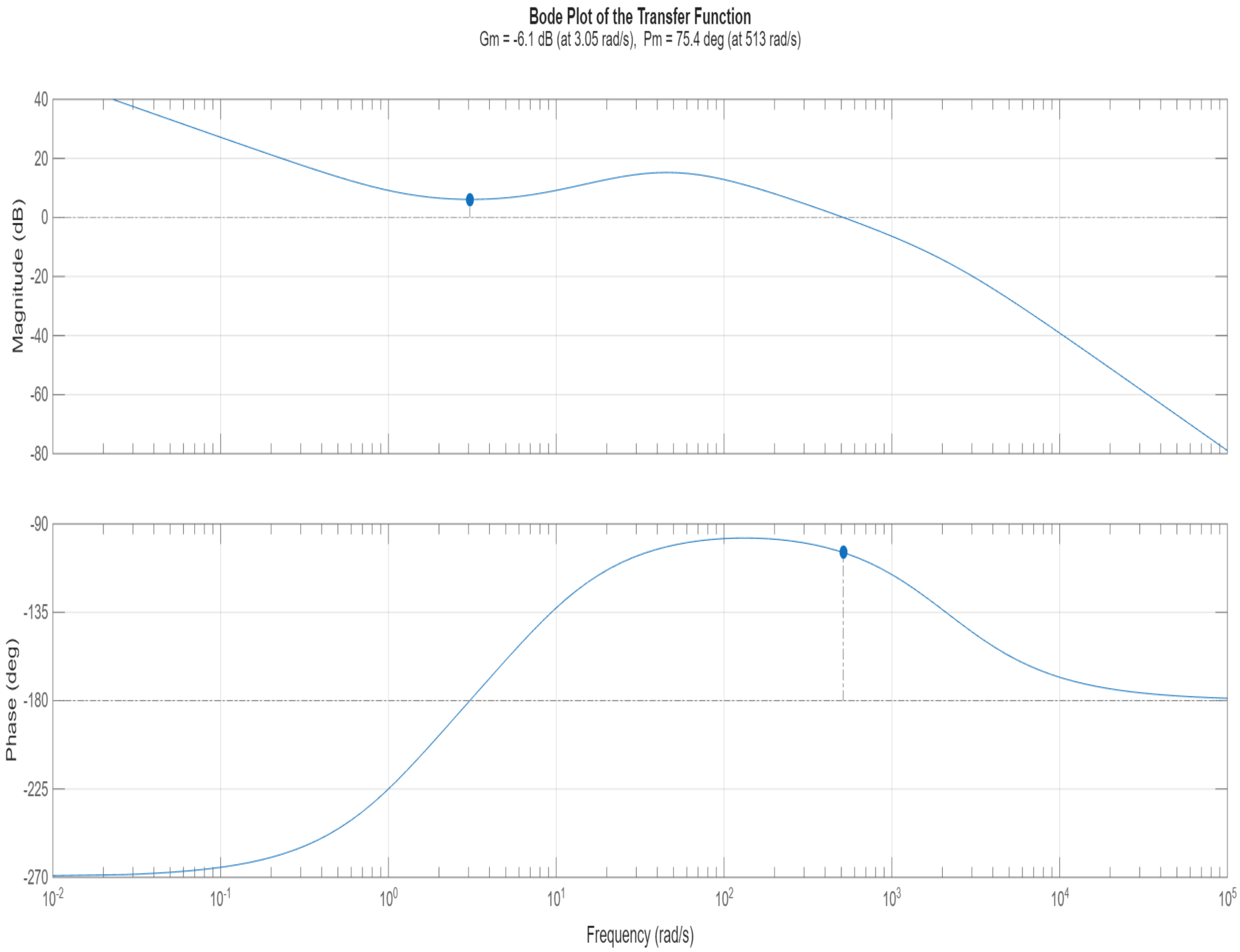
```
% Clear workspace and command window
clear; clc;

% Define symbolic variables and unknowns
syms s h2 h1 h0 k1

% Given parameters
zeta = 0.69;
w_n  = 3.0627;
kx   = 2180;
b    = 3694.61;

% Define alpha
alpha = 500 * (zeta * w_n);

% Define the polynomial delta(s)
del_s = expand((s^2 + 2*zeta*w_n*s + w_n^2) * (s + alpha)^2);

% Define gc_1(s) corresponding to:
% s^4 + k1*s^3 - s^2*(kx + b*h2) - s*(k1*kx + b*h1) - b*h0
gc1_s = expand(s^4 + k1*s^3 - s^2*(kx + b*h2) - s*(k1*kx + b*h1) - b*h0)
    ;

% Convert delta(s) to a coefficient vector (highest degree first)
p_del = sym2poly(del_s);
% p_del should be:
% [1, 2*(alpha + zeta*w_n), alpha^2 + 4*zeta*w_n*alpha + w_n^2, 2*(zeta*
    w_n*alpha^2 + w_n^2*alpha), w_n^2*alpha^2]

% In gc1_s, the coefficients are:
% s^4:    1
% s^3:    k1
% s^2:   -(kx + b*h2)
% s^1:   -(k1*kx + b*h1)
% s^0:   -b*h0

% Equate coefficients for corresponding powers of s
eq1 = k1 == p_del(2);                % Coefficient of s^3
eq2 = -(kx + b*h2) == p_del(3);     % Coefficient of s^2
eq3 = -(k1*kx + b*h1) == p_del(4); % Coefficient of s^1
eq4 = -b*h0 == p_del(5);            % Constant term

% Solve for k1, h2, h1, and h0
sol = solve([eq1, eq2, eq3, eq4], [k1, h2, h1, h0]);

```

```
% Simplify the solutions
k1_sol = simplify(sol.k1);
h2_sol = simplify(sol.h2);
h1_sol = simplify(sol.h1);
h0_sol = simplify(sol.h0);

% Convert symbolic solutions to numeric values
k1_val = double(vpa(k1_sol, 6));
h2_val = double(vpa(h2_sol, 6));
h1_val = double(vpa(h1_sol, 6));
h0_val = double(vpa(h0_sol, 6));

% Display the computed values
disp('Computed parameter values:');
fprintf('k1 = %f\n', k1_val);
fprintf('h2 = %f\n', h2_val);
fprintf('h1 = %f\n', h1_val);
fprintf('h0 = %f\n', h0_val);

% Define numerator and denominator coefficients of the transfer function
% Numerator: [-b*h2, -b*h1, -b*h0]
% Denominator: [1, k1, -kx, -k1*kx, 0]
num = [-b*h2_val, -b*h1_val, -b*h0_val];
den = [1, k1_val, -kx, -k1_val*kx, 0];

% Create the transfer function model
sys = tf(num, den);

% Plot the Bode plot with gain and phase margins
figure;
margin(sys);
grid on;
title('Bode Plot of the Transfer Function');
```

- **Choosing $\alpha = 500 * \zeta * \omega_n$ gives us the desired Phase and Gain Margins as can be seen from below Bode Plots.**

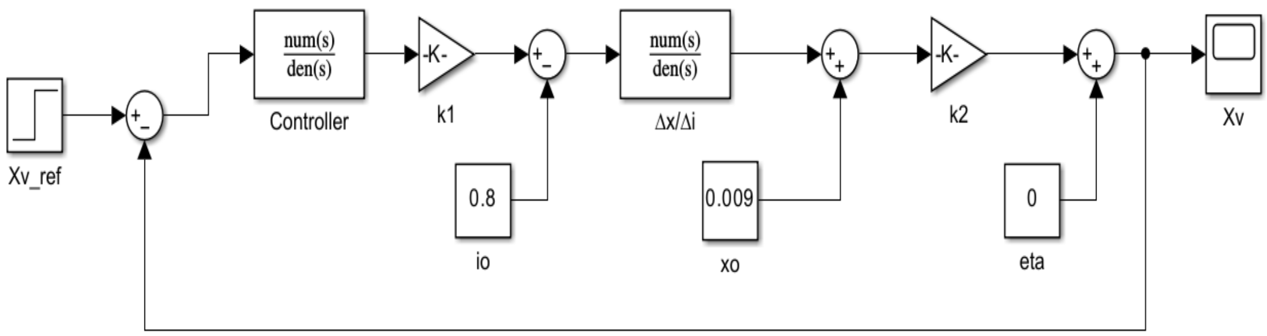
- **Which Gives us the value of Control Gains as k1 = 2117.489526 , h2 = -305.198997 , h1 = -2531.996576 , h0 = -2834.571544**

- **Bode Plot :**

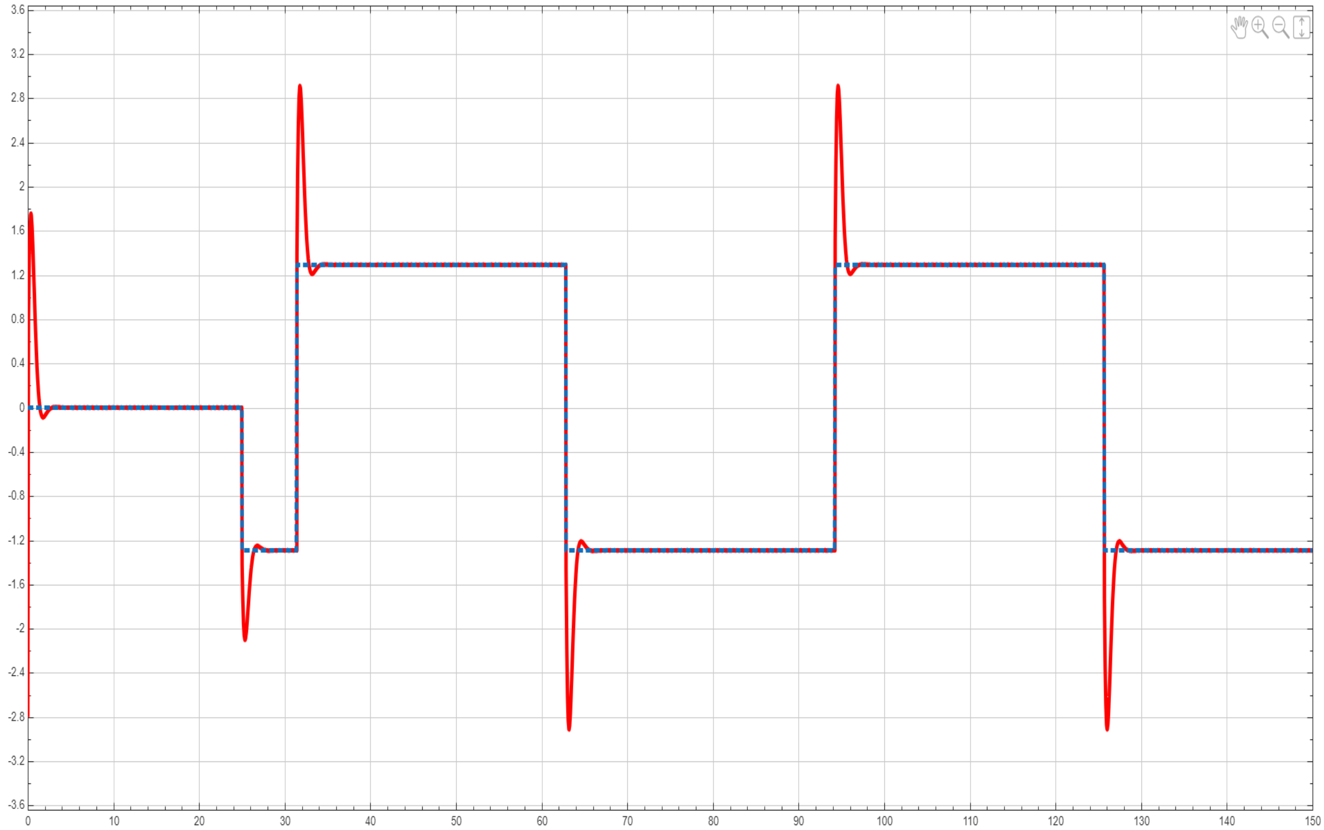


- **<u>1-DoF Controller :</u>**

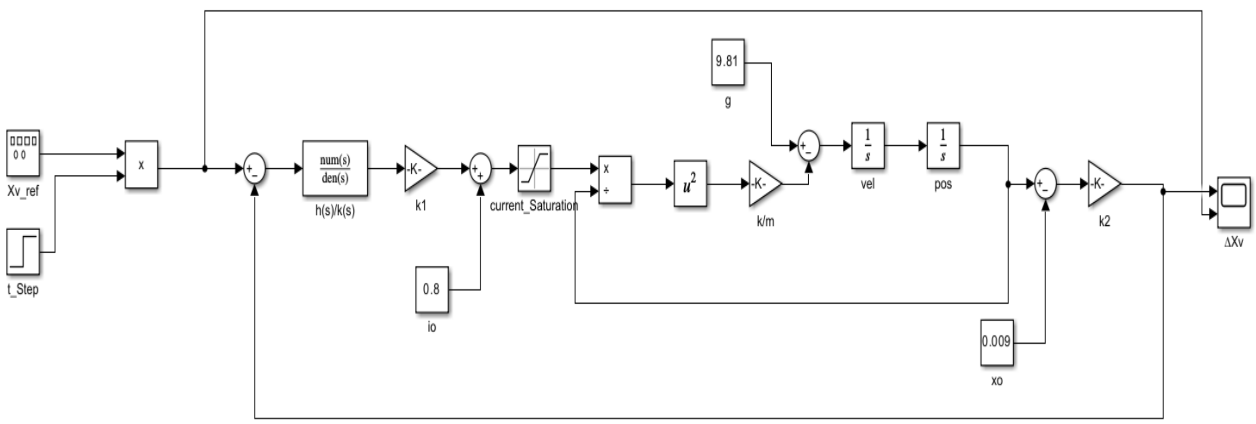
- **Block Diagram Linear Plant :**



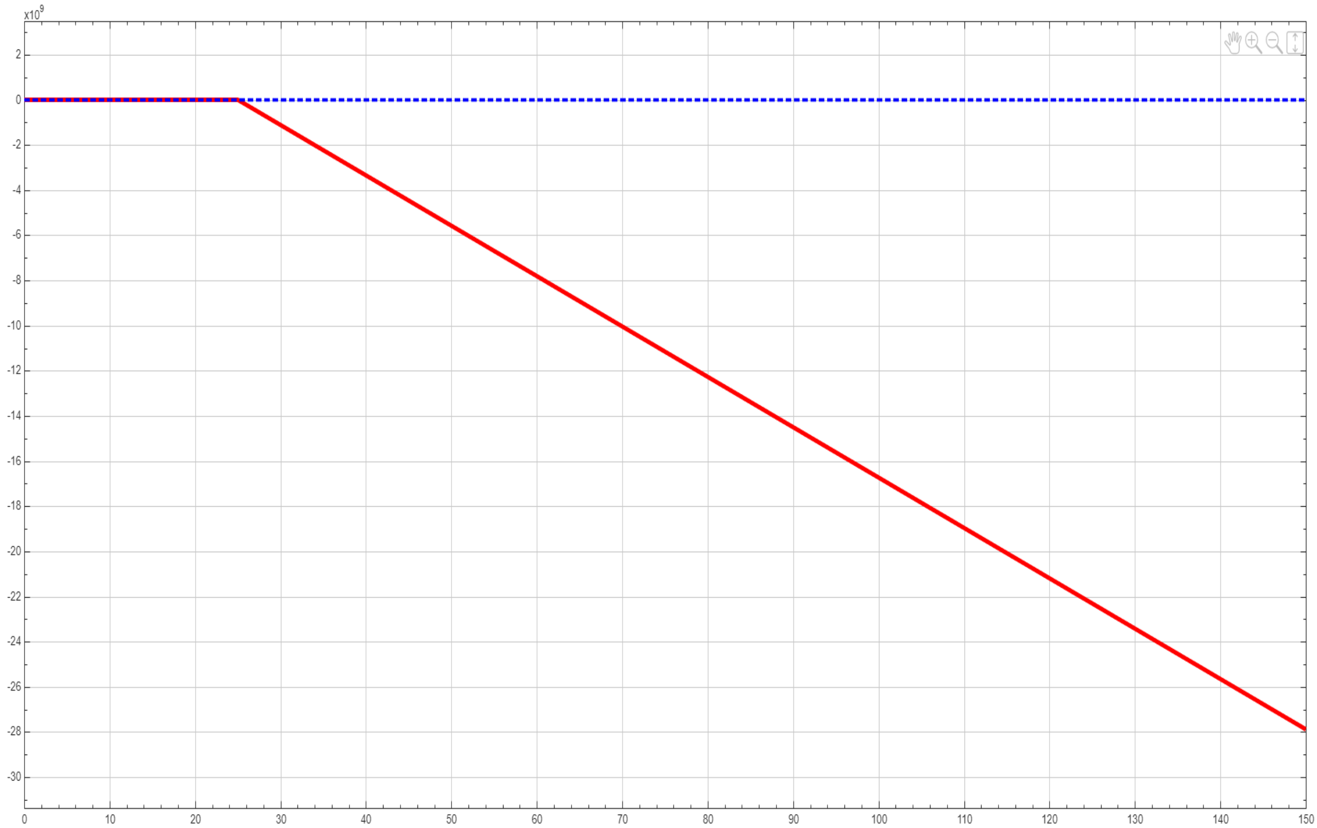
- **Output of Linear Plant :**



- **Block Diagram of Non-Linear Plant :**



- **Output of Non-Linear Plant :**



- **Observation :**

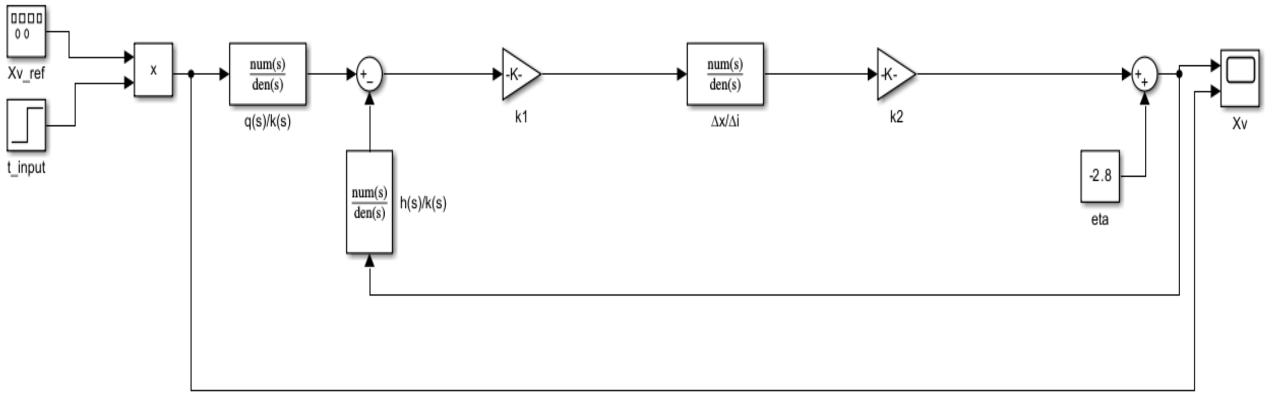
  - **Non-linear plant behavior:** Even after applying current saturation, the non-linear plant exhibits highly unstable output, indicating that its inherent non-linearities significantly affect stability.

  - **Linear plant performance:**
    * The linear plant tracks the reference position very effectively, although it shows some overshoot and undershoot, suggesting minor transient deviations during tracking.

    * The offset voltage induced in the plant leads to an initial overshoot that momentarily destabilizes the system; however, this transient disturbance is rapidly corrected, allowing the plant to swiftly settle into a stable operating state, which demonstrates the system's effective control mechanisms despite the early instability.

- **2-DoF Controller :**

- **Block Diagram Linear Plant :**



- **q(s) Calculation :**

  - We are free to choose $q(s)$ as desired. Here, we choose it to cancel out the non-dominant poles which, in our case, are $(s+\alpha)^2$, where

$$\alpha = 500 \cdot \zeta \cdot \omega_n = 500 \cdot 0.69 \cdot 3.0627 \approx 1056.632.$$

Since these non-dominant poles are far from the dominant poles, they have little impact on the transient specifications, and the gain and phase margins remain essentially unchanged. To reduce the steady-state error, we include a steady-state DC gain k'. Hence, we design a second-order $q(s)$ as

$$q(s) = k'\Big(q_2 s^2 + q_1 s + q_0\Big).$$

Comparing with

$$(s+\alpha)^2 = s^2 + 2\alpha s + \alpha^2,$$

we identify

$$q_2 = 1, \quad q_1 = 2\alpha \approx 2113.264, \quad q_0 = \alpha^2 \approx 1.1164 \times 10^6.$$

Given the plant

$$G(s) = \frac{b(s)}{a(s)} = \frac{-3694.61}{s^2 - 2180},$$

we have $b(0) = -3694.61$. We design

$$q(s) = k'\Big(q_2 s^2 + q_1 s + q_0\Big)$$

to cancel the non-dominant poles, choosing

$$(s+\alpha)^2 = s^2 + 2\alpha s + \alpha^2,$$

where

$$\alpha = 500 \cdot \zeta \cdot \omega_n = 500 \cdot 0.69 \cdot 3.0627 \approx 1056.632.$$

Thus, we set

$$q_2 = 1, \quad q_1 = 2\alpha \approx 2113.264, \quad q_0 = \alpha^2 \approx 1.1164 \times 10^6.$$

The closed-loop transfer function is

$$\frac{X_v(s)}{X_v r(s)} = \frac{b(s)\, q(s)}{\delta(s)},$$

with

$$\delta(s) = \Big(s^2 + 4.234s + 4.482\Big)(s + \alpha)^2.$$

To ensure unity DC gain, we require

$$\frac{X_v(0)}{X_v r(0)} = 1 \quad \Longrightarrow \quad b(0)\, q(0) = \delta(0).$$

At $s = 0$, since $q(0) = k'\, q_0$ and $\delta(0) = \omega_n^2\, \alpha^2$, this yields:

$$-3694.61\, (k'\, q_0) = \omega_n^2\, \alpha^2.$$

Noting that $q_0 = \alpha^2$, we cancel α^2 to obtain

$$-3694.61\, k' = \omega_n^2,$$

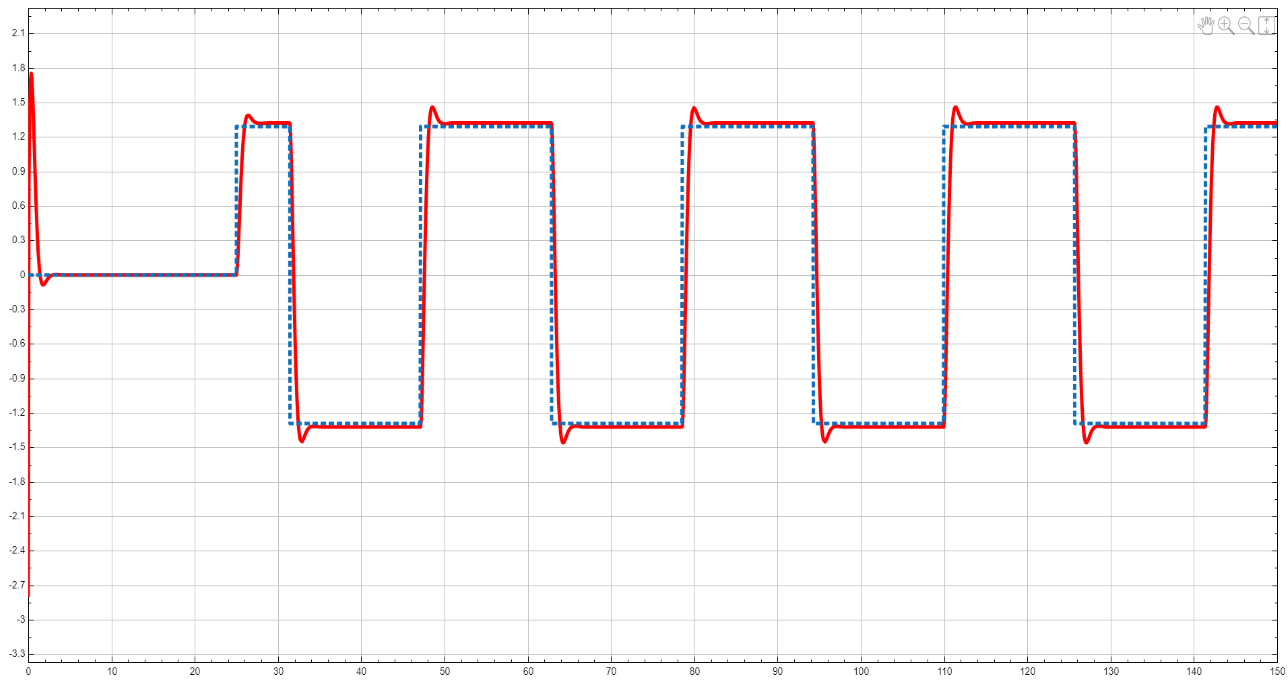
and hence

$$k' = -\frac{\omega_n^2}{3694.61} = -\frac{(3.0627)^2}{3694.61} \approx -\frac{9.379}{3694.61} \approx -0.00254.$$

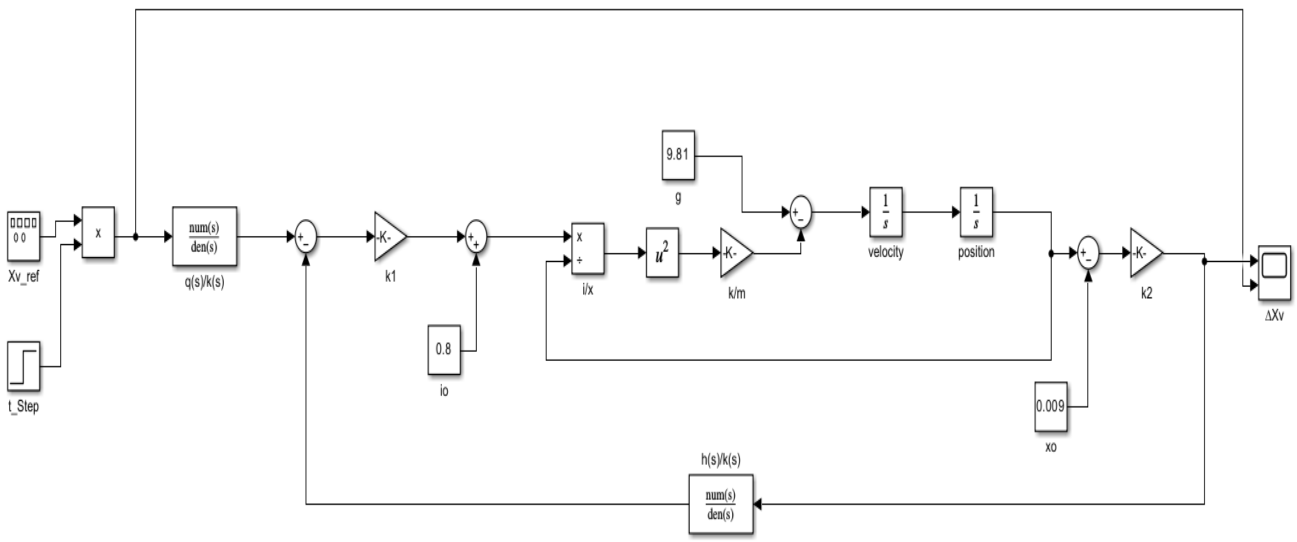
Therefore, the final expression for $q(s)$ is

$$q(s) = -0.00254\Big(s^2 + 2113.264\, s + 1.1164 \times 10^6\Big).$$

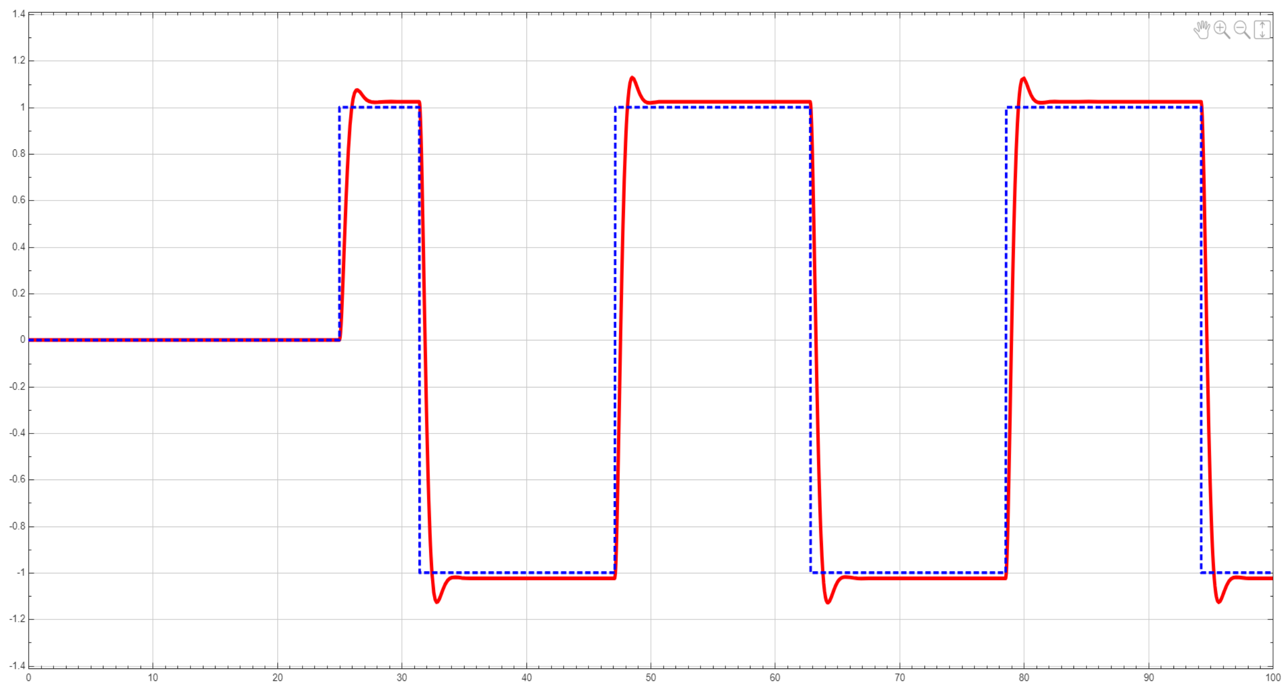
- **Output of Linear Plant :**



- **Block Diagram of Non-Linear Plant :**



- **Output of Non-Linear Plant :**



- **Observation :**

  – The plots clearly demonstrate that for both the non-linear and linear plants controller is accurately tracking the reference output. In particular, the transient response characteristics, such as the settling time and percentage overshoot, meet the desired specifications, thereby confirming the effectiveness of the implemented control strategy.

1.4 Comparison Between 1-DoF and 2-DoF Control Mechanism :-

- In the 2-DoF configuration, the overshoot and undershoot are significantly reduced compared to the 1-DoF case. This improvement is largely due to the controller's ability to separately manage the feedforward and feedback paths, which allows for a more refined tuning of the transient response. The reduced transient deviations result in a smoother tracking behavior, even when the system encounters rapid reference changes.

- The 1-DoF controller, when applied to the non-linear plant with current saturation, exhibits a pronounced instability. This instability is a consequence of the limited control action provided by a single degree of freedom, which is insufficient to effectively counteract the disturbances and non-linear effects introduced by current saturation, leading to a deterioration in the overall system stability.

- Conversely, the 2-DoF controller, implemented without current saturation, demonstrates a robust tracking performance. It achieves minimal steady-state error and a shorter settling time, highlighting its capability to handle dynamic variations more effectively. The separation of control actions in the 2-DoF design enables it to precisely adjust the system's response, ensuring that the reference signal is followed accurately under various operating conditions.

- These results clearly underscore the superiority of the 2-DoF controller over the 1-DoF configuration. By significantly enhancing tracking performance and reducing the required control effort, the 2-DoF approach offers a more reliable and efficient solution for controlling the non-linear plant, especially in scenarios where current saturation challenges the system's stability.