Calculus-2019-2

7.2 Series And Convergence pp.735-736

7°). Divergent since

$$\frac{n^3+1}{n^3+n^2} \to 1 \neq 0$$

by n-term test.

10°). Divergent since

$$\frac{(n+1)!}{5n!} \to \infty \neq 0$$

by n-term test.

17°).

$$\sum_{n=0}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right) = \sum_{n=0}^{\infty}\frac{1}{2^n} - \sum_{n=0}^{\infty}\frac{1}{3^n}$$

$$= \frac{1}{2-1} - \frac{1}{3-1} = \frac{1}{2}$$

18°).

$$\sum_{n=1}^{\infty}(\sin 1)^n = \frac{\sin 1}{1-\sin 1}$$

since it is a geometric series and the ratio, $\sin 1$, is smaller than 1.

19°).

$$\sum_{n=0}^{\infty} \frac{1}{9n^2 + 3n - 2} = \sum_{n=0}^{\infty} \left(\frac{3}{3n-1} - \frac{3}{3n+2} \right)$$

$$= \left(\frac{3}{2} - \frac{3}{5} \right) - \left(\frac{3}{5} - \frac{3}{8} \right) + \cdots =$$

25°). $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$ **is divergent since**

$$\frac{n+1}{2n-1} \to \frac{1}{2} \neq 0$$

by n-term test.

29°). $\sum_{n=0}^{\infty} \left(1 + \frac{k}{n}\right)^n$ **is divergent since**

$$\left(1 + \frac{k}{n}\right)^n \to e^k \neq 0$$

by n-term test.

55°). Fibonicci Sequence: 1. $a_1 = a_2 = 1$**; 2.** $a_{n+2} = a_{n+1} + a_n, n \geq 1$ **a°).**

$$\frac{1}{a_{n+1}a_{n+3}} + \frac{1}{a_{n+2}a_{n+3}} = \frac{a_{n+1} + a_{n+2}}{a_{n+1}a_{n+2}a_{n+3}}$$

$$= \frac{\cancel{a_{n+3}}}{a_{n+1}a_{n+2}\cancel{a_{n+3}}}$$

$$\Rightarrow \frac{1}{a_{n+1}a_{n+3}} = \frac{1}{a_{n+1}a_{n+2}} - \frac{1}{a_{n+2}a_{n+3}}$$

b°). Trivially, $\{a_n\}_n \to \infty$ **is increasing, and is telescoping; we have**

$$\sum_{n=0}^{\infty} \frac{1}{a_{n+1}a_{n+3}} = \left(\frac{1}{a_1a_2} - \frac{1}{a_2a_3}\right) + \left(\frac{1}{a_2a_3} - \frac{1}{a_3a_4}\right)$$

In [ ]: