Problem°. Find the Taylor's Series of $f(x) = \frac{1}{1+x}$ at $x = 1$.

1°. Geometric Series:

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n, \text{ for } |x| < 1.$$

2°. But we want to find:

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} a_n (x-1)^n, \text{ for } |x-1| < a.$$

3°. Since the Taylor's series is **unique**, we will expand $f(x)$ by the result from 1°.

4°. First, replace x by $x - 1$:

$$\frac{1}{1+x} = \frac{1}{2+(x-1)}$$

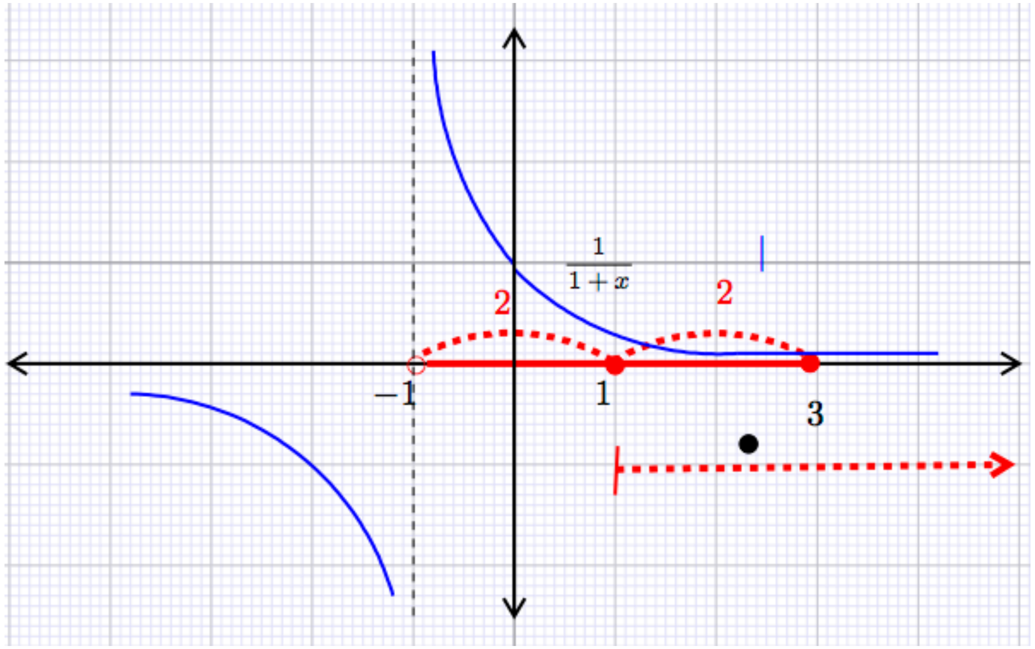
5°. Convert it similar to 1°.:

$$\frac{1}{2+(x-1)} = \frac{1}{2} \frac{1}{1+\left(\frac{x-1}{2}\right)}$$

6°. And by the result from 1°., $x \to \frac{x-1}{2}$ we have:

$$\frac{1}{1+x} = \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \left(\frac{x-1}{2}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} (x-1)^n$$

and convergent for $\left|\frac{x-1}{2}\right| < 1$, i.e. $|x-1| < 2$



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