Calculus

7.3 Integral and Comparison tests pp.745-748

8°). Take $f(x) = \frac{1}{x \ln x}$. Since

$$\int_2^{\infty} f(x)dx = \ln \ln x|_2^{\infty} = \infty$$

it is divergent by integral test.

10°). Take $f(x) = \frac{1}{(x+9)^{1/3}}$. Since

$$\int_1^{\infty} f(x)dx = \frac{3}{2}(x+9)^{2/3}\Big|_1^{\infty} = \infty$$

it is divergent by integral test.

18°). As usual, take $f(x) = \left(\frac{\sin x}{x}\right)^2$, then $a_n = f(n)$; but we can't integrate $f(x)$ directly. We can't determine its convergence by integral test. However,

$$|f(x)| \leq \frac{1}{x^2}$$

the latter is convergent by p-series test, $p = 2$ and by comparison test.

33°). Consider $f(x) = \frac{1}{x(\ln x)^p}$, then $a_n = f(n)$;

1. for $p > 1$:
$$\int_2^{\infty} f(x)dx = \frac{1}{(1-p)(\ln x)^{p-1}}\Big|_2^{\infty} < \infty$$
2. for $0 < p < 1$:
$$\int_2^{\infty} f(x)dx = \frac{(\ln x)^{1-p}}{(1-p)}\Big|_2^{\infty} = \infty$$
3. $p = 1$:
$$\int_2^{\infty} f(x)dx = \ln \ln x\Big|_2^{\infty} = \infty$$

   convergent for $1 < p$, but diverhgent for $0 < p \leq 1$, by integral test. Furthermore, $p = 0$ divergent since it is harmonic series, and divergent for $p < 0$ by n-term test.

36°). Consider $f(x) = \frac{1}{x \ln x (\ln \ln x)^p}$, then $a_n = f(n)$;

1. for $p > 1$:
$$\int_3^{\infty} f(x)dx = \frac{1}{(1-p)(\ln \ln x)^{p-1}}\Bigg|_3^{\infty} < \infty$$
2. for $0 < p < 1$:
$$\int_3^{\infty} f(x)dx = \frac{(\ln \ln x)^{1-p}}{(1-p)}\Bigg|_3^{\infty} = \infty$$
3. $p = 1$:
$$\int_3^{\infty} f(x)dx = \ln \ln x\Bigg|_3^{\infty} = \infty$$
convergent for $1 < p$, but diverhgent for $0 < p \leq 1$, by integral test.

72°). Divergent since
$$\frac{\sin(1/n)}{(1/n)} \to 1 \neq 0$$
and
$$\sum_{n=1}^{\infty} \frac{1}{n}$$
is divergent by integral test and $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ is also divergent by limit comparison test.

82°). Take another infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$. Since
$$\frac{\frac{3n^2+1}{4n^3+2}}{\frac{1}{n}} \to 3/4$$
it is divergent by limit comparison test and integral test.