Compute *y*[1,1] by using separable convolution method and compare the result with the output of normal convolution 2D.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Input

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Separable Kernel

i) Using convolution 2D

$$y[1,1] = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 1 + 4 \cdot 2 + 5 \cdot 4 + 6 \cdot 2 + 7 \cdot 1 + 8 \cdot 2 + 9 \cdot 1 = 80$$

ii) Using Separable Convolution 2D

First, do the vertical convolution 1D where the row is n=1, and the column is m=0,1,2;

$$y[*,1] = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 4 \cdot 2 + 7 \cdot 1 & 2 \cdot 1 + 5 \cdot 2 + 8 \cdot 1 & 3 \cdot 1 + 6 \cdot 2 + 9 \cdot 1 \end{bmatrix} = \begin{bmatrix} 16 & 20 & 24 \end{bmatrix}$$

Then, do the horizontal convolution with above result where column is m=1;

$$y[1,1] = \begin{bmatrix} 16 & 20 & 24 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} = 16 \cdot 1 + 20 \cdot 2 + 24 \cdot 1 = 80$$

從上面的結果可以發現，若是可以將 2D 的 convolution 拆成兩個 1D 的 convolution 來運算的話，每個像素的運算就可以從原本需要 n*m 次的乘法(假設為 n*m 的 filter)，變為 n+m 次的運算。