Ch14 Logistic regression

Y: binary response e.g. 1 or 0

yes or No

"success" or "Failure"

one predictor: X

~~SLR~~ $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \qquad E(\varepsilon_i)=0$

① If $Y_i = 1$, $\varepsilon_i = 1 - (\beta_0 + \beta_1 X_i)$
If $Y_i = 0$, $\varepsilon_i = 0 - (\beta_0 + \beta_1 X_i)$ } ε_i has two values

Non-Normality

② restriction on regression function

$$Y_i = \begin{cases} 1 \\ 0 \end{cases} \qquad \begin{aligned} P(Y_i = 1) &= \pi_i = P_i \\ P(Y_i = 0) &= 1 - \pi_i = 1 - P_i \end{aligned}$$

Bernoulli distribution

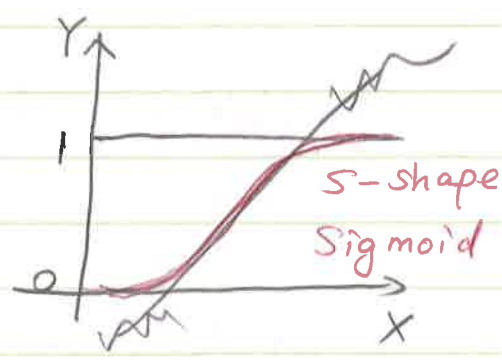
SLR, $E(Y_i) = \beta_0 + \beta_1 X_i$

$$E(Y_i) = 1 \cdot Pr(Y_i = 1) + \cancel{0 \cdot Pr(Y_i = 0)}$$

$$= \pi_i$$

$$0 \le \pi_i = \beta_0 + \beta_1 X_i \le 1$$

$0 \le \text{Prob} \le 1$



③. non-constant variance

$$Var(Y_i) = \pi_i(1 - \pi_i)$$ ✓

logistic regression work on $\log \frac{\pi_i}{1-\pi_i} = \log \text{Odds}$

$(-\infty, \infty)$

logit link (probit, log-log)

☆ $$\boxed{\log \frac{\pi_i}{1-\pi_i} = \log \frac{Pr(Y_i=1)}{1-Pr(Y_i=1)} = \beta_0 + \beta_1 X_i}$$

$$X_i = \begin{cases} 1 & \text{Male} \\ 0 & \text{Female} \end{cases}$$

$$\begin{cases} \log \text{Odds}_{\text{Female}} = \beta_0 \\ \log \text{Odd}_{\text{Male}} = \beta_0 + \beta_1 \end{cases}$$

$$\log \text{Odds}_{\text{male}} - \log \text{Odds}_{\text{female}} = \beta_1$$

$$\log \frac{\text{Odds}_{\text{male}}}{\text{Odds}_{\text{female}}} = \beta_1$$

$$\frac{\text{odds}_{\text{male}}}{\text{odds}_{\text{female}}} = \exp(\beta_1) = \exp(0.36) = 1.43$$

The odds that a man is a frequent binge drinker are 1.43 times the odds for a woman.