A practical a posteriori strategy to predict the optimal number of degrees of freedom for h-refinement in finite element methods

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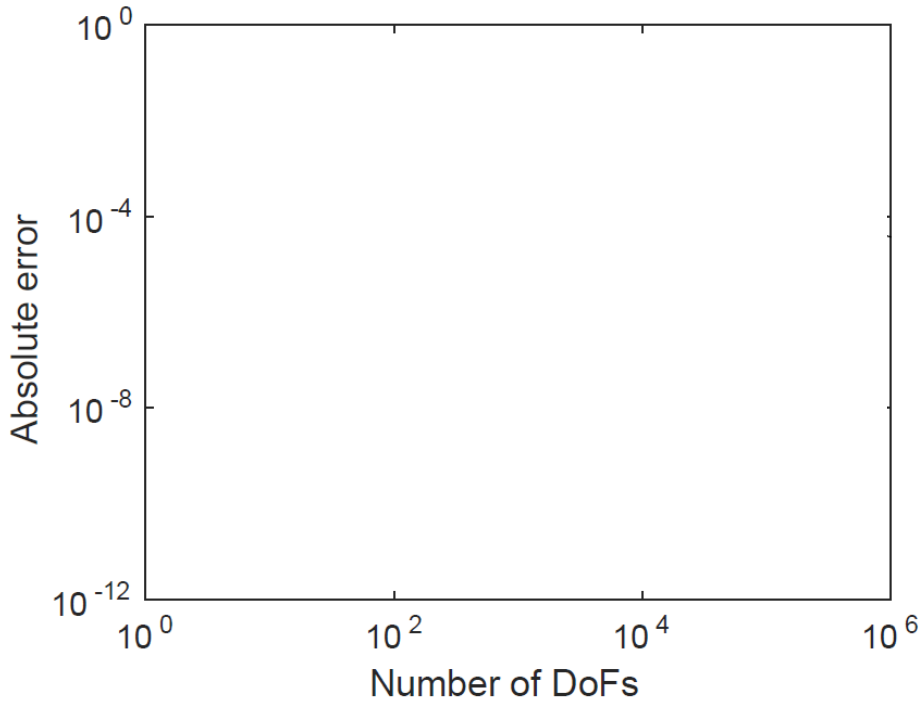

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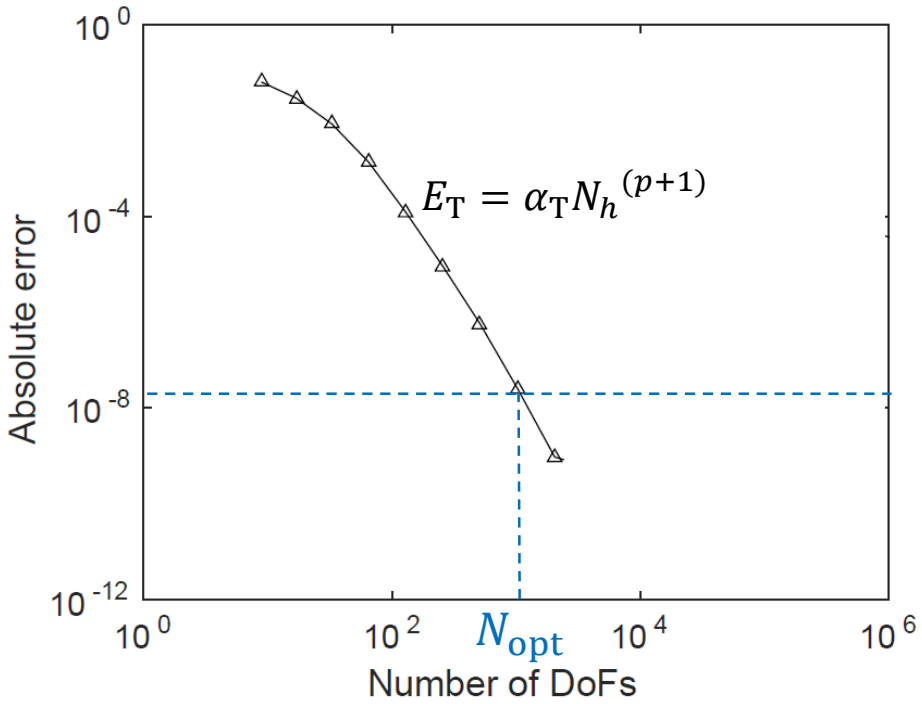

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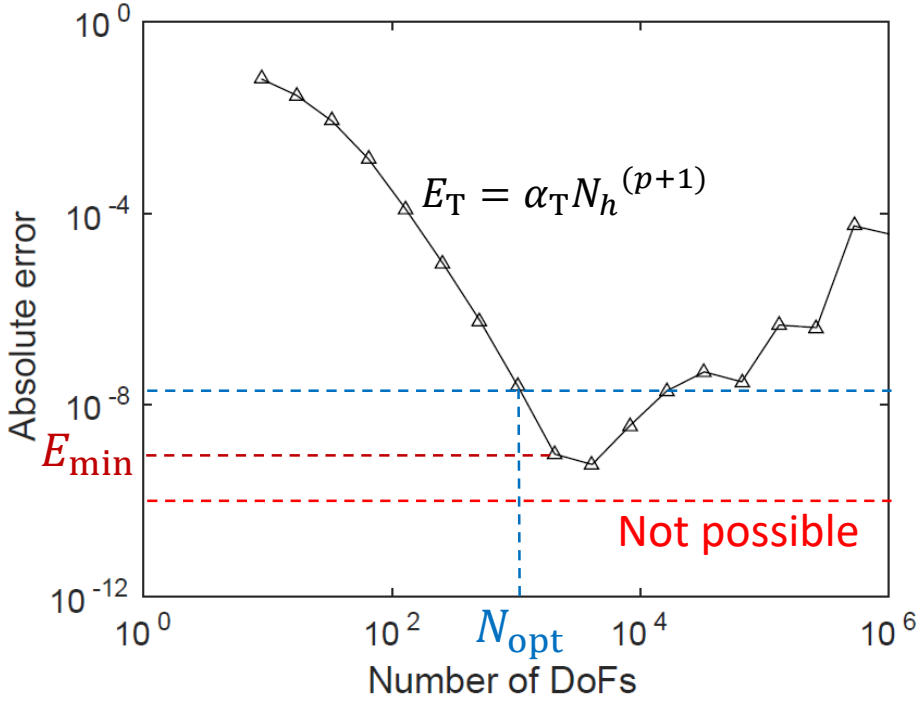

Can we know the highest attainable accuracy for a fixed p without performing h-refinements until the error starts increasing?

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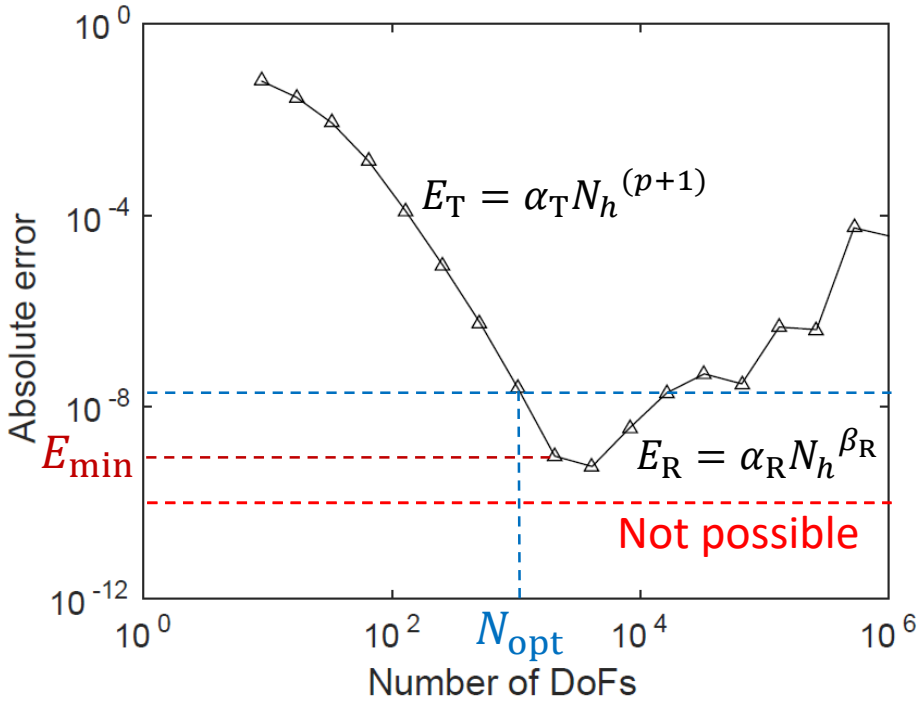

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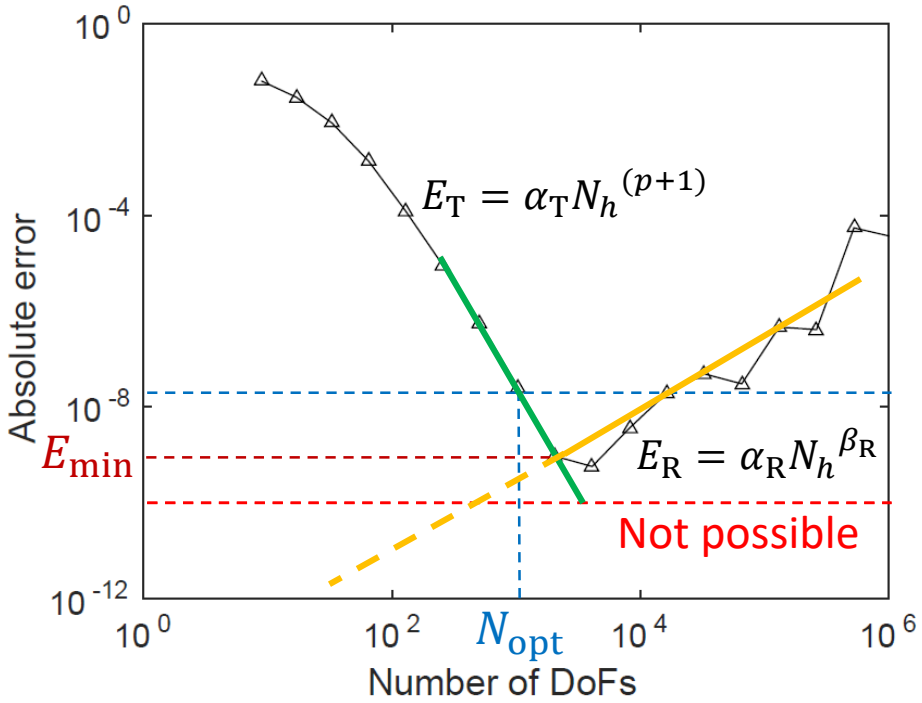

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$$(D(x)u_x)_x + r(x)u(x) = f(x), \qquad x \in I = (0,1)$$

[1]

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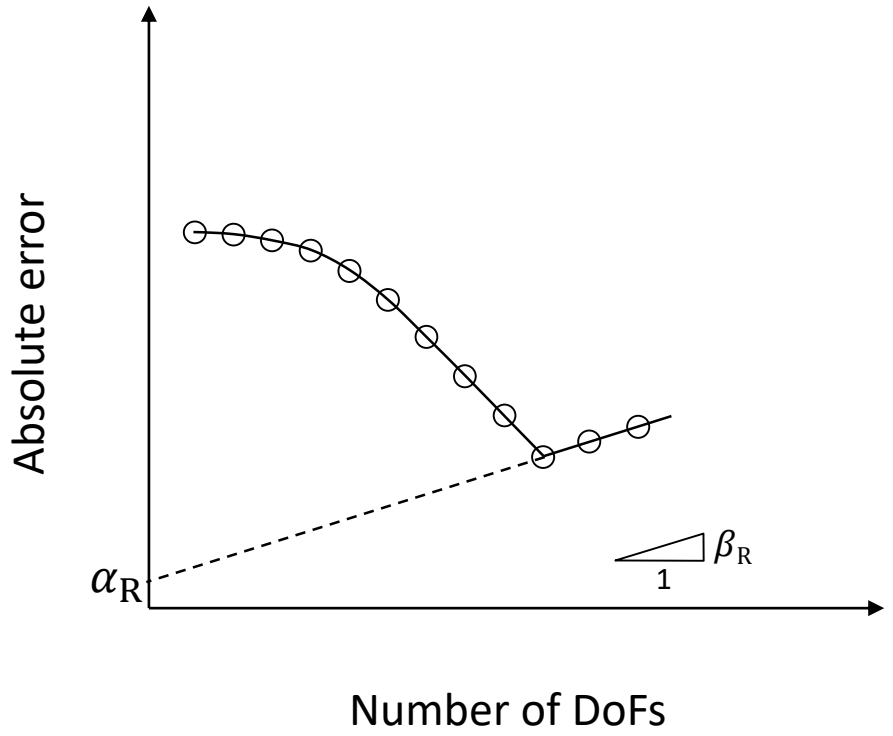
[1] J. H. Wilkinson Prize for numerical software in 2007



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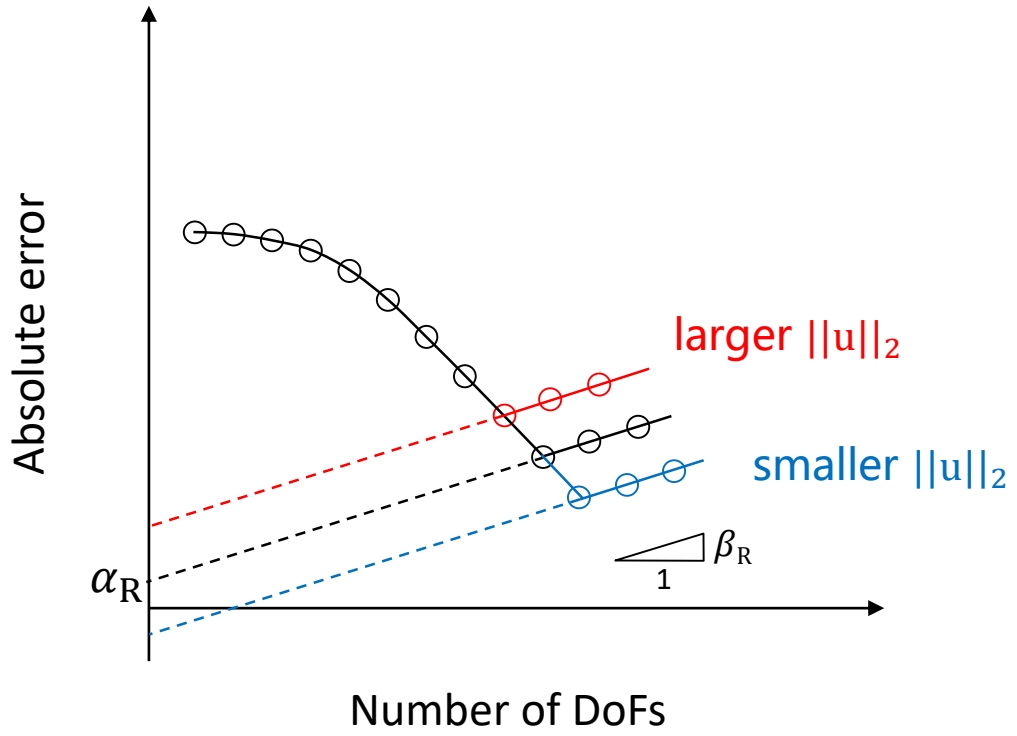
Absolute error
larger ||u||2
smaller ||u||2
α_R
β_R
1
Number of DoFs

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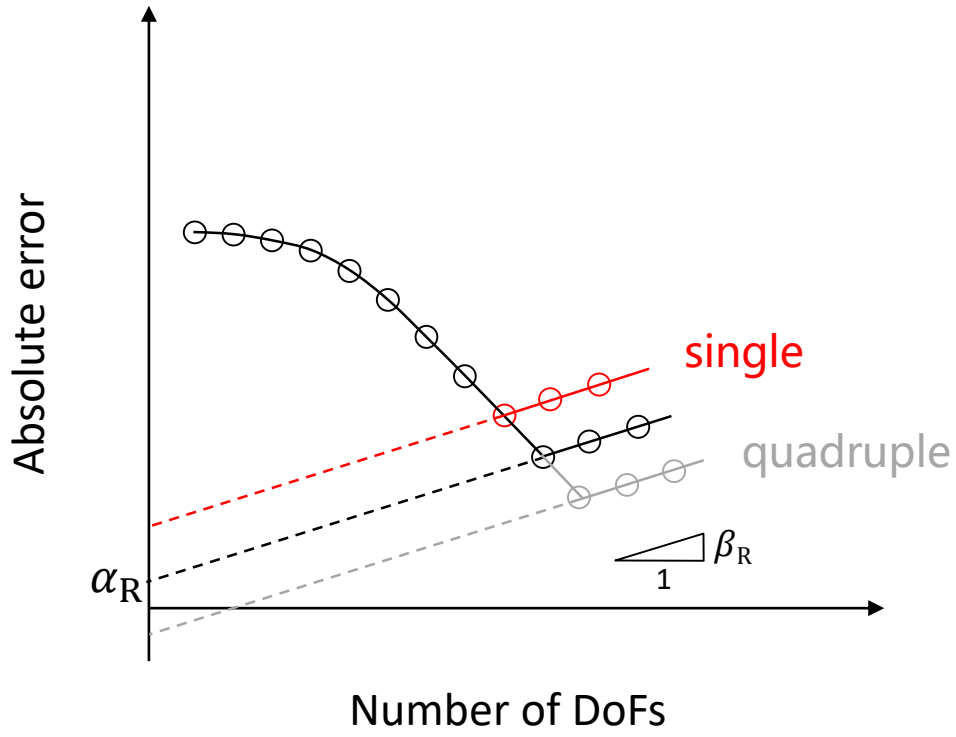
Absolute error
single
quadruple
α_R
β_R
1
Number of DoFs

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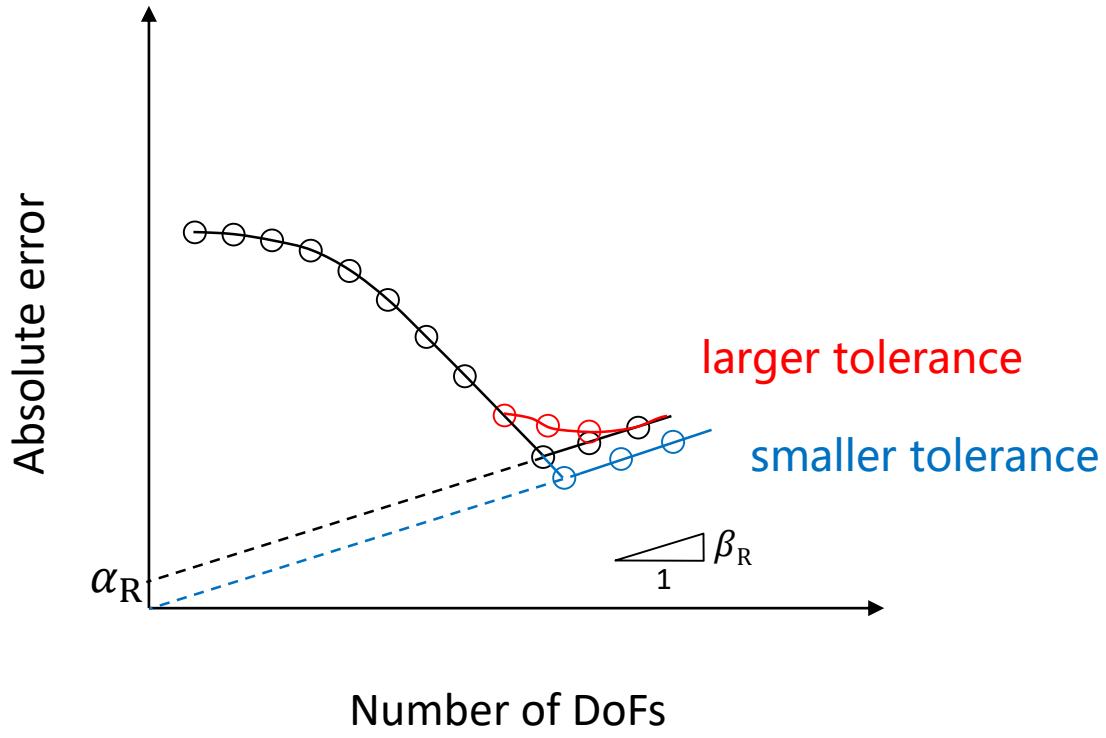
Absolute error
larger tolerance
smaller tolerance
α_R
β_R
1
Number of DoFs

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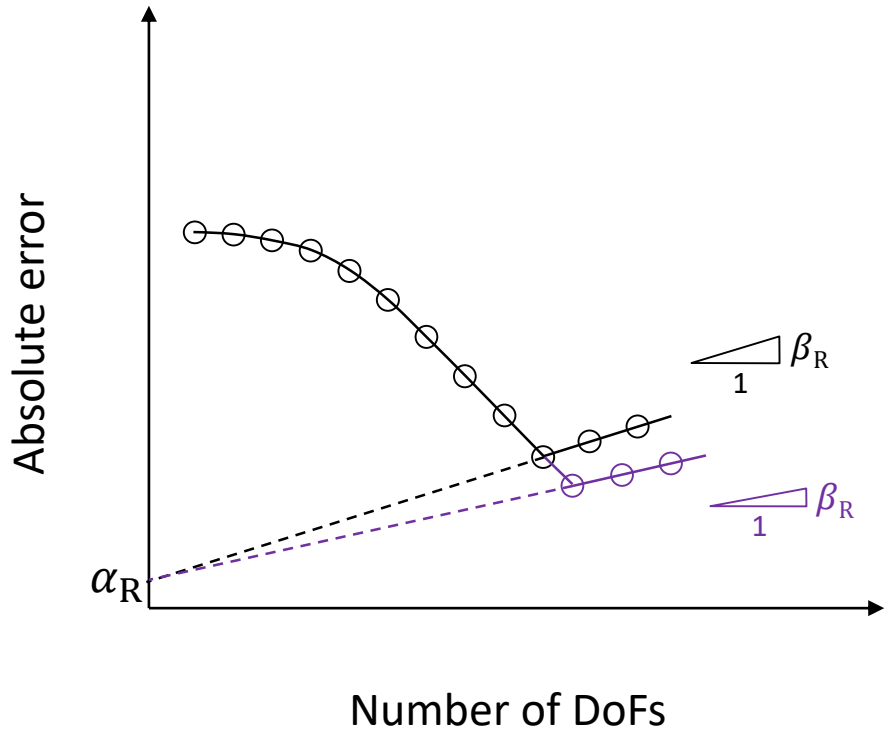

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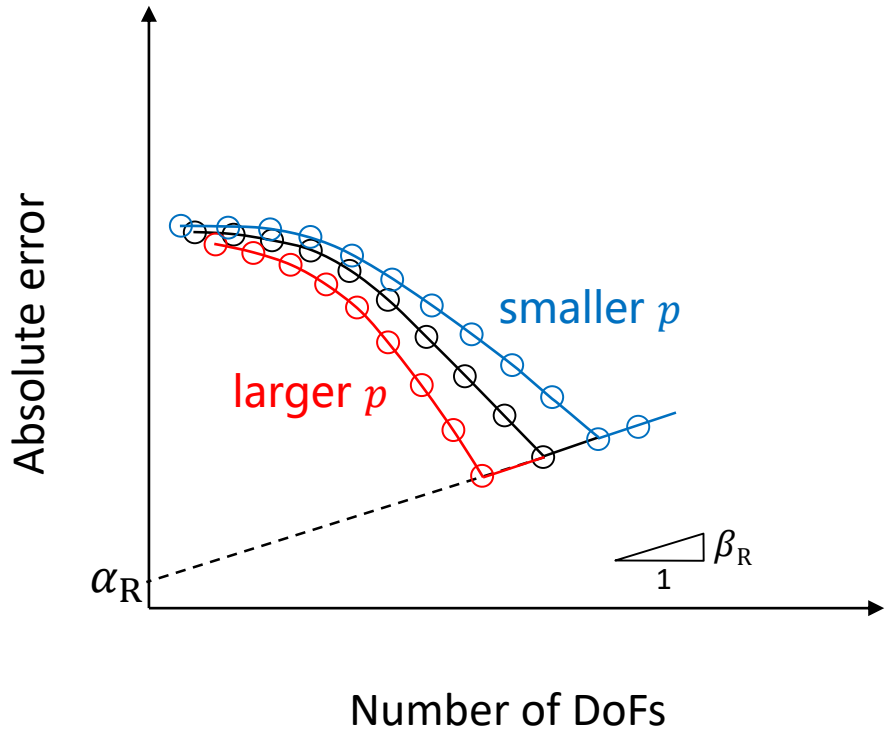
Absolute error
smaller p
larger p
α_{R}
β_{R}
1
Number of DoFs

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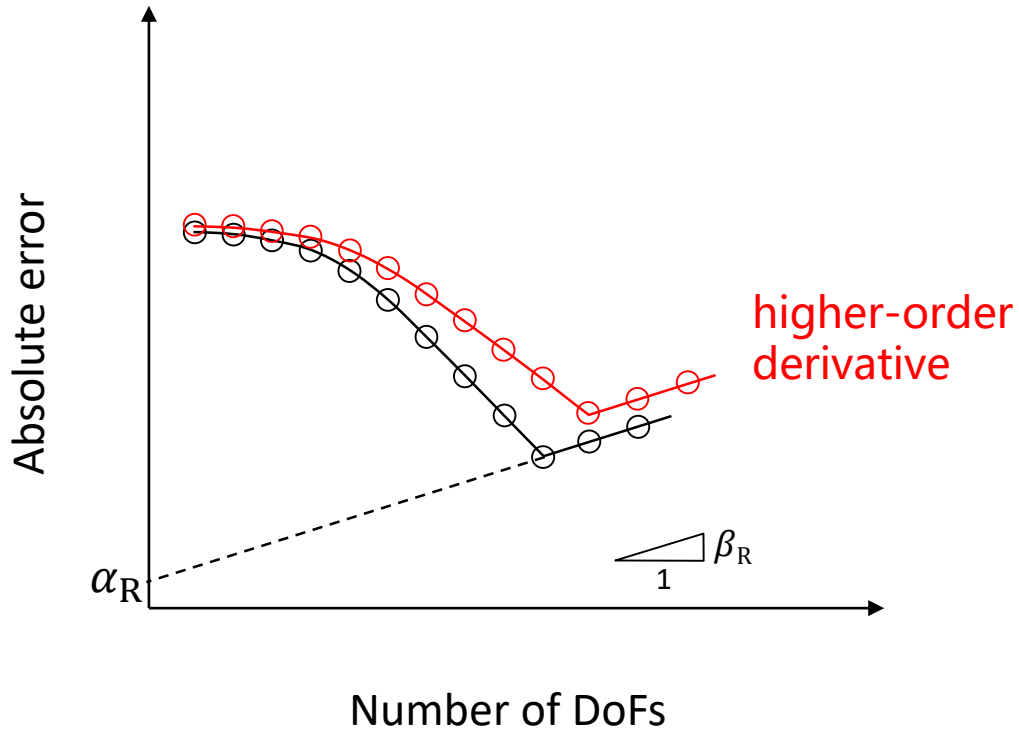
Absolute error
higher-order
derivative
α_R
β_R
1
Number of DoFs

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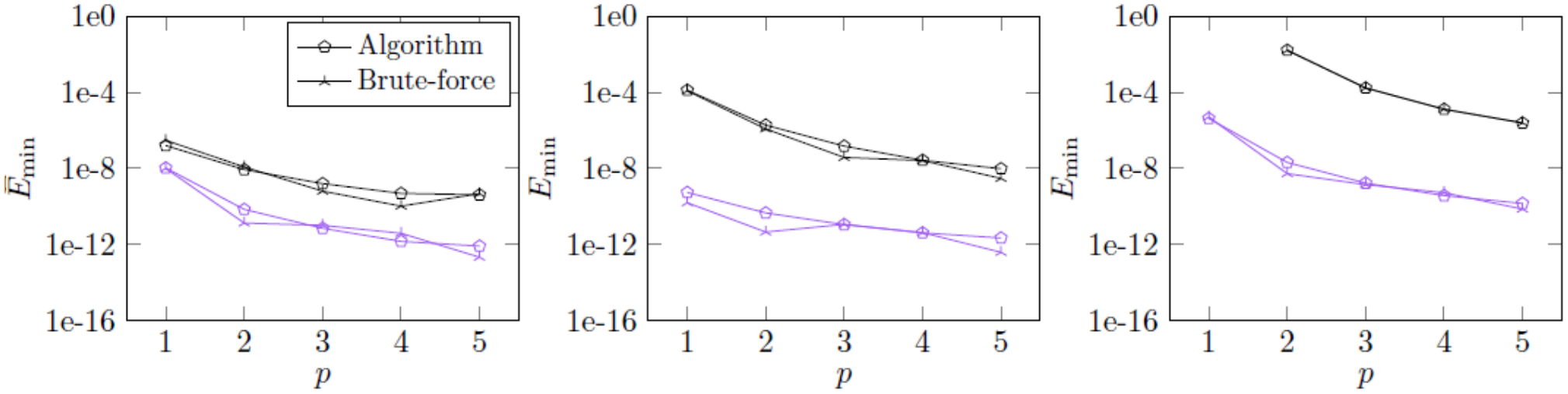
Validation

Example equation:

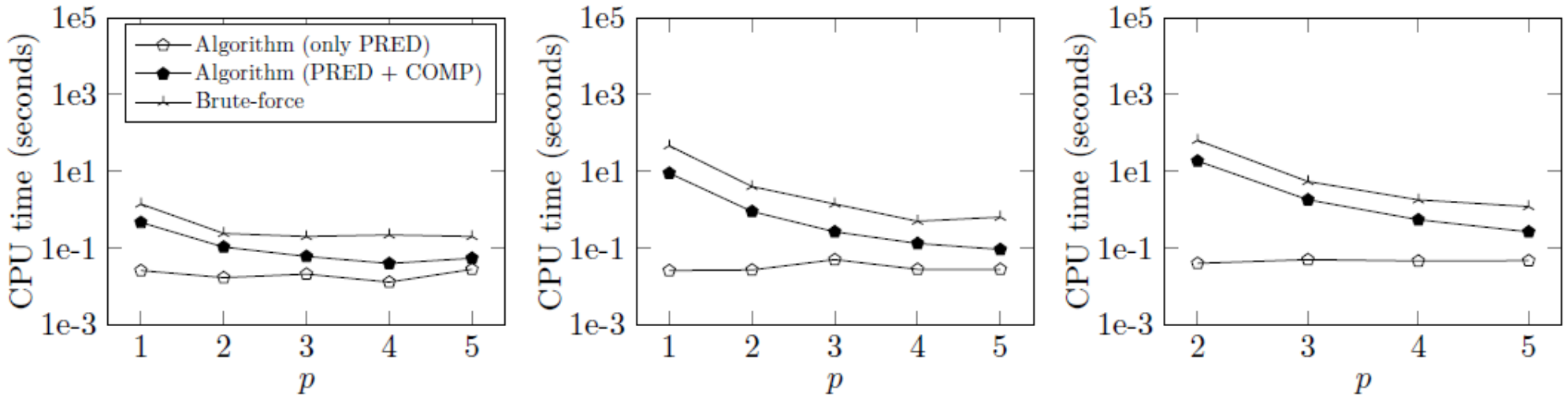
$$\big((0.01+x)(1.01-x)u_x\big)_x + (0.01i)u(x) = 1.0, \qquad x \in I = (0,1),$$
$$\text{with } u(0) = 0 \text{ and } u_x(1) = 0.$$

Variables: u, u_x and u_{xx}

Elements: P_p and $P_p/P_{p-1}^{\text{disc}}$ for p=1, 2, …, 5



(a) Solution (b) First derivative (c) Second derivative



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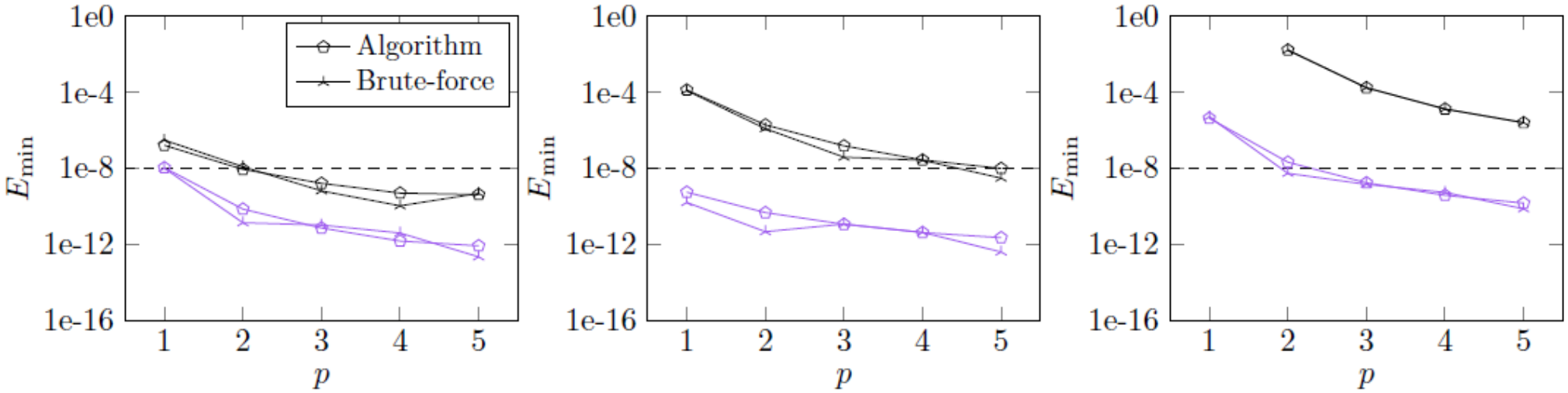

(a) Solution (b) First derivative (c) Second derivative

tol_{var}	FEM method	element degree	N_{opt}		
			u	u_x	u_{xx}
10^{-8}	mixed	3	12648	13502	372785

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Conclusions

- We propose a novel strategy to indicate whether the required accuracy can be reached and predict the optimal number of DoFs if it exists.
- This strategy makes use of the theoretical order of convergence of the truncation error and the bound of the round-off error.
- To obtain higher accuracy, higher-order elements or the mixed FEM are recommended.
- This strategy is able to indicate if the required accuracy can be reached efficiently and give the optimal number of DoFs accurately if it exists.

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Thank You!

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