1. Influence of $d(x)$ and $r(x)$ on α_R

If not stated otherwise, P_2 elements are used for the standard FEM, and P_4/P_3^{disc} elements are used for the mixed FEM.

1.1. $p = (2\pi c_1)^{-2}\sin(2\pi c_1 x)$

1.1.1. d=10, r=0

Table 1 Ratio of α_R for $d = 10$ to that for $d = 1$.

c	The standard FEM			The mixed FEM		
	u	u_x	u_{xx}	u	u_x	u_{xx}
10^{-2}	0.8	1.0	1.0	1.0	10.0	10.0
10^{-1}	0.8	1.0	1.0	1.0	10.0	10.0
10^{0}	1.0	0.5	1.0	0.1	10.0	10.0
10^{1}	1.3	0.5	1.0	1.0	10.0	10.0
10^{2}	1.3	1.3	1.0	1.0	10.0	10.0

1.2. $p = e^{-(x-0.5)^2}$

1.2.1. $d = 1 + 0.5\sin(cx)$, $r = 0$

For c ranging from 1 to 10^4, $\|d\|_2$ is of order 1, see Fig. 1. Note that, the oscillation of d magnifies when c increases.

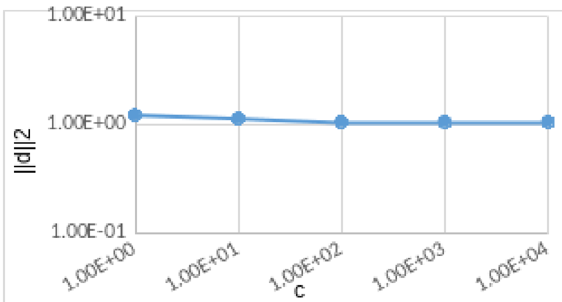
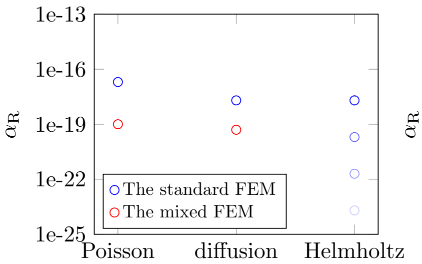
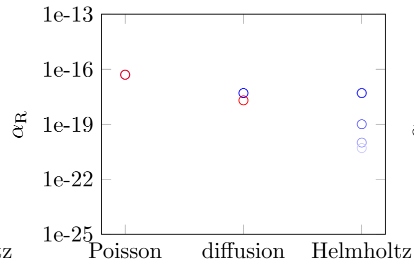


Fig. 1. Change of $\|d\|_2$ with the coefficient c for section 1.2.1.

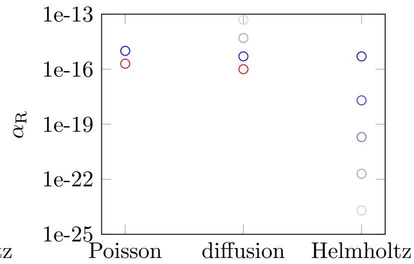
Using both the standard FEM and the mixed FEM, α_R are shown below.



(a) Solution



(b) First derivative



(c) Second derivative

Fig. 2. α_{R} for the Poisson, diffusion and Hemlholtz equations with $p = e^{-(x-0.5)^2}$.

The standard FEM. The truncation error increases when the d oscillates relatively large, i.e. when c is larger than 10.

The lines approximating the round-off error for the solution and first derivative are not affected by the oscillation of d, but that for the second derivative moves up a bit when c is larger than 10.

The mixed FEM. The truncation error increases when d oscillates relatively large. The lines approximating the round-off error are not affected.

Last but not least, the offsets α_{R} for the diffusion equation tend to be smaller than that for the Poisson equation.

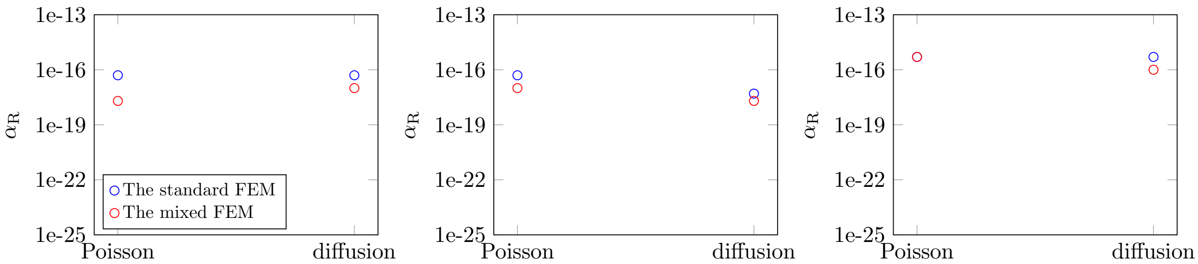
*1.2.2. d=1+0.5*sin*(x), r=c*

Here, c ranges from 1 to 10^8. For larger c, α_{R} is denoted by lighter color.

The standard FEM. α_{R} for different c are summarized in Fig. 2.

The mixed FEM.

1.3. p=sin($2\pi x$)



(a) Solution **(b)** First derivative **(c)** Second derivative

Fig. 3. α_R for the Poisson, diffusion and Hemlholtz equations with $p = \sin(2\pi x)$.

1.3.1. d=1+cx

c ranges from 10^{-4} to 10^4. The values of $\|d\|_2$ for different c are given in Fig. 4. It shows that when $c < 1$, $\|d\|_2$ are basically the same; when $c > 1$, $\|d\|_2$ increases quickly.

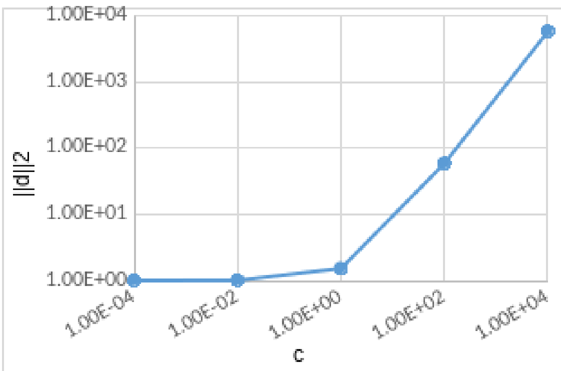


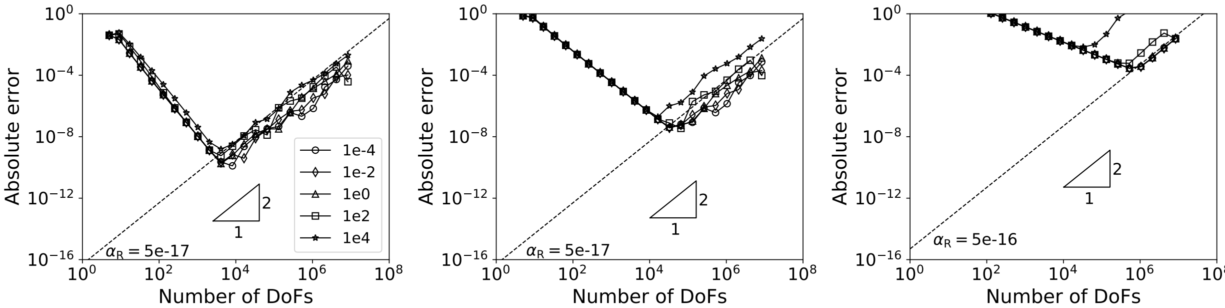
Fig. 4. Change of $\|d\|_2$ with the coefficient c for the diffusion equation.

The standard FEM. Using scheme S, the errors are shown in Fig. 5. When $c < 1$, like $\|d\|_2$, α_R for different c are basically the same; when $c > 1$, α_R increases with c, and the magnitude of the increase is larger for higher-order derivatives.

Fig. 6 proves that the element degree does not affect the round-off error. Fig. 7 proves that only using Dirichlet boundary conditions produces smaller α_R.

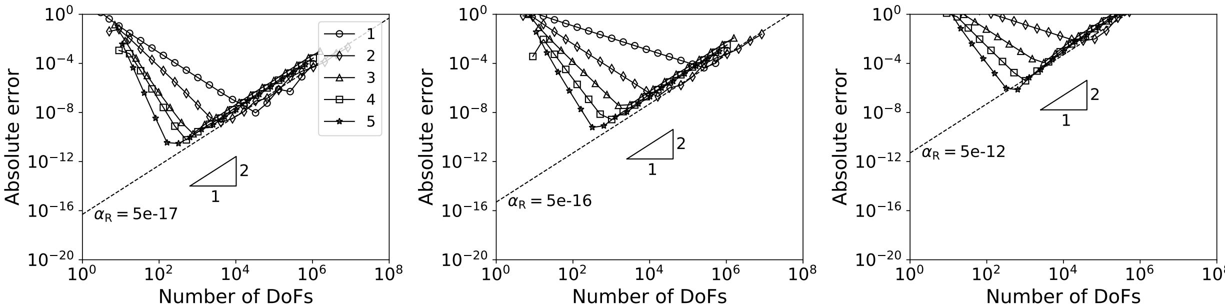
The mixed FEM. Not using scaling, the errors are shown in Fig. 8. It shows that when c is relatively large, α_R for the first and second derivatives increases. This is because the magnitude of the first derivative, which is $\|du_x\|_2$, increases with c when $c > 1$.

If we use scheme M_1 in [1], the errors are shown in Fig. 9, where the convergence behavior of α_R is observed. Note that, $\|v\|_2$ is $\|du_x\|_2$ for the diffusion equation.



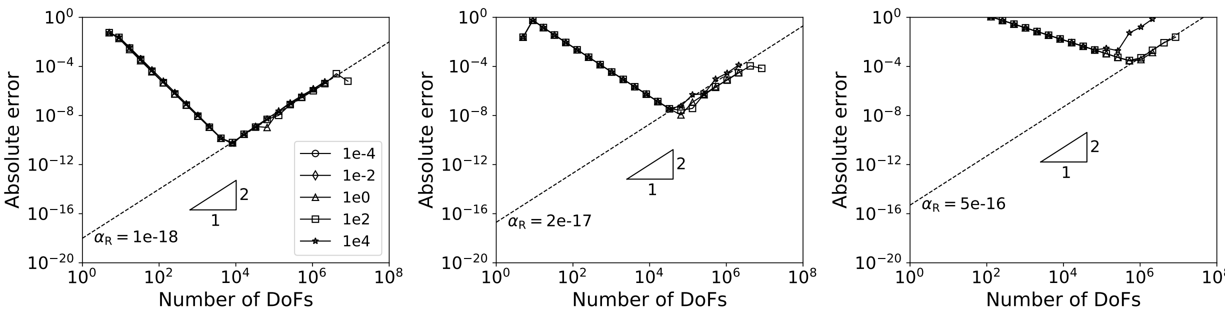
(a) Solution **(b)** First derivative **(c)** Second derivative

Fig. 5. Absolute errors for the benchmark diffusion equation using the standard FEM with scheme S, c variant.



(a) Solution **(b)** First derivative **(c)** Second derivative

Fig. 6. Absolute errors for the benchmark diffusion equation using the standard FEM with scheme S, c=1e4, degree variant.

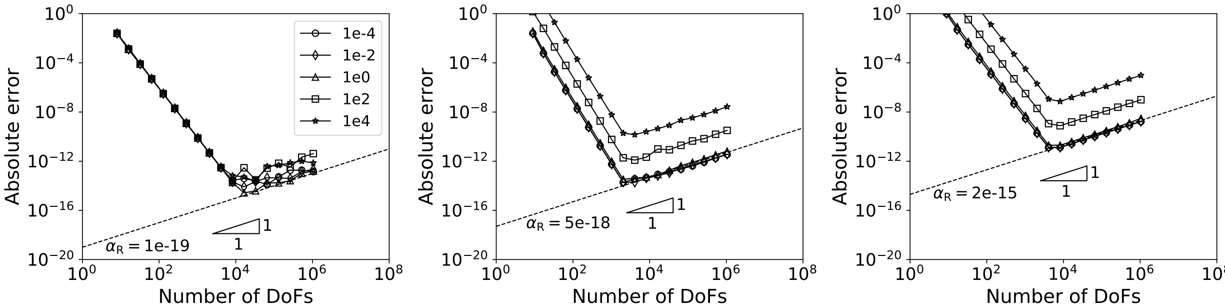


(a) Solution **(b)** First derivative **(c)** Second derivative

Fig. 7. Absolute errors for the benchmark diffusion equation but only imposed by Dirichlet boundary conditions using the standard FEM with scheme S, c variant.

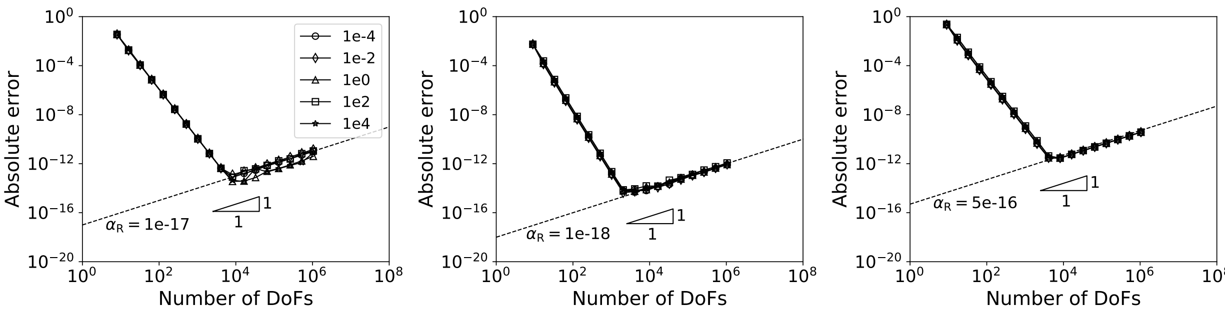
References

[1] Jie Liu, Matthias Möller, and Henk M Schuttelaars. Balancing truncation and round-off errors in practical fem: one-dimensional analysis. *arXiv preprint arXiv:1912.08004*, 2019.



(a) Solution **(b)** First derivative **(c)** Second derivative

Fig. 8. Absolute errors for the benchmark diffusion equation using the mixed FEM with P_4/P_3^{disc} elements, $v = -du_x$, coefficient variant, no scaling.



(a) Solution **(b)** First derivative **(c)** Second derivative

Fig. 9. Absolute errors for the benchmark diffusion equation using the mixed FEM with P_4/P_3^{disc} elements, $v = -du_x$, coefficient variant, scheme M_1.