A practical a posteriori strategy to ascertain the optimal number of degrees of freedom for hp-refinement in finite element methods

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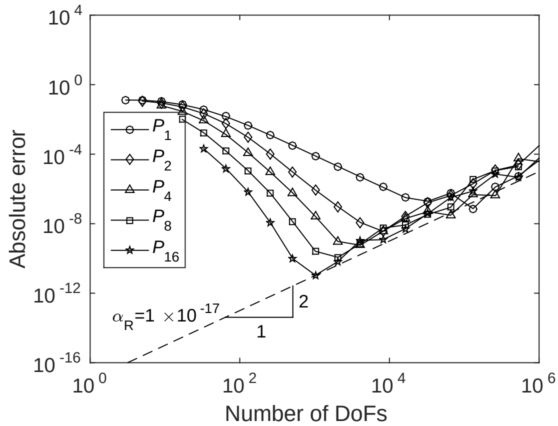
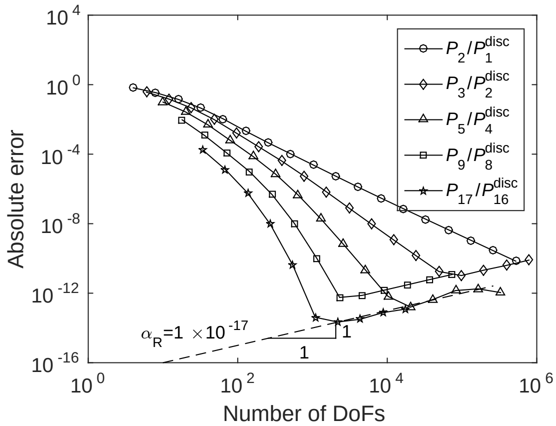
To improve the accuracy of solutions obtained with finite element methods, h-, p- and hp- refinements are widely used. They all aim at decreasing the truncation error by increasing the number of degrees of freedom ("DoFs"). However, when the number of DoFs becomes larger than a critical number N_{crit}, round-off errors accumulate and start to exceed the truncation error, and thus dominate the total error. Further refinements will even result in less accurate solutions. To illustrate a systematic method to identify N_{crit} a posteriori, we focus on the following one-dimensional model problem:

$$\frac{d}{dx}\left(D(x)\frac{du}{dx}\right) + r(x)u(x) = f(x), \qquad x \in I = (0,1),$$

with u denoting the unknown variable, $f(x) \in L^2(I)$ a prescribed right-hand side, and $D(x)$ and $r(x)$ coefficient functions. As an example, we consider the Helmholtz equation with $D(x) = (0.01+x)(1.01-x)$, $r(x)$=-0.01i, $f(x) = 1.0$; $u(0) = 0$ and $u_x(1) = 0$. The absolute errors for the real part of the solution obtained with the standard and mixed FEM are shown in the left and right bound of Fig. 1, respectively [1]. The deal.II finite element code [2] is used.



(a) The standard FEM.



(b) The mixed FEM.

Fig. 1. Absolute errors for the real part of the solution for the above equation. α_R denotes the offset of the line approximating the round-off error.

It shows that N_{crit} strongly depends on the order of the element, p, with N_{crit} decreasing for increasing p, both for the standard and mixed FEM. Thus, by taking higher-order elements, the round-off errors can be reduced, resulting in more accurate solutions. Furthermore, the type of FEM method also influences the accumulation of round-off errors. That is, the mixed FEM allows for more accurate solutions, compared to the most accurate solutions obtained with the standard FEM method.

References

[1] M. Liu, M. Möller, H. M. Schuttelaars, A practical a posteriori strategy to ascertain the optimal number of degrees of freedom for hp-refinement in finite element methods, in preparation.

[2] G. Alzetta, D. Arndt, W. Bangerth, V. Boddu, B. Brands, D. Davydov, R. Gassmöller, T. Heister, L. Heltai, K. Kormann, et al., The deal. ii library, version 9.0, Journal of Numerical Mathematics 26 (4) (2018) 173–183.