Report on the 2D Paper

Jie Liu

Delft University of Technology, the Netherlands

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Problem statement

Equation to be solved

$$\nabla \cdot (T_1 \nabla u) + T_2 \frac{\partial u}{\partial x} + T_3 \frac{\partial u}{\partial y} + T_4 u = f, \qquad (x, y) \in \Omega = [0, 1] \times [0, 1], \tag{1}$$

where T_1, T_2, T_3 and T_4 are coefficient functions[a]. The solution u can be both real-valued and complex-valued if not stated otherwise. By choosing different T_i, we can have Poisson, diffusion or Helmholtz problems.

[a] T_2 and T_3 have not been included practically in the code yet.

Aim of the second paper

1. To determine α_{R} and β_{R} for different FEM methods of different FEM packages for various 2D problems.
2. To choose FEM methods/elements that give smaller round-off error, i.e. α_{R} and β_{R}.
3. To apply the strategy in the 1D paper to find the optimal number of DoFs of 2D problems*.

FEM Status

The status of the application of FEM methods of different FEM packages, including deal.II and FEniCS, on various Eq. (1) is shown in Table 1.

	deal.II	FEniCS
Standard FEM (P_p)	☺[1]	☺
Mixed FEM (RT_p/P_p^{disc})	☺	–
Mixed FEM ($BDM_p/Q_{p-1}^{\text{disc}}$)[2]	☺	☺

Table 1: Status of application of FEM methods. The element degree p can be of different order if not stated otherwise.

[1] Working well.

[2] The notation Q is based on the notation in the deal.II code.

Progress

- Round-off errors of *BDM* elements of deal.II and FEniCS compared.
- The components of different error norms in deal.II are illustrated.

Discussion

1. Results of FEniCS or IGA as a supplement?
2. Based on the results of 2D problems, using the relative error instead of the absolute error for the 2D case?
3. Super convergence common when using RT_p/P_p^{disc} elements solving a problem with the solution only varying on the x direction in deal.II?

Future work

- To consider T_2 and T_3, i.e. first-order parts, of Eq. (1) in deal.II.

Possible topics of the third paper

1. Applying the mixed FEM on problems caused by boundary layers and/or constructing a method to avoid these boundary layers [1].
2. 2D lagrangian polynomials not the same order in each direction, which is a feature of the IGA analysis?


Mohit Kumar, Henk M Schuttelaars, Pieter C Roos, and Matthias Möller.
Three-dimensional semi-idealized model for tidal motion in tidal estuaries.
Ocean dynamics, 66(1):99–118, 2016.