# A practical a posteriori strategy to ascertain the optimal number of degrees of freedom for *hp*-refinement in finite element methods

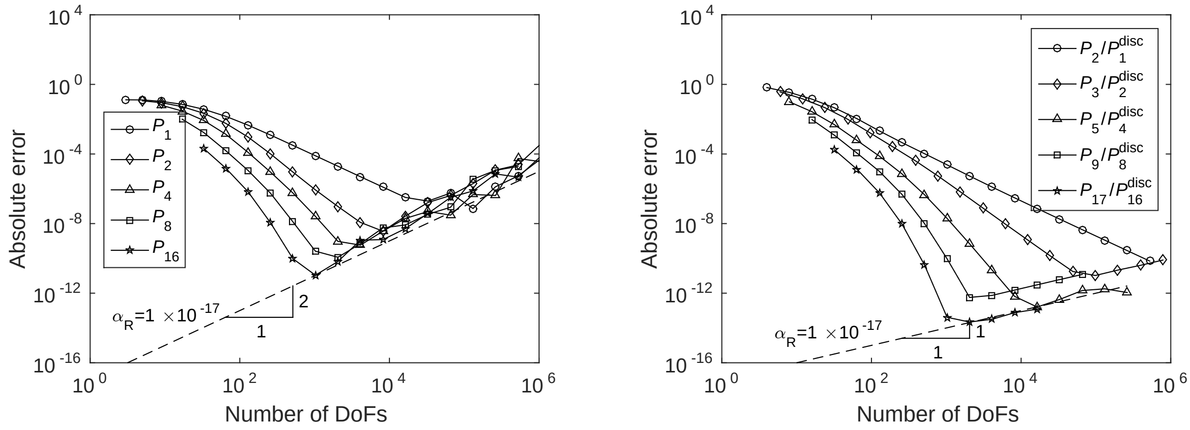
Monte Liu, Matthias Möller, Henk M. Schuttelaars

*Delft Institute of Applied Mathematics, Delft University of Technology,*
*Van Mourik Broekmanweg 6, 2628 XE Delft, The Netherlands*

To improve the accuracy of solutions obtained with finite element methods, *h*-, *p*- and *hp*- refinements are widely used. They all aim at decreasing the truncation error by increasing the number of degrees of freedom ("DoFs"). However, when the number of DoFs becomes larger than a critical number $N_{\text{crit}}$, round-off errors accumulate and start to exceed the truncation error, and thus dominate the total error. Further refinements will even result in less accurate solutions. To identify $N_{\text{crit}}$ a posteriori, we focus on the following one-dimensional model problem:

$$\frac{d}{dx}\left(D(x)\frac{du}{dx}\right) + r(x)u(x) = f(x), \qquad x \in I = (0,1),$$

with $u$ denoting the unknown variable, $f(x) \in L^2(I)$ a prescribed right-hand side, and $D(x)$ and $r(x)$ coefficient functions. For example, when $D(x) = (0.01 + x)(1.01 - x)$, $r(x)$=-0.01$i$, $f(x) = 1.0$; $u(0) = 0$ and $u_x(1) = 0$, the absolute errors for the real part of the solution are shown in Fig. 1 [1]. The deal.II finite element code [2] is used.




**(a)** The standard FEM. **(b)** The mixed FEM.

**Fig. 1.** Absolute errors for the real part of the solution for the above equation. $\alpha_R$ denotes the offset of the line approximating the round-off error.

It shows that $N_{\text{crit}}$ strongly depends on the order of the element, $p$, with $N_{\text{crit}}$ decreasing for increasing $p$. Thus, by taking higher-order elements, the round-off errors can be reduced, resulting in more accurate solutions. Furthermore, the type of FEM method also influences the accumulation of round-off errors. That is, the mixed FEM allows for more accurate solutions, compared to the most accurate solutions obtained with the standard FEM method.

## References

[1] M. Liu, M. Möller, H. M. Schuttelaars, A practical a posteriori strategy to ascertain the optimal number of degrees of freedom for *hp*-refinement in finite element methods, in preparation.

[2] G. Alzetta, D. Arndt, W. Bangerth, V. Boddu, B. Brands, D. Davydov, R. Gassmöller, T. Heister, L. Heltai, K. Kormann, et al., The deal. ii library, version 9.0, Journal of Numerical Mathematics 26 (4) (2018) 173–183.