

Fractal analysis

A fractal is an object or quantity that displays self-similarity on all scales, which means that it contains infinitely complex patterns that are self-similar across different scales. And the fractal analysis aims to measures complexity using the fractal dimension. In order to explain how it works, let's take the one and two dimension as examples. As we all known, line can be used to represent the one dimension and a square is one of the most common two dimension things. And both of them are self-similarity. We may break a line segment into 4 self-similar intervals, each with the same length, and each of which can be magnified by a factor of 4 to yield the original segment. We can also break a line segment into 7 self-similar pieces, each with magnification factor 7, or 20 self-similar pieces with magnification factor 20. In general, we can break a line segment into N self-similar pieces, each with magnification factor N. being different from the line, the square may be broken into N^2 self-similar copies of itself, each of which must be magnified by a factor of N to yield the original figure, such as decomposing a square into 4 self-similar sub-squares, and the magnification factor here is 2. As we can imply, the dimension is simply the exponent of the number of self-similar pieces with magnification factor N into which the figure may be broken. [1]

$$dimension = \frac{\log(\text{numbers of self - similar pieces})}{\log(\text{magnification factors})}$$

Back to the fractal analysis of the image drawn by Jackson Pollock, we have to calculate its numbers of self-similar pieces and magnification factors by counting the grids.

The code is attached as followed.

```
for (xi = x*resolution; xi < x*resolution + resolution; xi++){
    for (yi = y*resolution; yi < y*resolution + resolution; yi++){
        if (Image[yi][xi] != 0){
            sum ++;
            break; } }
    if (Image[yi][xi] != 0){
        break;} }
```

So we obtain the calculation results, and use Log function to transform original data.

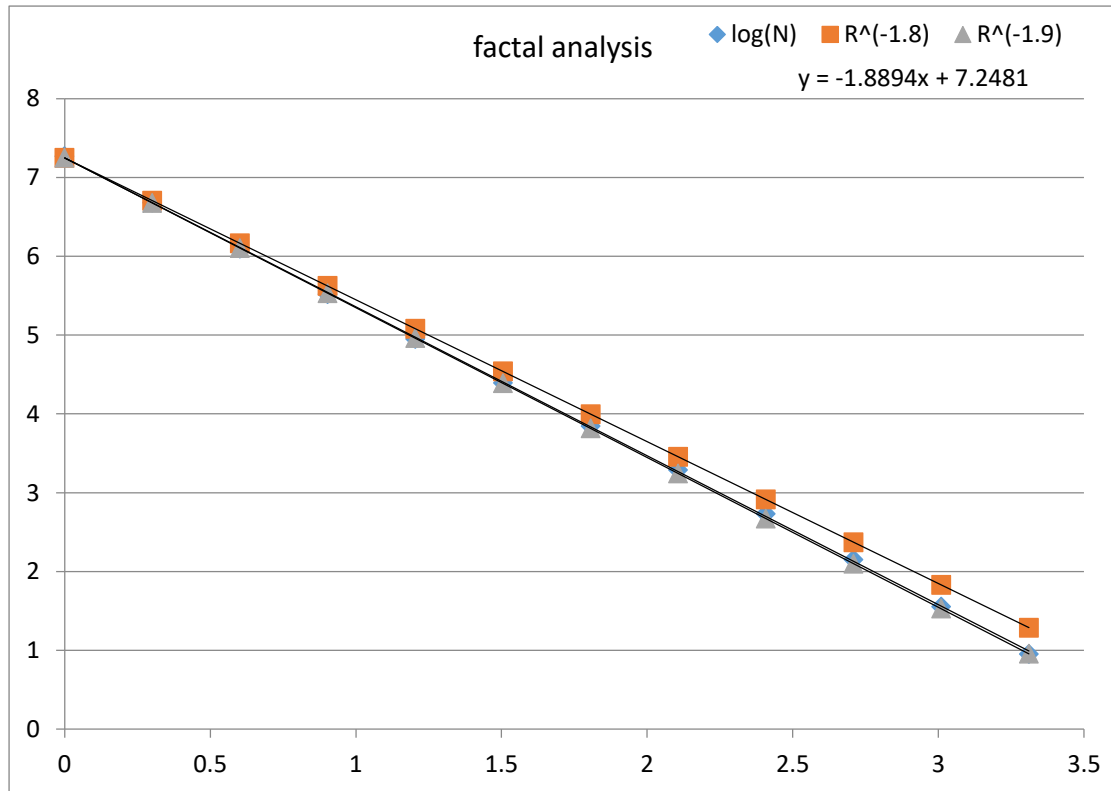
1	18675078
2	4782981
4	1248217
8	330985
16	89059
32	24728
64	6975
128	1941
256	542
512	142
1024	36
2048	9

Fig.2 the original calculation results

7.271262	0
6.679699	0.30103
6.09629	0.60206
5.519808	0.90309
4.949678	1.20412
4.393189	1.50515
3.843544	1.80618
3.288026	2.10721
2.733999	2.40824
2.152288	2.70927
1.556303	3.0103
0.954243	3.31133

Fig.3 The transformation in Log function

At last, we got its distribution and regression function, in which the value of the dimension is -1.8894.



Bibliography:

[1] The Chaos Game, Robert L. Devaney, Boston University