Homework 3 Question Math Formula Part

Poisson Distribution

Event: must happen in the limited space and time.

Example:

Eg1 One year time, the specific size of meteors which are dropped on the earth.

Eg2 Numbers of patients who visited the emergency room between specific times such as 10:00 pm - 11:00pm.

Eg3 Given an ellismate to the starbuck how many sandwiches should be prepared for the morning section in a specific time period.

Build Model Basic Requirement:

1. **K should be a variable which represents the events happening time, k range is 0 to n.**
2. **Each event should be an independently.event**
3. **The constant value which can represent a probability in the same time period should be the same.**
4. **Just one event happened at the same time.**
5. **There is a ratio into probability and time period.**
6. **The binomial distribution should exist in the Poisson Distribution.**

Calculate the probability for the lambda:

We can use lambda to represent the specific period the event happened probability.

Here is the Poisson Distribution Form:

$$p(X|\lambda) = \frac{\exp^{-\lambda} \lambda^{x_i}}{x_i!}$$

We need to derive the formula for this rule and Elismate the max likelihood from this formula.

Here are the steps for derivation of this formula.

Step1:

We writing out the likelihood function(L), let θ = λ

So the formula will change to

$$L(\theta) = \prod_{i=1}^{N} \frac{e^{-\lambda} \lambda^{x_i}}{x_i!}$$

Step2:

We write the log toe of the function by taking the log.

$$LL(\theta) = \log\left(\prod_{i=1}^{N} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}\right)$$

Step3:

Let us use some simplifications for the formula.

$$LL(\theta) = \log\left(\prod_{i=1}^{N} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}\right)$$

$$= \sum_{i=1}^{N} \log\left(\frac{e^{-\lambda}\lambda^{x_i}}{x_i!}\right)$$

Because we have some log properties:

$$\log \prod_{i}^{N} x_i = \sum_{i}^{N} \log x_i$$

.

Then we can use quotient rule, power rule to simplify the formula again.

$$= \sum_{i=1}^{N} \log\left(\frac{e^{-\lambda}\lambda^{x_i}}{x_i!}\right)$$

$$= \sum_{i=1}^{N} [\log(e^{-\lambda}) + \log(\lambda^{x_i}) - \log(x_i!)]$$

$$= \sum_{i=1}^{N} [-\lambda\log(e) + x_i\log(\lambda) - \log(x_i!)]$$

$$= \sum_{i=1}^{N} [-\lambda + x_i \log(\lambda) - \log(x_i!)]$$

Step4:

Let us use summation rules to simplify depth.

(Using online resources:

https://www.mathdoubts.com/constant-multiple-rule-of-derivatives-proof/

https://tutorial.math.lamar.edu/classes/calci/summationnotation.aspx

https://dwstockburger.com/Introbook/sbk10.htm

$$LL(\theta) = \sum_{i=1}^{N} [-\lambda + x_i \log(\lambda) - \log(x_i!)]$$

Rules we using(refer to the links):

$$= \sum_{i=1}^{N} -\lambda + \sum_{i=1}^{N} x_i \log(\lambda) - \sum_{i=1}^{N} \log(x_i!)$$

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \qquad \sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

$$= -\lambda N + \log(\lambda) \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} \log(x_i!)$$

$$\sum_{i=1}^{n} c a_i = c * \sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} c = n * c$$

Step5:

Calculate the derivative of the log likelihood function with the lambda.

$$LL(\theta) = -\lambda N + \log(\lambda) \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} \log(x_i!)$$

$$\frac{\partial LL(\theta)}{\partial \lambda} = -1 * N + \frac{1}{\lambda} \sum_{i=1}^{N} x_i$$

We treat derivative of λ with respect λ is 1. log(λ) with respect to λ is 1/λ. We can drop the last summation because there is no λ in there.