Probability Basics

ALL ABOUT LIKELIHOOD

What is the probability of finding a four-leaf clover?

The probability of finding a four-leaf clover is estimated to be about **1 in 5,000 to 1 in 10,000** clovers. The exact odds can vary depending on environmental conditions and the specific clover population you're examining.

Factors that can influence the probability:

- **Genetics:** Some clover strains are more prone to genetic mutations that lead to four leaves.
- **Soil and environment:** Stressful conditions can sometimes increase mutation rates.
- **Cultivation:** Some farms or gardens breed four-leaf clovers intentionally, which increases the odds in those areas.

Would you like tips on how to improve your chances of finding one?

What is the probability getting a head when flipping a coin?

The probability of getting a head when flipping a **fair** coin is:

$$\frac{1}{2} \text{ or } 50\%$$

This assumes the coin is perfectly balanced and not biased in any way. Real coins may have slight imperfections, but for most practical purposes, the 50% assumption is considered accurate.

Are you exploring probabilities for a specific project or just curious?

What is the probability that a meteorite will collide with Earth?

The probability of a **meteorite** (a space rock that survives its trip through Earth’s atmosphere and lands) colliding with Earth varies depending on the size of the object:

Small meteorites (pebble- to basketball-sized):

- **Very common** — Earth is hit by **about 17 meteors per day** that are large enough to reach the ground, according to NASA.
- **Probability**: Essentially **100%** — Earth is constantly being hit by small meteorites.

Larger impacts (city- or region-destroying):

- **1-kilometer-wide objects** (mass-extinction level):
  - Estimated to hit Earth **about once every 500,000 to 1 million years.**
  - **Annual probability**: ~**0.000001 to 0.000002** (or 0.0001% to 0.0002%)

Civilization-threatening impacts (like the one that killed the dinosaurs, ~10 km wide):

- Occur roughly **once every 100 million years.**
- **Annual probability**: ~**0.00000001** (or 0.000001%)

So, while Earth gets hit all the time by small meteorites, the chance of a catastrophic impact is very low — but **not zero**, which is why it's a focus of planetary defense efforts.

The Interpretations of Probabilities

AI Overview

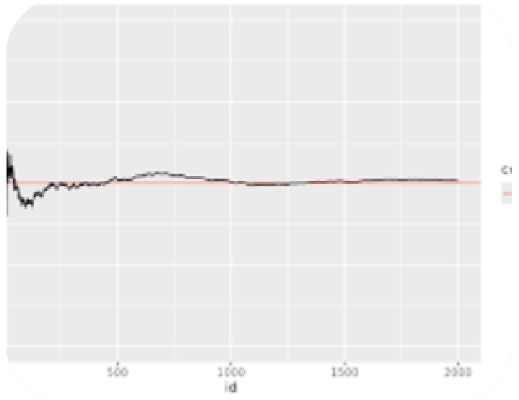
Probabilities can be interpreted in several ways, but two main approaches are the frequentist and Bayesian interpretations. The frequentist approach views probabilities as long-run relative frequencies, while the Bayesian approach interprets them as degrees of belief or partial belief.

Frequentist Interpretation:

- In this view, a probability represents the long-run proportion of times an event would occur if a random phenomenon were repeated many times.
- For example, the probability of flipping a fair coin and getting heads is 0.5, meaning that in a large number of coin flips, approximately half of them would result in heads.
- Frequentist probabilities are often associated with physical systems and repeatable experiments.
- Frequentists focus on the distribution of data and using statistical inference to draw conclusions about the underlying probability distribution.

AI Overview

The English mathematician John Kerrich, while imprisoned by the Germans during World War II, flipped a coin 10,000 times. He obtained 5067 heads. This experiment was conducted to study empirical probability.



Bayesian Interpretation:

- Bayesian probabilities are interpreted as degrees of belief or credences that an agent assigns to the occurrence of an event.
- These beliefs are updated based on new evidence or information.
- Bayesian probabilities can also be applied to the parameters of a statistical model, allowing for probability statements about these parameters.
- Bayesian approaches are more flexible and can handle situations where prior knowledge or beliefs are relevant.

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Civilization-threatening impacts (like the one that killed the dinosaurs, ~10 km wide):

- Occur roughly **once every 100 million years.**
- **Annual probability**: ~**0.00000001** (or 0.000001%)

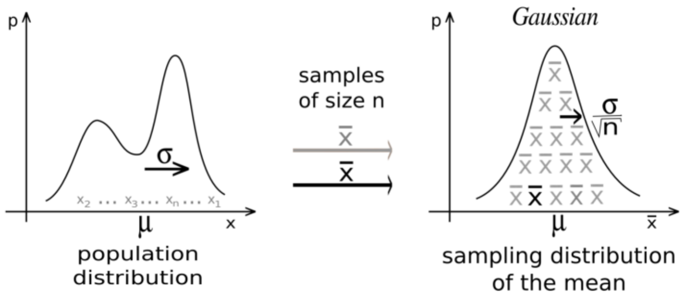
What do we want to learn?

Sets of creatures with two legs, and creatures that fly

- Probabilities of an EVENT and its complement
- Probabilities of Intersection or Union of Events
- Joint Probability and Marginal Probability
- Conditional Probabilities and Bayes' Rule

Why do we learn these?

- Understand the Central Limit Theorem
- Understand the behaviors of sample means.
- Understand the meaning of P-values



Example: Comparing Test Scores

Suppose you want to compare the average test scores of two different teaching methods.

🎓 Study Setup:

- **Group A (Traditional Method):**
  - Sample size: 10 students
  - Scores: 78, 85, 82, 88, 75, 79, 84, 80, 77, 83
- **Group B (New Method):**
  - Sample size: 10 students
  - Scores: 85, 89, 90, 92, 88, 91, 87, 86, 90, 89

🧪 Question:

Is there a statistically significant difference in average test scores between the two groups?

Step 4: Look up p-value

A t-statistic of approximately **-5.12** with ~15 degrees of freedom gives a **very small p-value** (< 0.001).

✅ Conclusion:

Since the p-value is much smaller than 0.05, you **reject the null hypothesis**. There **is a statistically significant difference** in test scores between the two teaching methods.

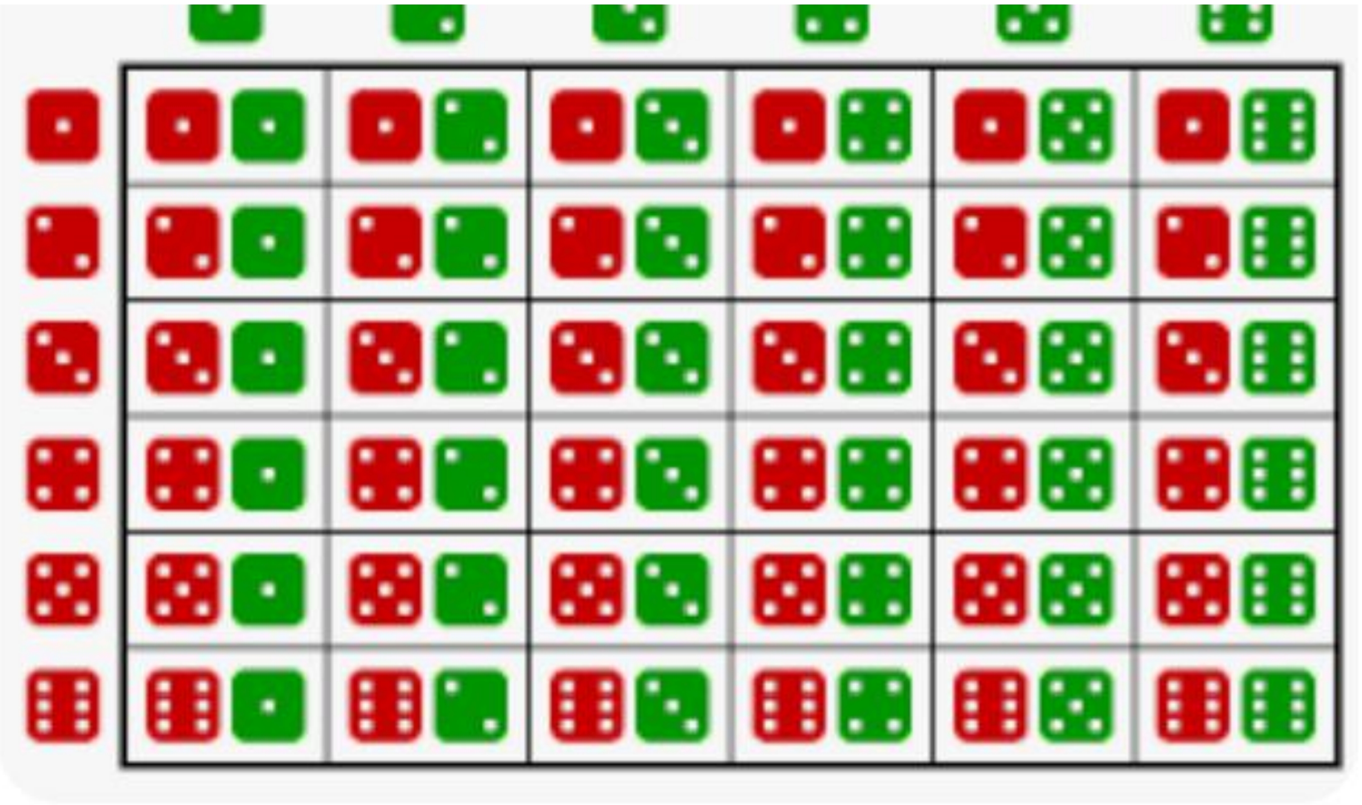
Would you like me to retry the code to give you the exact numbers later, or continue with another example?

Let's get started!

- We will use a commonly used example to introduce sample space and events.

Sample Space

THE COLLECTION OF ALL POSSIBLE OUTCOMES



Medium

Intro to Probability. It's easy but ...

Sample Space

- The collection of all possible outcomes

  It is often denoted as $\mathcal{S}$.

  In this case,

  $$\mathcal{S} = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),\\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6),\\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6),\\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6),\\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6),\\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

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  In this case,

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Events

- Subsets of a sample space

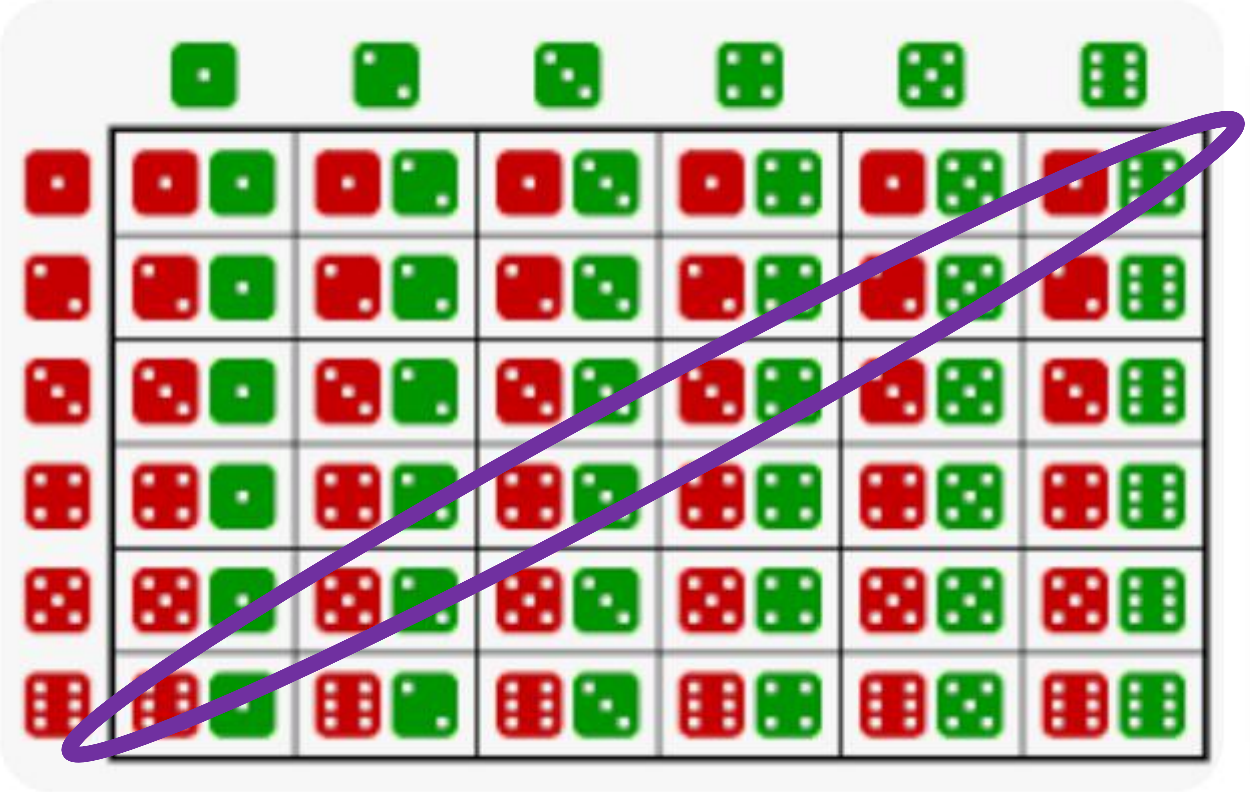
For example: Getting a 7

This can be denoted as an event

A = obtaining a 7

Or more precisely

A={(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)}



Medium

Intro to Probability. It's easy but ...

Probabilities of Events

- $Pr(\ \{(1,1\}\ ) = Pr(\ \{(1,2)\}\ ) = Pr(\ \{(1,2)\}\ ) = Pr(\ \{(1,2)\}\ ) = \ldots = 1/36$
- Each rolling outcome can be viewed as an event, sometimes called a simple event.
- These outcomes/events are called equally likely events.

- If $A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$, then $Pr(A)=6/36=1/6$.
- A is not a simple event; it is the union of a group of simple events.

Complement, Union and Intersection of Events

For convenience, let's define another event called B.

- The complement of an event A is the set of outcomes that are not in A.
- The union of two events A and B is the set of outcomes that are in A, or B, or both.
- The intersection of two events A and B is the set of outcomes that are in both A and B.

Examples

- Consider two events:

  A = {(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)},

  B = Getting a double
  ={(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

Then it is reasonable to obtain probabilities as :

- Pr(A^c)=1-1/6=5/6
- Pr($A \cup B$)=12/36=1/3
- Pr($A \cap B$)= 0

- The complement of A is

$$A^c = \{(1,1),(1,2),(1,3),(1,4),(1,5),\\ (2,1),(2,2),(2,3),(2,4),(2,6),\\ (3,1),(3,2),(3,3),(3,5),(3,6),\\ (4,1),(4,2),(4,4),(4,5),(4,6),\\ (5,1),(5,3),(5,4),(5,5),(5,6),\\ (6,2),(6,3),(6,4),(6,5),(6,6)\}$$

- The union of A and B is

$$A \cup B = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),\\ (1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$$

- The intersection of A and B is

$$A \cap B = \{\ \} = \emptyset$$

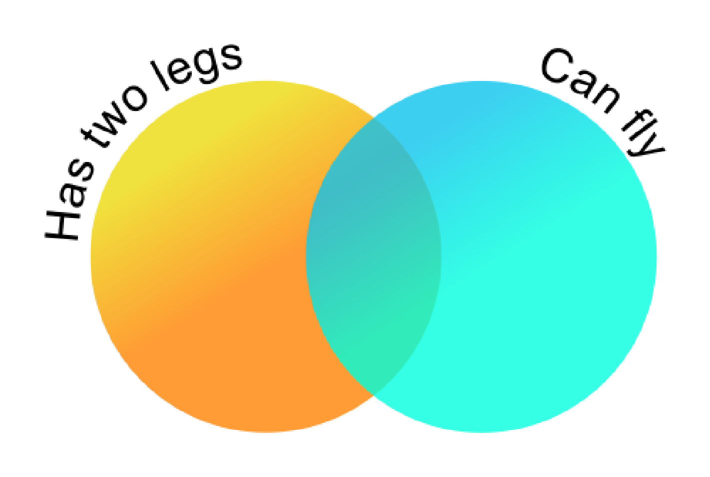
Remarks

- Please note that in a process, the sample space can be defined differently depending on the measurement of interest.
- For example, if the only interest is in the total dots obtained when rolling two dice. The sample space can be written as $\mathcal{S}$={2,3,4,5,6,7,8,9,10,11,12}.
- The simple events are $\{i\}$, where i=2,…,12. These are not equally likely events. But the probabilities of these simple events can still be written out as shown in the table below:

	2	3	4	5	6	7	8	9	10	11	12
prob	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

- Event A, described in the previous example, is then A = {7}.
- Event B, described in the previous example, cannot be written out easily in this case.

Venn Diagram



Sets of creatures with two legs, and creatures that fly

Venn diagram

Article Talk

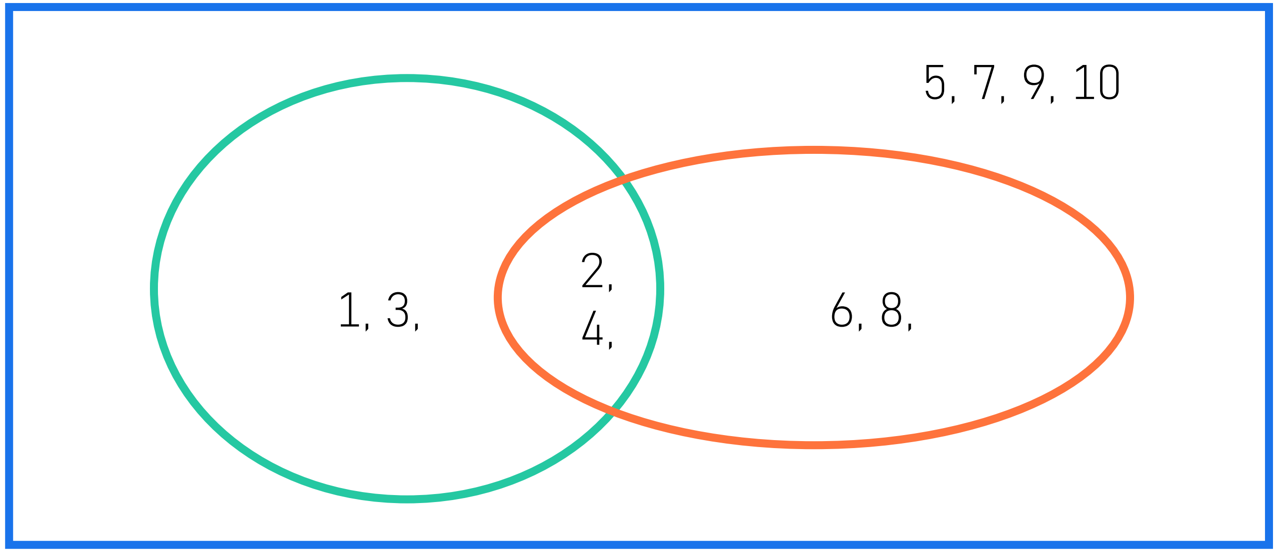
From Wikipedia, the free encyclopedia

A **Venn diagram** is a widely used diagram style that shows the logical relation between sets, popularized by John Venn (1834–1923) in the 1880s. The diagrams are used to teach elementary set theory, and to illustrate simple set relationships in probability, logic, statistics, linguistics and computer science. A Venn diagram uses simple closed curves on a plane to represent sets. The curves are often circles or ellipses.

Similar ideas had been proposed before Venn such as by Christian Weise in 1712 (*Nucleus Logicoe Wiesianoe*) and Leonhard Euler in 1768 (*Letters to a German Princess*). The idea was popularised by Venn in *Symbolic Logic*, Chapter V "Diagrammatic Representation", published in 1881.

Example

- Let's consider a set of integers $\mathcal{S}$={1:10}, and events A={1,2,3,4}, B={2,4,6,8}.
- Then, a Venn Diagram can be drawn as follows:



An Example from Sec. Math 2 Extended

- Amara thinks that chocolate ice cream is the greatest! She cannot even imagine that someone saying that bland vanilla is better. She claims that chocolate is the favorite ice cream around the world. Her friend, Isla, thinks that vanilla is much better and more popular. To settle the argument, they create a survey asking people to choose their favorite ice cream flavor between chocolate and vanilla. After completing the survey, the following results came back.

	Chocolate	Vanilla	Total
Teens	4732	4024	8756
Adults	2420	3590	6010
Total	7152	7614	14766

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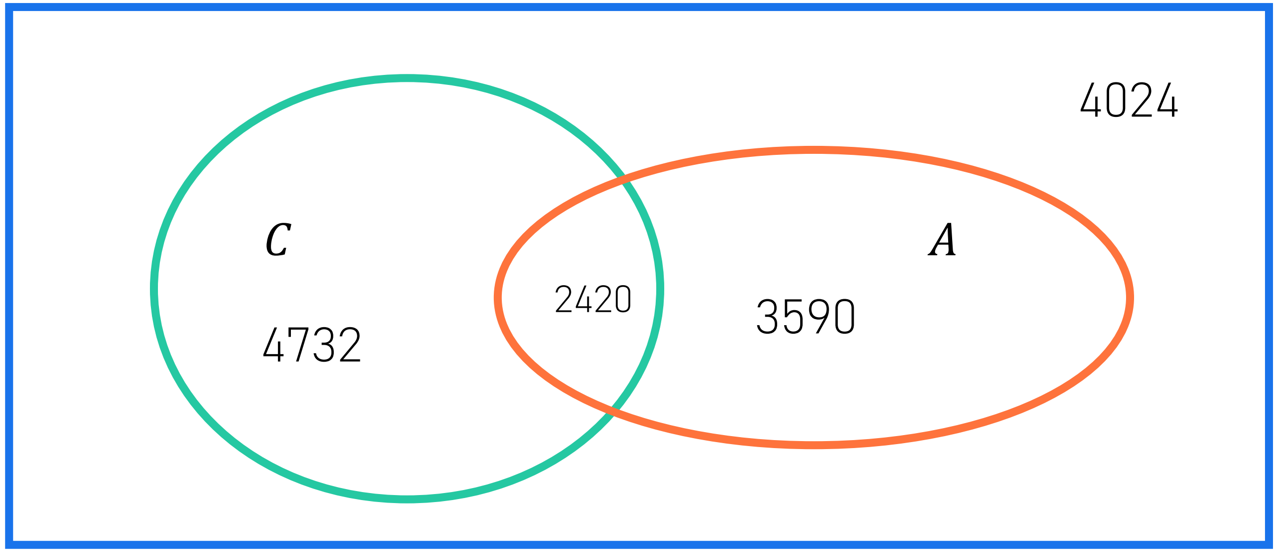
Example: Chocolate or Vanilla

- Events $C = \{Favor\ Chocolate\}\ \ and\ C^C = \{Favor\ Vanilla\}$
- Events $\mathrm{A} = \{Being\ an\ Adult\}\ \ and\ A^C = \{Being\ a\ Teen\}$



	Chocolate	Vanilla	Total
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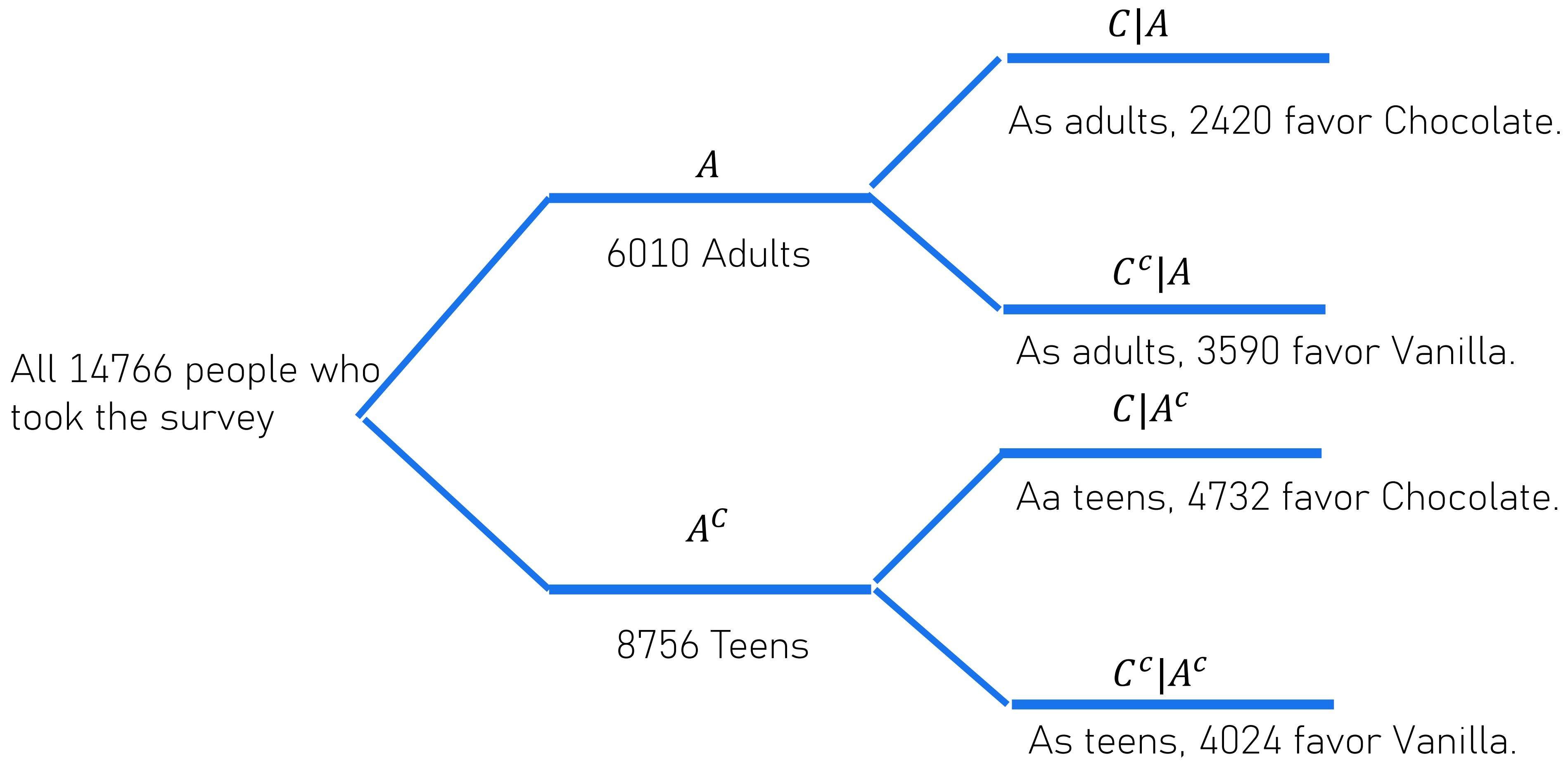
Tree Diagram



Remarks

- A survey is often a special kind of observational study.
- The chocolate-vanilla example data came from a survey.
- The study units are people who took the survey.
- The counts and proportions are sample statistics.
- We cannot draw a cause-and-effect relationship, but we can discuss the association.
- Because we have no information on how the 14766 people were recruited, this may not be a representative sample of the targeted population. Be cautious when generalize the results to a bigger population.

Back to the Discussion of Probabilities

- Suppose that we do not have a bigger population of interest.
- Suppose that the whole group of 14766 people is the population we are interested in.
- Then, the counts and proportions are the summary of the population.
- Then, the population proportions can be interpreted as probabilities.
- The sample space is the collection of outcomes(responses) of all people

Note:

- A **sample statistic** is a numerical summary of a sample.
- A **population parameter** is a numerical summary of a population.

	Chocolate	Vanilla	Total
Teens	4732	4024	8756
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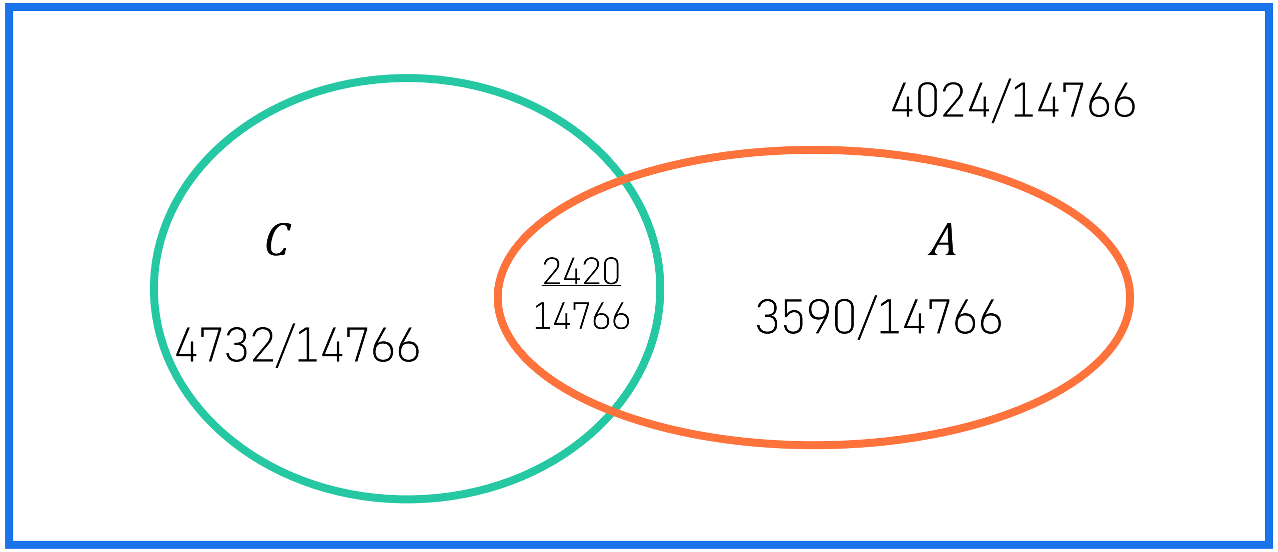
Revisit Example: Chocolate or Vanilla

- Events $C = \{Favor\ Chocolate\}\ \ and\ C^C = \{Favor\ Vanilla\}$
- Events $\mathrm{A} = \{Being\ an\ Adult\}\ \ and\ A^C = \{Being\ a\ Teen\}$



	Chocolate	Vanilla	Total
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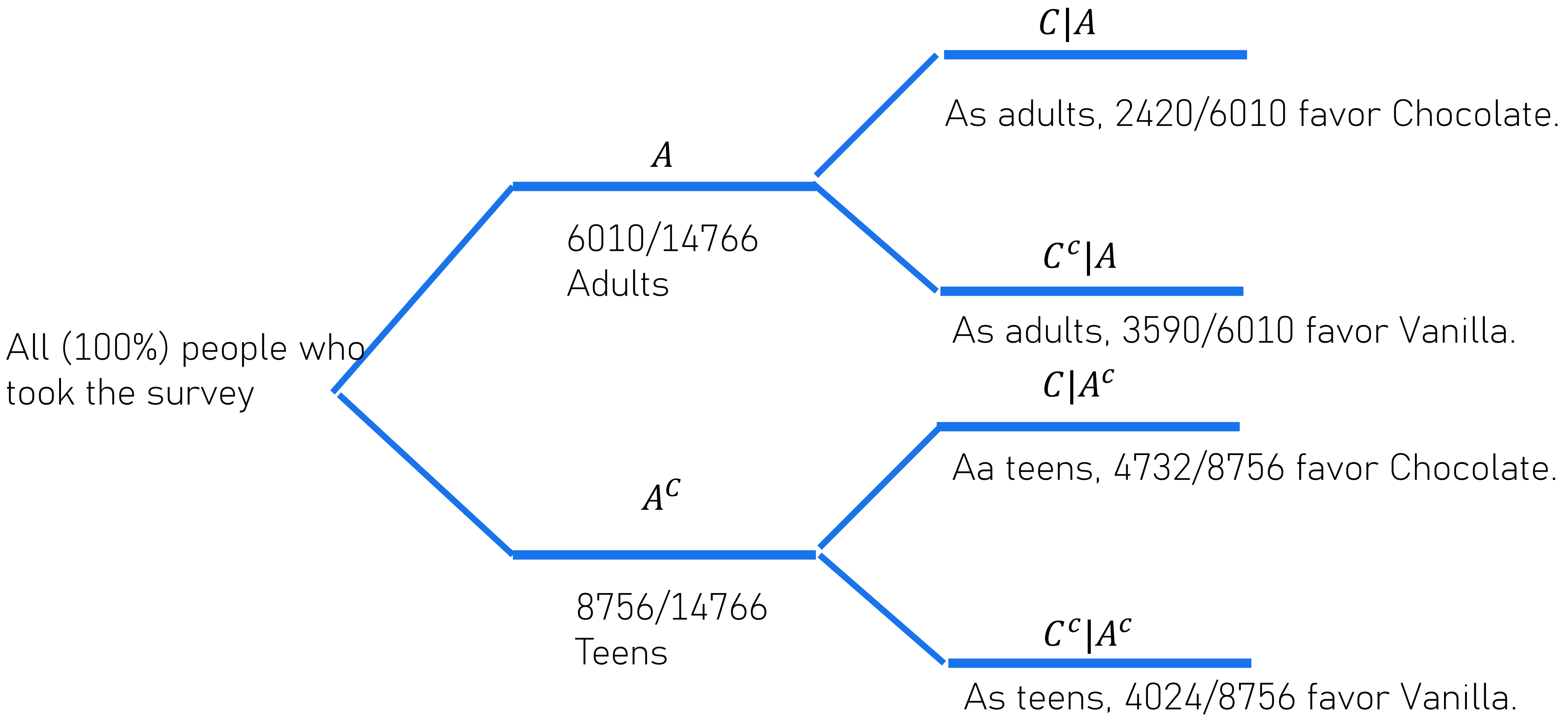
Revisit Tree Diagram



More Remarks

- We use proportions as probabilities because we suppose that each person in the population has the same chance to be selected, and each of their responses (outcomes) has the same likelihood to appear in the sample space.

Interesting Probabilities

- What is the probability that a randomly selected adult favors the chocolate flavor?
- What is the probability that a randomly selected person is an adult and favors the chocolate flavor?
- What is the probability that a randomly selected person is an adult?

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Conditional Probabilities

- What is the probability that a randomly selected adult favors the chocolate flavor?

$$\Pr(C|A) = \frac{\Pr(A \ \cap C)}{P(A)}$$

Note: Pr(C|A) is read as “probability of C given A”.

Joint Probability

	Chocolate	Vanilla	Total
Teens	4732	4024	8756
Adults	2420	3590	6010
Total	7152	7614	14766

- What is the probability that a randomly selected person is an adult and favors the chocolate flavor?

$$\Pr(A \cap C)$$

	Chocolate	Vanilla	Total
Teens	4732	4024	8756
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Total	7152	7614	14766

Marginal Probability

- What is the probability that a randomly selected person is an adult?

$$\Pr(A)$$

	Chocolate	Vanilla	Total
Teens	4732	4024	8756
Adults	2420	3590	6010
Total	7152	7614	14766

Let's try more

- $\Pr(C)$
- $\Pr(A^c)$
- $\Pr(C^c)$
- $\Pr(A^c \cap C)$
- $\Pr(A \cap C^c)$
- $\Pr(A^c \cap C^c)$

- $\Pr(C^c|A)$
- $\Pr(C|A^c)$
- $\Pr(C^c|A^c)$

- $\Pr(A|C)$
- $\Pr(A^c|C)$
- $\Pr(A|C^c)$
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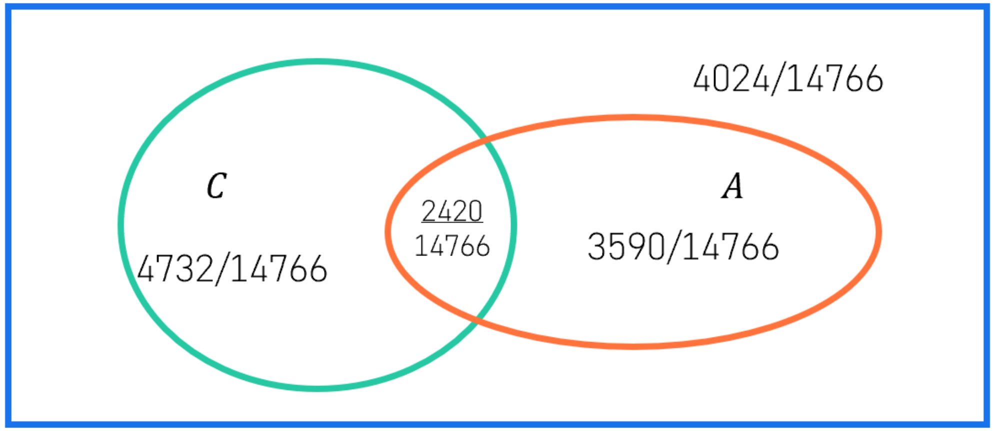
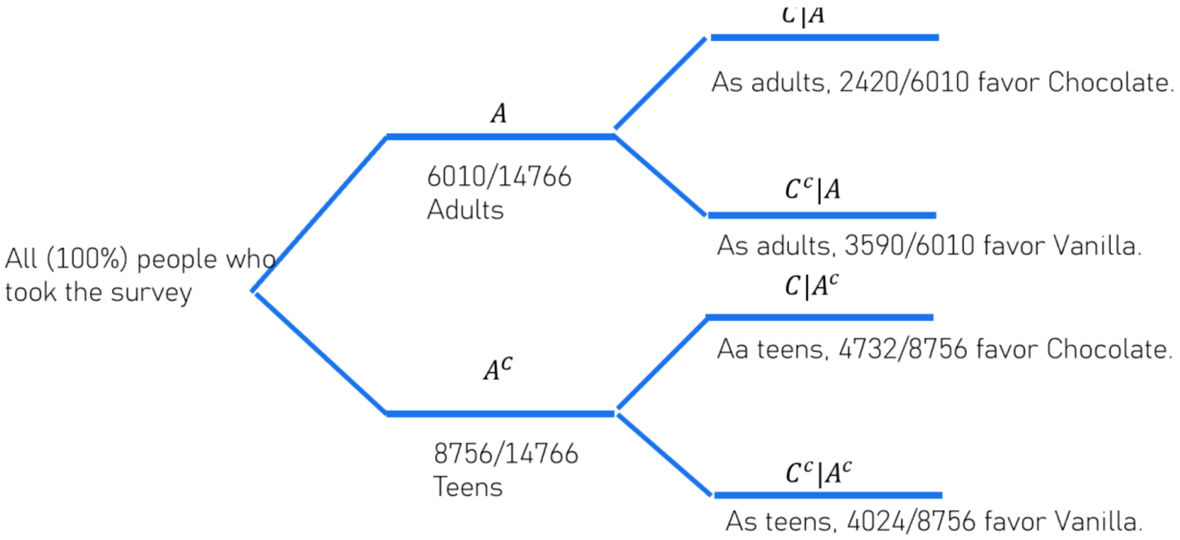




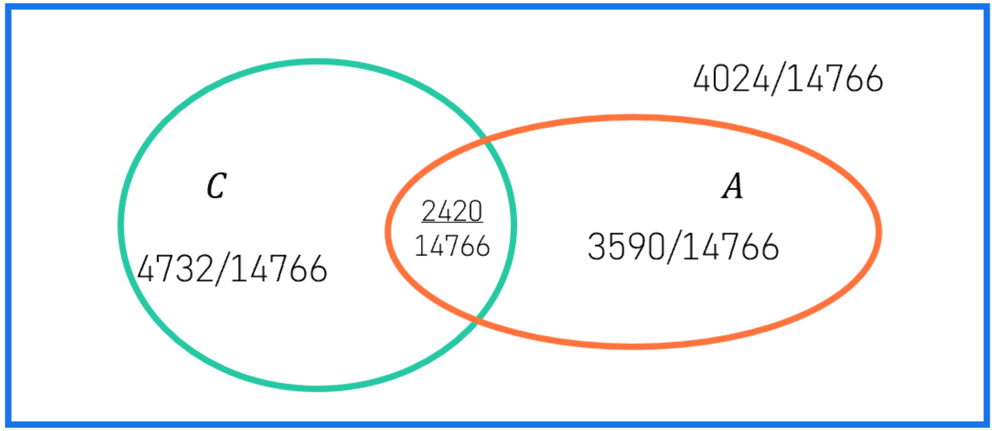
Let's try more

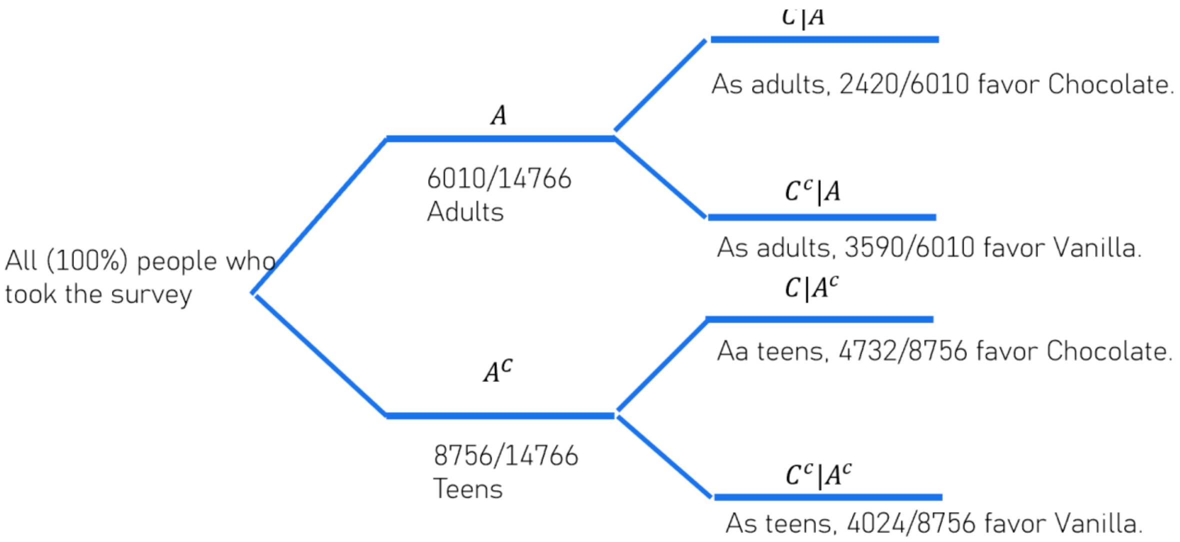
- $\Pr(C) = 0.484$
- $\Pr(A^c) = 0.593$
- $\Pr(C^c) = 0.516$
- $\Pr(A^c \cap C) = 0.320$
- $\Pr(A \cap C^c) = 0.243$
- $\Pr(A^c \cap C^c) = 0.273$

- $\Pr(C^c|A) = 0.597$
- $\Pr(C|A^c) = 0.540$
- $\Pr(C^c|A^c) = 0.460$

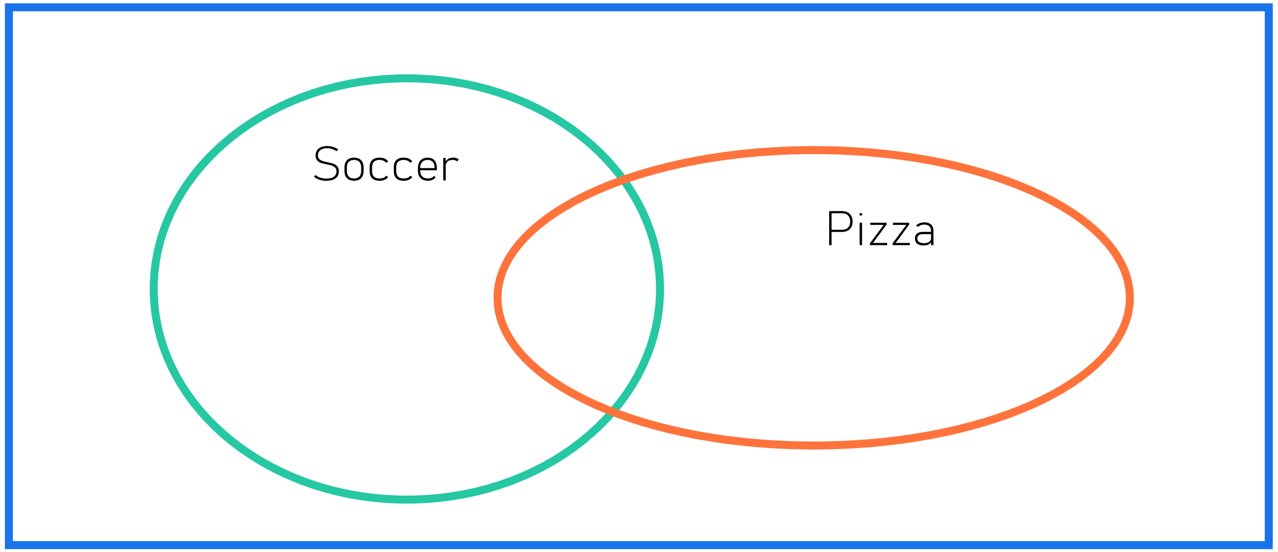
- $\Pr(A|C) = 0.338$
- $\Pr(A^c|C) = 0.662$
- $\Pr(A|C^c) = 0.471$
- $\Pr(A^c|C^c) = 0.529$

	Chocolate	Vanilla	Total
Teens	4732	4024	8756
Adults	2420	3590	6010
Total	7152	7614	14766





Let's try another example



Pr(Soccer)=0.4
Pr(Pizza)=0.25
Pr(Soccer and Pizza)=0.1

$$\Pr(Soccer \cup Pizza) = ?$$

Pr(Soccer)=0.4
Pr(Pizza)=0.25
Pr(Soccer and Pizza)=0.1

Another way to solve

	Soccer	Not Soccer	Total
Pizza	0.1		0.25
Not Pizza			
Total	0.4		1.00

$\Pr(Soccer \cup Pizza) = ?$

Pr(Soccer or Pizza) = (0.4-0.1) + (0.25 – 0.1)+0.1 = 0.4 + 0.25 – 0.1

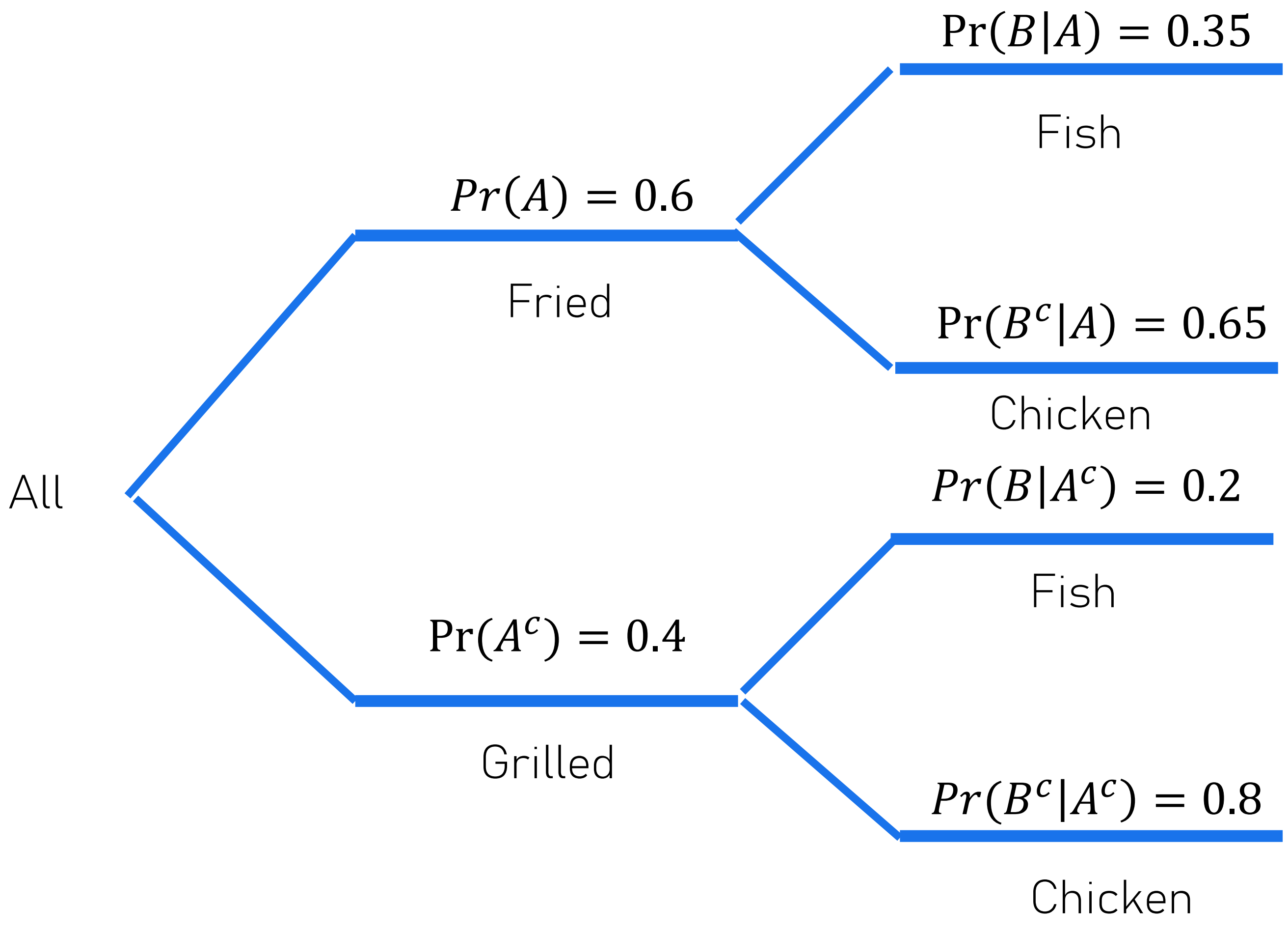
Addition Rule

For any two events A and B,

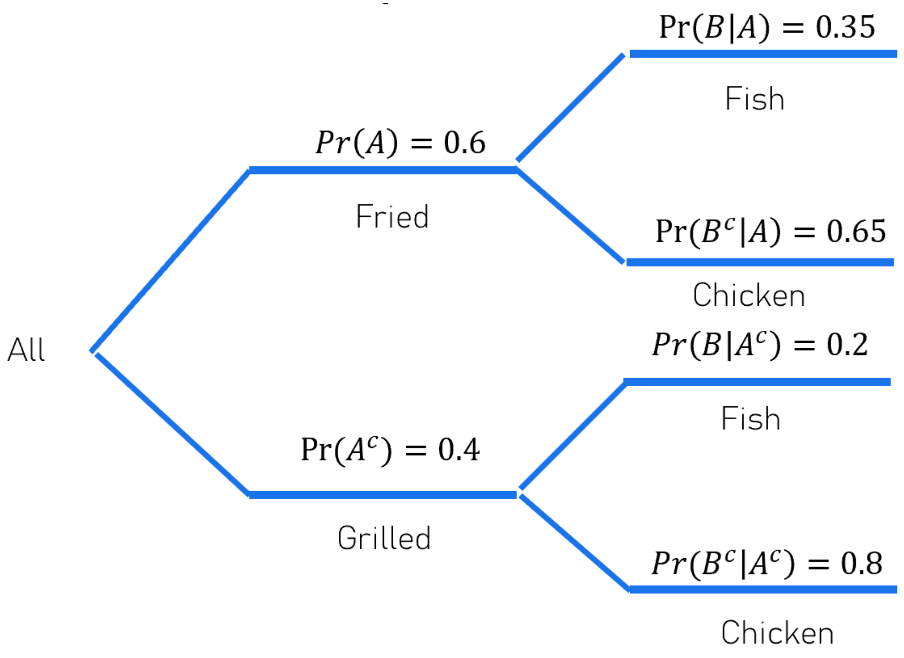
$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

One More Example

$\Pr(A|B) = ?$



My Intuition



	Fish	Chicken	Total
Fried	0.35*0.60	0.65*0.60	0.60
Grilled	0.2*0.40	0.8*0.40	0.40
Total			1.00

$\Pr(A|B) = ?$

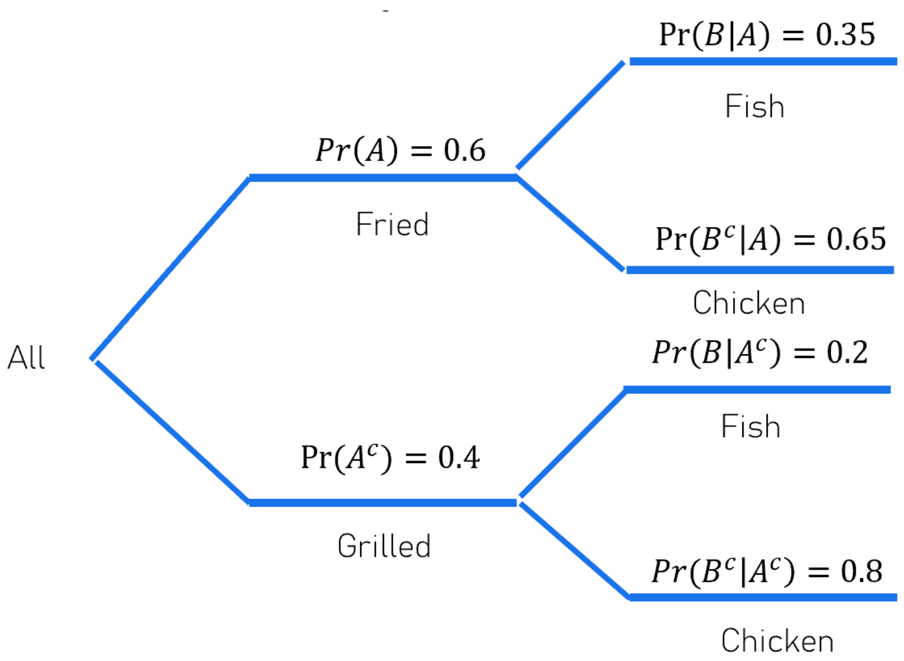
$$\Pr(Fired \mid Fish) = \frac{0.35 * 0.60}{0.35 * 0.60 + 0.2 * 0.40}$$

Bayes' Rule

Bayes rule

$$P(A|B) = \boxed{\frac{P(A \cap B)}{P(B)}} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)}$$

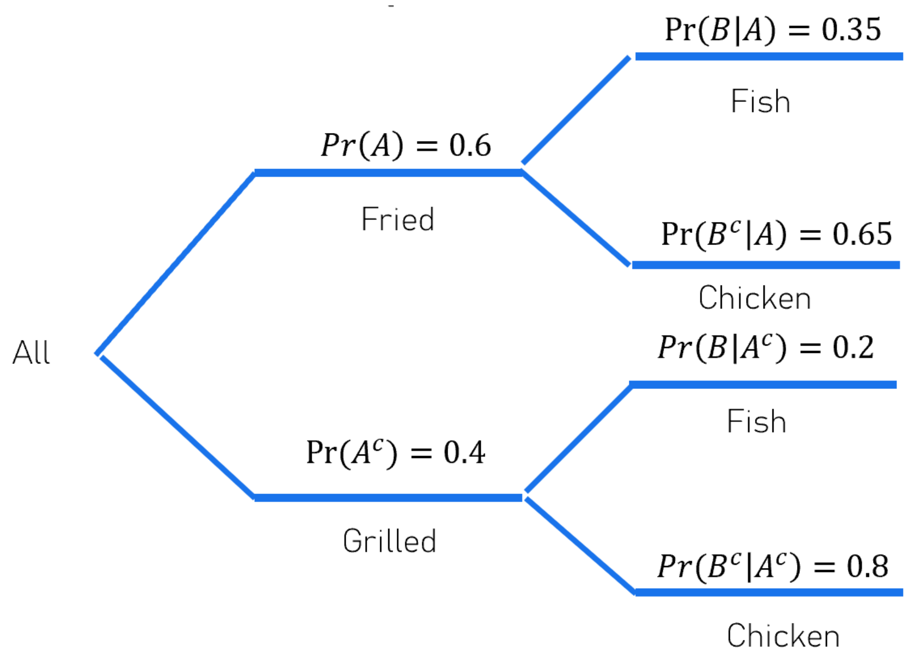
Pr(Fried|Chicken)



	Fish	Chicken	Total
Fried	0.35*0.60	0.65*0.60	0.60
Grilled	0.2*0.40	0.8*0.40	0.40
Total			1.00

$$\Pr(Fired \mid Chicken) = \frac{\qquad * \qquad}{\qquad + \qquad}$$

Pr(Grillled|Chicken)



	Fish	Chicken	Total
Fried	0.35*0.60	0.65*0.60	0.60
Grilled	0.2*0.40	0.8*0.40	0.40
Total			1.00

$$\Pr(Fired \mid Chicken) = \frac{*}{+}$$

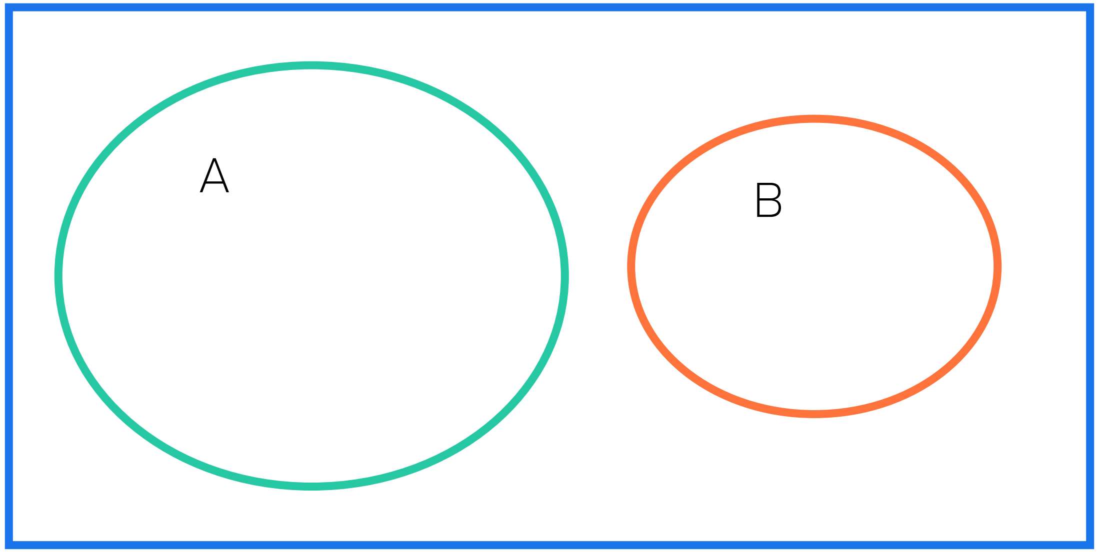
Application of the Bayes Rule – This is too much for us, neglect if not interested in

- I believe the probability of obtaining a head when flipping a regular quarter coin is θ, that is $\Pr(H) = \theta$.
- I don't know the actual value of θ. But I know it could be any value between 0 and 1 with an equal likelihood.
- Then I flip the coin 10 times, obtaining "HTTHTHTTHH"
- Now using the Bayes theory, I know the posterior distribution of θ is Beta(6,6). The best estimate of the value of θ is $\mathbf{0.5}$.

$$f(\theta|x_1, \dots, x_n) = \frac{f(\theta) f(x_1, \dots, x_n|\theta)}{f(x_1, \dots, x_n)}$$

$$= \frac{1 \cdot \theta^{\sum x_i} \cdot (1-\theta)^{n-\sum x_i}}{\int_0^1 1 \cdot \theta^{\sum x_i} \cdot (1-\theta)^{n-\sum x_i} \, d\theta}$$

Relationship: Mutually Exclusive



If A and B are mutually exclusive, then

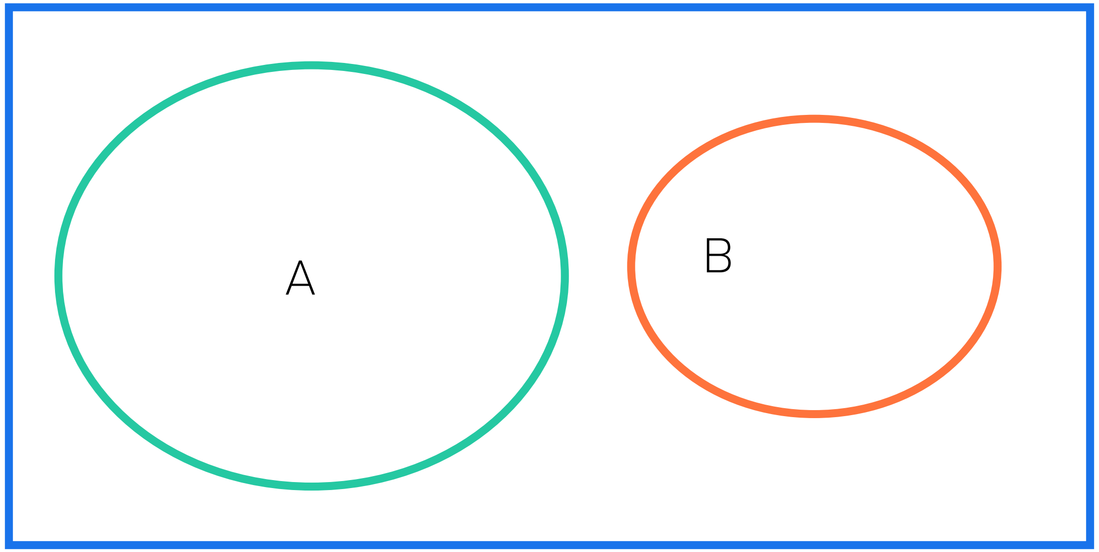
$$P(A \cap B) = 0.$$

The converse is true too.

This relationship is also called **disjoint**.

As a result, $P(A \cup B) = P(A) + P(B)$

Example: Rolling One Die

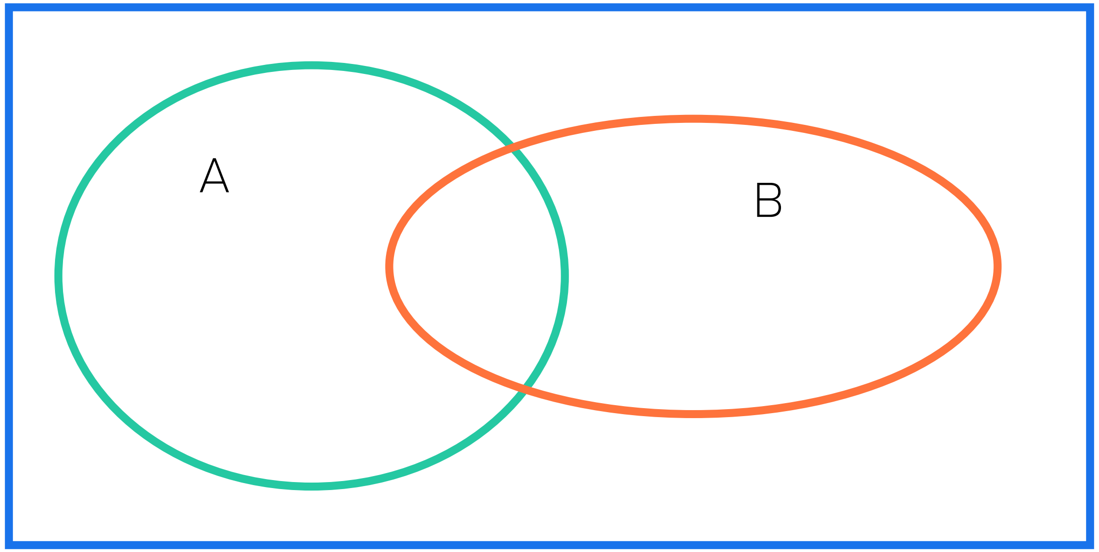


A=getting a 6 = {6}

B=getting a 1 = {1}

A and B are mutually exclusive.

Relationship: Independent

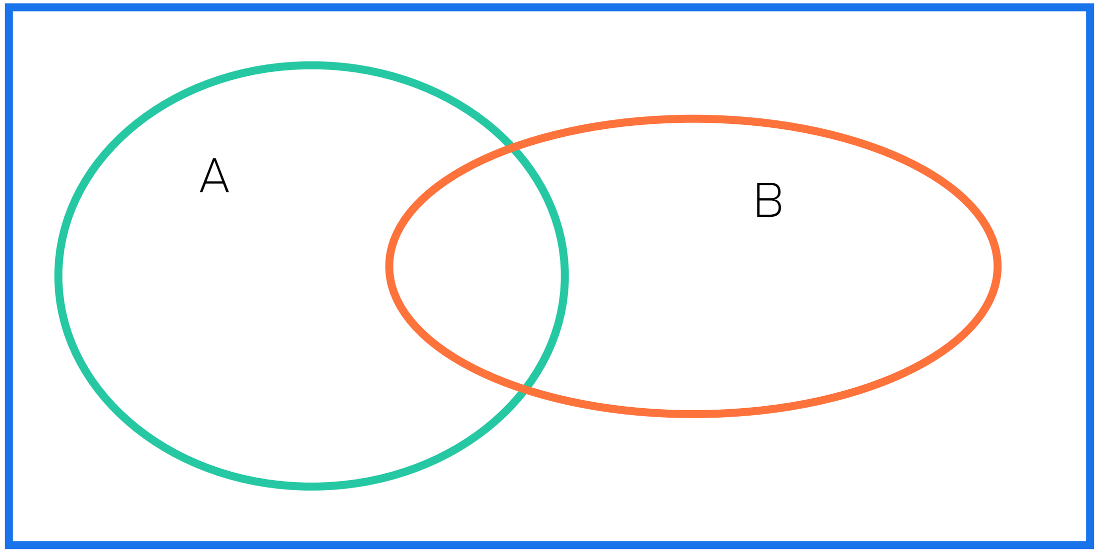


If A and B are independent, then

$P(B|A) = P(B)$ and $P(A \cap B) = P(A)P(B)$.

The converse is true too.

Example: Rolling Two Dice



A= The first die shows a 6
= {(6, 1), (6, 2), (6, 3), (6,4), (6, 5), (6, 6) }

B= The second die shows a 1
= {(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)}

$A \cap B$=A and B=AB={(6,1)}

Pr(A)=1/6 and Pr(B)=1/6
Pr(A and B)=Pr(AB)=1/36

Thus, events A and B are independent events.

Remarks

- Recall $\Pr(C|A) = \frac{\Pr(A \cap C)}{\Pr(A)}$. A and C can be replaced by generic notation A and B, thus, for two events A and B, the formula can be replaced by

$$\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}$$

- It is required that the denominator probability isn't zero. The equation can be re-written as

$$\Pr(\mathrm{A})\Pr(B|A) = \Pr(A \cap B)$$

to avoid the restriction.

- If A and B are independent, then

$$\Pr(\mathrm{A})\Pr(B) = \Pr(A \cap B)$$

Remarks

- If events A and B are independent, then every of all of the following is true.

$$\Pr(B|A) = P(B)$$
$$\Pr(B|A^C) = P(B)$$
$$\Pr(B^c|A) = P(B^c)$$
$$\Pr(B^c|A^c) = P(B^c)$$

Because A and B are generical, thus, if you switch A and B notation above, all will still be true.

Remarks

Given two non-empty events A and B.

Then,

- If A and B are independent, they cannot be mutually exclusive.
- If A and B are mutually exclusive, they cannot be independent.

	Chocolate	Vanilla	Total
Teens	4732	4024	8756
Adults	2420	3590	6010
Total	7152	7614	14766

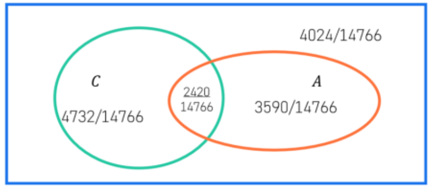
Note: Obviously, A and C are not mutually exclusive. There are people who are adults and who love the Chocolate flavor.

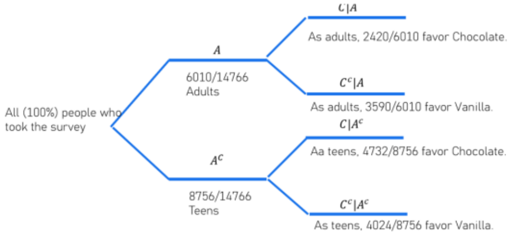
Let's try more

- $\Pr(C) = 0.484$
- $\Pr(A^c) = 0.593$
- $\Pr(C^c) = 0.516$
- $\Pr(A^c \cap C) = 0.320$
- $\Pr(A \cap C^c) = 0.243$
- $\Pr(A^c \cap C^c) = 0.273$

- $\Pr(C^c|A) = 0.597$
- $\Pr(C|A^c) = 0.540$
- $\Pr(C^c|A^c) = 0.460$

- $\Pr(A|C) = 0.338$
- $\Pr(A^c|C) = 0.662$
- $\Pr(A|C^c) = 0.471$
- $\Pr(A^c|C^c) = 0.529$





- A and C are not independent. Pr(C)=0.484 but Pr(C|A)=0.403
- Overall, 48.4% of people chose the Chocolate flavor.
- But if we only look at the adults, only 40.3% chose the Chocolate flavor.
- The probability of liking the Chocolate flavor changes/depends on a person's age.
- Please remember that the interpretation of the above results is based on the assumption that we treat all 14766 people as a population.

Probability Axioms

Kolmogorov axioms [edit]

The assumptions as to setting up the axioms can be summarised as follows: Let (Ω, F, P) be a measure space such that $P(E)$ is the probability of some event E, and $P(\Omega) = 1$. Then (Ω, F, P) is a probability space, with sample space Ω, event space F and probability measure P.[1]

First axiom [edit]

The probability of an event is a non-negative real number:

$$P(E) \in \mathbb{R}, P(E) \geq 0 \qquad \forall E \in F$$

where F is the event space. It follows (when combined with the second axiom) that $P(E)$ is always finite, in contrast with more general measure theory. Theories which assign negative probability relax the first axiom.

Second axiom [edit]

This is the assumption of unit measure: that the probability that at least one of the elementary events in the entire sample space will occur is 1.

$$P(\Omega) = 1$$

Third axiom [edit]

This is the assumption of σ-additivity:

Any countable sequence of disjoint sets (synonymous with *mutually exclusive* events) $E_1, E_2, \ldots$ satisfies

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i).$$

Some authors consider merely finitely additive probability spaces, in which case one just needs an algebra of sets, rather than a σ-algebra.[5] Quasiprobability distributions in general relax the third axiom.

Probability (**Axioms**) · Determinism (System) · Indeterminism · Randomness

Probability space · Sample space · Event (Collectively exhaustive events · Elementary event · Mutual exclusivity · Outcome · Singleton) · Experiment (Bernoulli trial) · Probability distribution (Bernoulli distribution · Binomial distribution · Exponential distribution · Normal distribution · Pareto distribution · Poisson distribution) · Probability measure · Random variable (Bernoulli process · Continuous or discrete · Expected value · Variance · Markov chain · Observed value · Random walk · Stochastic process)

Complementary event · Joint probability · Marginal probability · Conditional probability

Independence · Conditional independence · Law of total probability · Law of large numbers · Bayes' theorem · Boole's inequality

Venn diagram · Tree diagram

V · T · E

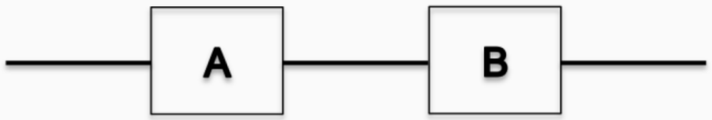
Translation:

1. $P(\mathcal{S}) = 1, \textit{where } \mathcal{S} \textit{ is the sample space}$
2. $P(A) \geq 0, \text{where } A \subseteq \mathcal{S}$
3. $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$, *where* A_i's are disjoint events

In English:

1. At least one outcome will occur.
2. The probability is any event is between 0 and 1.
3. The probability of either A or B occurs is the sum of their probabilities if they are mutually exclusive events.

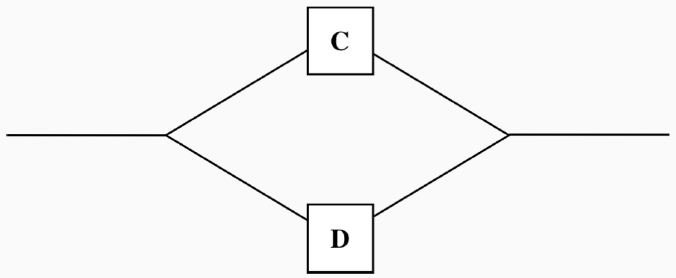
Some Additional Examples for HW: Part I



When components are connected *in series* the system functions only if BOTH components function.

Suppose $\mathrm{P}(A \text{ functions}) = 0.9$, $\mathrm{P}(B \text{ functions}) = 0.6$, and that A and B are independent. What is the probability that the system functions?

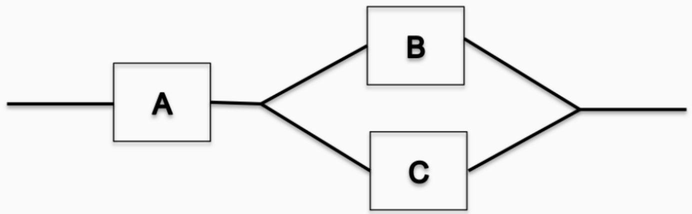
Some Additional Examples for HW: Part II



When components are connected *in parallel* the system functions only if EITHER of the components function.

Suppose $\mathrm{P}(C \text{ functions}) = 0.9$, $\mathrm{P}(D \text{ functions}) = 0.85$, and that C and D are independent. What is the probability that the system functions?

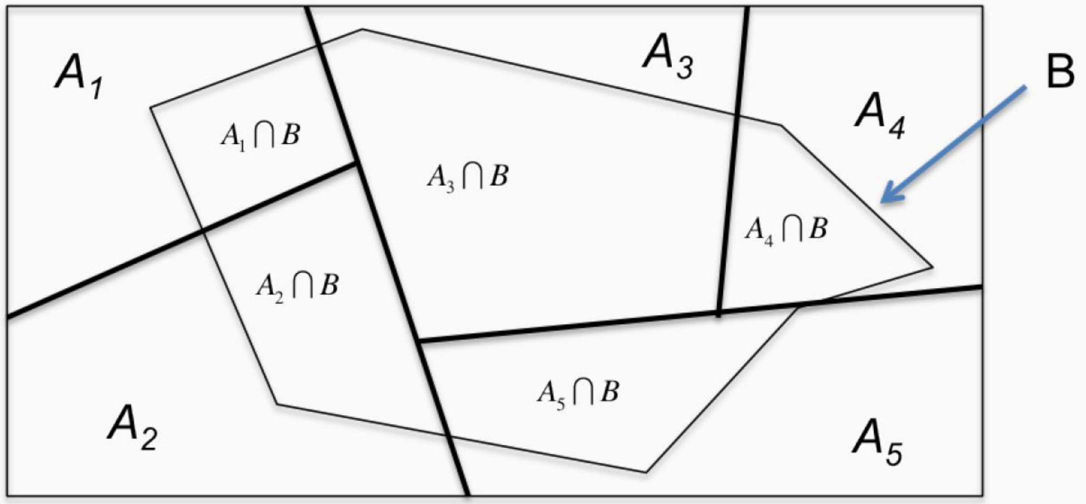
Some Additional Examples for HW: Part III



Suppose the individual components function independently with the following probabilities: $\mathrm{P}(A) = 0.9$, $\mathrm{P}(B) = 0.6$, $\mathrm{P}(C) = 0.5$. What is the probability the system functions?

Honorable Mention: The Law of Total Probability

This is a result derived by Axiom 3 and the conditional probability equation.



$$B = (A_1 \cap B) \cup (A_2 \cap B) \cup \cdots \cup (A_5 \cap B)$$

$$\begin{aligned} \mathrm{P}(B) &= \mathrm{P}(A_1 \cap B) + \cdots + \mathrm{P}(A_5 \cap B) \\ &= \mathrm{P}(B|A_1)\mathrm{P}(A_1) + \cdots + \mathrm{P}(B|A_5)\mathrm{P}(A_5) \end{aligned}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B|A)}{\boxed{P(A)P(B|A) + P(A^c)P(B|A^c)}}$$

Summary

- Sample Space and Events
- Probabilities of Events
- Probabilities of Complement, Union, and Intersection of Events
- Venn Diagram and Tree Diagram
- Conditional, Joint, and Marginal Probabilities
- Addition Rule
- Bayes Rule
- Two Types of Relationship – Mutually Exclusive and Independent
- Probability Axioms
- Honorable Mention: The Law of Total Probability

Quote of the Day

Doctrine and Covenants 46:26

And all these gifts come from God, for the benefit of the children of God.

The scripture verses I shared may not show the signs of connections to our lessons about statistics. These are just the scriptures I read by following "Come, Follow Me" Weekly readings. You are also welcome to recommend any verse you want to share. Just let me know.

Typos?

- Please email me any typos and errors you found in this lesson.
- Thank you!