CS589: Machine Learning - Fall 2017

Quiz 1

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Instructions: Only the final answer for each question will be graded, with no partial credit. If you do any intermediate calculations, please draw a box around your final answer. For each minute late the quiz is turned in, 20% credit will be subtracted from your final score.

1. Matrix Inverse

What is the inverse of the matrix

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}?$$

$$A^{-1} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

2. Matrix Algebra

Suppose that

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad y = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

What is $x^T Ay$?

$$x^T Ay = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= 7$$

3. Calculus

Given the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 9$ what is the derivative, $\frac{df(x)}{dx}$ evaluated at the point $x = 1$? (Give an exact answer, as a number/fraction.)

$$\frac{df}{dx} = x^2 + x, \quad \left.\frac{df}{dx}\right|_{x=1} = 1^2 + 1 = 2.$$

4. Probability.

Suppose that A is a binary random variable, that is 1 with probability $\frac{1}{2}$ and 0 with probability $\frac{1}{2}$. Suppose that B is a binary random variable such that

$$P(B=1|A=1)=1$$
$$P(B=0|A=1)=0$$
$$P(B=1|A=0)=\frac{1}{2}$$
$$P(B=0|A=0)=\frac{1}{2}.$$

(Part A) Are A and B independent? Just answer "**yes**" or "**no**".

No.

(Part B) What is the expected value of B? Give an **exact answer, as a number/fraction.**

$$E[B] = 1 \cdot P(B=1) + 0 \cdot P(B=0)$$
$$= P(B=1)$$
$$= P(B=1|A=1)P(A=1) + P(B=1|A=0)P(A=0)$$
$$= 1 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$$

5. More Probability

Suppose 1% of University of Massachusetts students have a harmless genetic mutation. There is a test for this mutation that is 90% accurate. (It will make errors 10% of the time, both for people who have the mutation, and for those who don't.) Suppose we pick a random UMass student, administer the test, and recieve a test result positive for the mutation. What is the probability that the student *actually* has the mutation? Give an **exact answer, as a number/fraction.**

The probability is $\frac{1}{12}$.