Dynamic Weighing Under Nonzero Initial Conditions

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***Abstract*—This paper presents a new method of dynamic weighing under nonzero initial conditions. The discrete output error (OE) model of second order is derived for the weighing system dynamics. Using the model and the recursive least squares procedure (RLS), model parameters and then the mass being weighed can be estimated from the dynamic measurement signal of very short duration. The validity and the accuracy of this method are illustrated by digital simulation studies and real-life measurements.**

I. Introduction

The requirement for dynamic or quick weighing is often met in packing departments and post offices during check weighing. When a package is put onto the weighing platform, the package and the dynamic mass of the weigher can vibrate together for a long time. Dynamic weighing means the ability to ascertain the static weight of the package while the package is still in vibration. Several solutions to this problem have been proposed in the last few years [1], [2]. But none of them was simple enough for most real applications.

In this paper, a weigher of the spring-damper-mass type is considered which belongs to, or can be simplified as, a second-order system. We first derive the discrete model with unknown parameters, from the continuous model of the weighing system. The parameters can then be estimated by fitting the model to the measured transient weighing signal so that the fit is optimal in the sense of least-squared errors. It will then be shown, according to the end-value theorem of the z-transforms, that the mass being weighed can be determined from the model parameters, and that the algorithm is independent of the initial conditions. Finally, results from both digital simulation studies and real measurements will be presented.

II. Modeling and Estimation Algorithm

A typical weighing system is shown in Fig. 1, where m, c, and d represent the dynamic mass, the spring constant and the damping coefficient of the weigher, respectively. M is the mass being weighed, $x(t)$ is the common displacement of $(m + M)$ or the step response of the weigher, as M is placed onto the weigher. $x(k) = x(kT)$ represents the sampled and digitized $x(t)$, at discrete time intevals $t = kT$.

The weighing process of such a system can also be represented as a block diagram (Fig. 2(a)), where $u(t)$ is the unit step function, $H(s)$ is the "s-transfer function" of the continuous subsystem with $Mgu(t)$ as the input and $x(t)$ as the output. $x(0+)$ and $\dot{x}(0+)$ indicate the initial position and the initial velocity of the platform, respectively. They are not equal to zero in most cases, because by quick weighing the weigher will often still not have reached its steady-state position before the package is removed and replaced by another one. Besides, the package can't be placed with zero speed on the weigher in general.

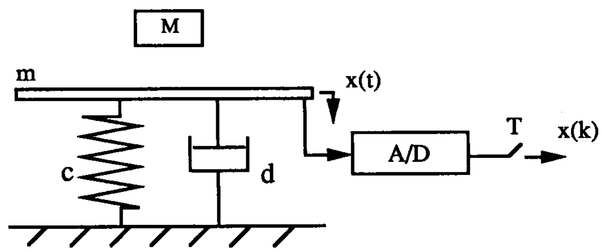

Fig. 1. A weighing system of the second order.

According to the following equation:

$$(m + M)\ddot{x}(t) + d\dot{x}(t) + cx(t) = Mgu(t) \qquad (1)$$

where g is the acceleration of gravity.

It is easy to show that the Laplace transformation of the analog output $x(t)$ can be written as

$$X(s) = MgU(s)\frac{1}{c}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \frac{x(0+)s + \dot{x}(0+) + 2\zeta\omega_n x(0+)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (2)$$

where

$$U(s) = L\{u(t)\} = \frac{1}{s}, \quad \zeta = \frac{d}{2\sqrt{c(m + M)}},$$

$$\omega_n = \sqrt{\frac{c}{m + M}}.$$

Obviously, the second term of the right side in (2) is created by the initial conditions.

Since the whole system in Fig. 2(a) is a continuous-discrete hybrid with a continuous input but a discrete output, it can't be used directly for our purpose. According to a well known theory, continuous-time system excited with a piecewise constant signal can be exactly described

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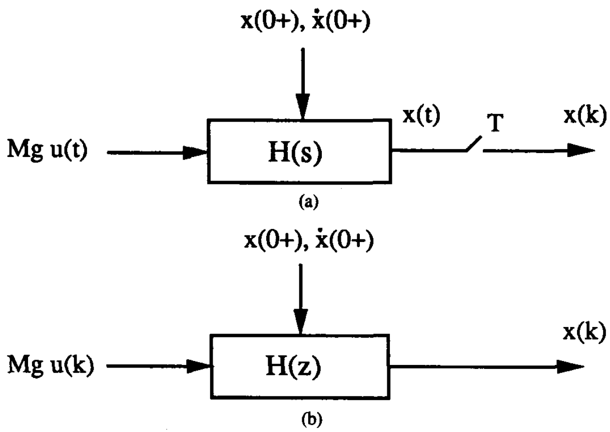

Fig. 2. (a) The continuous-discrete system; (b) the equivalent discrete system.

by a discrete-time model (Fig. 2(b)) at the sampling instants [3].

From (2) the z-transformation of the output series $x(kT)$ can be derived as

$$X(z) = MgU(z)\,\frac{1}{c}\,\frac{b_1^* z^{-1} + b_2^* z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} + x(0+)\,\frac{1 + \alpha z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \qquad (3)$$

where $U(z) = Z\{u(k)\} = 1/1 - z^{-1}$; a_1, a_2, b_1^*, and b_2^* are parameters related to ζ and ω_n; and α is a relatively complicated expression of $x(0+)$, $\dot{x}(0+)$, a_1, and a_2. The second term of the right side of (3) represents the response due to the initial conditions.

However, (3) still can not be used directly due to the unknown mass M. It can be rewritten in the following form:

$$X(z) = U(z)\,\frac{x(0+) + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}, \qquad (4)$$

where

$$b_1 = \frac{Mg}{c}\, b_1^* + (\alpha - 1)x(0+)$$

and

$$b_2 = \frac{Mg}{c}\, b_1^* - \alpha x(0+).$$

From the end-value theorem of the z-transformation, it follows that

$$\lim_{k \to \infty} x(kT) = \lim_{z \to 1} (z - 1)X(z) = \frac{x(0+) + b_1 + b_2}{1 + a_1 + a_2} \qquad (5)$$

which is also equal to Mg/c from (2).

Because the spring constant c can be obtained by static calibration once, the mass M can be determined by

$$M = \frac{c}{g}\,\frac{x(0+) + b_1 + b_2}{1 + a_1 + a_2} \qquad (6\text{-}1)$$

or

$$M = \frac{c}{g}\,\frac{b}{1 + a_1 + a_2}, \qquad (6\text{-}2)$$

where $b = (x(0+) + b_1 + b_2)$, which is a function of the structural parameters ζ, ω_n, and the initial conditions $x(0+)$ and $\dot{x}(0+)$.

Now, the dynamic weighing problem is transformed into a problem of identifying a_1, a_2, $x(0+)$, b_1, and b_2, or a_1, a_2, and b, according to measurements $x(k)$.

Equation (4) can be written in the form of a difference equation:

$$x(k) = -a_1 x(k-1) - a_2 x(k-2) + x(0+)\,u(k) + b_1 u(k-1) + b_2 u(k-2) \qquad (7\text{-}1)$$

and simplified as

$$x(k) = -a_1 x(k-1) - a_2 x(k-2) + b \quad \text{for } k \geq 2. \qquad (7\text{-}2)$$

From (6-2) and (7-2) we come to a very interesting and useful conclusion that to determine the mass M we do not have to identify the initial conditions $x(0+)$ and $\dot{x}(0+)$ individually, but only the combination parameter b, and in this sense the method is independent of the initial conditions. This conclusion has also been proved by both digital simulations and real measurements.

An additional explanation for it is the following: although the initial condition changes the appearance of the weighing signal formally through changing the values of $x(0)$, $x(1)$, and $x(2)$ in (7-1), it influences neither the end value of the weighing process nor the forming law of the weighing signal after $k \geq 2$ (see (7-1) and (7-2)).

The two curves shown in Fig. 4 in Section III are due to different initial conditions but the same forming law. It can be found that one can get the second curve from the first curve by first moving the latter along the time axis in the right direction, and then extending it in the left direction, according to the "forming law."

The method proposed in this paper is based only on the forming law of the weighing signal, i.e., the dynamics of the system. This is the very reason why the method is independent of the initial conditions. The cost to be paid is that the first two points of the data stream are not used.

Of course, measurements inevitably contain many types of noise. Real measurements showed the noise exists at both the input side and the output side of the system. The noise at the input side is chiefly caused by the sliding or springing of the package on the platform, as the package is placed onto the weigher with nonzero speed. This type of noise is often of very short duration, and its influence

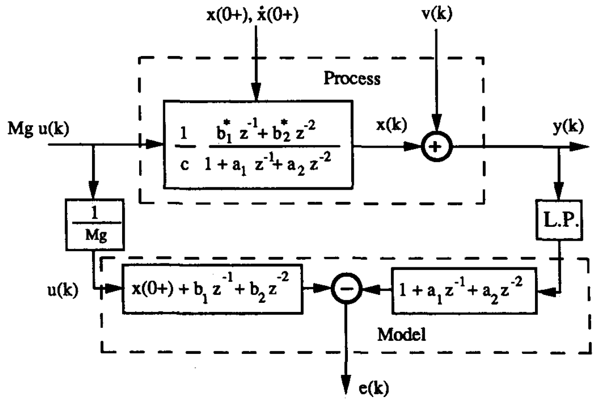

Fig. 3. The OE model for identification.

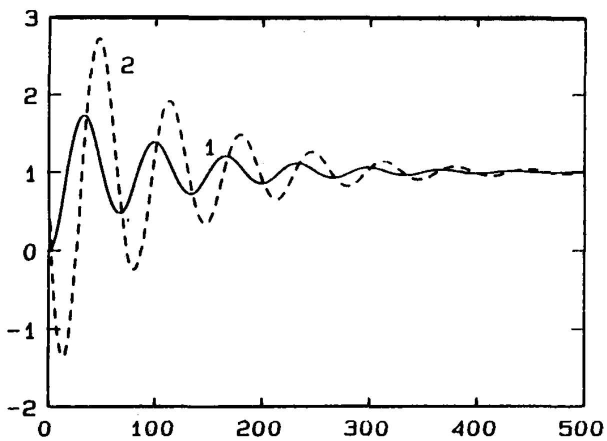

Fig. 4. Two simulated weighing signals under different initial conditions.

can be eliminated by giving up several data points at the starting phase of the measurement. For this reason one need not consider it by modeling. On the contrary, the noise at the output side exists in the whole weighing process, and it is chiefly caused by electrical elements such as the sensor, amplifier, and so on. In a well designed weigher, the output noise is always stochastic and stationary.

In Fig. 3, $v(k)$ represents the measurement noise at the output side; $y(k) = x(k) + v(k)$ is the disturbed output and $e(k)$ the residual of the model.

From (7-2) and Fig. 3 we have

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b + v(k) + a_1 v(k-1) + a_2 v(k-2). \quad (8)$$

Since $e(k) = v(k) + a_1 v(k-1) + a_2 v(k-2)$, the residual $e(k)$ is colored, even if $v(k)$ is white.

In order to obtain an unbiased estimation, we can rewrite (8) as

$$y(k) = a_1[v(k-1) - y(k-1)] + a_2[v(k-2) - y(k-2)] + b + v(k). \quad (9)$$

In that case, $e(k) = v(k)$, i.e., the residual is white if $v(k)$ is white. Now we can treat it as a least square problem using the well known RLS algorithm:

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + \boldsymbol{L}(k)[y(k) - \boldsymbol{h}^T(k)\,\hat{\Theta}(k-1)]$$
$$\boldsymbol{L}(k) = \boldsymbol{P}(k-1)\,\boldsymbol{h}(k)[\boldsymbol{h}^T(k)\,\boldsymbol{P}(k-1)\,\boldsymbol{h}(k) + 1]^{-1}$$
$$\boldsymbol{P}(k) = [\boldsymbol{I} - \boldsymbol{L}(k)\,\boldsymbol{h}^T(k)]\boldsymbol{P}(k-1), \quad (10)$$

where $\hat{\Theta}(k) = [\hat{a}_1(k), \hat{a}_2(k), \hat{b}(k)]^T$, which is the parameter vector to be determined, and $\boldsymbol{I}$ is the unit matrix. The data vector $\boldsymbol{h}(k)$ has the following form:

$$\boldsymbol{h}(k) = [v(k-1) - y(k-1), v(k-2) - y(k-2), 1]^T. \quad (11\text{-}1)$$

Since the noise $v(k-1)$ and $v(k-2)$ are never observed, we can only use their estimations to replace them:

$$\hat{v}(k-i) = y(k-i) - \boldsymbol{h}^T(k-i)\,\hat{\Theta}(k-i)$$
$$i = 1, 2.$$

In this sense, the algorithm is somewhat different from a standard RLS.

If the measurement noise $v(k)$ is fundamentally eliminated by a hardware or a software low-pass filter at the output side (see Fig. 3), then we can use the RLS algorithm (10) with the simple data vector

$$\boldsymbol{h}(k) = [-y(k-1), -y(k-2), 1]^T. \quad (11\text{-}2)$$

The cutoff frequency of the low-pass filter can be reasonably selected to be slightly greater than the highest possible natural frequency of the weigher

$$\omega_0 > \omega_{\text{nmax}} = \sqrt{\frac{c}{m}}. \quad (12)$$

After every step of recursion we have

$$\hat{M}(k) = \frac{c}{g}\,\frac{\hat{b}(k)}{1 + \hat{a}_1(k) + \hat{a}_2(k)}. \quad (13)$$

III. Simulation Studies

Fig. 4 shows two simulated noise-free weighing signals. They were created from the same physical structural parameters: $m = 10$ kg, $M = 100$ kg, $c = 1000$ N/mm, $d = 12$ N/(mm/s), but under different initial conditions: $x(0+) = 0$ mm, $\dot{x}(0+) = 0$ mm/s for signal one and $x(0+) = 0.6$ mm, $\dot{x}(0+) = -250$ mm/s for signal two.

Although the two signals appear quite different, they yielded similar results by using the method discussed in Section II with the data vector (11-2) (see Table I).

The second signal was then corrupted with a white-Gaussian noise $v(k)$ and processed using the data vector (11-1). The results (see Table II) show that the algorithm converges quite slowly even if an LP filter of order three is used. The same signal was then processed using the data vector (11-2). The results reported in Table III indicate that the algorithm converges quickly, especially when the LP filter is cascaded in.

TABLE I
SIMULATION RESULTS

	Real Weight: 100 kg, $\sigma_v/y(\infty) = 0\%$	
k	$x(0+) = 0$ mm, $\dot{x}(0+) = 0$ mm/s, Est. Weight	$x(0+) = 0.6$ mm, $\dot{x}(0+) = -250$ mm/s, Est. Weight
5	68.62838	188.9175
10	84.93655	109.8219
15	95.42455	100.8318
20	98.55861	100.0611
25	99.46874	99.98022

TABLE II
SIMULATION RESULTS

Real Weight: 100 kg, $\sigma_v/y(\infty) = 5\%$
$x(0+) = 0.6$ mm, $\dot{x}(0+) = -250$ mm/s
Using Data Vector (11-1)

k	Without Filter Est. Weight	With a LP Filter Est. Weight
80	12.99705	19.226289
160	89.84571	92.42928
240	104.0041	103.6265
320	102.2299	101.9777
400	99.48176	100.0755

TABLE III
SIMULATION RESULTS

Real Weight: 100 kg, $\sigma_v/y(\infty) = 5\%$
$x(0+) = 0.6$ mm, $\dot{x}(0+) = -250$ mm/s
Using Data Vector (11-2)

k	Without Filter Est. Weight	With a LP Filter Est. Weight	k
60	105.1959	111.2408	10
90	99.91519	100.1327	20
120	103.912	98.95274	30
150	102.972	99.22825	40
180	102.5539	99.77757	50

Many simulation studies with other noise realizations show similar results. In most cases, it is far better to use the algorithm with the data vector (11-2), supported by a low-pass data filter. This conclusion was also proved by real measurements. If the SNR at the output side of the filter is high enough, then the bias of the estimation is very small.

IV. REAL MEASUREMENTS

The method proposed in Section II was verified by means of the experimental setup shown in Fig. 5. Three pieces of iron with weights 36.2 g, 100 g, and 173.8 g, respectively, were used as the test weights; the analog weighing signal was produced by a strain gauge and a differential amplifier. A digital IIR low-pass filter with the order three according to the principle in (12) was also arranged, and it can be connected to the system or bypassed as desired; but experiments showed that satisfactory results can be obtained only when the filter is cascaded in. To eliminate the input noise caused by sliding or springing of the weight on the platform, the first 40 points of measurements were cut off.

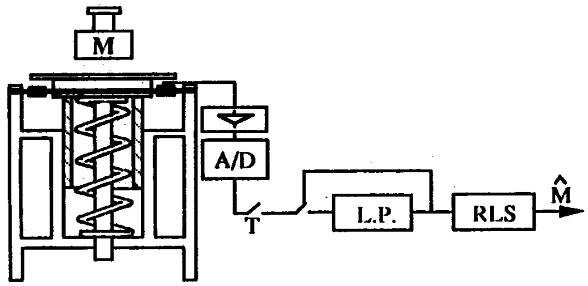

Fig. 5. Experimental arrangement.

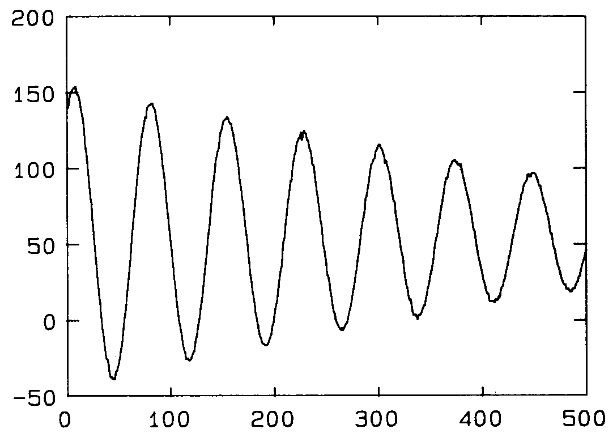

Fig. 6. Real weighing signal for weight of 36.2 g.

In order to examine the robustness of the algorithm, the weighing signal in Fig. 9 was produced by placing the iron piece onto the weighing platform with a deliberate jerk.

The experimental resutls are reported in Figs. 6–9 (signals before the filter), and in Table IV–Table VII, where k represents the recursion step of the calculation, which also represents the number of data points which were used.

In order to judge the efficiency of the algorithm, a time ratio can be defined as

$$R = t_d/t_s \tag{14}$$

where t_d and t_s are the measurement times needed by the new method and the traditional static method respectively, in order to obtain the same accuracy.

As shown in Table IV–Table VII, if the accuracy is defined as 1%, then the time ratio R is smaller than 20% in all measurements, because a static weighing needs at least more than 1000 data points (see Figs. 6–9).

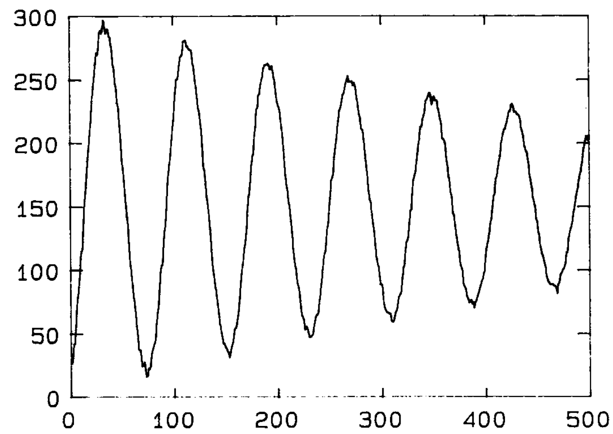

Fig. 7. Real weighing signal for weight of 100 g.

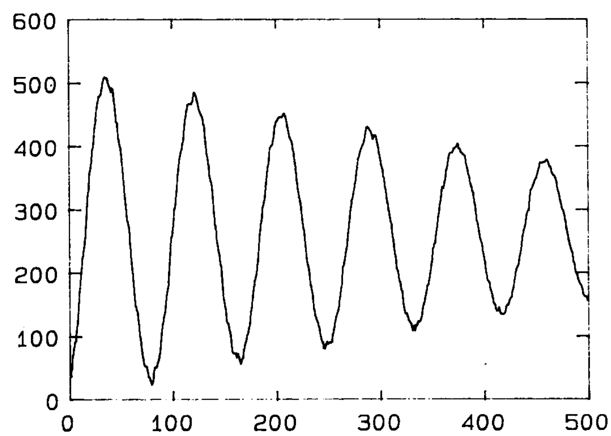

Fig. 8. Real weighing signal for weight of 173.8 g.

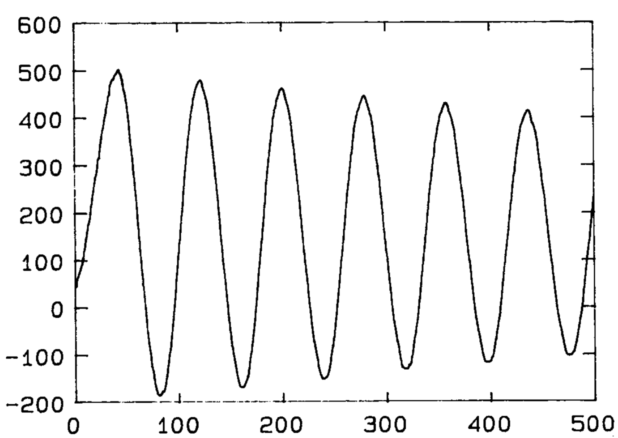

Fig. 9. Another real weighing signal for weight of 100 g.

TABLE IV
REAL MEASUREMENT RESULTS

Real Weight: 36.2 g			
k	Est. Weight	Est. Weight	k
20	−49.49819	35.78548	100
40	19.76887	35.77178	120
60	33.58292	36.01601	140
80	35.46608	36.16902	160

TABLE V
REAL MEASUREMENT RESULTS

Real Weight: 100 g			
k	Est. Weight	Est. Weight	k
20	−87.31718	101.0762	120
40	90.71598	100.7804	140
60	105.9952	100.3036	160
80	101.802	100.1003	180
100	101.5199	100.3367	200

TABLE VI
REAL MEASUREMENT RESULTS

Real Weight: 173.8 g			
k	Est. Weight	Est. Weight	k
20	368.1601	175.7477	120
40	238.7305	175.2259	140
60	183.1414	175.3933	160
80	181.2774	175.3169	180
100	174.9943	175.1681	200

TABLE VII
REAL MEASUREMENT RESULTS

Real Weight: 100 g			
k	Est. Weight	Est. Weight	k
20	239.4366	101.7656	120
40	795.2604	101.4153	140
60	114.7643	101.6628	160
80	107.2259	101.0563	180
100	102.5148	100.6883	200

V. Final Remarks

The algorithm proposed in this paper can be used to effectively reduce the weighing time in a spring-damper-mass type weigher. In most cases, a hardware or a software low-pass filter should be cascaded into the system to improve the data quality; otherwise, a satisfactory result is not guaranteed.

The algorithm with the data vector (11-1) converges slowly so that it is, in general, not the proper selection for dynamic weighing.

Although theoretically RLS with the data vector (11-2) can't provide the unbiased estimation, the possible bias is often acceptable for most real applications.

In addition, the choice of the sampling interval T was proved to be not critical, and the starting states of the algorithm, i.e., $\hat{\boldsymbol{\Theta}}(0)$ and $\boldsymbol{P}(0)$, can be selected quite arbitrarily.

Finally, it is worth pointing out that the method described in this paper is not restricted to the weighing system of second order. If (7-2) is extended into the following form:

$$x(k) = -a_1x(k-1) - a_2x(k-2) - \cdots a_nx(k-n) + b \qquad \text{for } k \geq n \tag{15}$$

then RLS with the corresponding parameter vector and data vector can also be used to solve the dynamic weighing problem for weighers of higher order.

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References

[1] T. Ono, "Dynamic weighing of mass," *Instrum. and Automation*, in Japanese, vol. 12, no. 2, pp. 35–41, 1984.

[2] J. Stöckle, "Improvement of the dynamical properties of checkweighers using digital signal processing," in *IMEKO Symp. Meas. Estimation*, Bressanone, Italy, 1984, pp. 103–108.

[3] L. Ljung, *System Identification: Theory for the User.* Englewood Cliffs, NJ: Prentice-Hall, 1987, section 2.1, pp. 20–21.