GP1 HW7

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Problem 1 (Angular momentum conservation)
(1) The angular momentum is conserved

$$mv_1r_1 = mv_2r_2. \tag{1.1}$$

Thus,

$$v_2 = \frac{r_1}{r_2}v_1. \tag{1.2}$$

(2) Work-energy theorem:

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}mv_1^2\left(1 - \frac{r_1^2}{r_2^2}\right). \tag{1.3}$$

(3)

$$F = m\frac{v^2}{r} = m\frac{v_1^2r_1^2}{r_2^3}. \tag{1.4}$$

Problem 2 (A toy of yo-yo)
(1) Suppose the friction be f, acceleration be a, and then the angular acceleration is $\beta = \frac{a}{R}$ since no slipping.

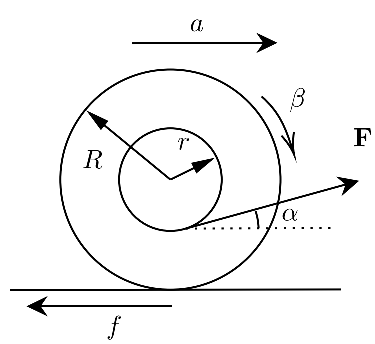

Figure 1: Toy of yo-yo

We have

$$F\cos\alpha - f = ma, \tag{2.1}$$

$$fR - Fr = I\frac{a}{R}. \tag{2.2}$$

So,

$$a = F\frac{R\cos\alpha - r}{mR + I/R}. \tag{2.3}$$

(2) When $F > \frac{mg}{\sin\alpha}$, the yo-yo can be lift off the table.

Problem 3 (Bounce back)
Suppose the force in horizontal given by table during the collision is F. Then,

$$F = ma, \tag{3.1}$$

$$F(h - r) = \frac{2}{5}mr^2\beta. \tag{3.2}$$

Since it is still a pure rolling,

$$r\beta = a. \tag{3.3}$$

Therefore,

$$\frac{h}{r} = \frac{7}{5}. \tag{3.4}$$

Problem 4 (Euler equations)
(1)

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\tilde{\mathrm{d}}\mathbf{L}}{\mathrm{d}t} + \boldsymbol{\omega} \times \mathbf{L} = 0. \tag{4.1}$$

So the angular momentum is conserved.
(2)

$$\mathbf{L} = \mathbf{I} \cdot \boldsymbol{\omega}. \tag{4.2}$$

$$T = \frac{1}{2}\boldsymbol{\omega} \cdot \mathbf{L}. \tag{4.3}$$

Since $\boldsymbol{I}$ is a symmetric second order tensor,

$$\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} \cdot \boldsymbol{I} \cdot \boldsymbol{\omega} = \boldsymbol{\omega} \cdot \boldsymbol{I} \cdot \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} \tag{4.4}$$

$$2\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} \cdot \boldsymbol{L} + \boldsymbol{\omega} \cdot \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = 2\boldsymbol{\omega} \cdot \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = 0. \tag{4.5}$$

So the kinetic energy of rotation is a constant.