

GP1 HW4

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Problem 1 (The hyperbolic orbit)
(1)

Problem 2 (A parabolic orbit)

Problem 3 (Proof of Kepler's First Law)
We start from Binet's equation:

$$h^2 u^2 \left(\frac{d^2 u}{d\theta^2} + u \right) = -\frac{F}{m}. \quad (3.1)$$

Where $h = r^2 \dot{\theta}$ is a constant, $u = \frac{1}{r}$, $F = -\frac{GMm}{r^2} = -GMmu^2$. Hence,

$$\frac{d^2 u}{d\theta^2} + u = \frac{Gm}{h^2}. \quad (3.2)$$

$$u = A \cos(\theta + \theta_0) + \frac{GM}{h^2}. \quad (3.3)$$

$$r = \frac{1}{A \cos(\theta + \theta_0) + \frac{GM}{h^2}}. \quad (3.4)$$

We can use the initial condition to determine A and θ_0 . Anyway, we know it is a quadratic curve.