GP1 HW5

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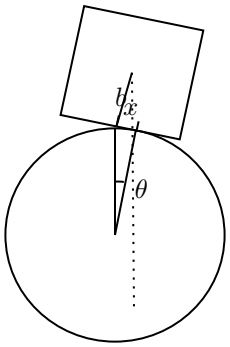
Problem 1 (Kinetic Energy Machine)

Problem 2 (Total Energy of a Many-particle System)

Problem 3 (Stability of a cube balanced on a cylinder)
Suppose the cube has a displacement.

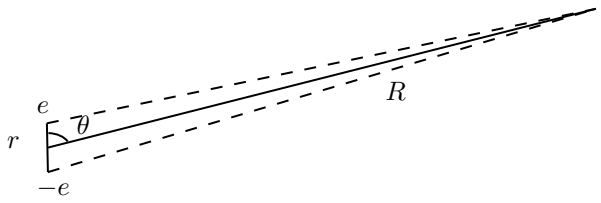


The dotted line needs to be on the left side of the contact point. By observing the picture, we can get: (1) If $b < r$, then it is stable. (2) If $b > r$, then it is unstable.

Problem 4 (The electric potential of an electric dipole)

$$U = \frac{e}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{R^2 + \frac{r}{2}^2 - rR\cos(\theta)}} - \frac{1}{\sqrt{R^2 + \frac{r}{2}^2 + rR\cos(\theta)}}\right). \tag{4.1}$$

$$U \approx \frac{e}{4\pi\varepsilon_0 R}\frac{r}{R}\cos\theta = \frac{\vec{p}\cdot\hat{R}}{4\pi\varepsilon_0 R^2}. \tag{4.2}$$



Problem 5 (The tide potential)
(1)Let

$$U_{\text{tide}}(\mathbf{r}) = -GMm\left(\frac{1}{|\mathbf{d_0}+\mathbf{r}|} - \frac{1}{|\mathbf{d_0}|}\right), \tag{5.1}$$

then,

$$\mathbf{F}_{\text{tide}}(\mathbf{r}) = -\nabla U_{\text{tide}}(\mathbf{r}). \tag{5.2}$$

(2) (We also consider the centrifugal potential energy). Since $\frac{|\mathbf{r}|}{|\mathbf{d_0}|} \ll 1$,

$$\varphi_{\text{tide}} \approx -\frac{GMr^2}{d_0^3}\frac{3\cos\theta^2 - 1}{2}, \tag{5.3}$$

where r is the distance to the center of the earth. Sea level is a equalpotential plain.

$$\Delta h = \frac{3GMr^2}{2d_0^3 g}. \tag{5.4}$$