

1. We initiate an induction on  $n \in \mathbb{N}$ .
2. Note that  $a_{n+1} = \left(1 + \frac{1}{n+1}\right)^{\frac{n+1}{n} \cdot n}$ . By (1), (in rational case)  $a_{n+1} \geq \left(1 + \frac{1}{n}\right)^n = a_n$ .
3.  $b_n = \left(1 + \frac{1}{n}\right)^{\frac{n}{n+1} \cdot \frac{(n+1)^2}{n}} \geq \left(1 + \frac{1}{n+1}\right)^{\frac{(n+1)^2}{n}} \geq \left(1 + \frac{1}{n+1}\right)^{n+2} = b_{n+1}$
4. Easy to prove  $\forall k \in \mathbb{N}, b_k \geq a_k$ , by (2) and (3),...