

1 Basic Logic

1. truth value:

P	Q	$P \wedge \neg P$	$P \vee \neg P$	$(P \vee Q) \Rightarrow (P \wedge Q)$	$(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$
T	T	F	T	T	T
F	T	F	T	F	F
T	F	F	T	F	T
F	F	F	T	T	T

Table 1: truth value table

2. (1) $Q \wedge \neg Q = F, P \Rightarrow (Q \wedge \neg Q) = \neg P \vee F = \neg P$
 (2) $(P \wedge \neg Q) \Rightarrow Q = \neg P \vee Q \vee Q = \neg P \vee Q = P \Rightarrow Q$
3. (1) $P \wedge Q \Rightarrow R$
 (2) $Q \Rightarrow P$
 (3) $P \Leftarrow Q$
4. We denote that "bear is smart" as P , "bear is lazy" as Q , then "bear is not smart" can be denoted as $\neg P$. We have $(P \wedge Q \vee (\neg P)) \wedge P$, it's equivalent to $P \wedge Q$, then Q must be true .
6. We denote "At door 1,2,3" as P, Q, R , one of them is true ,while we can get another infomation:one of $\neg P, \neg Q, Q$ is true. Due to "not Q then $\neg Q$ ",we can infer that $\neg P$ is false.(We can confirm while $Q = R = \text{false}$,it can satisfies the requirements of the question)
 so the treasure is behind the Door 1!
7. We denote ...can leads to the capital as P, Q, R , then $P \wedge (R \Rightarrow Q) = (\neg P) \wedge (\neg R) = P \wedge (\neg Q) = \text{False}$. Combine the first and the third formula $P \wedge (\neg R \vee Q \vee \neg Q) = P = \text{False}$, then from the second $\neg R = \text{False}$. We are not sure about the stone path ,but we are sure that the dirt path can lead to capital.
8. Denote " $a + 1 == 0$ " as P , " $b + 1 == 0$ " as Q , then $ab + a + b \neq -1 = (a + 1)(b + 1) == 0 = \neg P \wedge \neg Q$
9. (1) Use the proof by coontrodiction. Not losing generaity ,we assume that $a = 1$