Thinking and Method of FAA

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Contents

1 Basic Logic

Iff. $P = Q = \neg R$ =True, $P \Rightarrow (Q \Rightarrow R)$ is False. This is equivalent to $(P \wedge Q) \Rightarrow R$. So in `LEAN 4`, you can see a goal in the form

$$a \to b \to c \to \ldots,$$

then you can use *intro* to get props. They have the relation *and* logically.

2 Set Theory

Definition 2.4.1 defines quantifiers, by 2.6.3 and 2.7.4 , we can use set to understand quantifiers. Let us first consider

$$\forall x \in X, \forall y \in Y, P(x, y). \tag{2.1}$$

That is

$$X = \{x \in X \mid \forall y \in Y, P(x, y)\} = \bigcap_{y \in Y} \{x \in X \mid P(x, y)\}. \tag{2.2}$$

That means

$$\forall y \in Y,\ X \subseteq \{x \in X \mid P(x, y)\}. \tag{2.3}$$

Thus,

$$\forall y \in Y, X = \{x \in X \mid P(x, y)\}, \tag{2.4}$$

equivalent to

$$\forall y \in Y, \forall x \in X, P(x, y). \tag{2.5}$$

But if we consider

$$\forall x \in X, \exists y \in Y, P(x, y), \tag{2.6}$$

the situation becomes

$$X = \bigcup_{y \in Y} \{x \in X \mid P(x, y)\} \tag{2.7}$$

The union equals to X does not give enough information. Similarly, $\exists, \forall, \ldots$ can't go farther, too[1]. But

$$\exists x \in X, \exists y \in Y, P(x, y) \tag{2.8}$$

is equivalent to

$$\bigcup_{y \in Y} \{x \in X \mid P(x, y)\} \neq \varnothing. \tag{2.9}$$

That means

$$\exists y \in Y, \{x \in X \mid P(x, y)\} \neq \varnothing. \tag{2.10}$$

Thus,

$$\exists y \in Y, \exists x \in X, P(x, y). \tag{2.11}$$

[1] The intersection is not empty leads to any sets is not empty, but it is not equivalent, $\exists x \in X, \forall y \in Y, P(x, y) \Rightarrow \forall y \in Y, x \in X, P(x, y)$.

3 Correspondence

For the similar reason, if f is a correspondence, then

$$f\left(\bigcup_{i\in I} A_i\right) = \bigcup_{i\in I} f\left(A_i\right), \tag{3.1}$$

$$f\left(\bigcap_{i\in I} A_i\right) \subseteq \bigcap_{i\in I} f\left(A_i\right). \tag{3.2}$$

If in addition, f is injective, then

$$f\left(\bigcap_{i\in I} A_i\right) = \bigcap_{i\in I} f\left(A_i\right). \tag{3.3}$$

A conclusion: Let f, g be correspondences, if $f \circ g = \mathrm{Id},\ g \circ f = \mathrm{Id}$, then f is a bijection and $f^{-1} = g$.

4 Ordering

Forgettable concepts: Well-ordered set 4.7.1, Order-complete 4.8.1

Problem 4.1 (Eg.)

$$m := \inf(A^{\mathrm{u}}) \in A^{\mathrm{u}}.$$

Proof. By definition, we only need to prove $\forall x \in A,\ x \leq m$. m is the max element in $(A^{\mathrm{u}})^{l}$, then we only need to prove $\forall x \in A,\ x \in (A^{\mathrm{u}})^{\mathrm{l}}$. It is easy to check.

□

The power set with $\subseteq$ forms a order-complete partially ordered set. If we want to construct a order-complete partially ordered set, we may consider build a relation between them. Knaster-Tarski fixed point theorem tell us a property of monotonic functions, and Dedekind-MacNeille theorem tell us how to do in detail.