QM HW1

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Problem 1 (Canonical Transformation)
(1)

$$\mathrm{d}\Phi(q, P, t) = p\mathrm{d}q + Q\mathrm{d}P + (H' - H)\mathrm{d}t \tag{1.1}$$

$$\frac{\partial \Phi}{\partial q} = p, \frac{\partial \Phi}{\partial P} = Q, \frac{\partial \Phi}{\partial t} + H = H'. \tag{1.2}$$

(2) We have proved:

$$\frac{\partial S}{\partial q_t} = -p_t, \frac{\partial S}{\partial q_{t+\tau}} = p_{t+\tau}, \tag{1.3}$$

First, we can check:

$$\frac{\partial \Psi}{\partial q_t} = -\frac{\partial S}{\partial q_t} = p_t \tag{1.4}$$

Second ,by chain rule,we obtain

$$\frac{\partial \Psi}{\partial p_{t+\tau}} = q_{t+\tau} + \left(p_{t+\tau} - \frac{\partial S}{\partial p_{t+\tau}}\right)\frac{\partial q_{t+\tau}}{\partial p_{t+\tau}} = q_{t+\tau} \tag{1.5}$$

(3)

$$\mathrm{d}p_t \wedge \mathrm{d}q_t = \mathrm{d}p_{t+\tau} \wedge \mathrm{d}q_{t+\tau}, \frac{\partial(q_{t+\tau}, p_{t+\tau})}{\partial(q_t, p_t)} = 1 \tag{1.6}$$

Problem 2 (Hamilton-Jacobi equation)
(1)

$$\frac{\partial S}{\partial t} = \frac{\mathrm{d}S}{\mathrm{d}t} - \frac{\partial S}{\partial q}\dot{q} = L - p\dot{q} = -H \tag{2.1}$$

(2)

$$\beta = \frac{\partial S}{\partial \alpha} \tag{2.2}$$

$$H' = H + \frac{\partial S}{\partial t} = 0 \tag{2.3}$$

Problem 3 (Harmonic Oscillator)
(1) easy to check:

$$\frac{\partial S}{\partial x} = p, \frac{\partial S}{\partial t} = -E \tag{3.1}$$

(2)

$$\frac{\partial S}{\partial t_f} = -E \tag{3.2}$$

$$\frac{\partial S}{\partial x_f} = p = \pm m\omega\sqrt{A^2 - x^2} \tag{3.3}$$

(3)

$$S = \pm m\omega \int \sqrt{\frac{2E}{m\omega^2} - x^2}\,\mathrm{d}x - Et + \mathrm{const.} \tag{3.4}$$

$$\frac{\partial S}{\partial E} = \pm\frac{\arcsin(\sqrt{\frac{m\omega^2}{2E}}x)}{\omega} - t \tag{3.5}$$

New Hamitonian:

$$H' = H + \frac{\partial S}{\partial t} = 0 \tag{3.6}$$

By Hamilton equation

$$\dot{\beta} = \frac{\partial H'}{\partial E} = 0 \tag{3.7}$$

Therefore β is a constant.It means the initial phase of oscillator.

Problem 4 (Planck's derivation of black body radiation)
Confused.

Problem 5 (Heisenberg's magic)