GP1 HW5

Jiete XUE

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Problem 1 (Kinetic Energy Machine)
If there exisis a heigher speed than reversible machine, than we can use the rest energy to do work. Then we can use the reversible machine to restore the higher speed. That means we can use this system to get energy forever. That contradicts to the impossibility of perpetual motion machine.

Similarly, if two reversible machine have different speed, then we can use the differece of the speed to do work. That also leads to a contradiction.

Problem 2 (Total Energy of a Many-particle System)
Since the interaction only depens on their distance, we can write the forcw into the form of

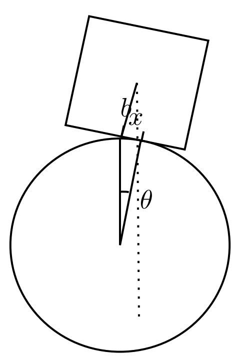
$$\mathbf{F}_{ij} = -\nabla_i U\left(|\mathbf{r}_{ij}|\right). \tag{2.1}$$

Then, the total energy can be weitten as

$$E = \frac{1}{2}\sum_{ij} U_{ij} + \sum_i m_i g z_i, \tag{2.2}$$

where z_i is the height of the ith particle.

Problem 3 (Stability of a cube balanced on a cylinder)
Suppose the cube has a displacement.

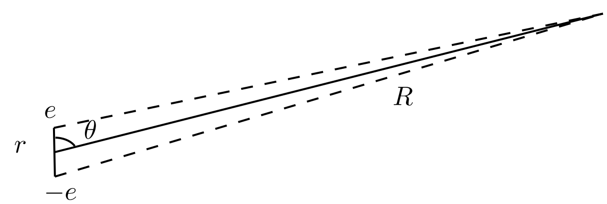


The dotted line needs to be on the left side of the contact point. By observing the picture, we can get: (1) If $b < r$, then it is stable. (2) If $b > r$, then it is unstable.

Problem 4 (The electric potential of an electric dipole)

$$U = \frac{e}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{R^2 + \frac{r}{2}^2 - rR\cos(\theta)}} - \frac{1}{\sqrt{R^2 + \frac{r}{2}^2 + rR\cos(\theta)}}\right). \tag{4.1}$$

$$U \approx \frac{e}{4\pi\varepsilon_0 R}\frac{r}{R}\cos\theta = \frac{\vec{p}\cdot\hat{R}}{4\pi\varepsilon_0 R^2}. \tag{4.2}$$



Problem 5 (The tide potential)
(1)Let

$$U_{\text{tide}}(\mathbf{r}) = -GMm\left(\frac{1}{|\mathbf{d_0} + \mathbf{r}|} - \frac{1}{|\mathbf{d_0}|}\right), \tag{5.1}$$

then,

$$\mathbf{F}_{\text{tide}}(\mathbf{r}) = -\nabla U_{\text{tide}}(\mathbf{r}). \tag{5.2}$$

(2) (We also consider the centrifugal potential energy). Since $\frac{|\mathbf{r}|}{|\mathbf{d_0}|} \ll 1$,

$$\varphi_{\text{tide}} \approx -\frac{GMr^2}{d_0^3}\frac{3\cos\theta^2 - 1}{2}, \tag{5.3}$$

where r is the distance to the center of the earth. Sea level is a equalpotential plain.

$$\Delta h = \frac{3GMr^2}{2d_0^3 g}. \tag{5.4}$$