Family name: ____________ Given name: ____________ Student ID: ____________

Westlake University
Fundamental Algebra and Analysis I

Test of September 8th 2025

1. Let x be a real number such that $x > -1$. Prove that, for any natural number n, one has

$$(1+x)^n \geqslant 1 + nx.$$

You could reason by induction on n.

2. Prove that the mapping from $\mathbb{N}_{\geqslant 1}$ to $\mathbb{Q}$ that sends $n \in \mathbb{N}_{\geqslant 1}$ to

$$a_n = \left(1 + \frac{1}{n}\right)^n$$

is increasing. You could use the previous question to prove that $a_{n+1}/a_n \geqslant 1$.

3. Prove that the mapping from $\mathbb{N}_{\geqslant 1}$ to $\mathbb{Q}$ that sends $n \in \mathbb{N}_{\geqslant 1}$ to

$$b_n = \left(1 + \frac{1}{n}\right)^{n+1}$$

is decreasing.

4. Prove that, for any $(n, m) \in \mathbb{N}_{\geqslant 1}^2$, one has $a_n \leqslant b_m$.