Introduction to Computer Graphics

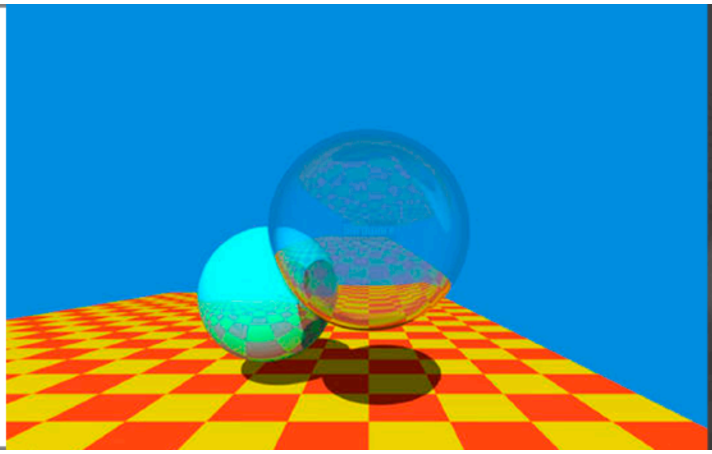
GAMES101, Lingqi Yan, UC Santa Barbara

Lecture 15:

Ray Tracing 3

(Light Transport & Global Illumination)






Announcements

- Homework 5 — 240 submissions so far
- Next two homeworks (1.5 weeks each)
  - Homework 6 — acceleration
  - Homework 7 — path tracing (new!)
- Course website has been updated
  - Two more lectures, one more homework (hw7)

Announcements Cont.

- Why am I always extending the lecture length
  - My CS180 was designed to last 1h to 1.25h
- On the BBS
  - I'd welcome more questions on concepts
- My real-time rendering course
  - Unfortunately has to be internal
  - But will deliver it to GAMES later (maybe summer 2020)
- Again, today's lecture won't be easy

Last Lectures

- Basic ray tracing
  - Ray generation
  - Ray object intersection
- Acceleration
  - Ray AABB intersection
  - Spatial partitions vs object partitions
  - BVH traversal
- Radiometry

Today

- Radiometry cont.
- Light transport
  - The reflection equation
  - The rendering equation
- Global illumination
- Probability review

Reviewing Concepts

Radiant energy Q $[\mathrm{J} = \mathrm{Joule}]$ (barely used in CG)

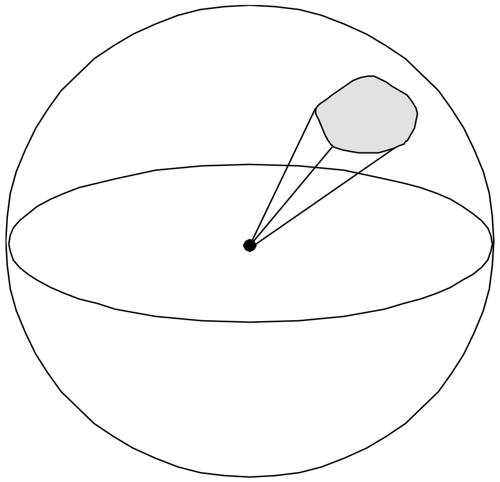
- the energy of electromagnetic radiation

Radiant flux (power) $\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t}$ $[\mathrm{W} = \mathrm{Watt}]$ $[\mathrm{lm} = \mathrm{lumen}]$

- Energy per unit time

Radiant intensity $I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$

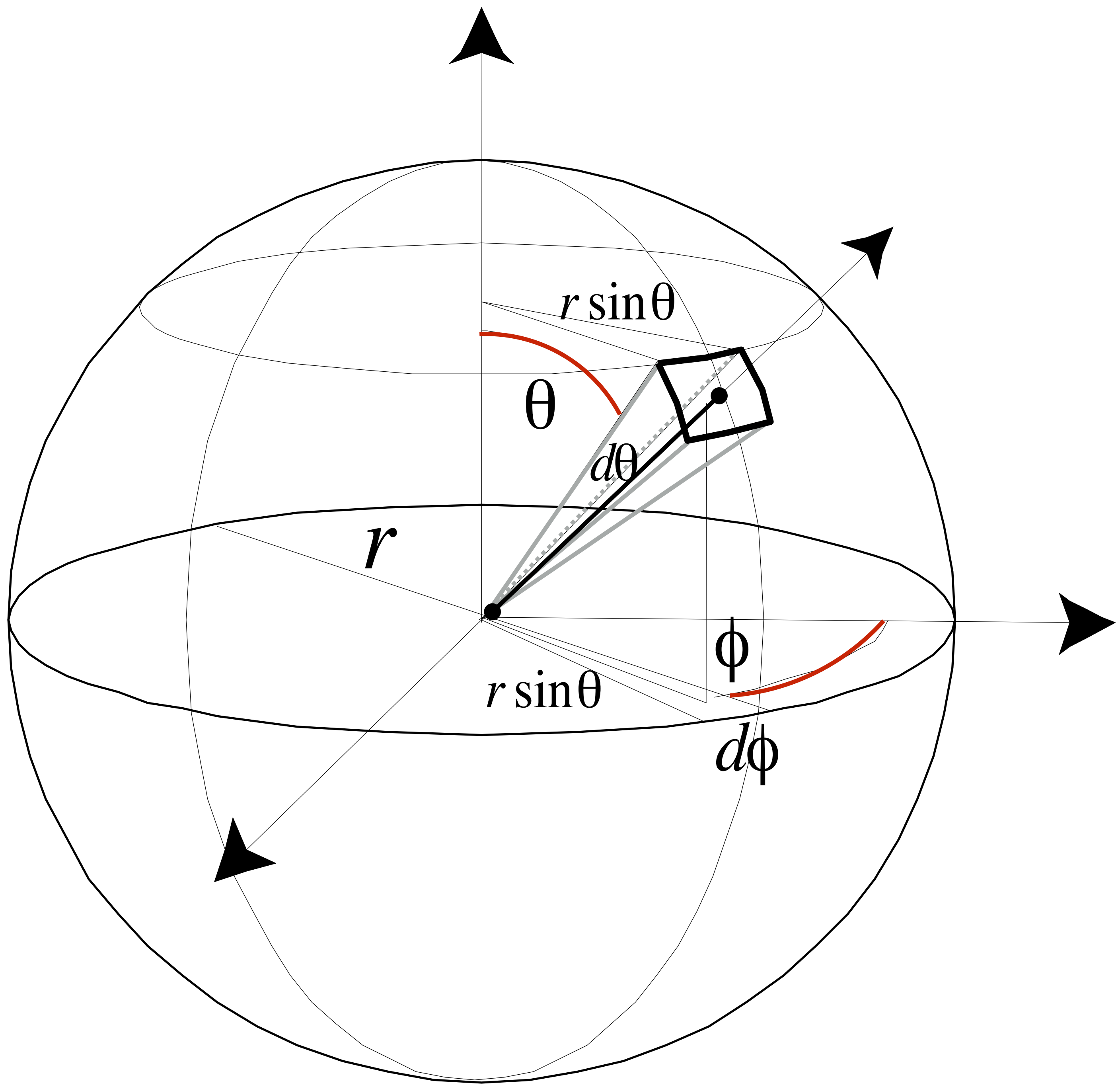
- power per unit solid angle



Solid Angle $\Omega = \frac{A}{r^2}$

- ratio of subtended area on sphere to radius squared

Differential Solid Angles



$$\begin{aligned}\mathrm{d}A &= (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)\\ &= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\end{aligned}$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

微分立体角实际上是 衡量一个方向的光

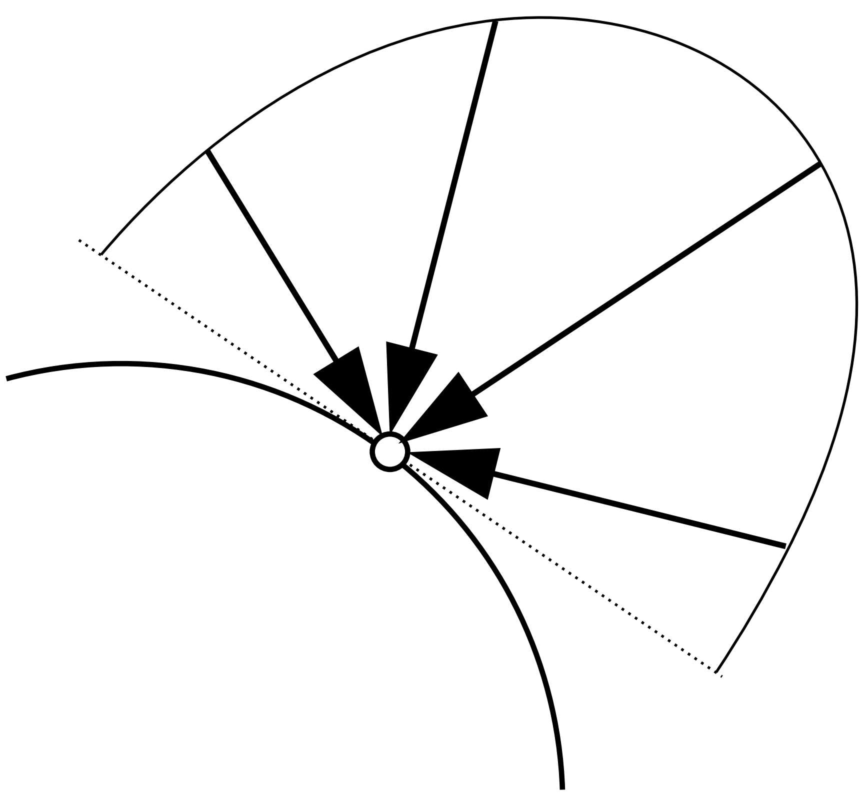
Irradiance

Irradiance

Definition: The irradiance is the power per unit area incident on a surface point.

$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

$$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \left[\frac{\mathrm{lm}}{\mathrm{m}^2} = \mathrm{lux}\right]$$

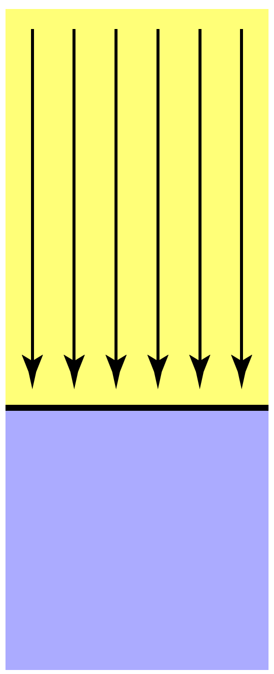


Lambert's Cosine Law

dA 必须得是和光线垂直的面，如果光线不垂直则就要取投影方向上垂直光线的面积

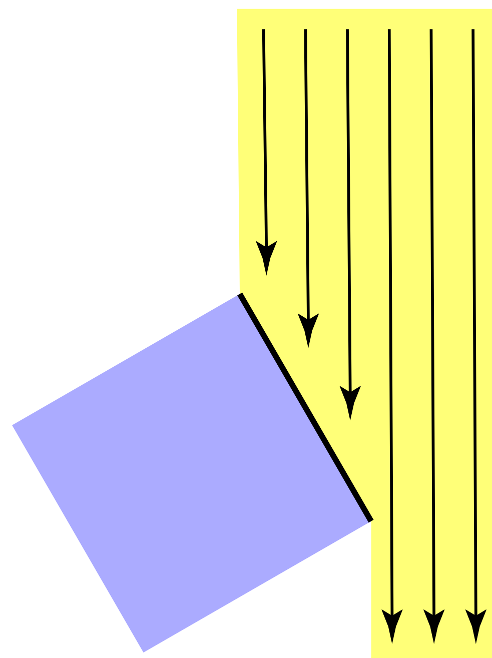
Irradiance at surface is proportional to cosine of angle between light direction and surface normal.

(Note: always use a unit area, the cosine applies on Φ)



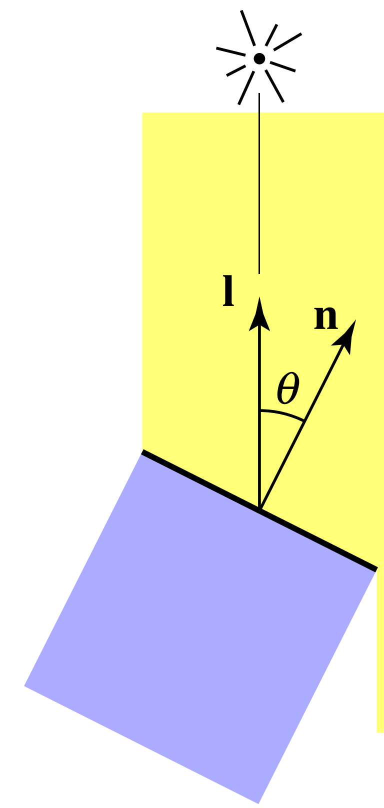
Top face of cube receives a certain amount of power

$$E = \frac{\Phi}{A}$$



Top face of 60° rotated cube receives half power

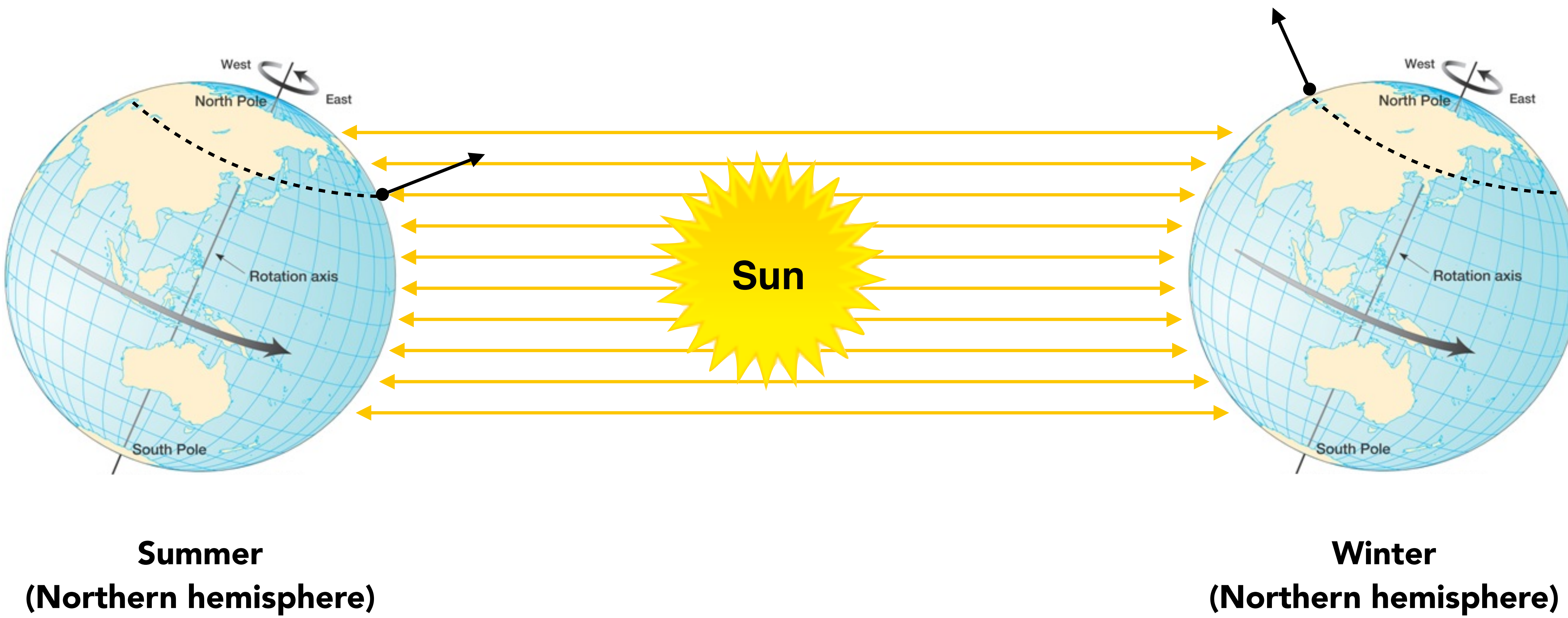
$$E = \frac{1}{2}\frac{\Phi}{A}$$



In general, power per unit area is proportional to $\cos\theta = l \cdot n$

$$E = \frac{\Phi}{A}\cos\theta$$

Why Do We Have Seasons?



[Image credit: Pearson Prentice Hall]

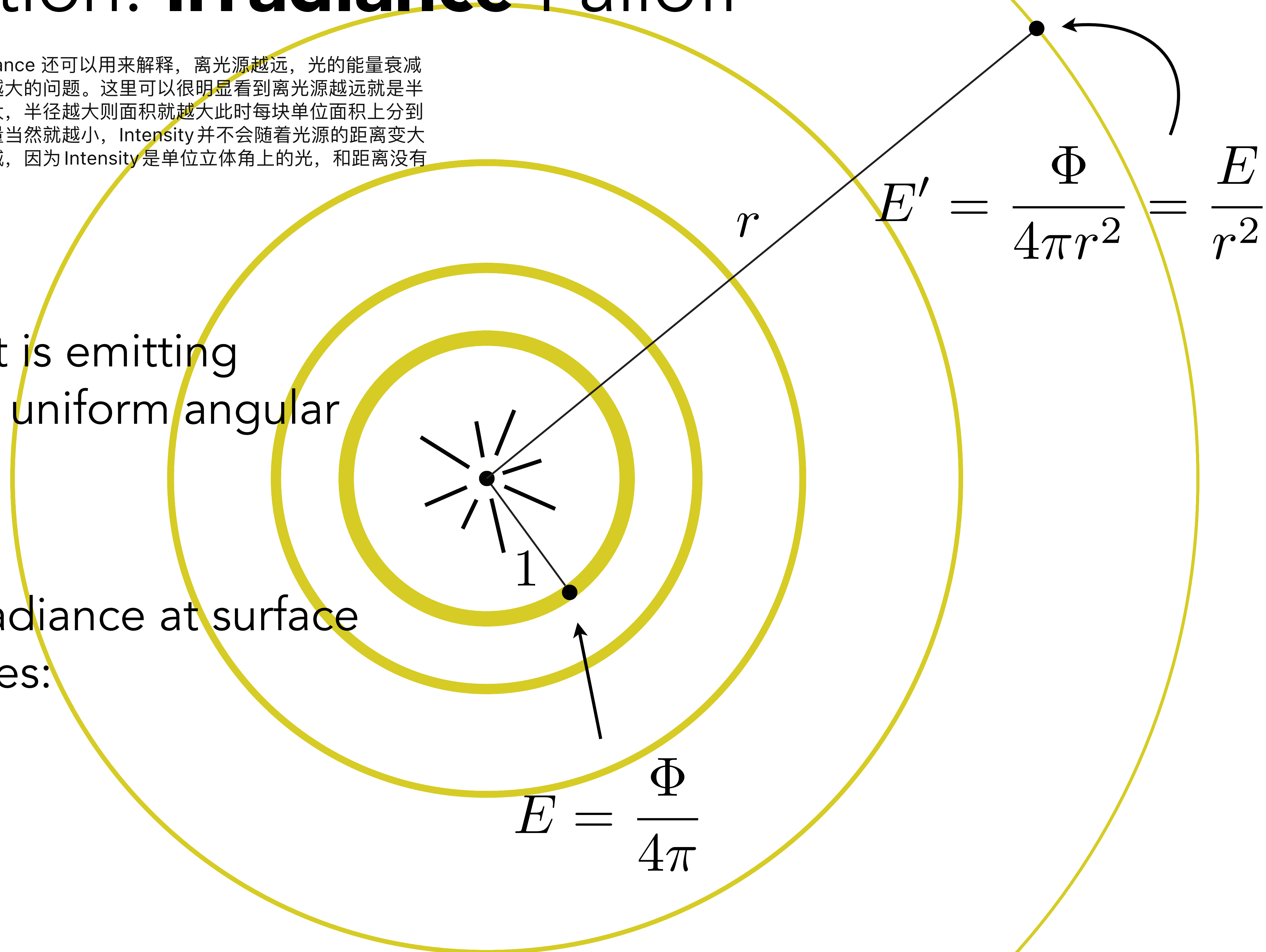
Earth's axis of rotation: ~23.5° off axis

Correction: **Irradiance** Falloff

Irradiance 还可以用来解释，离光源越远，光的能量衰减程度越大的问题。这里可以很明显看到离光源越远就是半径越大，半径越大则面积就越大此时每块单位面积上分到的能量当然就越小，Intensity 并不会随着光源的距离变大而衰减，因为 Intensity 是单位立体角上的光，和距离没有关系

Assume light is emitting power Φ in a uniform angular distribution

Compare irradiance at surface of two spheres:



$$E' = \frac{\Phi}{4\pi r^2} = \frac{E}{r^2}$$

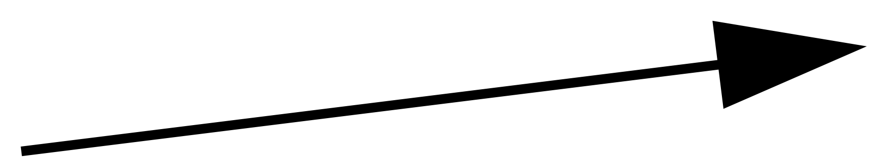
$$E = \frac{\Phi}{4\pi}$$

Radiance

Radiance

Radiance is the fundamental field quantity that describes the distribution of light in an environment

- Radiance is the quantity associated with a ray
- Rendering is all about computing radiance

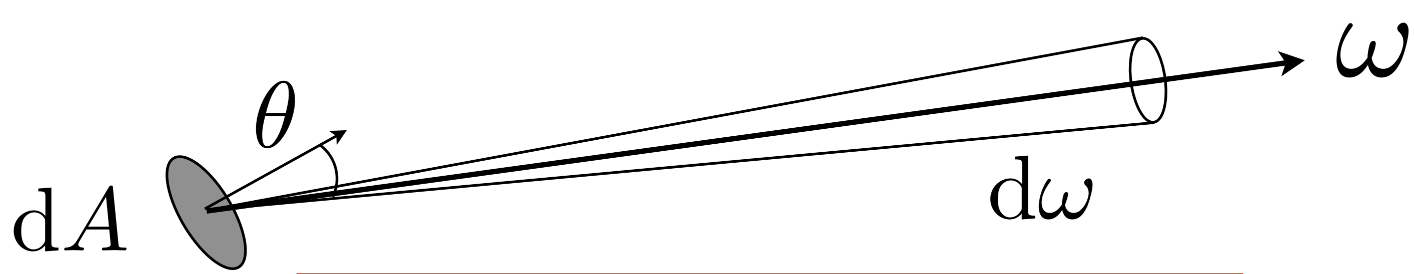


Light Traveling Along A Ray

Radiance

Radiance 从定义上看就是 Intensity 和 Irradiance 的结合（考虑了单位立体角和单位面积），Irradiance 表示的是立体角 w 的所有 Power 在接受光照的表面上的单位面积能量，而 radiance 表示的就是单位立体角 dw 在 dA 上的 Power:

Definition: The radiance (luminance) is the power emitted, reflected, transmitted or received by a surface, per unit solid angle, per projected unit area.



$$\boxed{L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega\, \mathrm{d}A \cos\theta}}$$

$\cos\theta$ accounts for projected surface area

$$\left[\frac{\mathrm{W}}{\mathrm{sr\,m^2}}\right] \left[\frac{\mathrm{cd}}{\mathrm{m^2}} = \frac{\mathrm{lm}}{\mathrm{sr\,m^2}} = \mathrm{nit}\right]$$

Radiance

Definition: power per unit solid angle per projected unit area.

$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega \, \mathrm{d}A \cos\theta}$$

Recall

- Irradiance: power per projected unit area
- Intensity: power per solid angle

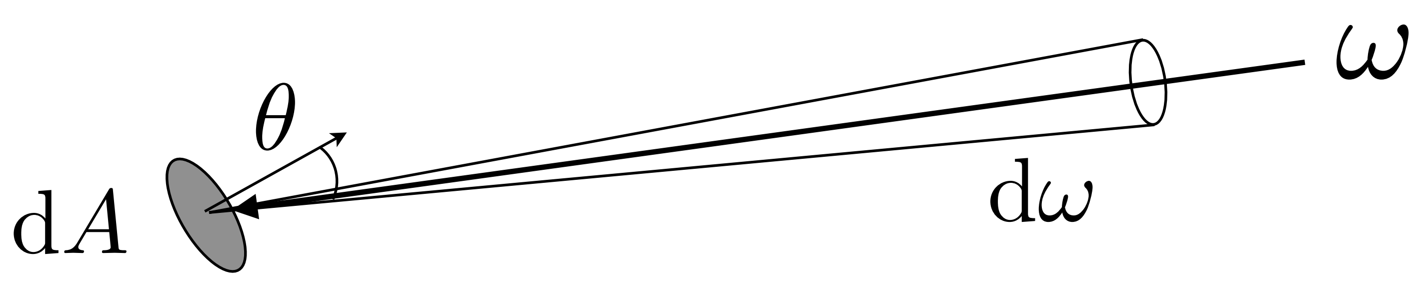
So

Radiance是某个单位面积向某个单位立体角辐射出去的能量，Irradiance是某个单位面积上接受到来自四面八方的能量
区别就在于辐亮度Radiance有方向的概念，而辐照度Irradiance没有

- Radiance: Irradiance per solid angle
- Radiance: Intensity per projected unit area

Incident Radiance

Incident radiance is the irradiance per unit solid angle arriving at the surface.

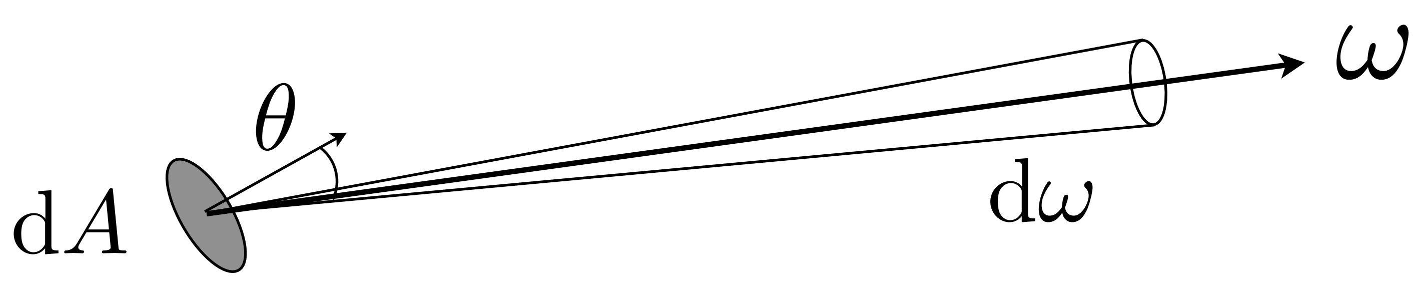


$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}E(\mathrm{p})}{\mathrm{d}\omega \cos\theta}$$

i.e. it is the light arriving at the surface along a given ray (point on surface and incident direction).

Exiting Radiance

Exiting surface radiance is the intensity per unit projected area leaving the surface.



$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}I(\mathrm{p}, \omega)}{\mathrm{d}A \cos\theta}$$

e.g. for an area light it is the light emitted along a given ray (point on surface and exit direction).

Irradiance vs. Radiance

Irradiance: total power received by area dA

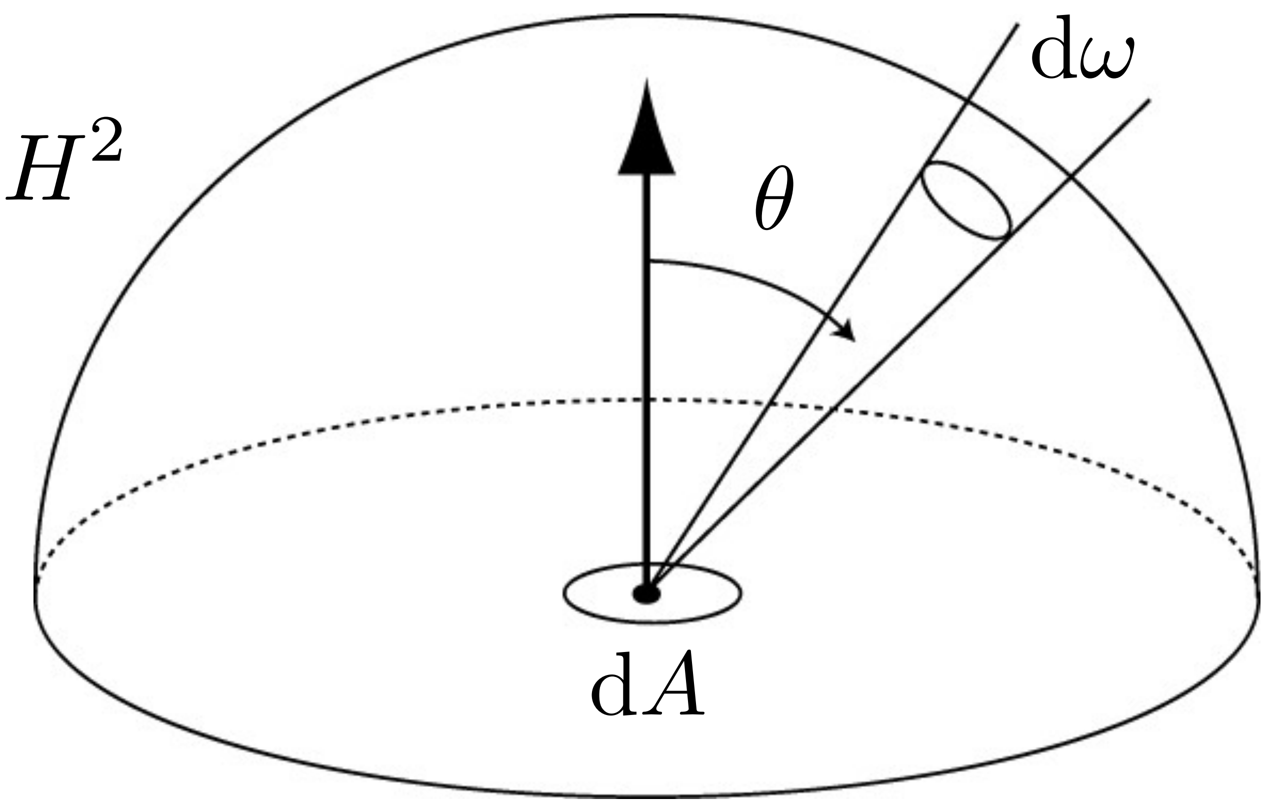
Radiance: power received by area dA from "direction" dω

$$dE(\mathrm{p}, \omega) = L_i(\mathrm{p}, \omega) \cos\theta \, \mathrm{d}\omega$$

$$E(\mathrm{p}) = \int_{H^2} L_i(\mathrm{p}, \omega) \cos\theta \, \mathrm{d}\omega$$

把半球面上的所有Radiance积分起来得到的就是Irradiance

Unit Hemisphere: H^2



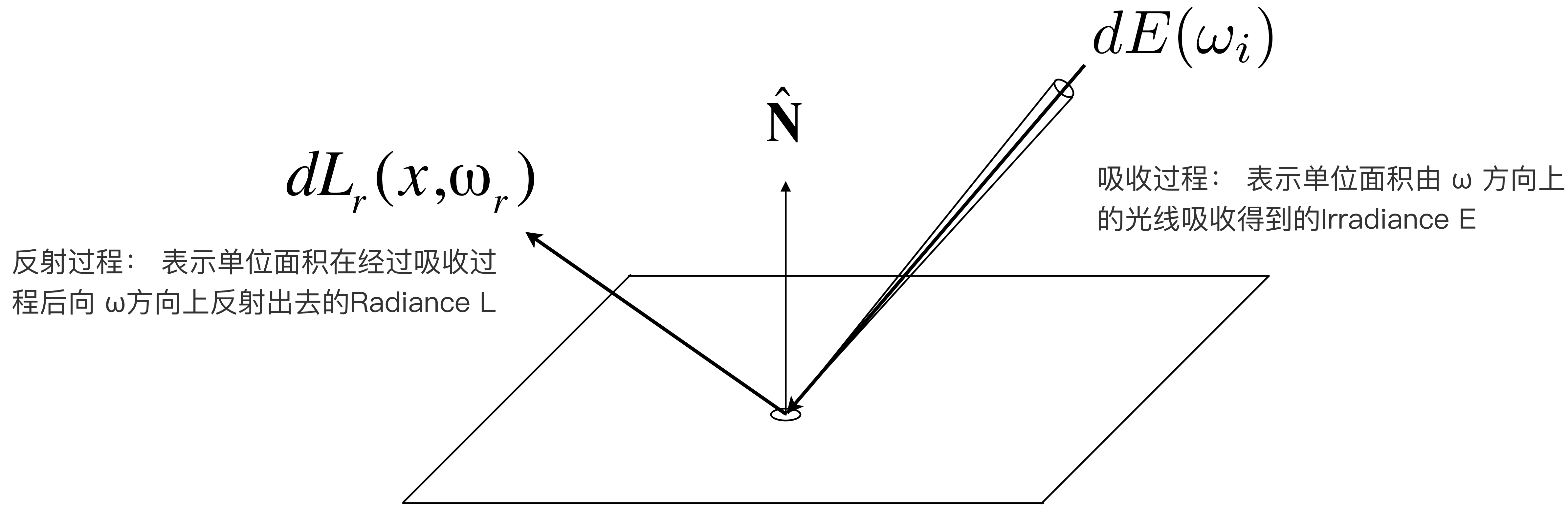
Bidirectional Reflectance Distribution Function (BRDF)

Reflection at a Point

Radiance from direction ω_i turns into the power E that dA receives

Then power E will become the radiance to any other direction ω_o

空间中仅有一束光，从方向打在单位面积 dA 上，则 dA 吸收的全部能量就是 Irradiance，然后再向其他方向散射，比如方向 wr，则这部分能量就是 radiance



Differential irradiance incoming: $dE(\omega_i) = L(\omega_i)\cos\theta_i\, d\omega_i$

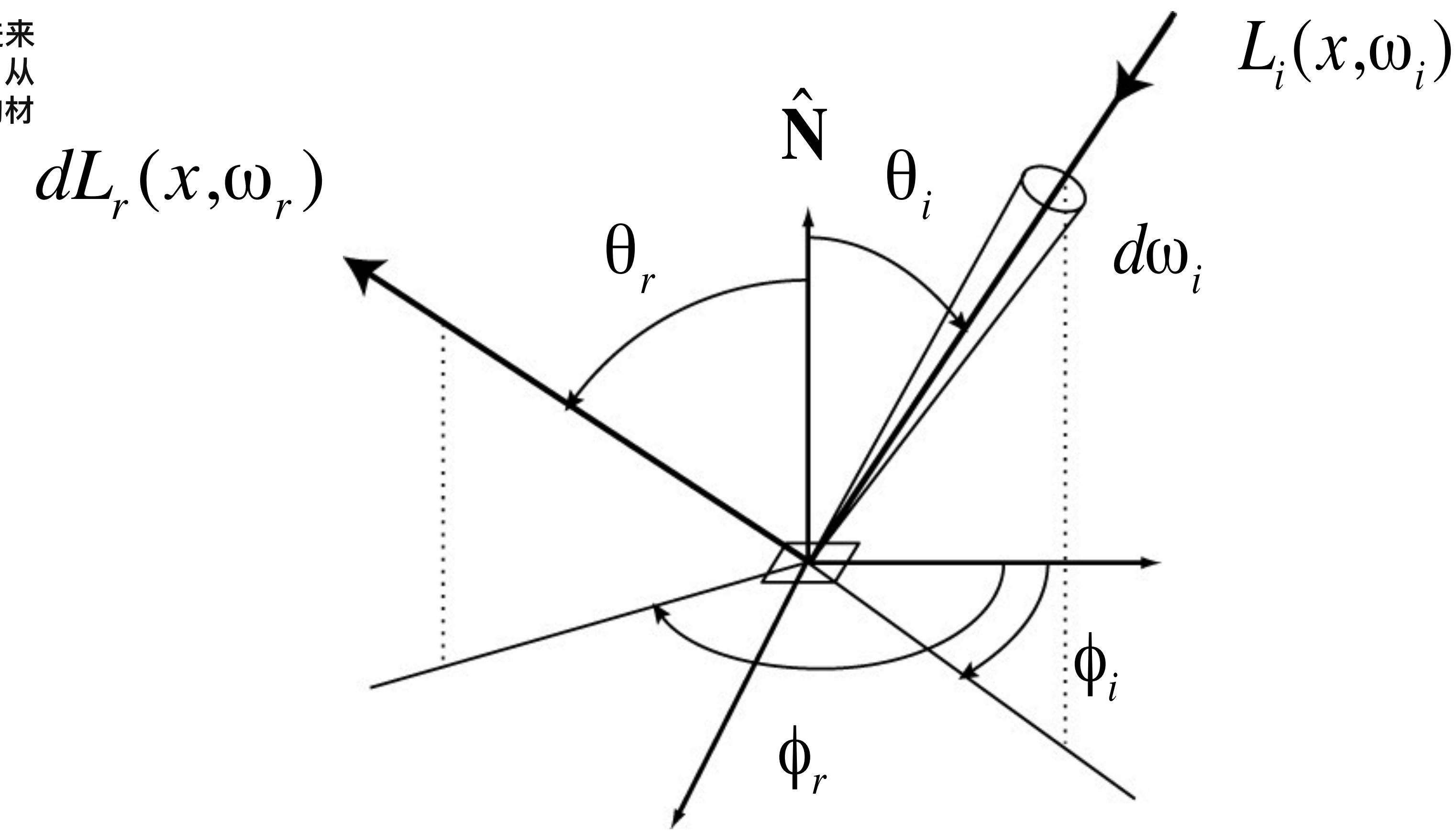
Differential radiance exiting (due to $dE(\omega_i)$): $dL_r(\omega_r)$

BRDF

BRDF 其实描述的就是有一束光从某一个方向辐射进来，则应该向其他方向散射多少能量

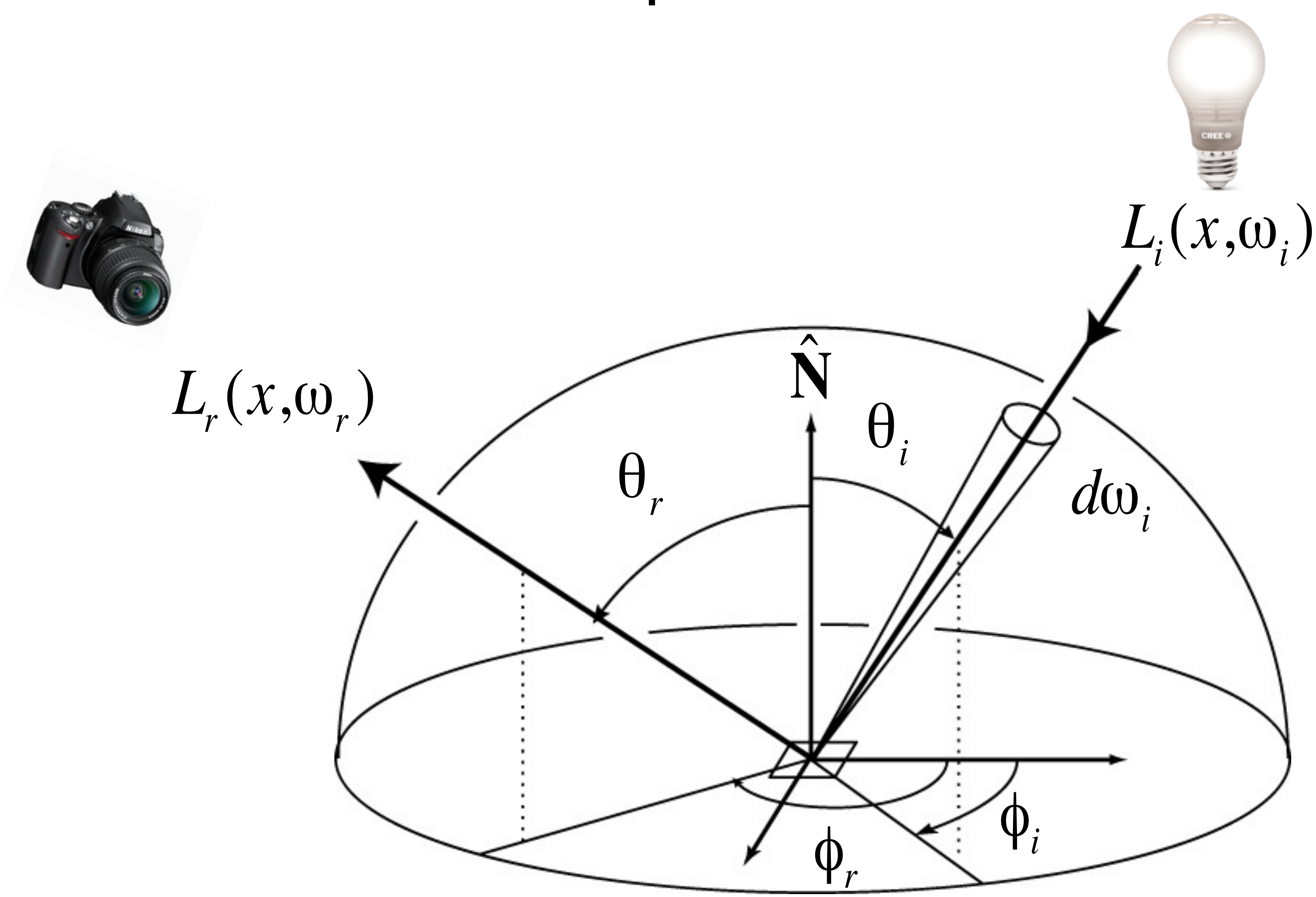
The Bidirectional Reflectance Distribution Function (BRDF) represents how much light is reflected into each outgoing direction ω_r from each incoming direction

BRDF 的公式实际上就是描述：从某个立体角 辐射进来的光束能量会按什么规律被分配到其他立体角上去。从物理意义上看，决定光束散射规律的就是物体表面的材质，因此 BRDF 描述的就是物体表面的材质。



$$f_r(\omega_i \to \omega_r) = \frac{\mathrm{d}L_r(\omega_r)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\,\mathrm{d}\omega_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$

The Reflection Equation



$$L_r(\mathrm{p},\omega_r) = \int_{H^2} f_r(\mathrm{p},\omega_i \rightarrow \omega_r)\, L_i(\mathrm{p},\omega_i)\, \cos\theta_i \,\mathrm{d}\omega_i$$

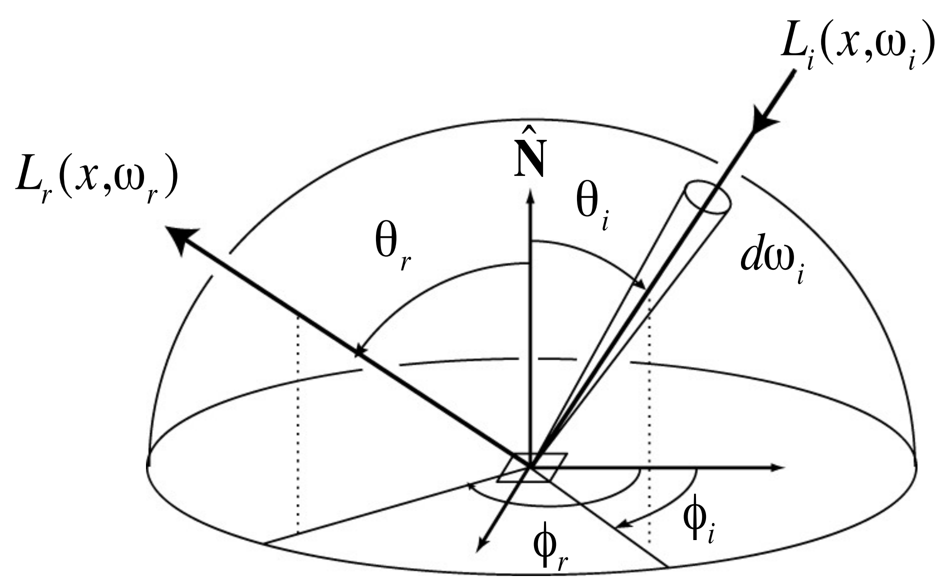
面光源用积分，点光源用累加 这个出射的radiance还有可能作为其他点所接收到的irradiance（n次反射），所以这是一个递归的方程

Challenge: Recursive Equation

反射方程研究的是： 某个接受的单位面积从各个方向上吸收了能量而在 ωr 上总共辐射的能量

Reflected radiance depends on incoming radiance

$$\boxed{L_r(\mathrm{p}, \omega_r)} = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r) \boxed{L_i(\mathrm{p}, \omega_i)} \cos\theta_i \, \mathrm{d}\omega_i$$



But incoming radiance depends on reflected radiance (at another point in the scene)

The Rendering Equation

Re-write the reflection equation:

$$L_r(\mathrm{p}, \omega_r) = \int_{H^2} f_r(\mathrm{p}, \omega_i \to \omega_r)\, L_i(\mathrm{p}, \omega_i)\, \cos\theta_i\, \mathrm{d}\omega_i$$

by adding an Emission term to make it general!

The Rendering Equation

渲染方程与反射方程相比只是多加了一个自发光的项
渲染效果 = 反射光 + 自发光

$$\boxed{L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega^+} L_i(p, \omega_i) f_r(p, \omega_i, \omega_o)(n \cdot \omega_i)\, \mathrm{d}\omega_i}$$

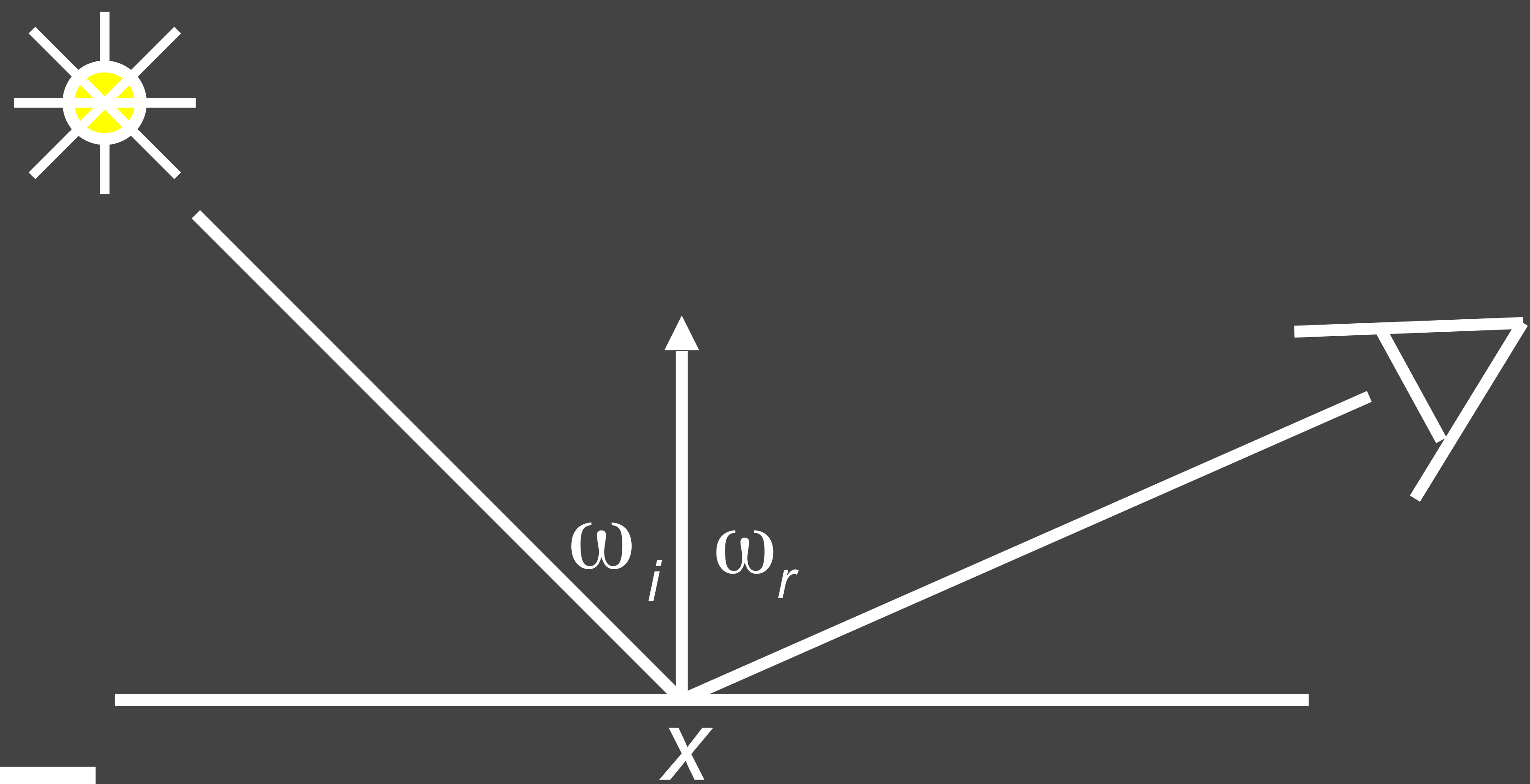
How to solve? Next lecture!

Note: now, we assume that all directions are pointing outwards!

表示半球面 积分区域，并且在渲染方程中，假设入射出射所有的向量都是由内指向外的

Understanding the rendering equation

Reflection Equation



只有一个点光源

$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i,n)$$

Reflected Light (Output Image)

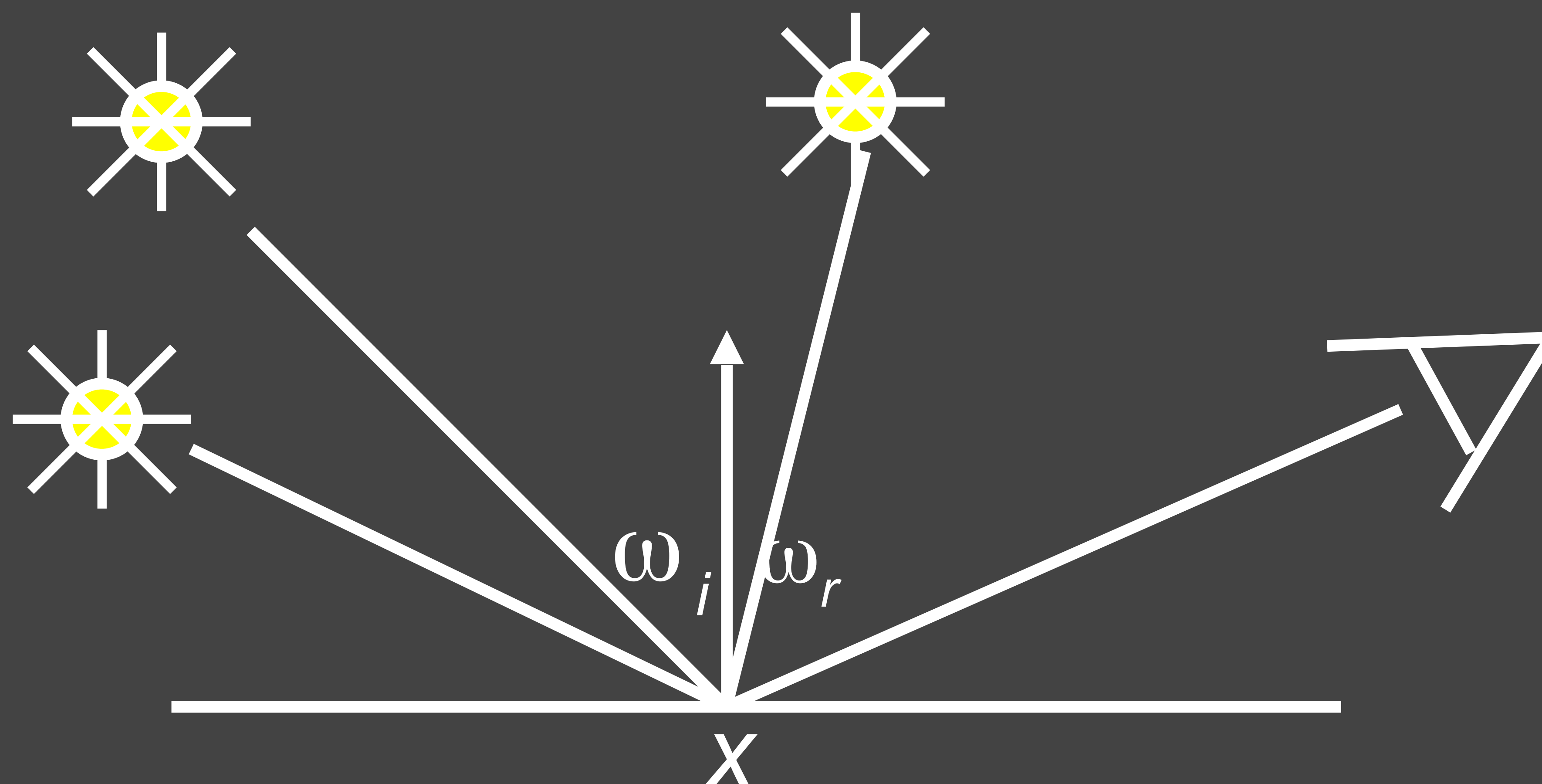
Emission

Incident Light (from light source)

BRDF

Cosine of Incident angle

Reflection Equation



如果有很多个点光源，那么反射光就是把所有点光源的反射光能量加起来

Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ (\omega_i,n)$$

Reflected Light (Output Image)

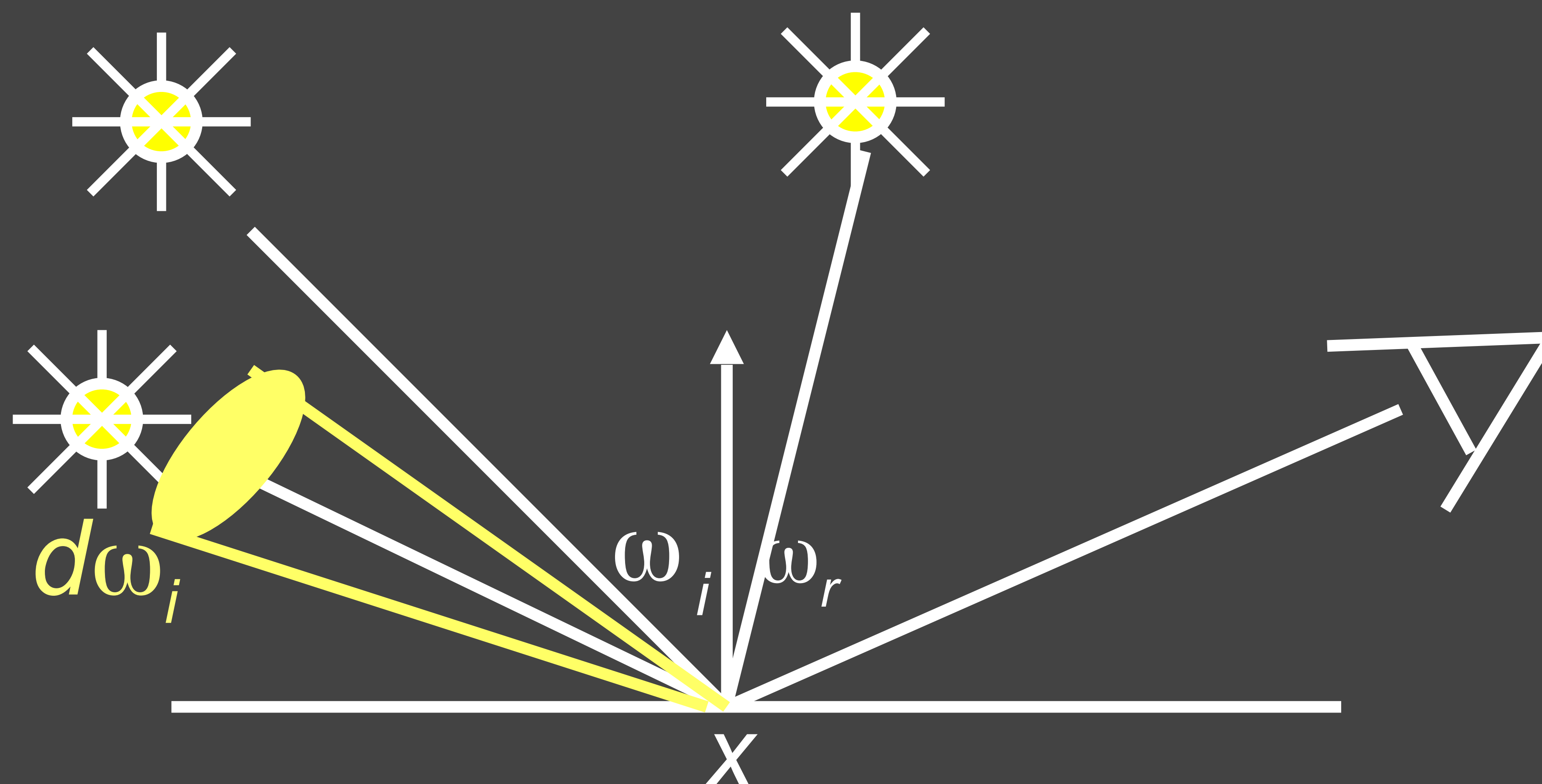
Emission

Incident Light (from light source)

BRDF

Cosine of Incident angle

Reflection Equation



如果存在面光源，那么将这个面光源当成点光源的集合，求积分

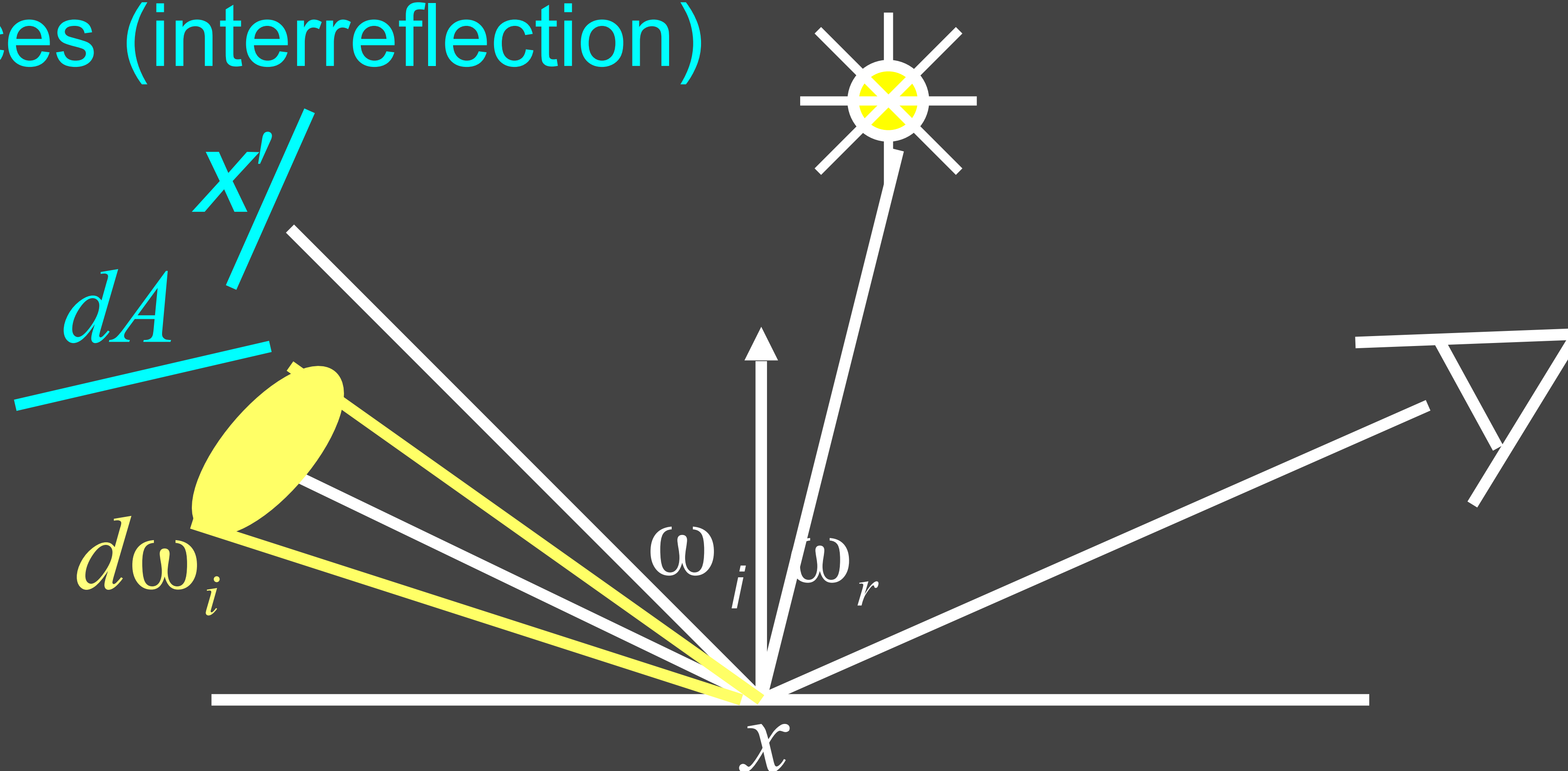
Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i)\ f(x,\omega_i,\omega_r)\ \cos\theta_i\,\mathrm{d}\omega_i$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Rendering Equation

Surfaces (interreflection)



如果不止光源，还有其他物体反射来的光，则把其他物体的反射面当成光源，递归

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_r(x',-\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i \, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Rendering Equation (Kajiya 86)

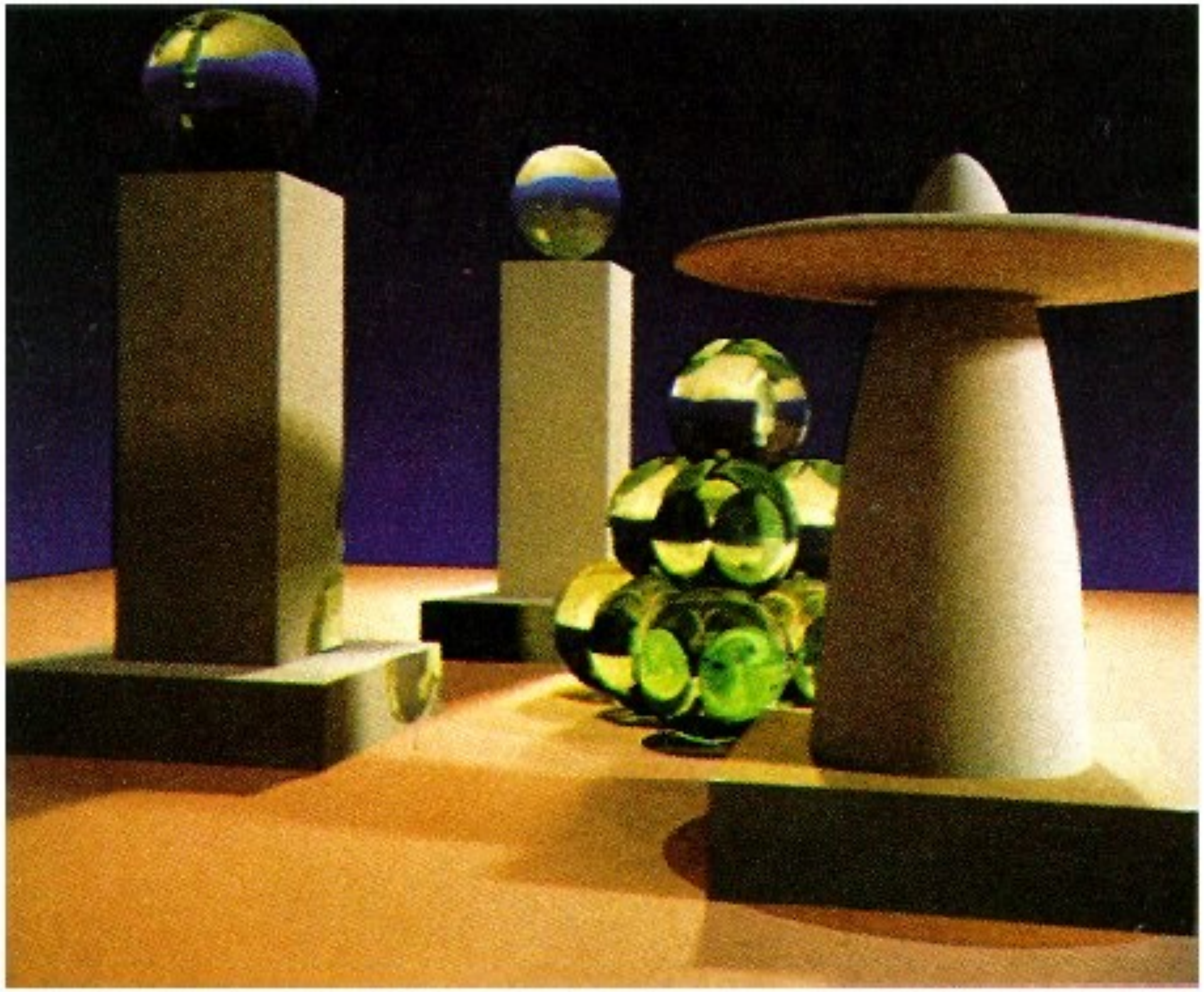


Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Rendering Equation as Integral Equation

全局光照渲染方程的简化

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\boxed{f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i}$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v)\,\boxed{K(u,v)dv}$$

Kernel of equation

Linear Operator Equation

$$l(u) = e(u) + \int l(v) \boxed{K(u,v)dv}$$

Kernel of equation
Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation
[or system of simultaneous linear equations]
(L, E are vectors, K is the light transport matrix)

Ray Tracing and extensions

- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1}E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + \ldots)E$$

$$L = E + KE + K^2E + K^3E + \ldots$$

Ray Tracing

物理意义就是人眼或相机，接受到的光弹射0次(直接光照)，和弹射1次、2次、3次......的间接光照

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect illum.)

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

如果用渲染方程来理解光栅化，可以发现光栅化只做了全局光照的前两步，即自发光和直接光照

Direct Illumination
on surfaces

Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

Shading in
Rasterization

(Two bounce indirect illum.)

p
Direct illumination


p
One-bounce global illumination (dir+indir)


p
Two-bounce global illumination

p
Four-bounce global illumination


p
Eight-bounce global illumination


p
Sixteen-bounce global illumination

Probability Review

Random Variables

X random variable. Represents a distribution of potential values

$X \sim p(x)$ probability density function (PDF). Describes relative probability of a random process choosing value x

Example: uniform PDF: all values over a domain are equally likely

e.g. A six-sided die

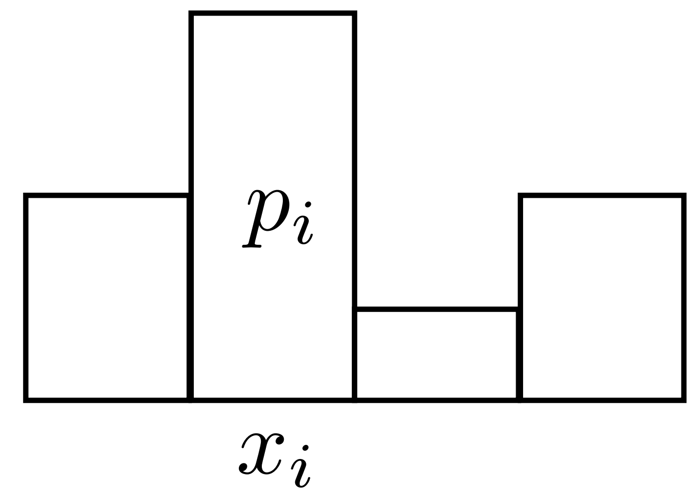
X takes on values 1, 2, 3, 4, 5, 6

$p(1) = p(2) = p(3) = p(4) = p(5) = p(6)$

Probabilities

n discrete values x_i

With probability p_i



Requirements of a probability distribution:

$$p_i \geq 0$$

$$\sum_{i=1}^{n} p_i = 1$$

Six-sided die example: $p_i = \dfrac{1}{6}$

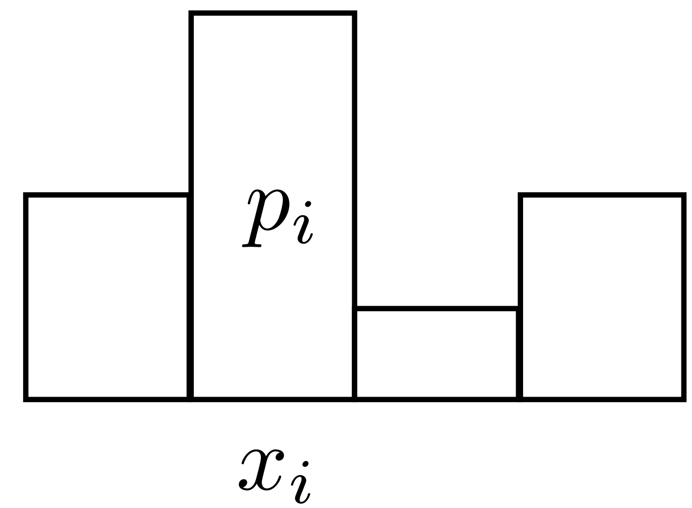
Expected Value of a Random Variable

The average value that one obtains if repeatedly drawing samples from the random distribution.

X drawn from distribution with

n discrete values x_i

with probabilities p_i



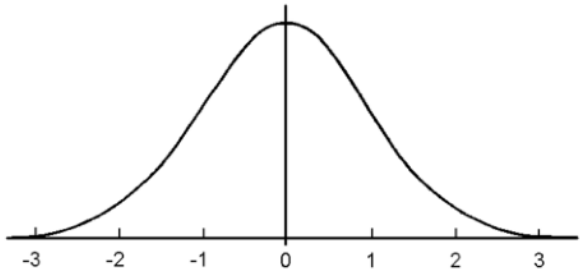
Expected value of X: $$E[X] = \sum_{i=1}^{n} x_i p_i$$

Die example: $$E[X] = \sum_{i=1}^{n} \frac{i}{6}$$

$$= (1+2+3+4+5+6)/6 = 3.5$$

Continuous Case: **Probability Distribution Function (PDF)**

$$X \sim p(x)$$



A random variable X that can take any of a continuous set of values, where the relative probability of a particular value is given by a continuous probability density function p(x).

Conditions on p(x): $p(x) \geq 0 \text{ and } \int p(x)\,dx = 1$

Expected value of X: $E[X] = \int x\,p(x)\,dx$

Function of a Random Variable

A function Y of a random variable X is also a random variable:

$$X \sim p(x)$$
$$Y = f(X)$$

Expected value of a function of a random variable:

$$E[Y] = E[f(X)] = \int f(x)\, p(x)\, dx$$

Thank you!

(And thank Prof. Ravi Ramamoorthi and Prof. Ren Ng for many of the slides!)