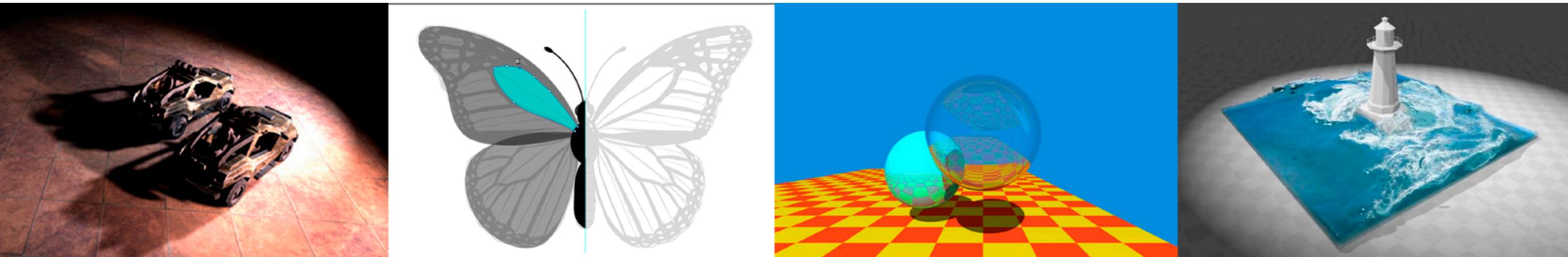


# Introduction to Computer Graphics

AMES101, Lingqi Yan, UC Santa Barbara

## Lecture 5: Rasterization 1 (Triangles)



# Announcements

- Homework 0 – 188 submissions
  - No worries if you did not submit
- Homework 1 will be released today
  - Containing basic and advanced requirements (graded separately)
  - Pass or not pass depends on basic requirements only
- Asking on BBS
  - Please try to describe your question more clearly
- Today's lecture is pretty easy

# Last Lecture

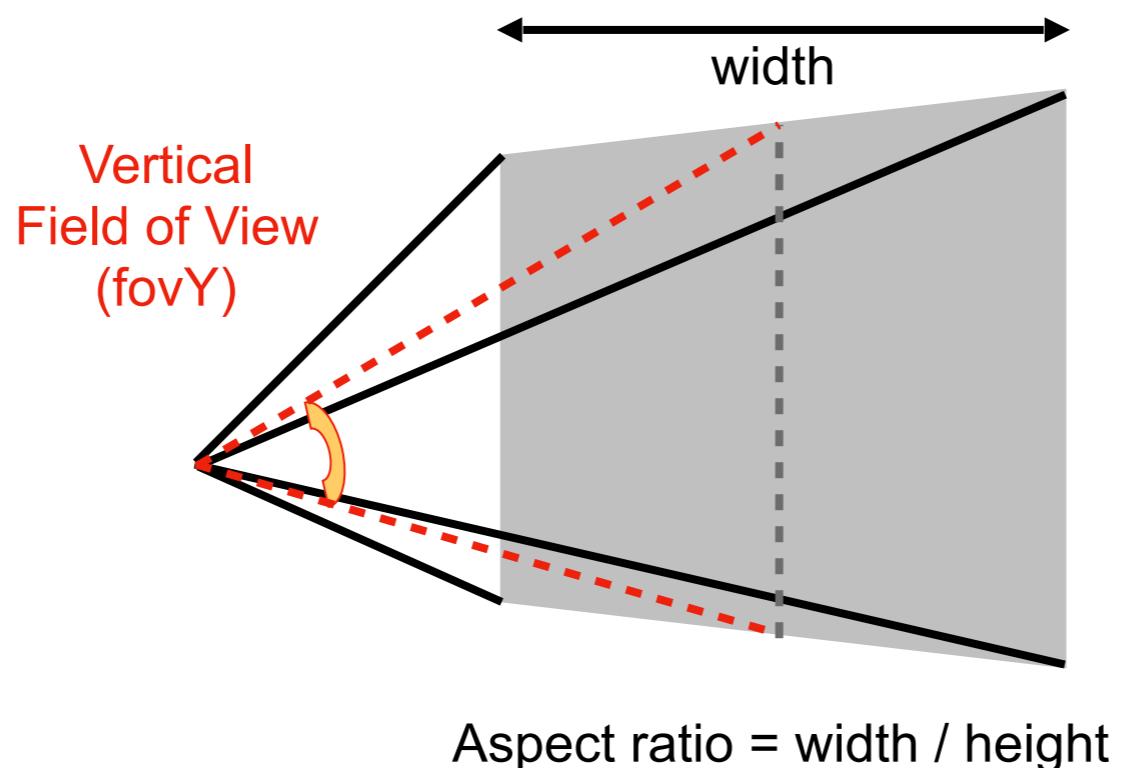
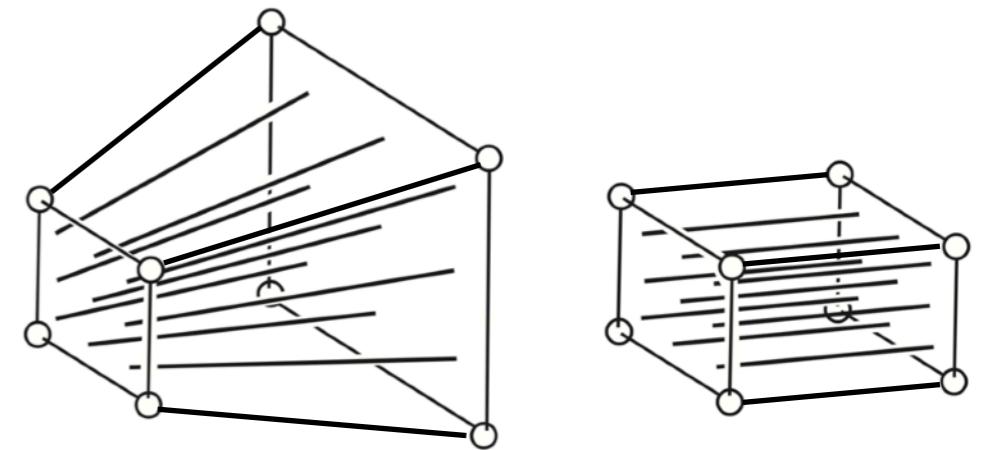
- Viewing (观测) transformation
  - View (视图) / Camera transformation
  - Projection (投影) transformation
    - Orthographic (正交) projection
    - Perspective (透视) projection

# Today

- Finishing up Viewing
  - Viewport transformation
- Rasterization
  - Different raster displays
  - Rasterizing a triangle
- Occlusions and Visibility

# Perspective Projection

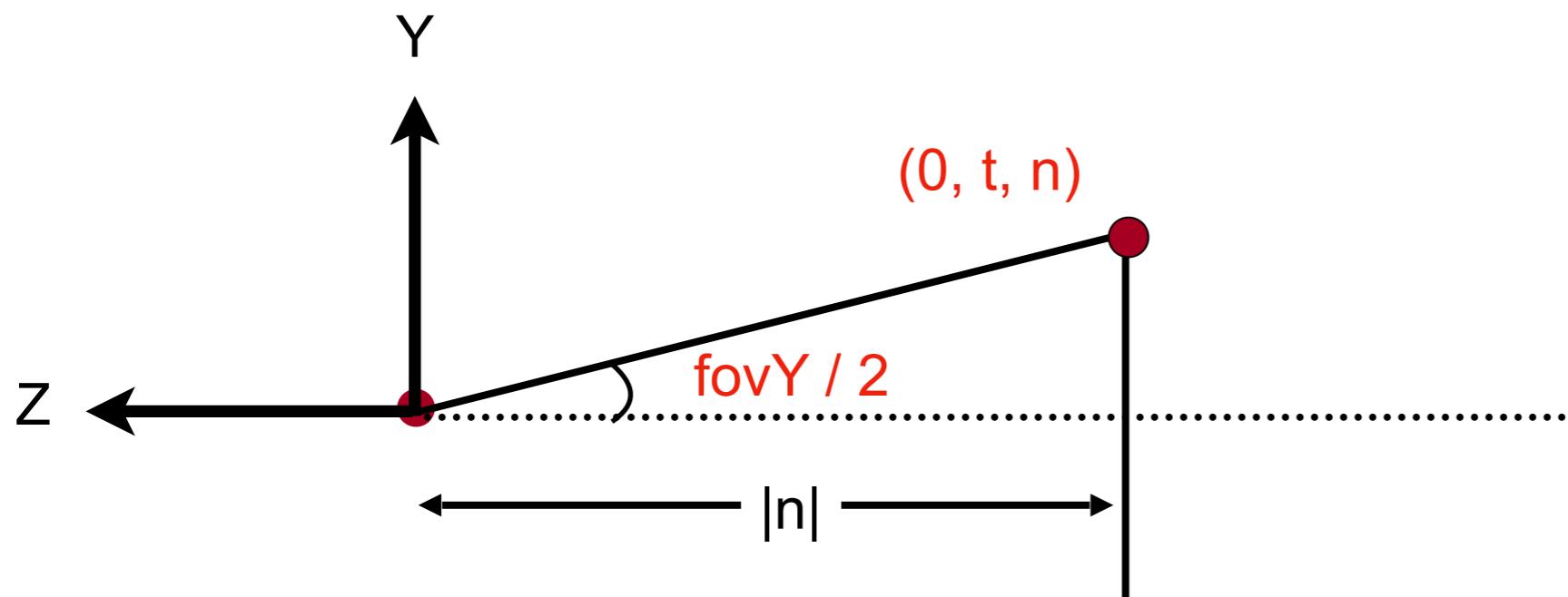
- What's near plane's  $l, r, b, t$  then?
  - If explicitly specified, good
  - Sometimes people prefer:  
vertical **field-of-view** (fovY) and  
**aspect ratio**  
(assume symmetry i.e.  $l = -r, b = -t$ )



定义视点的可视范围：  
- 长宽比，衡量可视面积  
- 垂直视角，衡量上下可视点的角度

# Perspective Projection

- How to convert from  $\text{fovY}$  and aspect to  $l, r, b, t$ ?
  - Trivial



$$\tan \frac{\text{fovY}}{2} = \frac{t}{|n|}$$

$$\text{aspect} = \frac{r}{t}$$

# What's after MVP?

- Model transformation (placing objects)
- View transformation (placing camera)
- Projection transformation
  - Orthographic projection (cuboid to “canonical” cube  $[-1, 1]^3$ )
  - Perspective projection (frustum to “canonical” cube)
- Canonical cube to ?

MVP这三个变换之后，所有东西都会停留在一个 $1, 1, 1$ 的位于原点的标准立方体中 下一步就要把这立方体画在屏幕上

# Canonical Cube to Screen

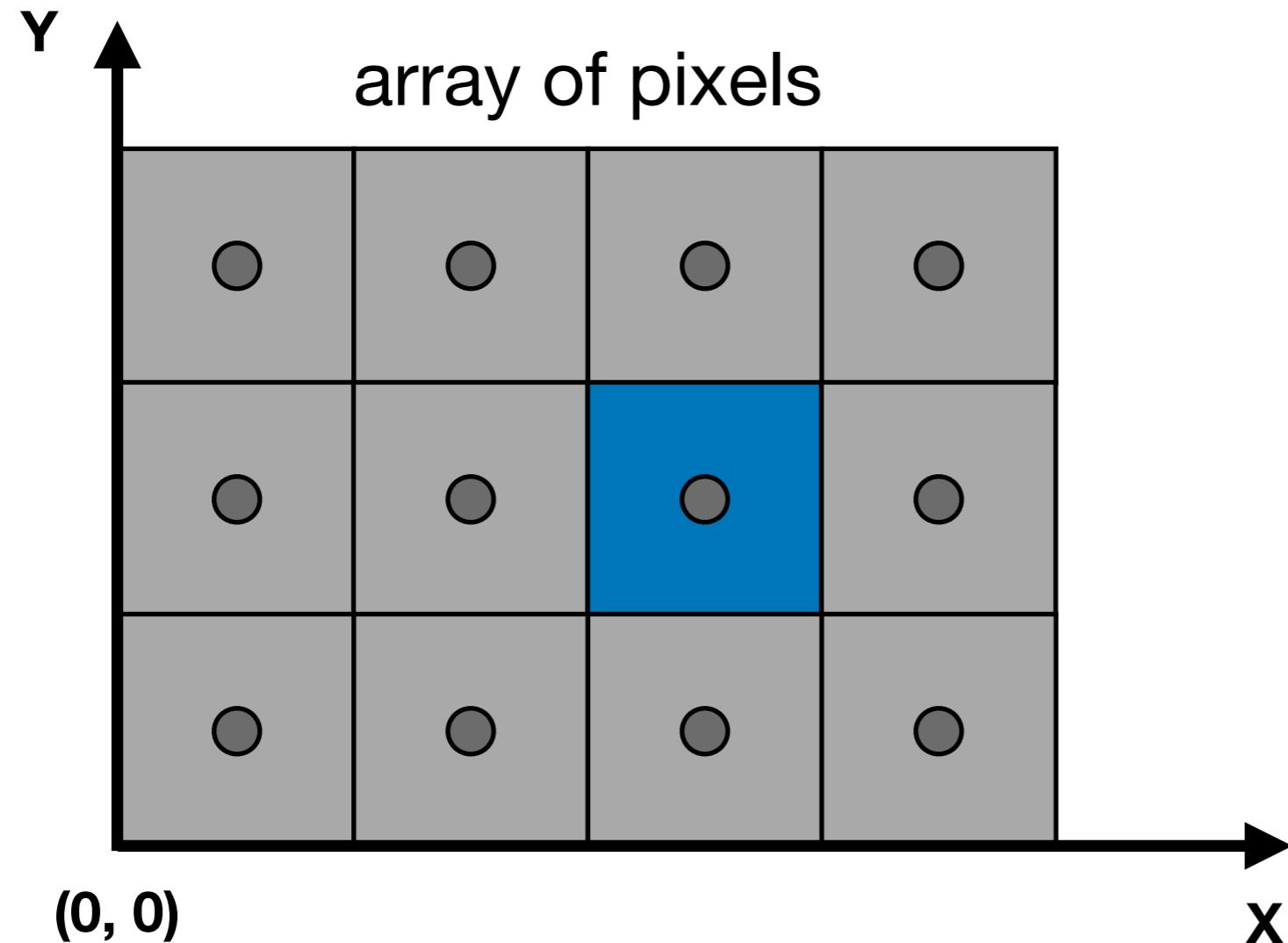
- What is a screen?
  - An array of pixels
  - Size of the array: resolution
  - A typical kind of raster display  
光栅化
- Raster == screen in German
  - Rasterize == drawing onto the screen  
光栅化就是将图形画到屏幕上
- Pixel (FYI, short for “picture element”)
  - For now: A pixel is a little square with uniform color
  - Color is a mixture of (red, green, blue)

# Canonical Cube to Screen

- Defining the screen space

- Slightly different from the “tiger book”

屏幕空间:就是给屏幕定义一个坐标系 比如, 可以定义左下角是原点



Pixels' indices are in the form of  $(x, y)$ , where both  $x$  and  $y$  are integers

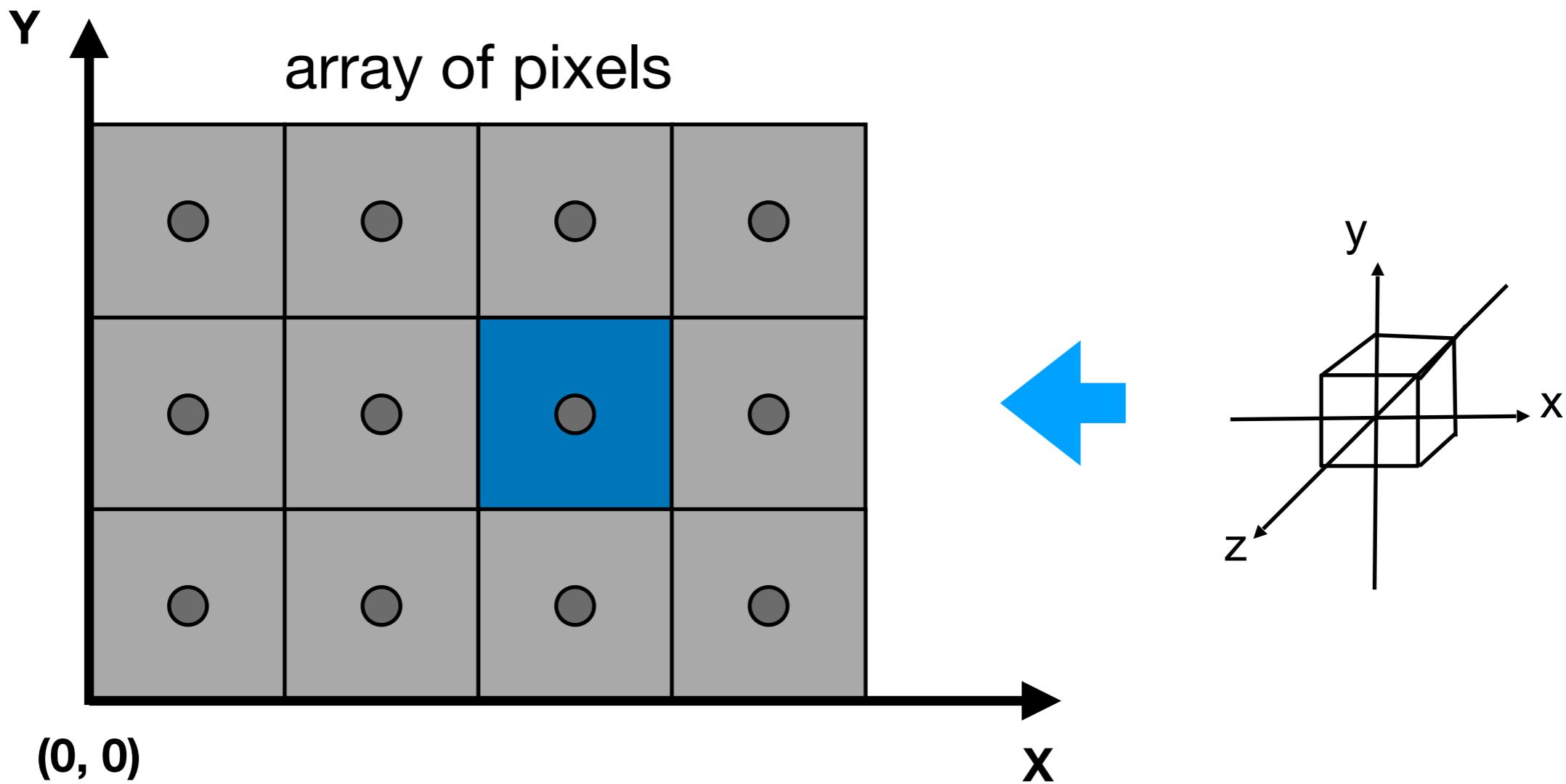
Pixels' indices are from  $(0, 0)$  to  $(\text{width} - 1, \text{height} - 1)$

Pixel  $(x, y)$  is centered at  $(x + 0.5, y + 0.5)$

The screen covers range  $(0, 0)$  to  $(\text{width}, \text{height})$

# Canonical Cube to Screen

- Irrelevant to  $z$  drop  $z$
- Transform in  $xy$  plane:  $[-1, 1]^2$  to  $[0, \text{width}] \times [0, \text{height}]$



# Canonical Cube to Screen

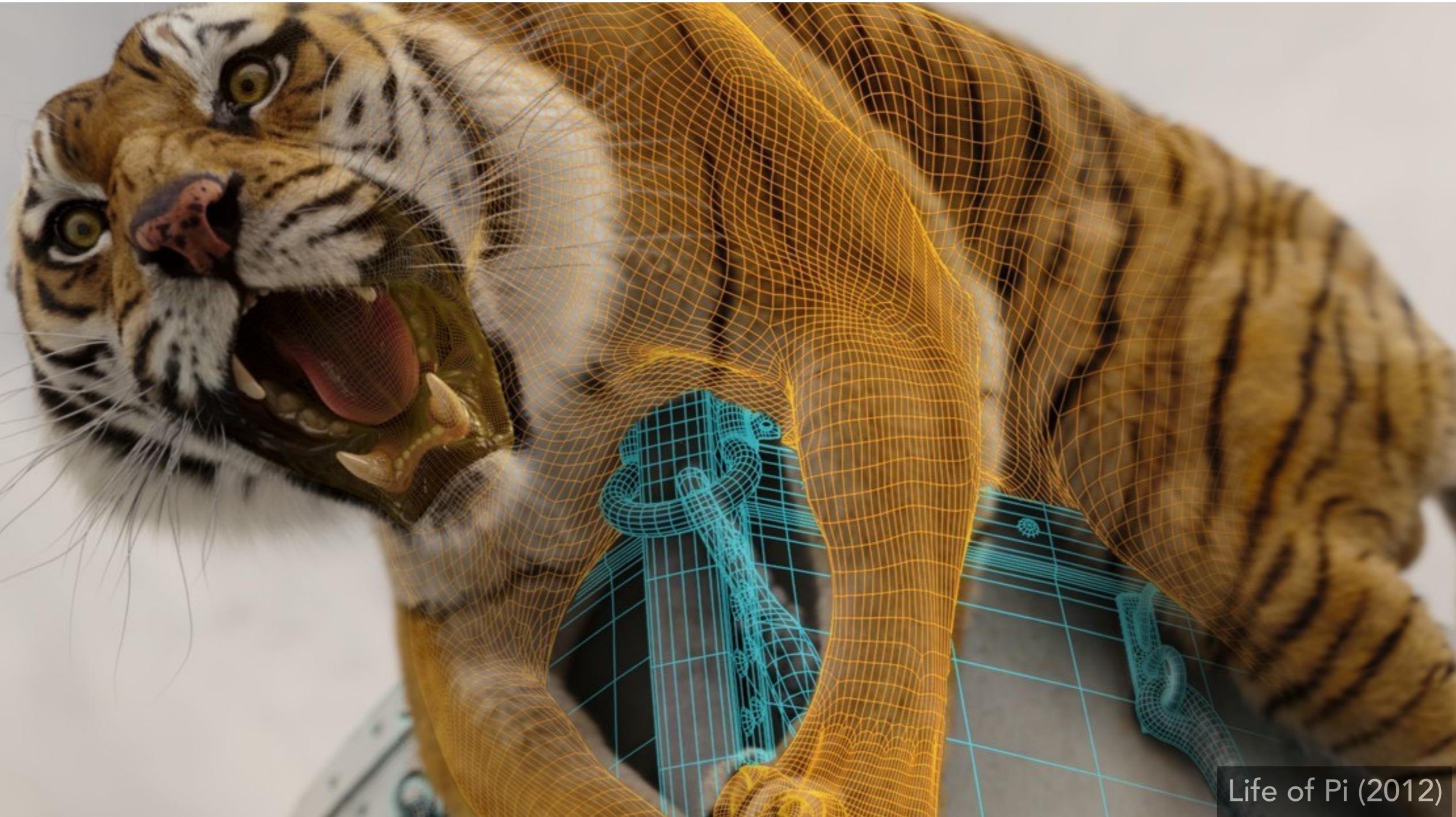
- Irrelevant to z 3D 转 2D 坐标的方式
- Transform in xy plane:  $[-1, 1]^2$  to  $[0, \text{width}] \times [0, \text{height}]$
- Viewport transform matrix:

$$M_{viewport} = \begin{pmatrix} \frac{\text{width}}{2} & 0 & 0 & \frac{\text{width}}{2} \\ 0 & \frac{\text{height}}{2} & 0 & \frac{\text{height}}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

通过这个矩阵把标准立方体空间映射到屏幕这个二维世界中去

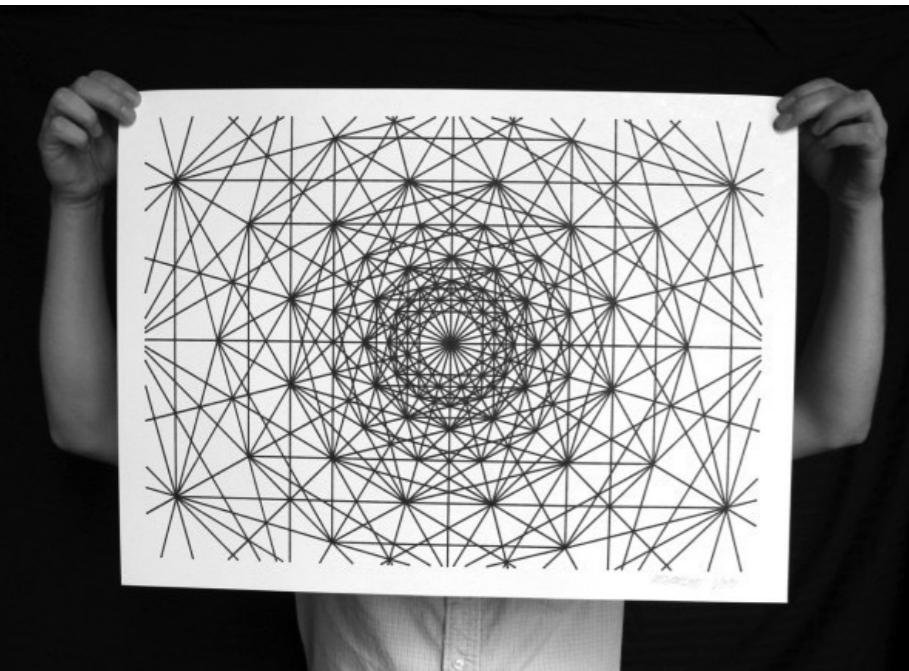
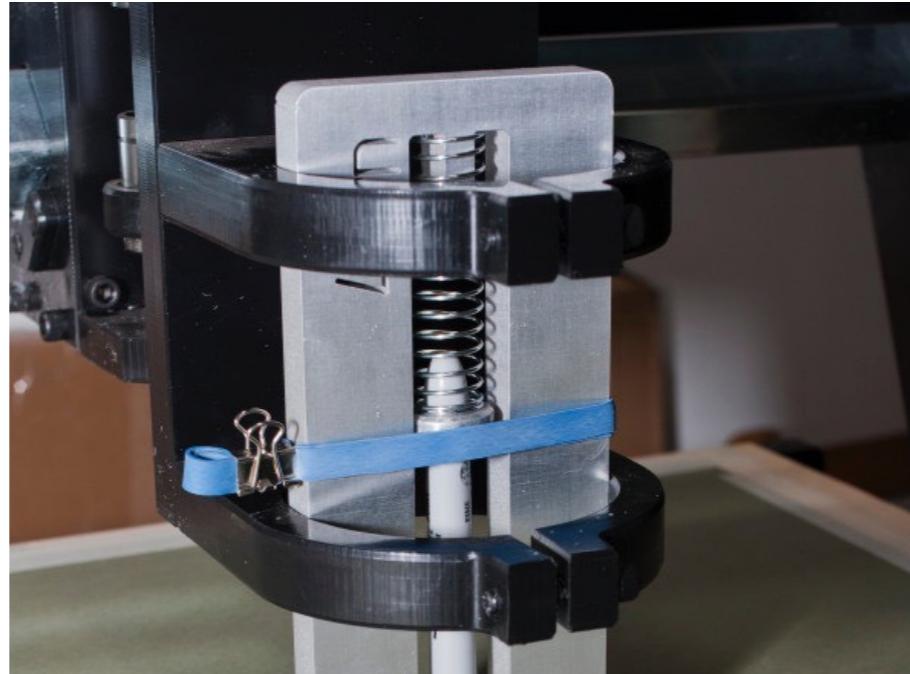
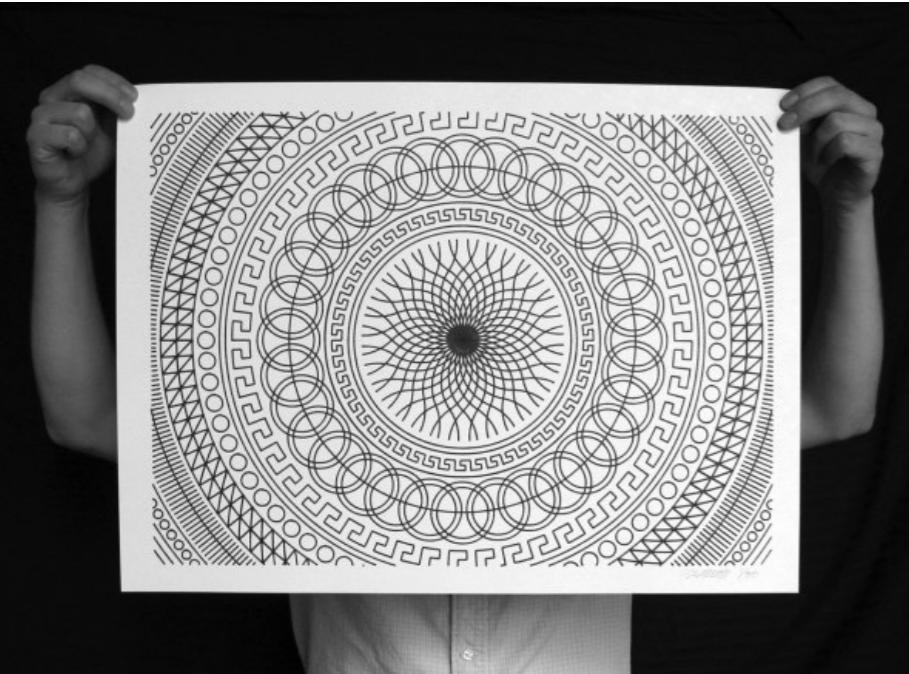
# Next: Rasterizing Triangles into Pixels

将三角形打散为像素



# Drawing Machines

# CNC Sharpie Drawing Machine



Aaron Panone with Matt W. Moore

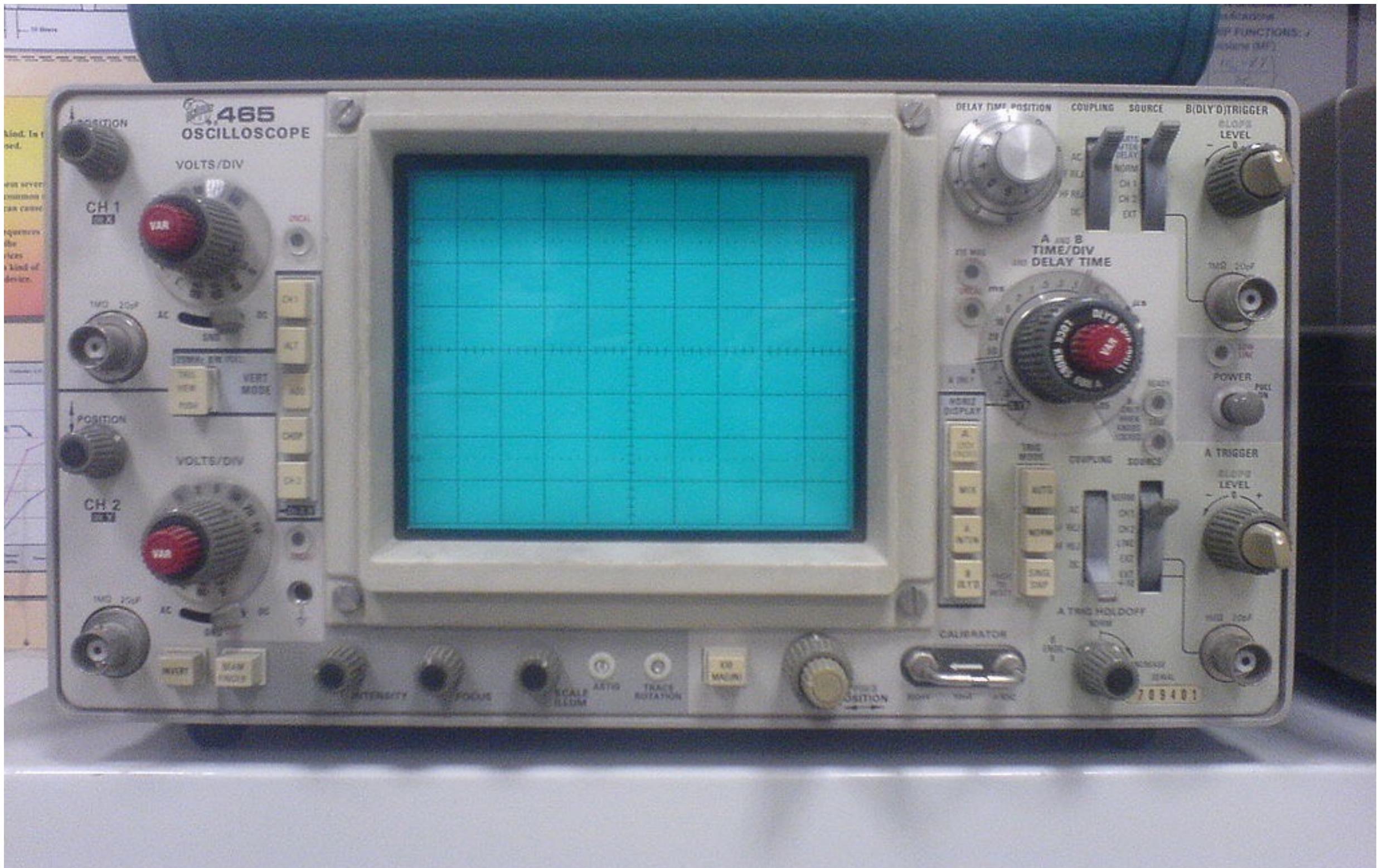
<http://44rn.com/projects/numerically-controlled-poster-series-with-matt-w-moore/>

# Laser Cutters

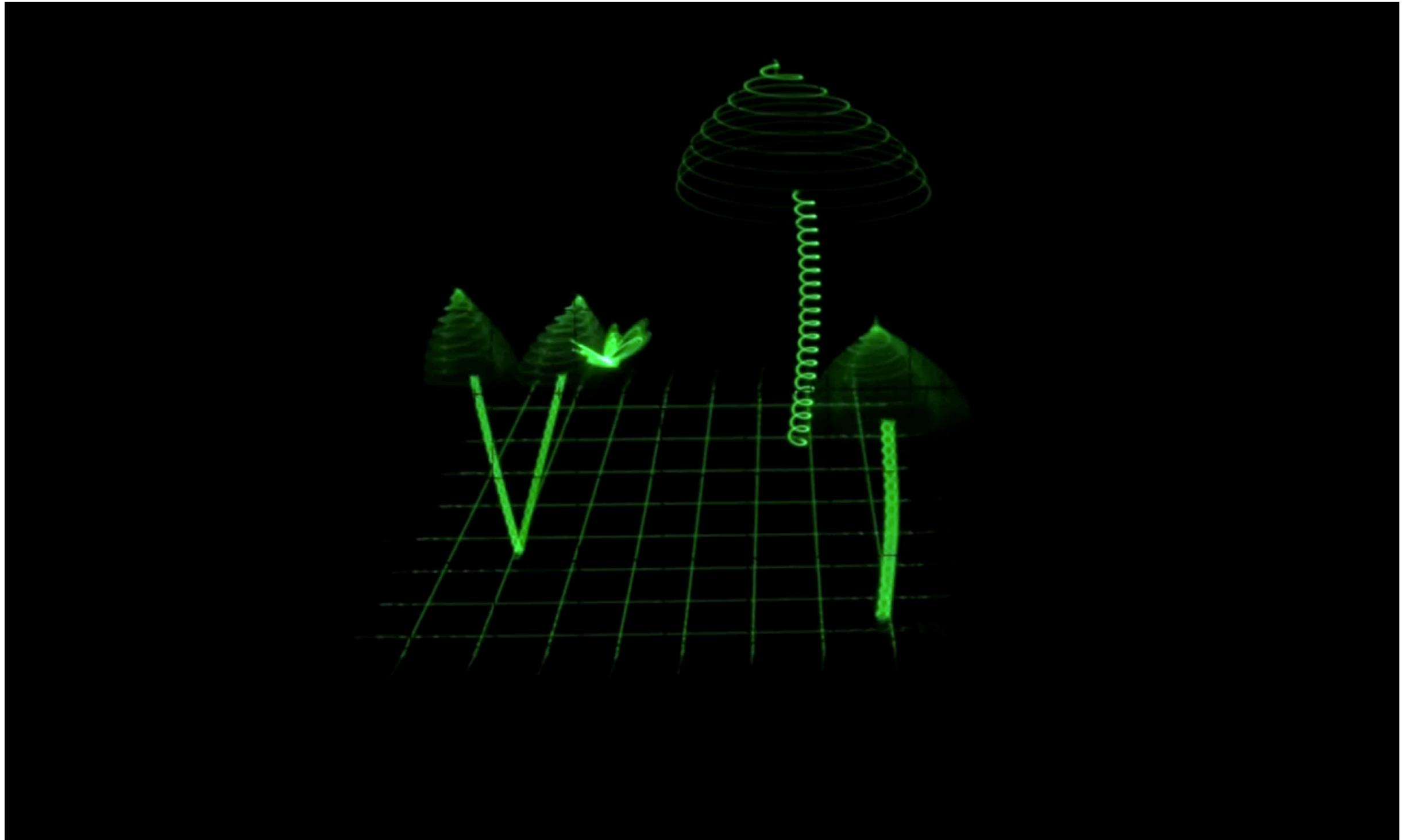


# Different Raster Displays

# Oscilloscope



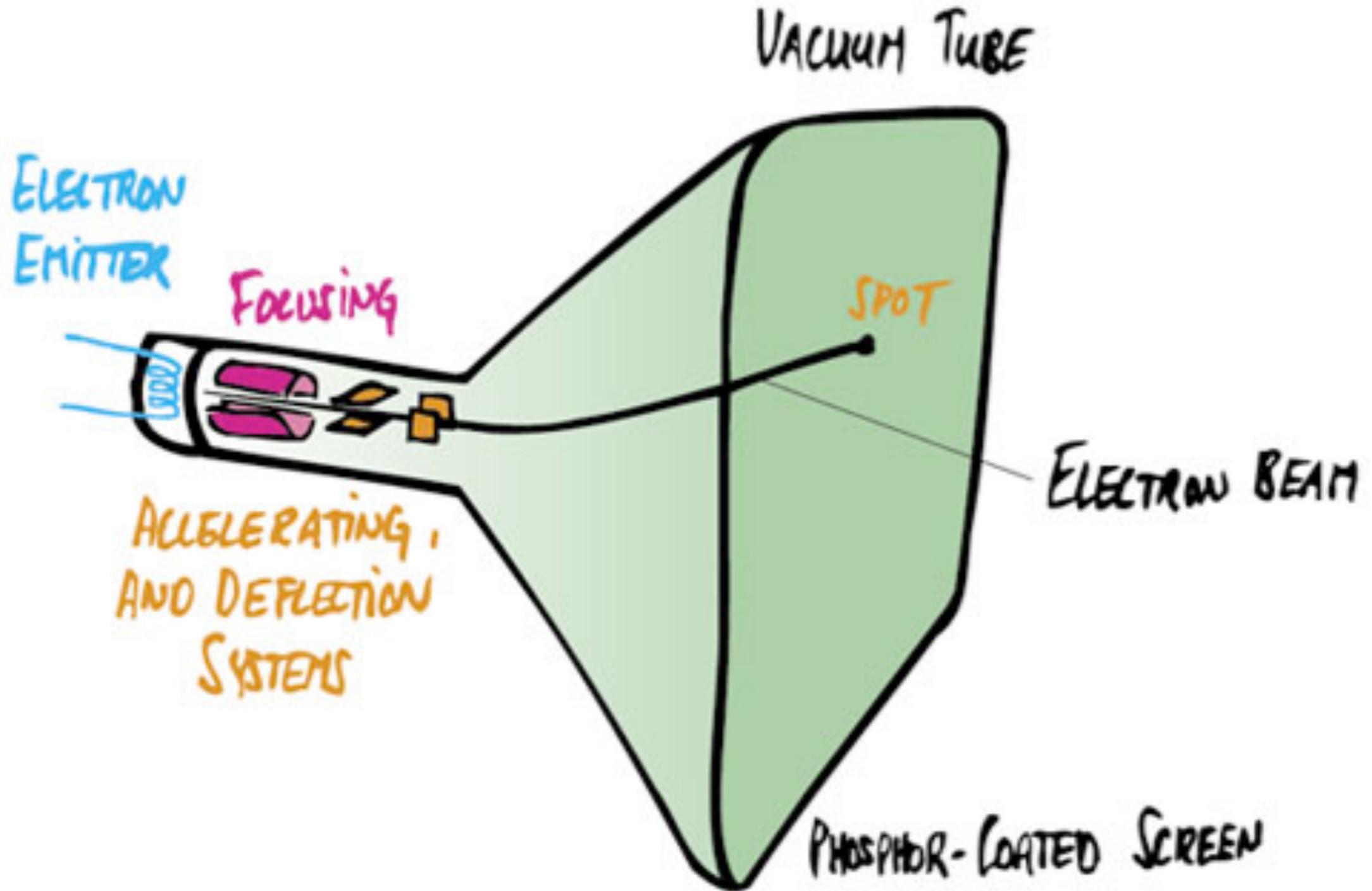
# Oscilloscope Art



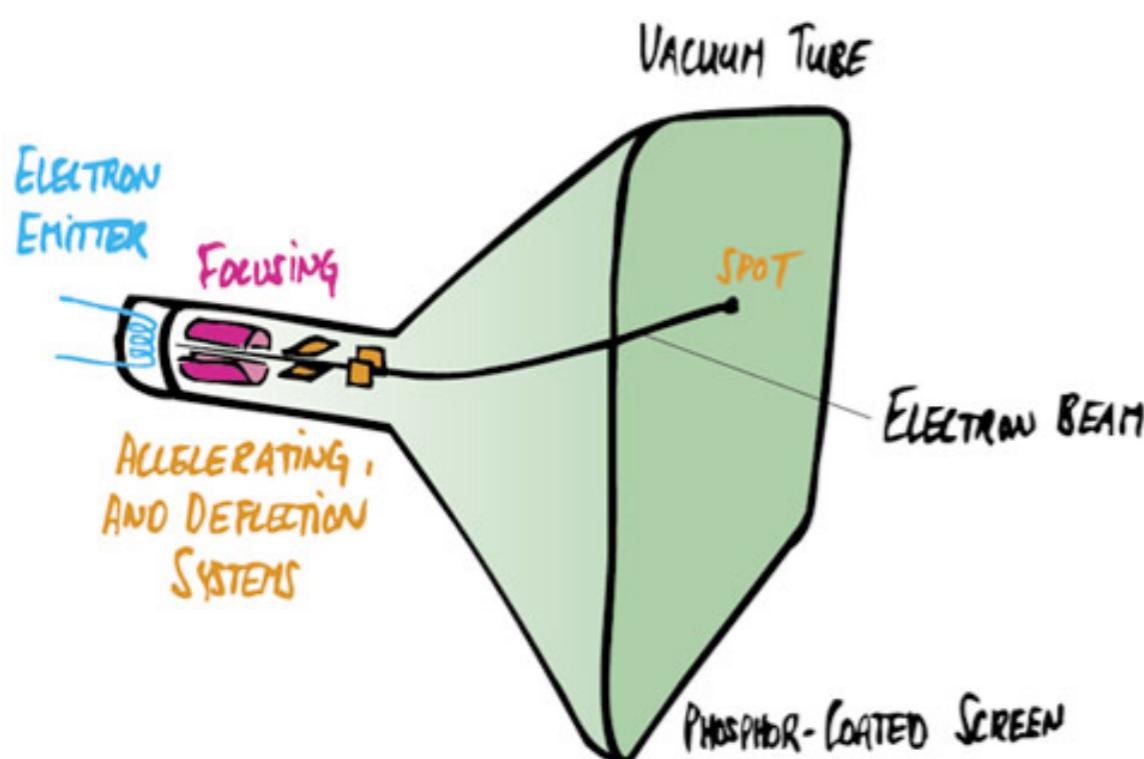
Jerobeam Fenderson

<https://www.youtube.com/watch?v=rtR63-ecUNo>

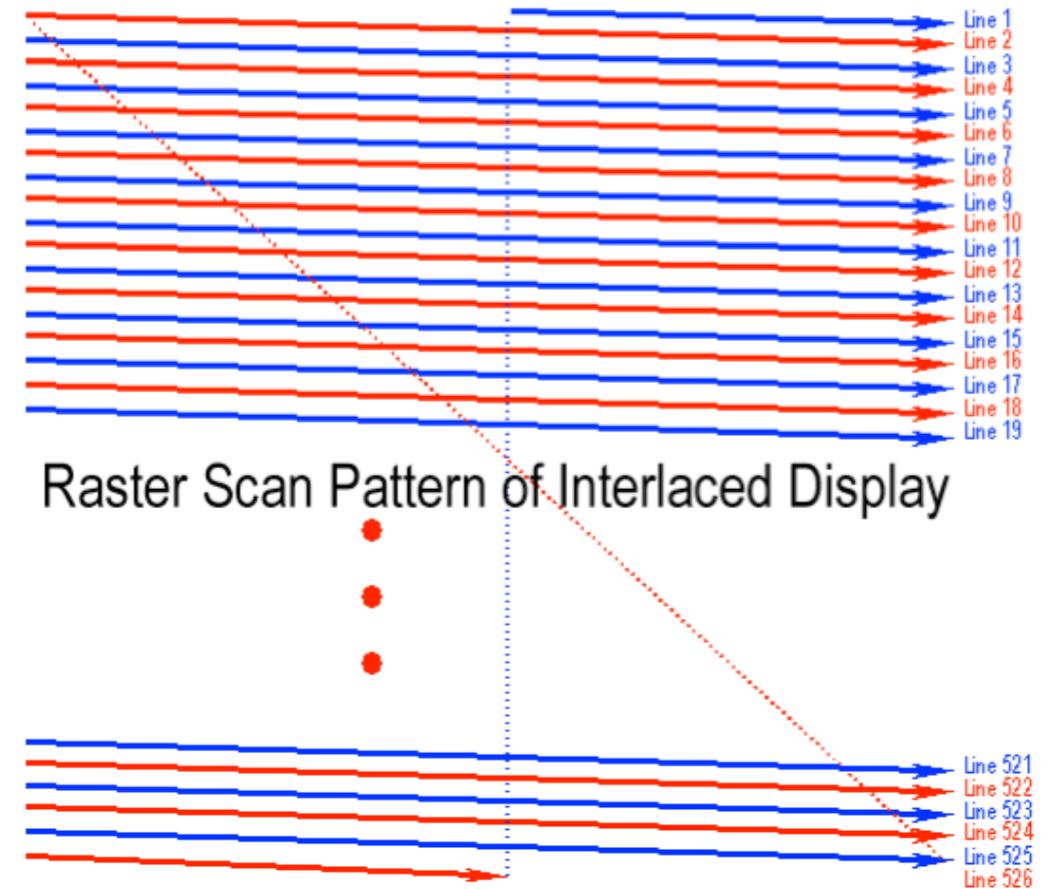
# Cathode Ray Tube



# Television - Raster Display CRT

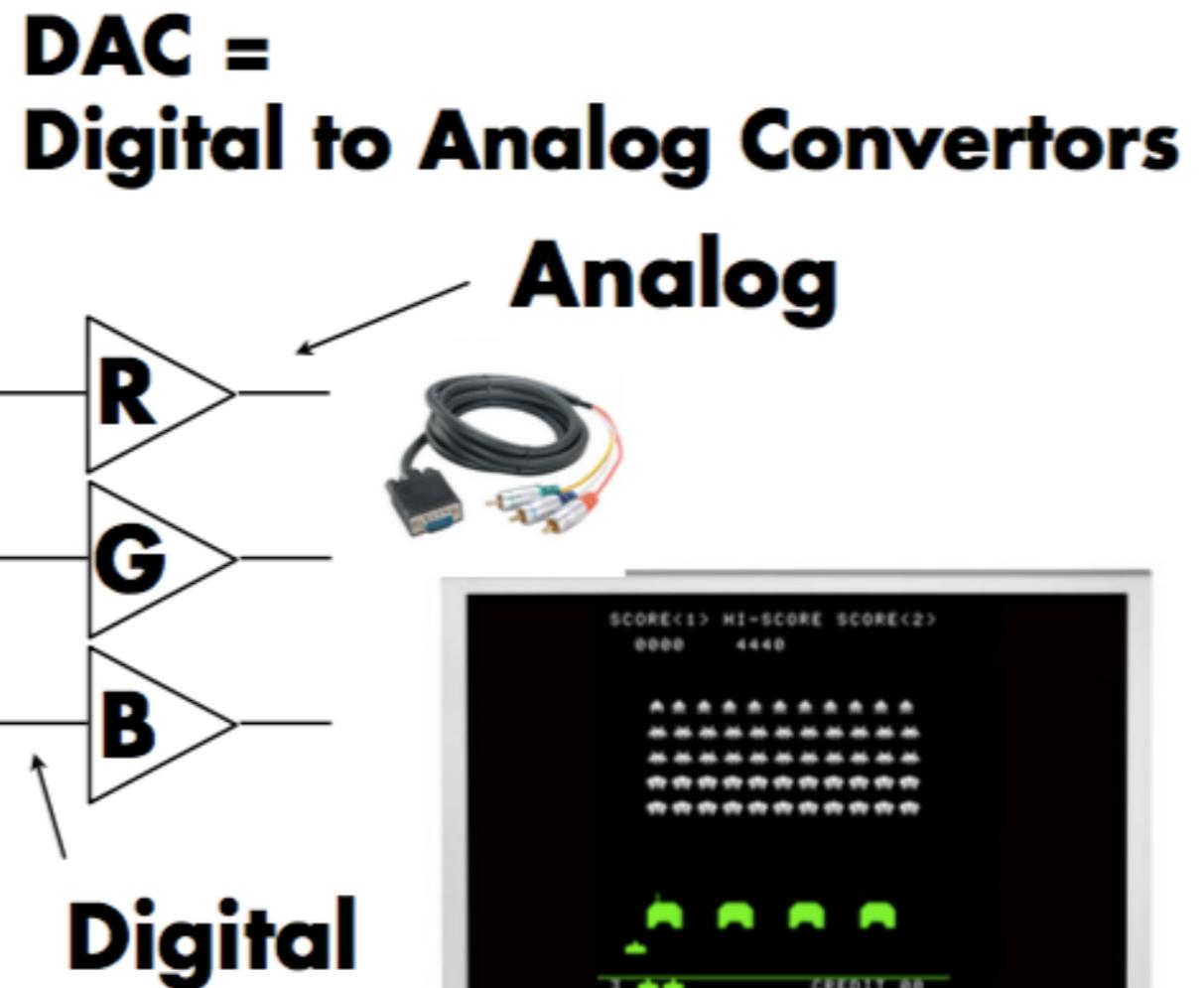


Cathode Ray Tube



Raster Scan  
(modulate intensity)

# Frame Buffer: Memory for a Raster Display



**Image = 2D array of colors**

# Flat Panel Displays



Low-Res LCD Display



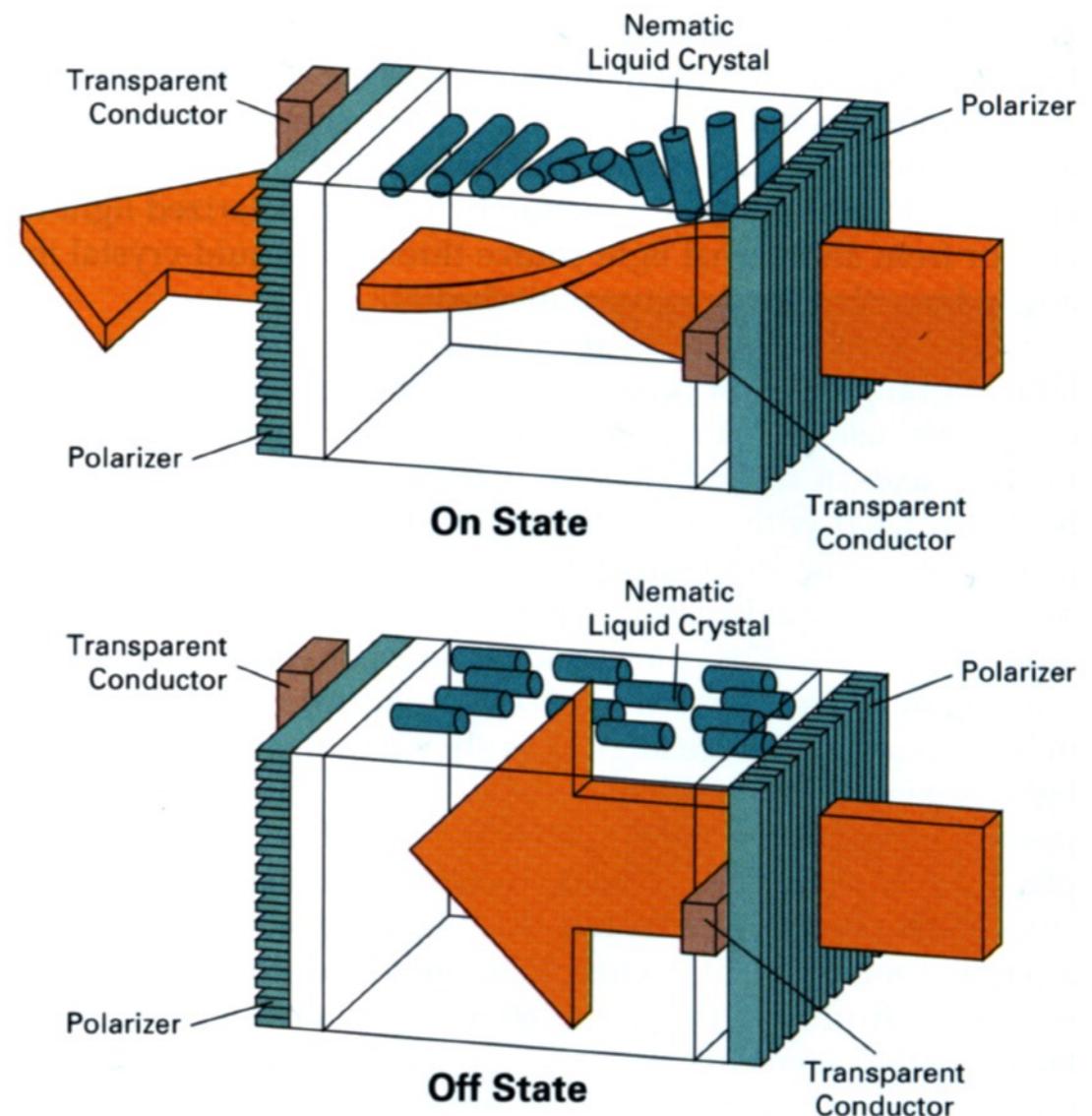
Color LCD, OLED, ...

# LCD (Liquid Crystal Display) Pixel

Principle: block or transmit light by twisting polarization

Illumination from backlight  
(e.g. fluorescent or LED)

Intermediate intensity levels by partial twist



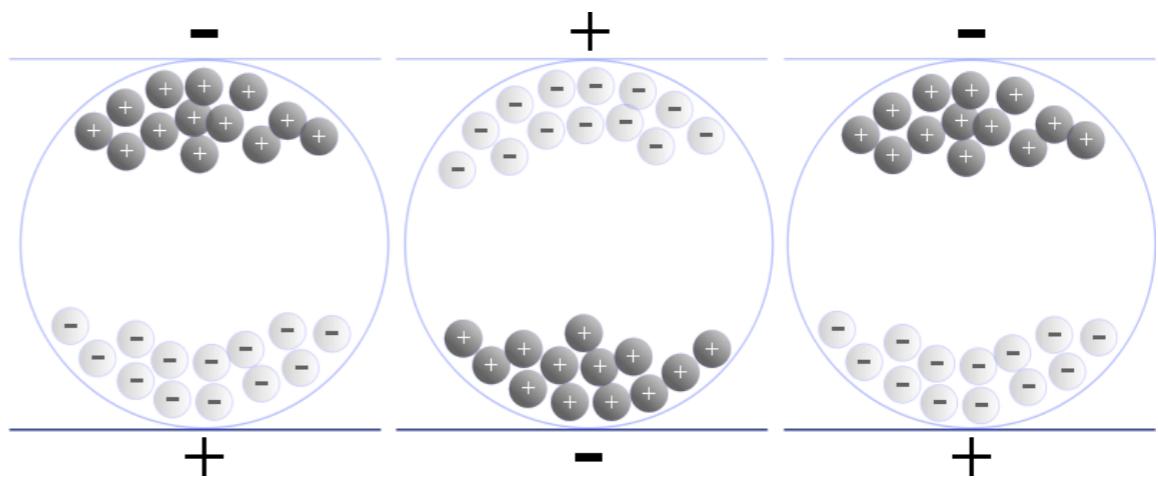
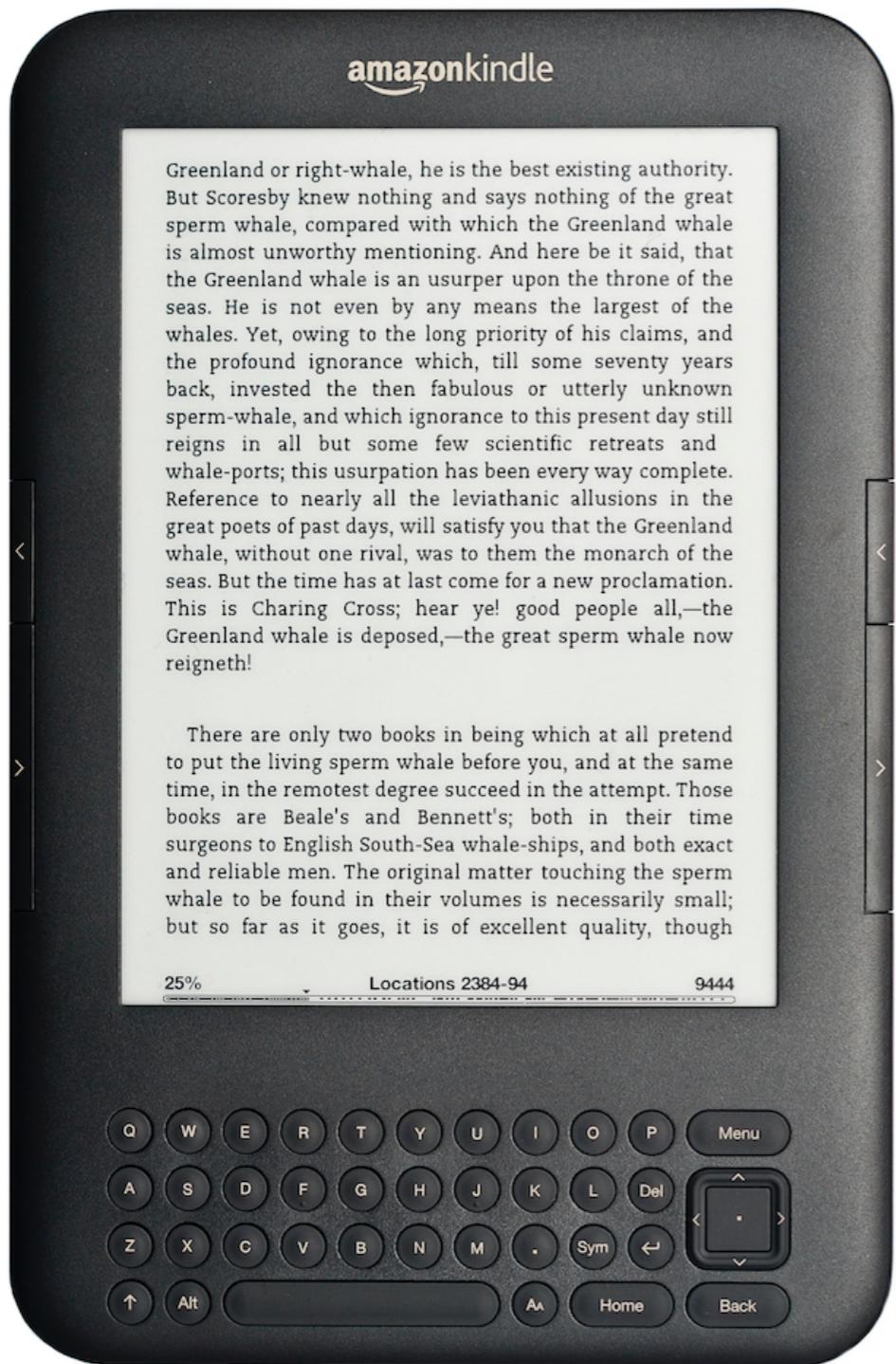
[H&B fig. 2-16]

# LED Array Display



Light emitting diode array

# Electrophoretic (Electronic Ink) Display



# Rasterization: Drawing to Raster Displays

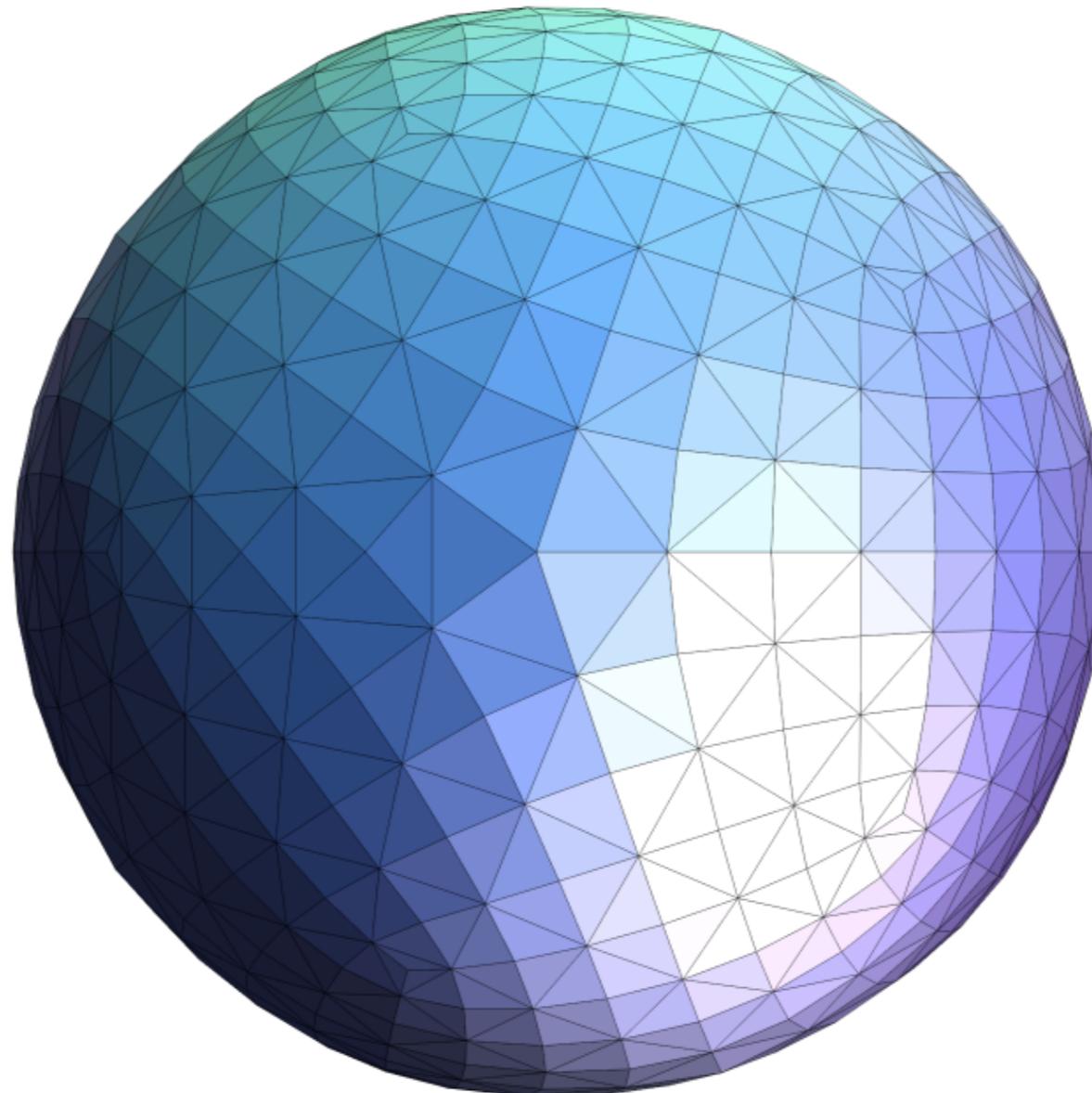
# Polygon Meshes



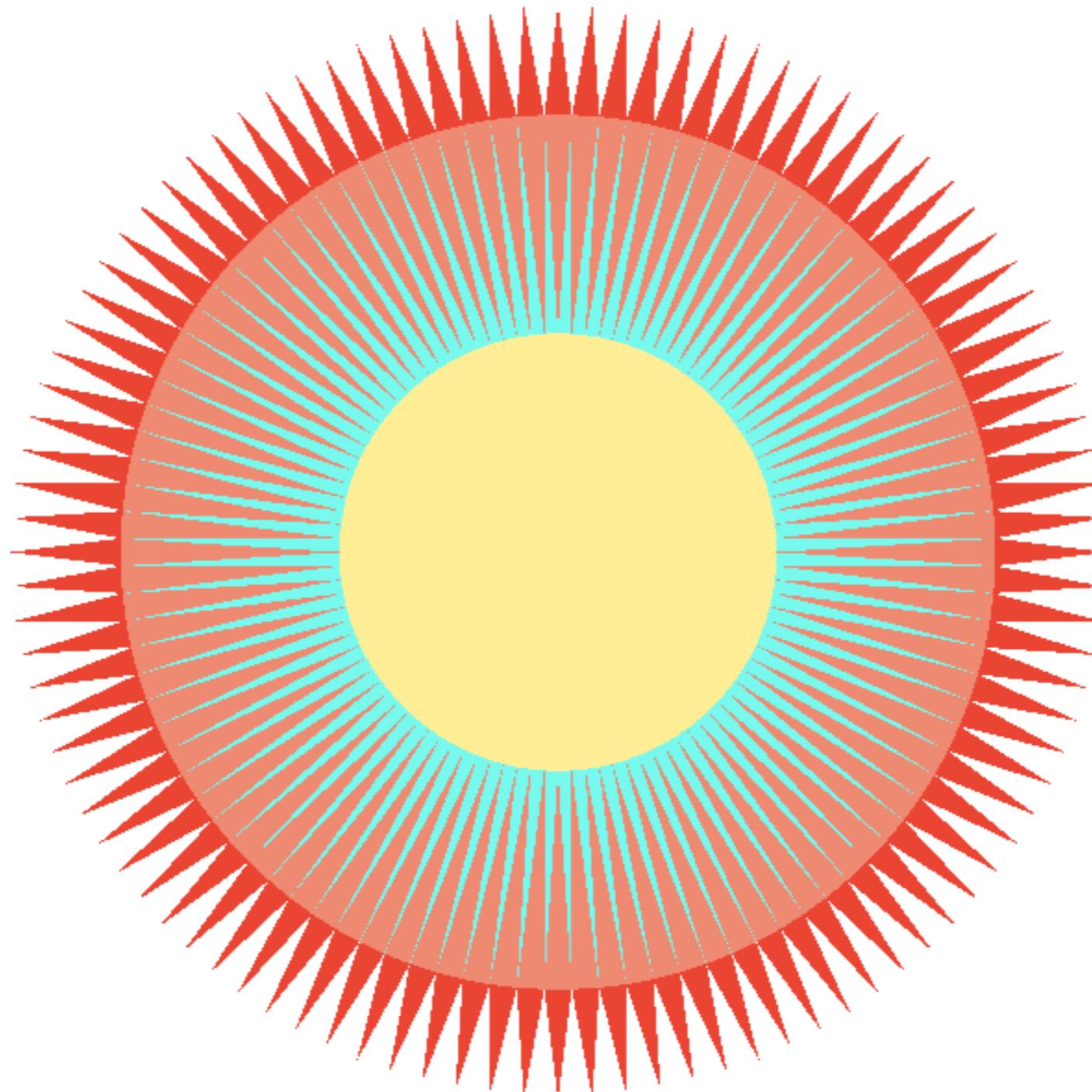
Life of Pi (2012)

# Triangle Meshes

三角形网格



# Triangle Meshes

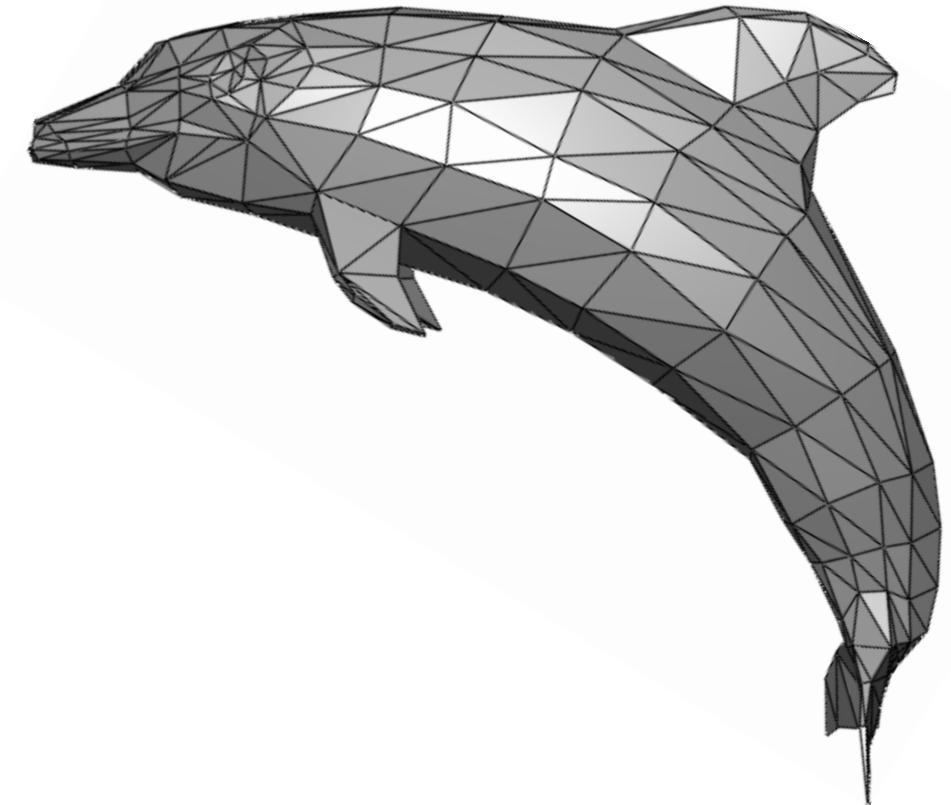


# Triangles - Fundamental Shape Primitives

形状基元

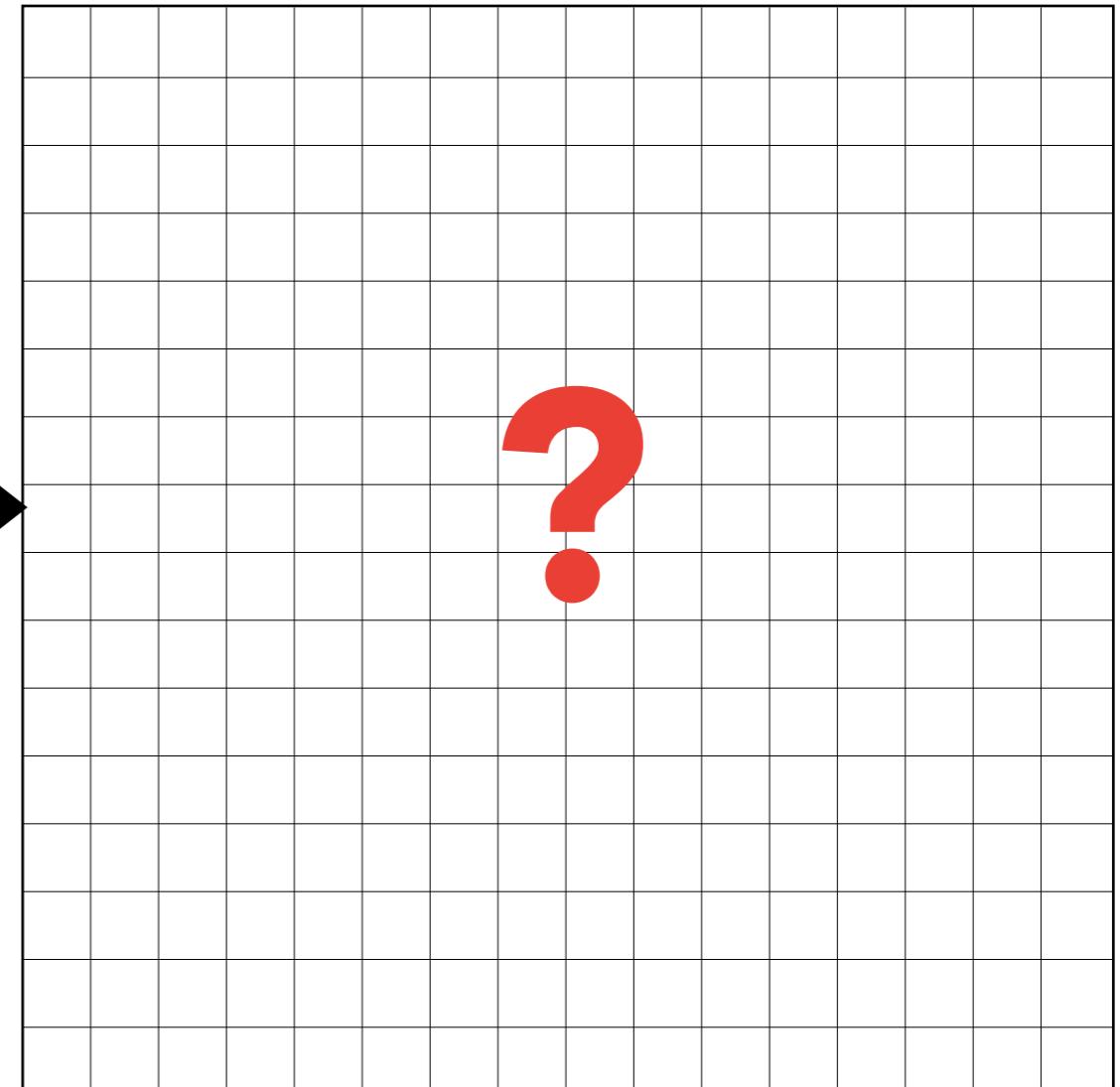
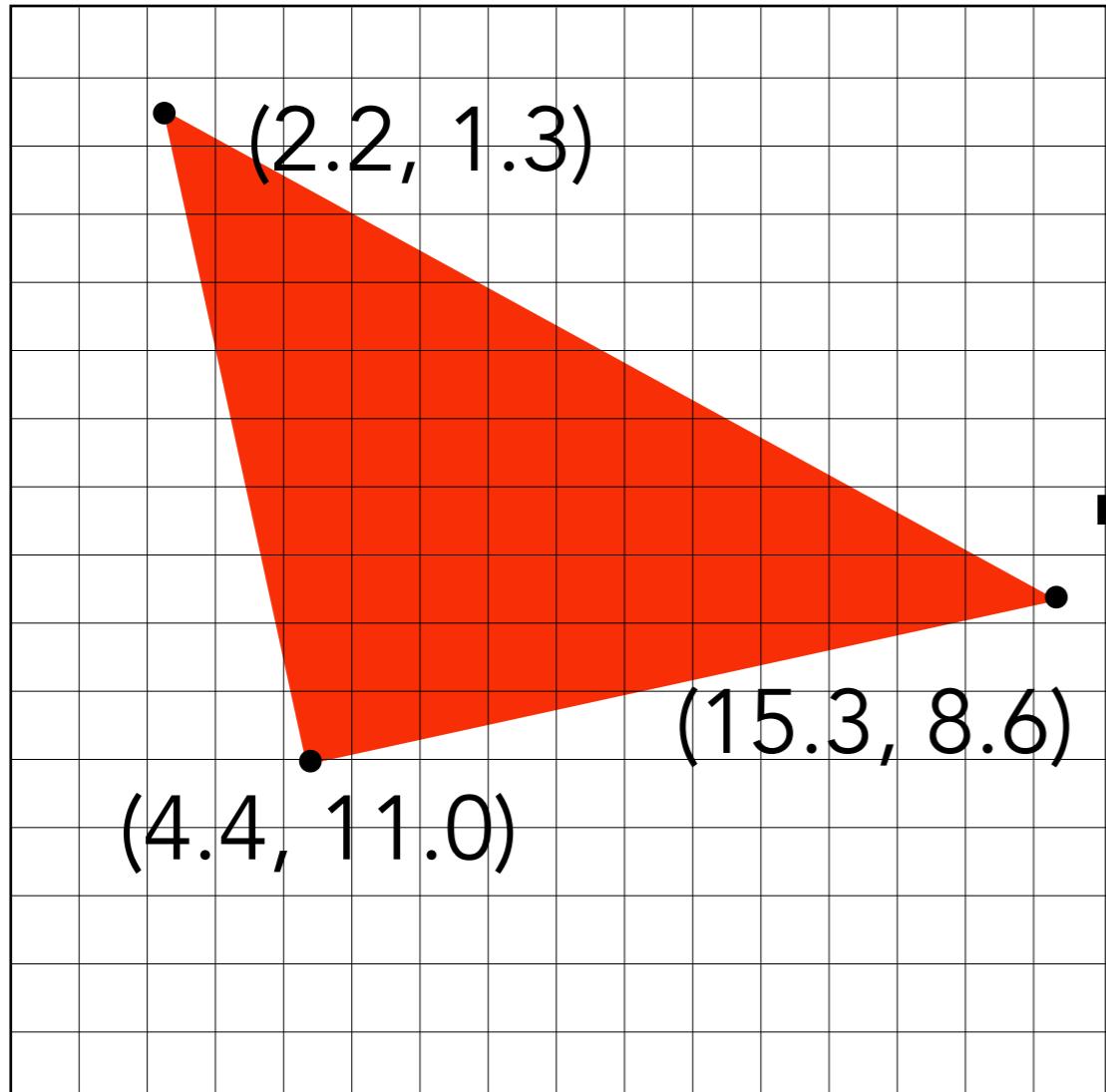
Why triangles?

- Most basic polygon
- Break up other polygons
- Unique properties
  - Guaranteed to be planar
  - Well-defined interior
  - Well-defined method for interpolating values at vertices over triangle (barycentric interpolation)



# What Pixel Values Approximate a Triangle?

将连续的三角形光栅化，就是离散 + 采样



Input: position of triangle  
vertices projected on screen

Output: set of pixel values  
approximating triangle

# A Simple Approach: Sampling

# Sampling a Function

Evaluating a function at a point is sampling.

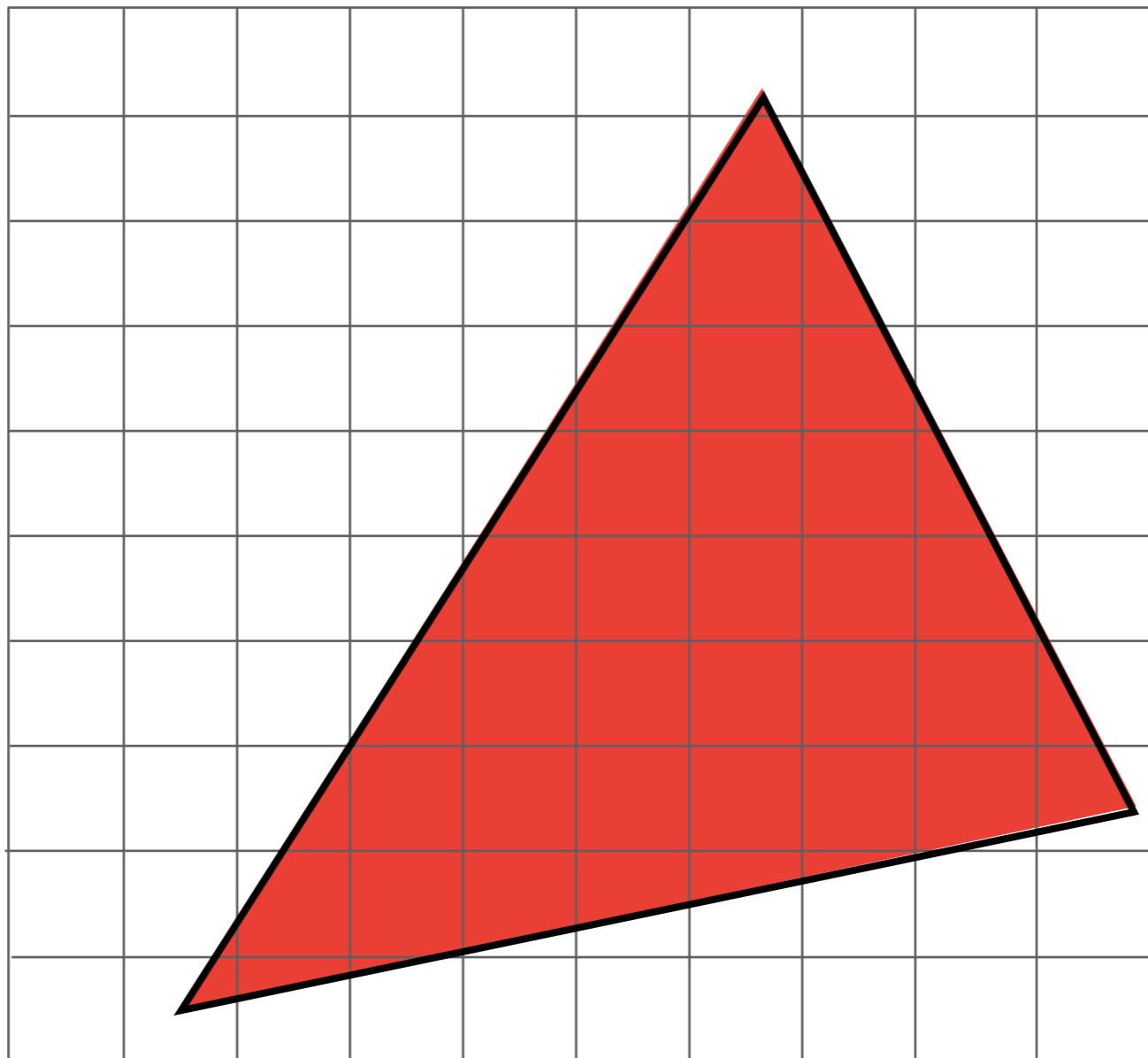
We can **discretize** a function by sampling.

```
for (int x = 0; x < xmax; ++x)
    output[x] = f(x);
```

Sampling is a core idea in graphics.

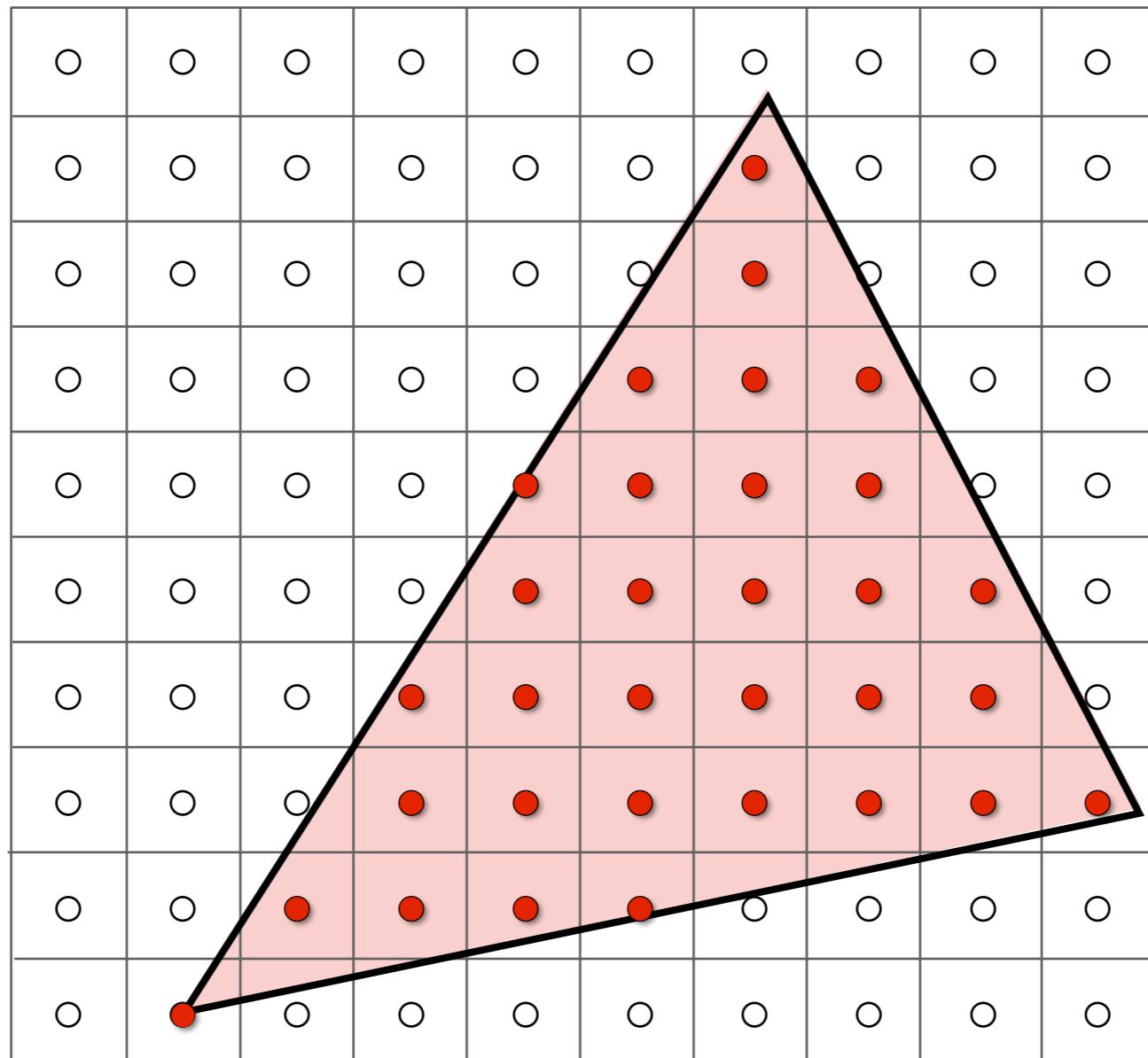
We sample time (1D), area (2D), direction (2D), volume (3D) ...

# Rasterization As 2D Sampling

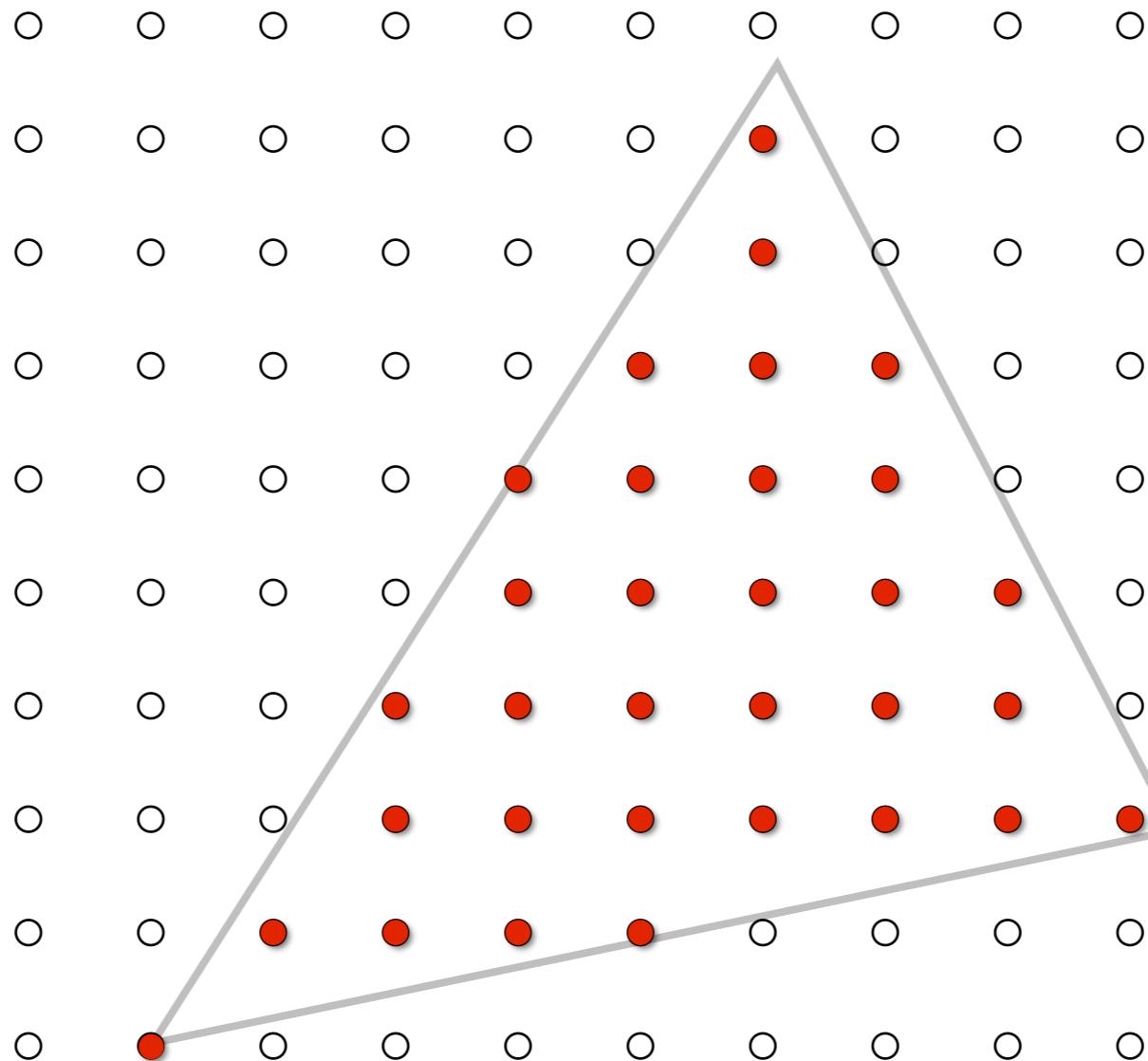


# Sample If Each Pixel Center Is Inside Triangle

对三角形内的像素点进行采样，给定相同的 pixel 值

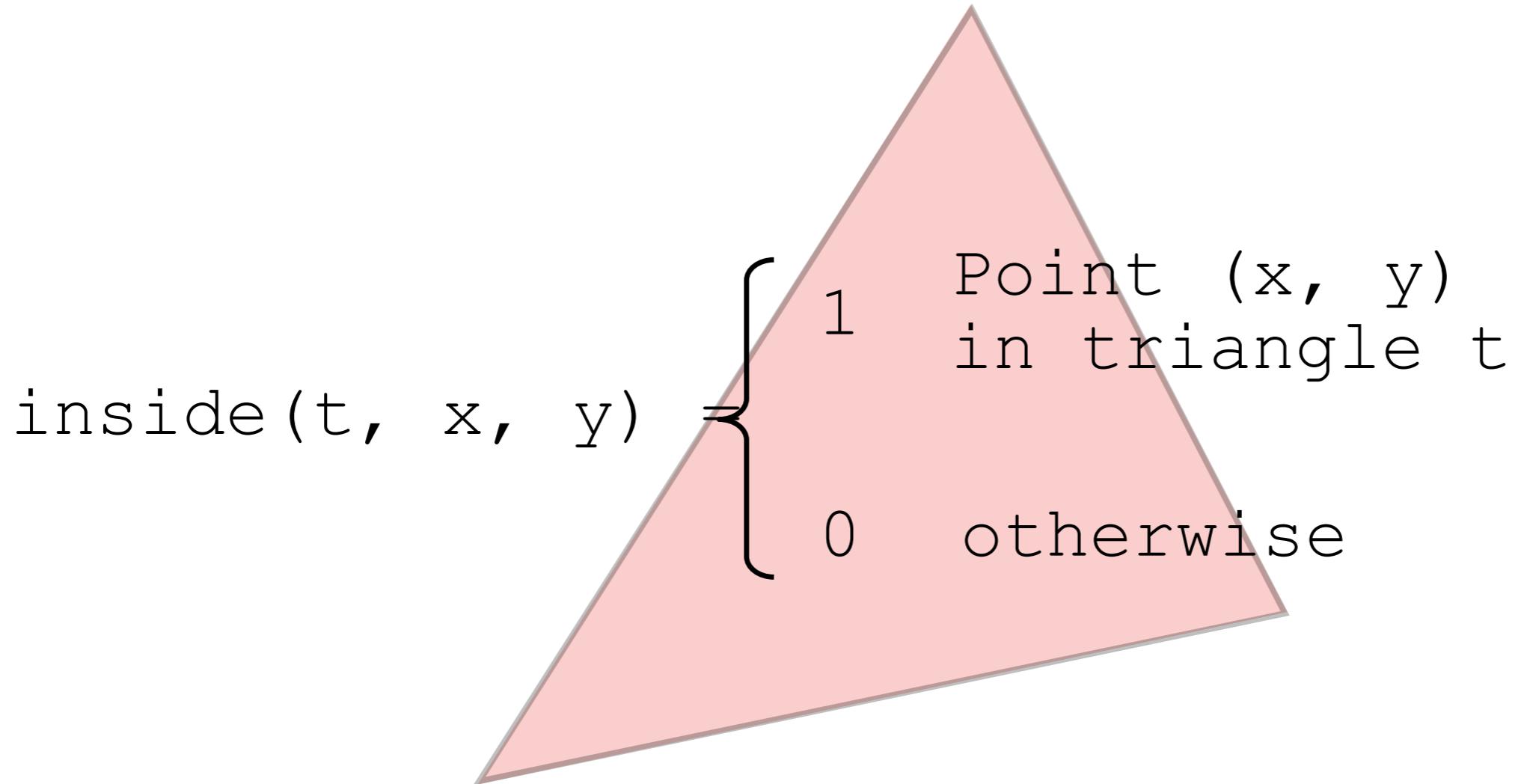


# Sample If Each Pixel Center Is Inside Triangle



Define Binary Function: `inside(tri, x, y)`

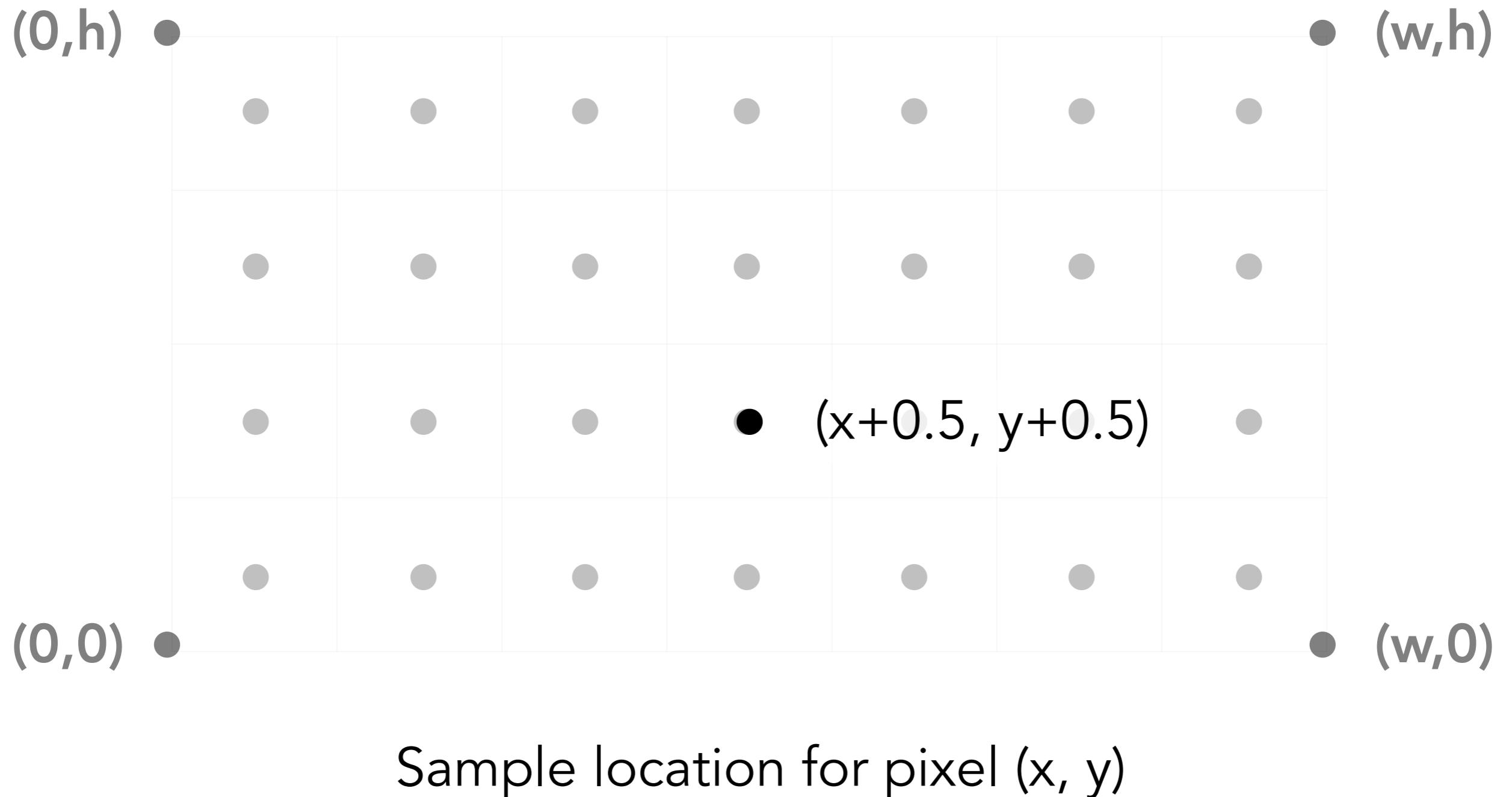
$x, y$ : not necessarily integers



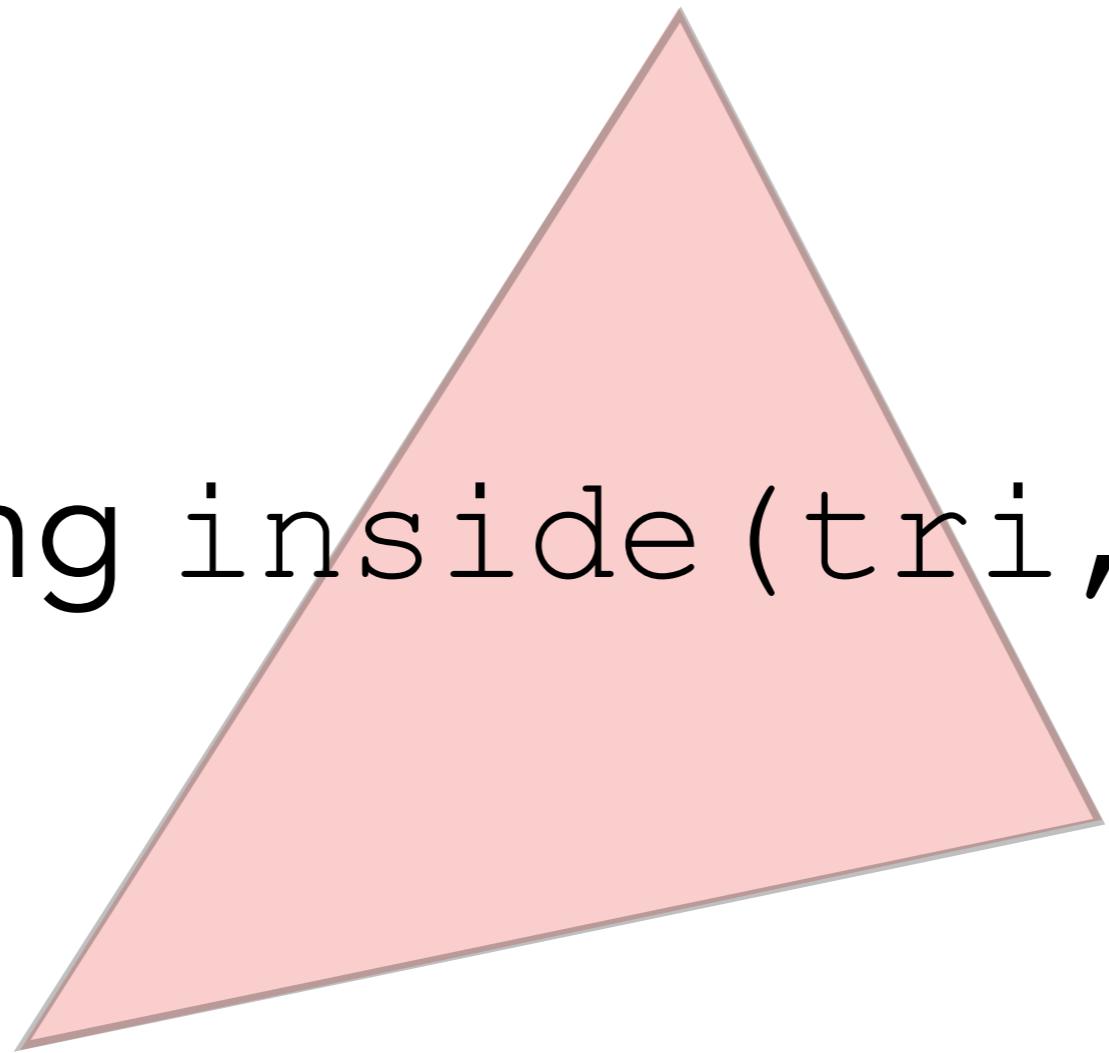
# Rasterization = Sampling A 2D Indicator Function

```
for (int x = 0; x < xmax; ++x)
    for (int y = 0; y < ymax; ++y)
        image[x][y] = inside(tri,
                                x + 0.5,
                                y + 0.5);
```

# Recall: Sample Locations

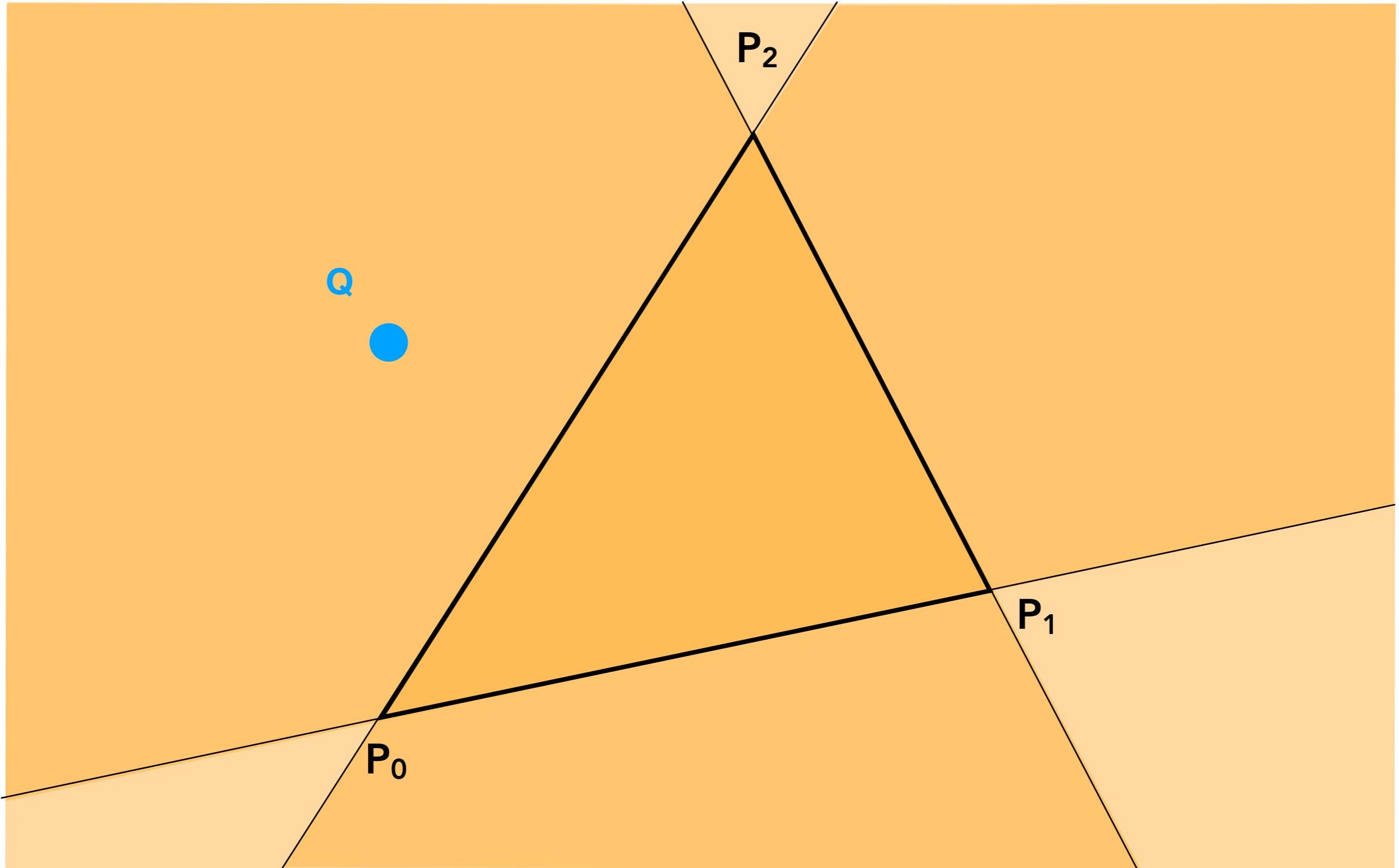


Evaluating `inside(tri, x, y)`



# Inside? Recall: Three Cross Products!

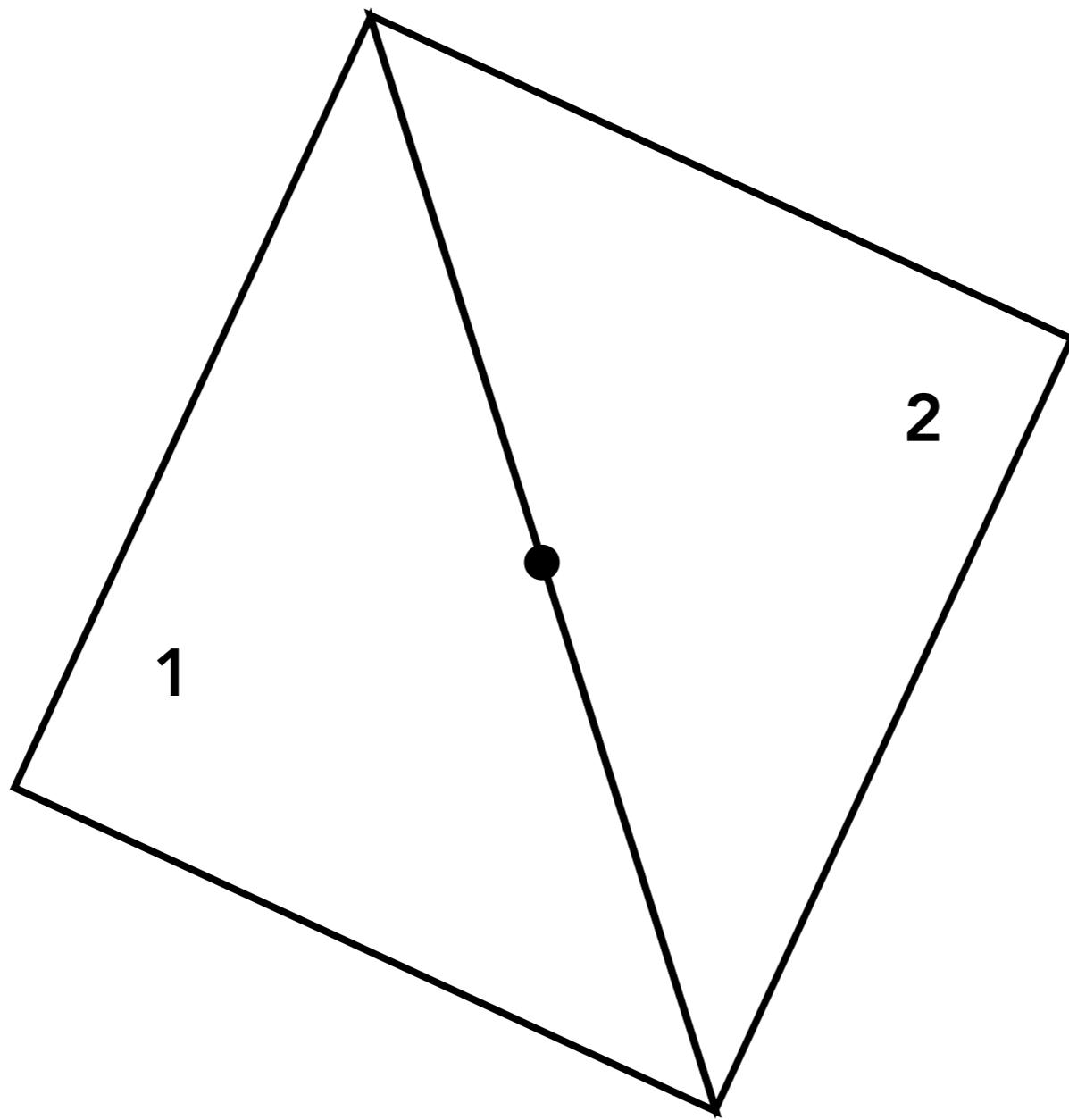
基于叉积可以获得点和三角形的关系



# Edge Cases (Literally)

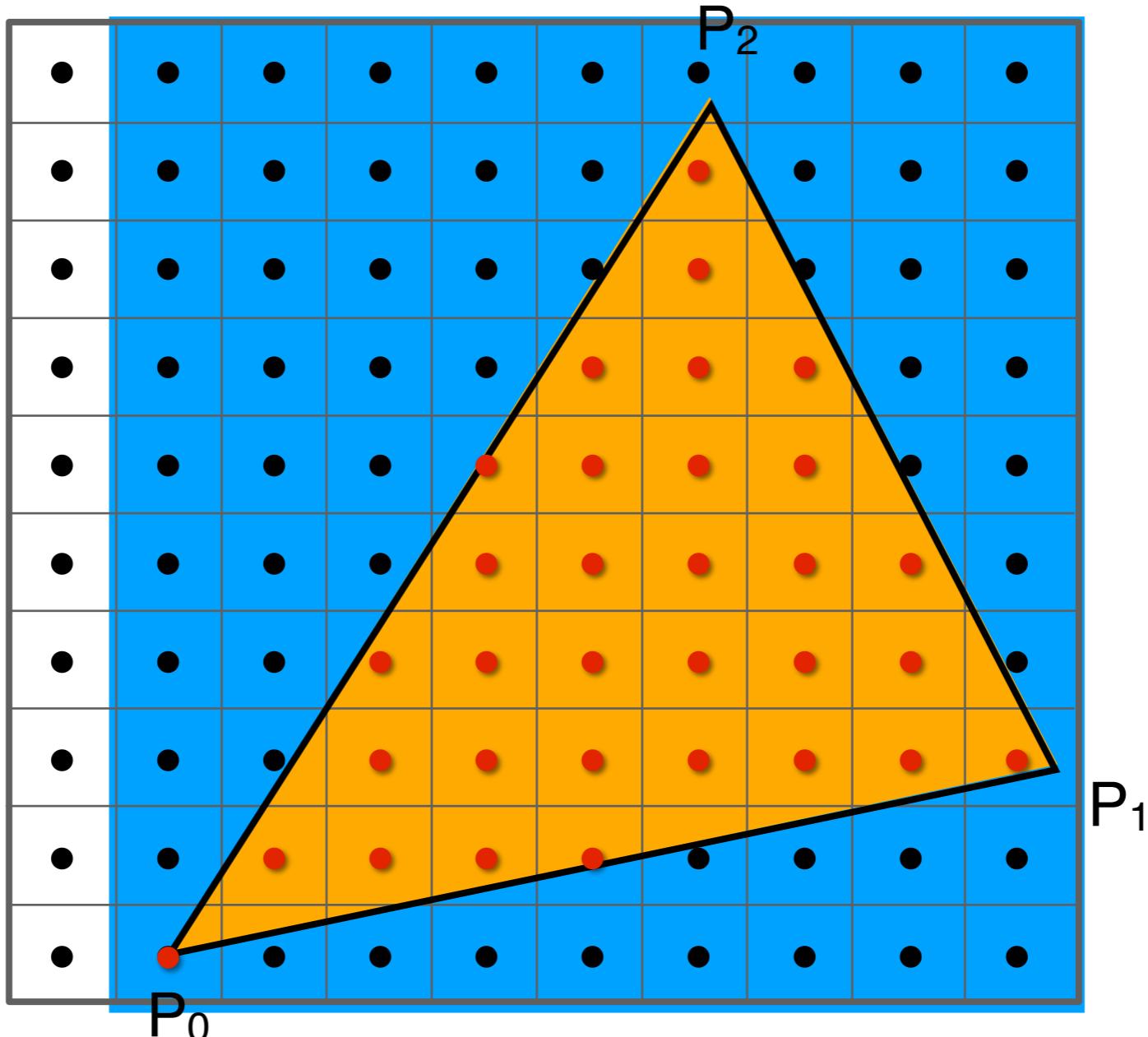
像素点在三角形边上的处理方式

Is this sample point covered by triangle 1, triangle 2, or both?



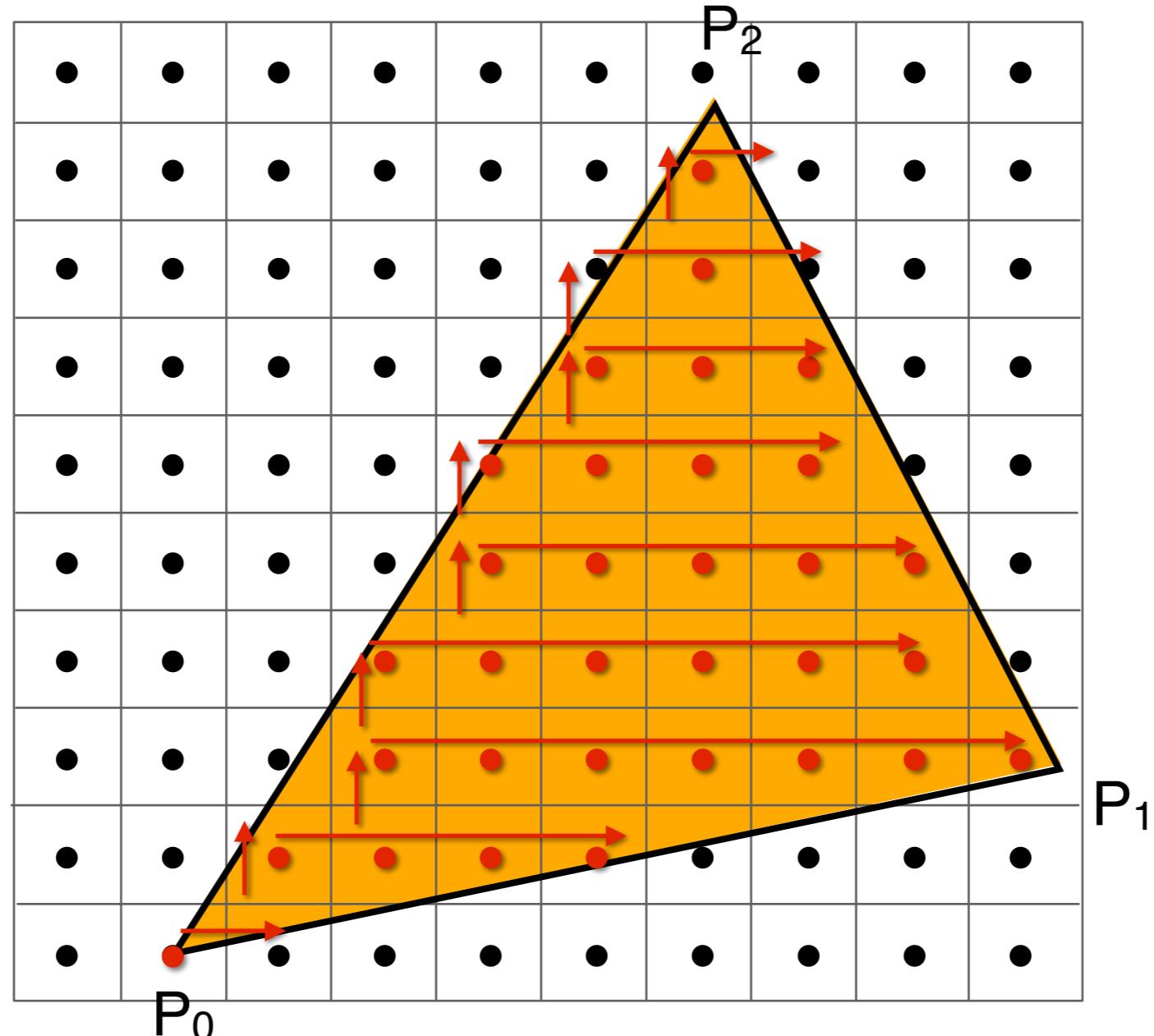
# Checking All Pixels on the Screen?

采用 bounding box 防止对所有像素进行遍历检查，只需要获取三角形的最小外接矩形就可以了



Use a **Bounding Box!**

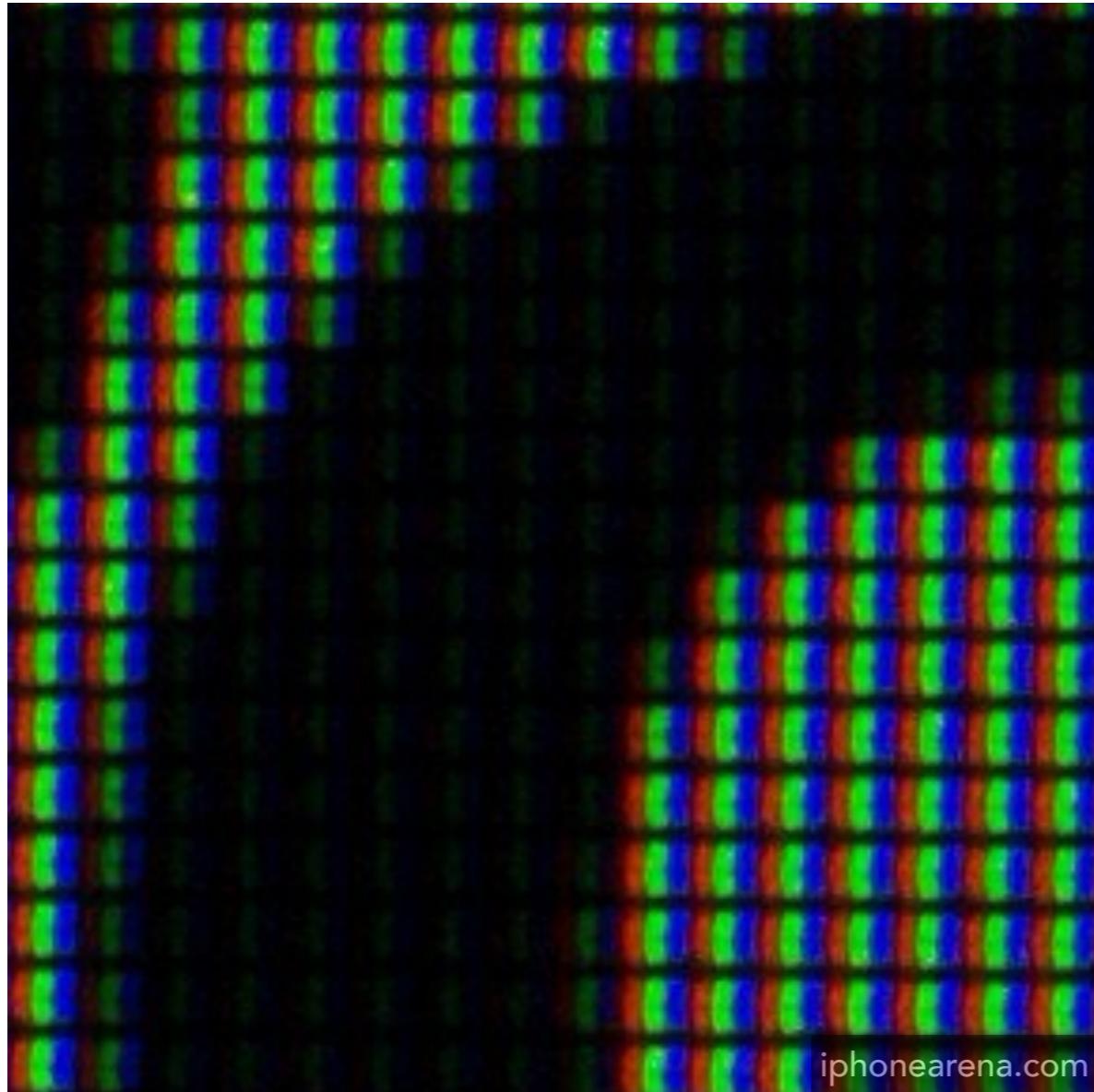
# Incremental Triangle Traversal (Faster?)



suitable for thin and rotated triangles

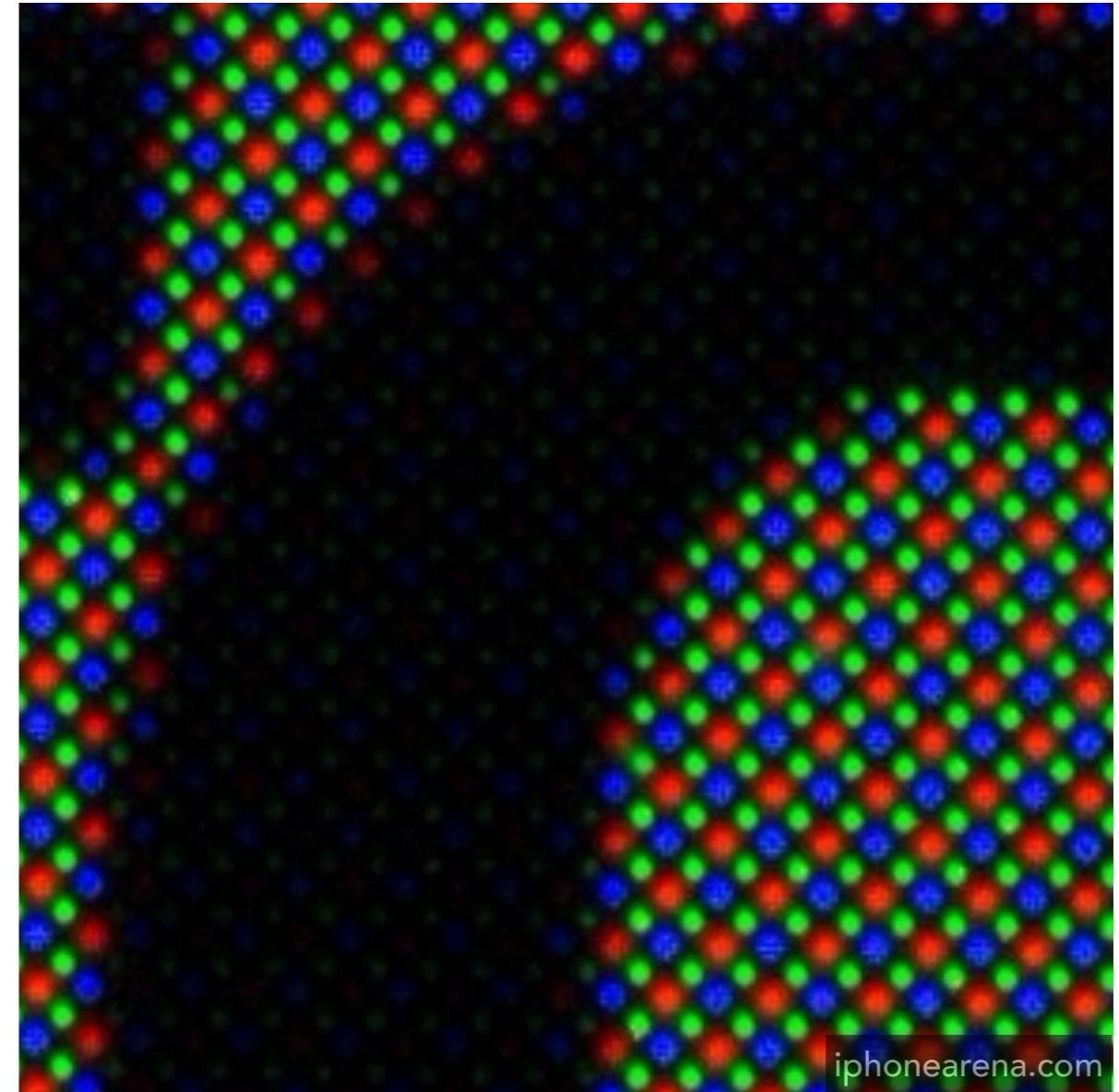
# Rasterization on Real Displays

# Real LCD Screen Pixels (Closeup)



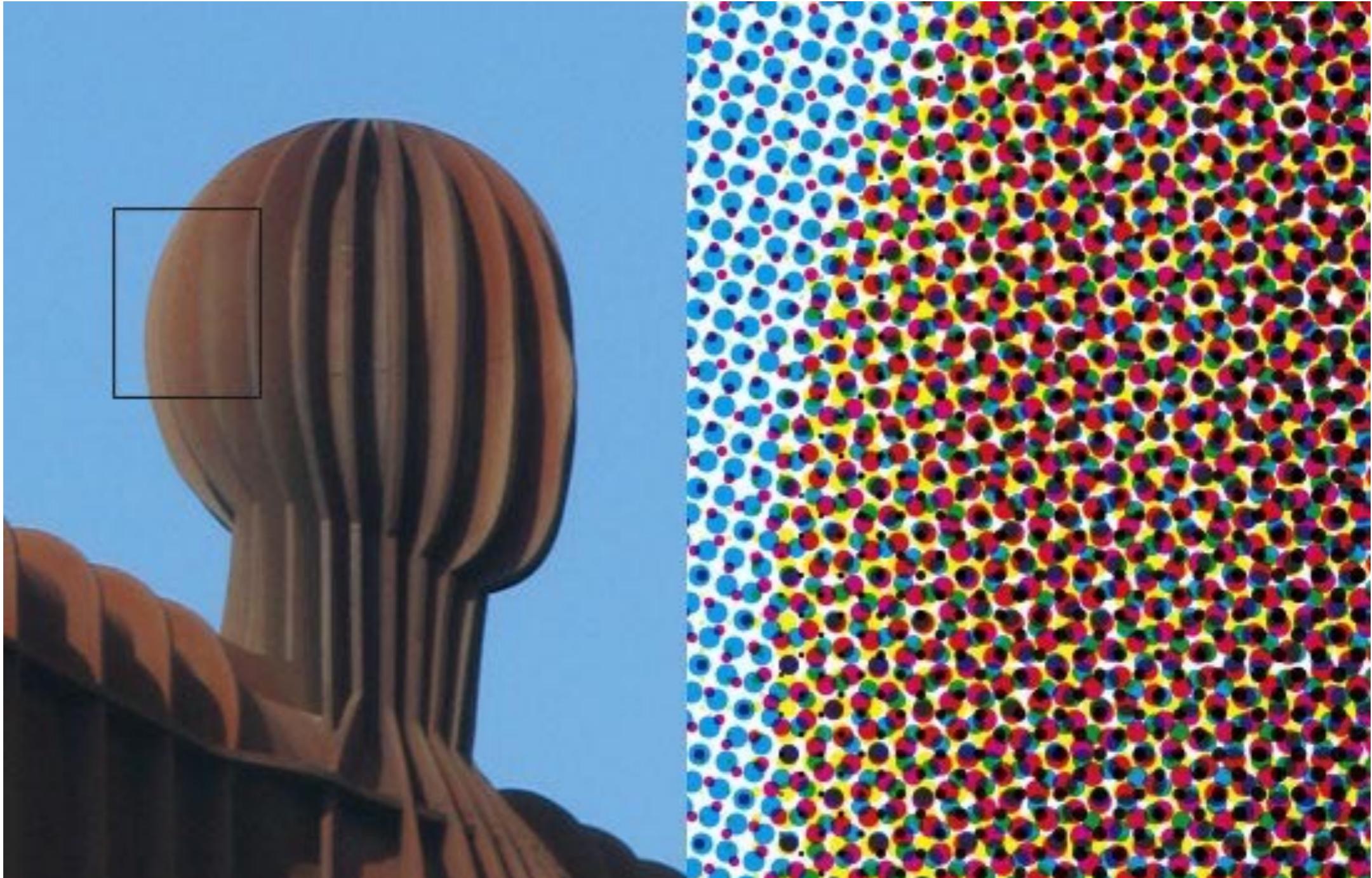
iPhone 6S

Notice R,G,B pixel geometry! But in this class, we will assume a colored square full-color pixel.



Galaxy S5

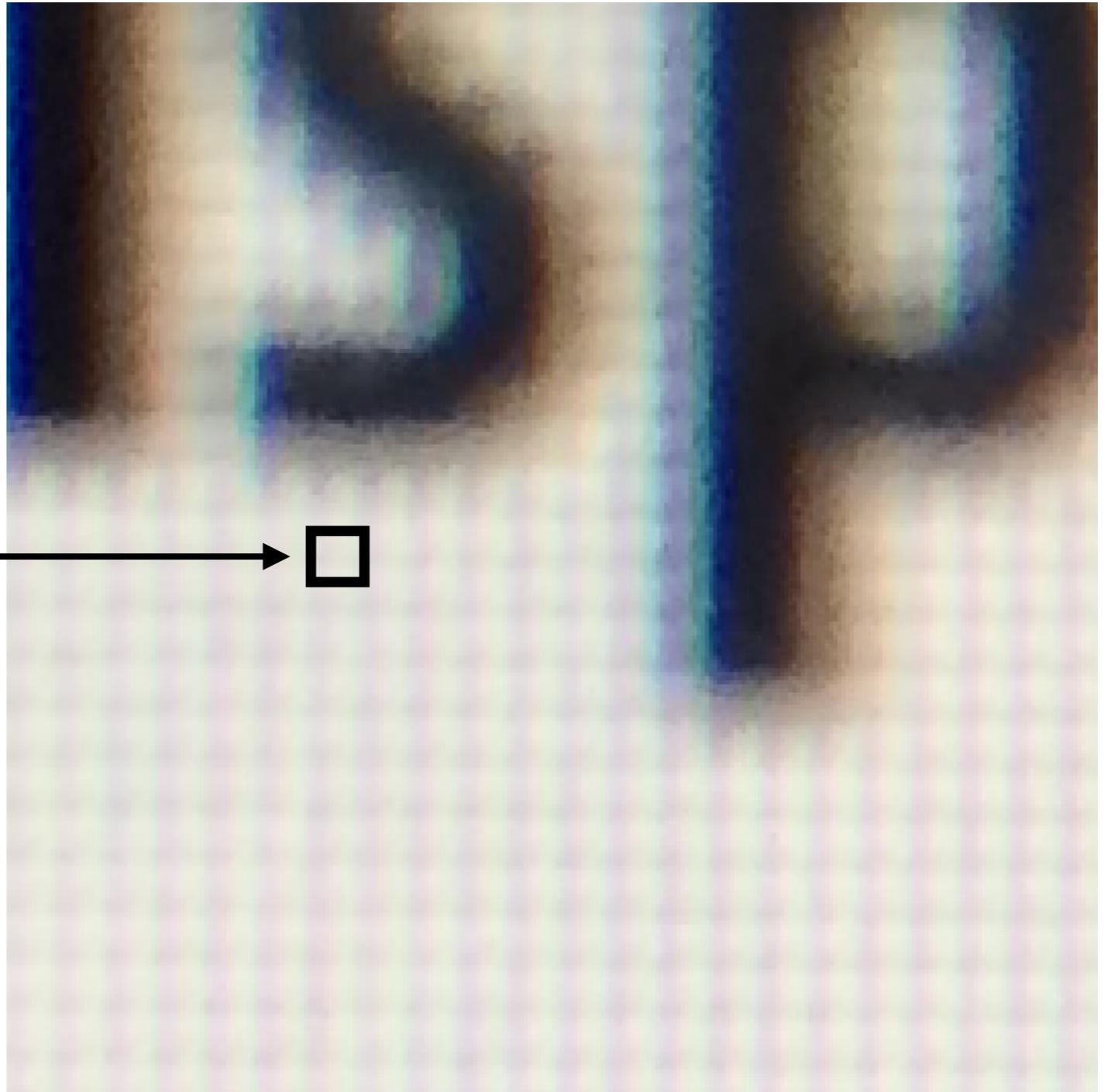
# Aside: What About Other Display Methods?



Color print: observe half-tone pattern

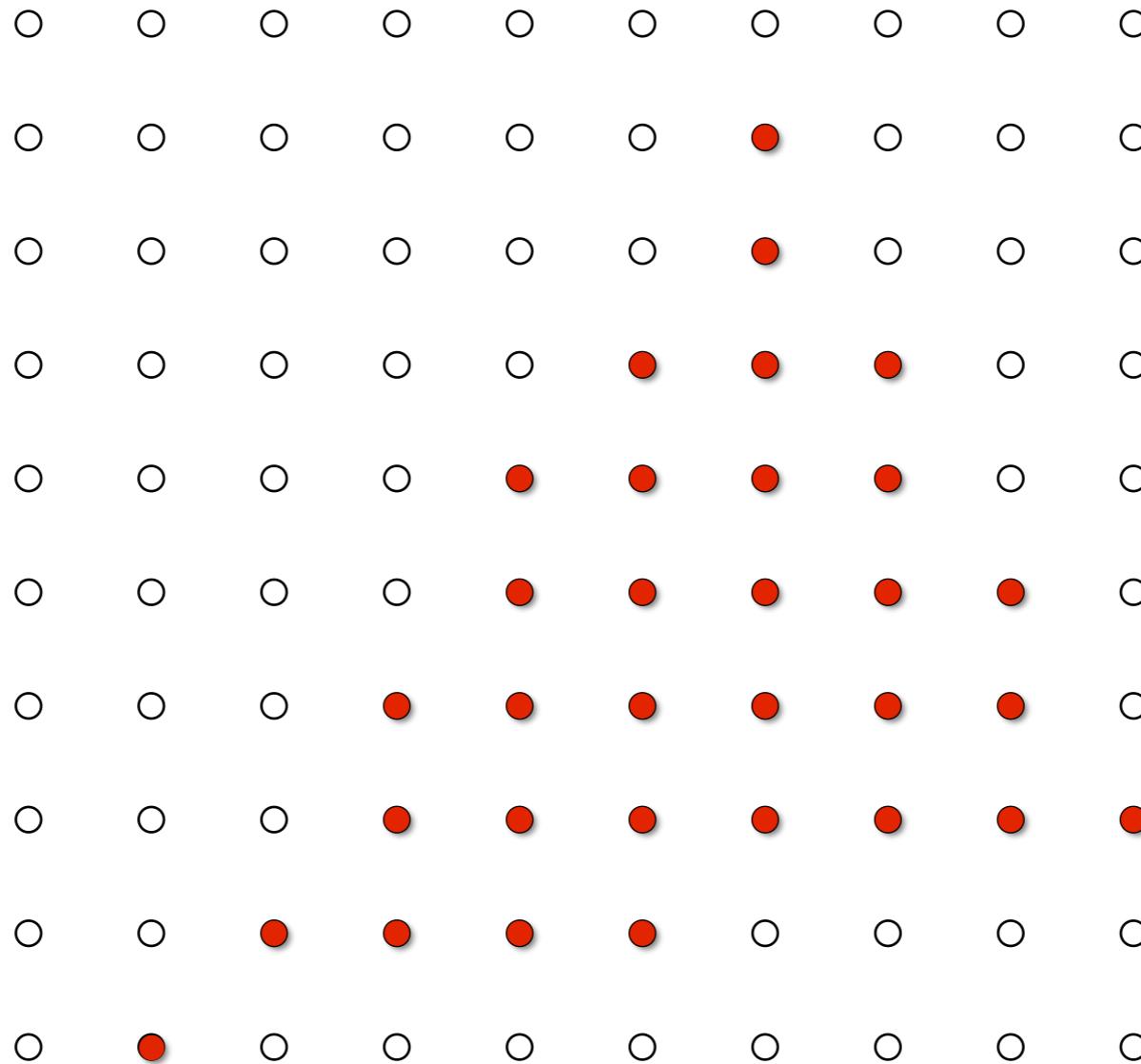
# Assume Display Pixels Emit Square of Light

LCD pixel  
on laptop

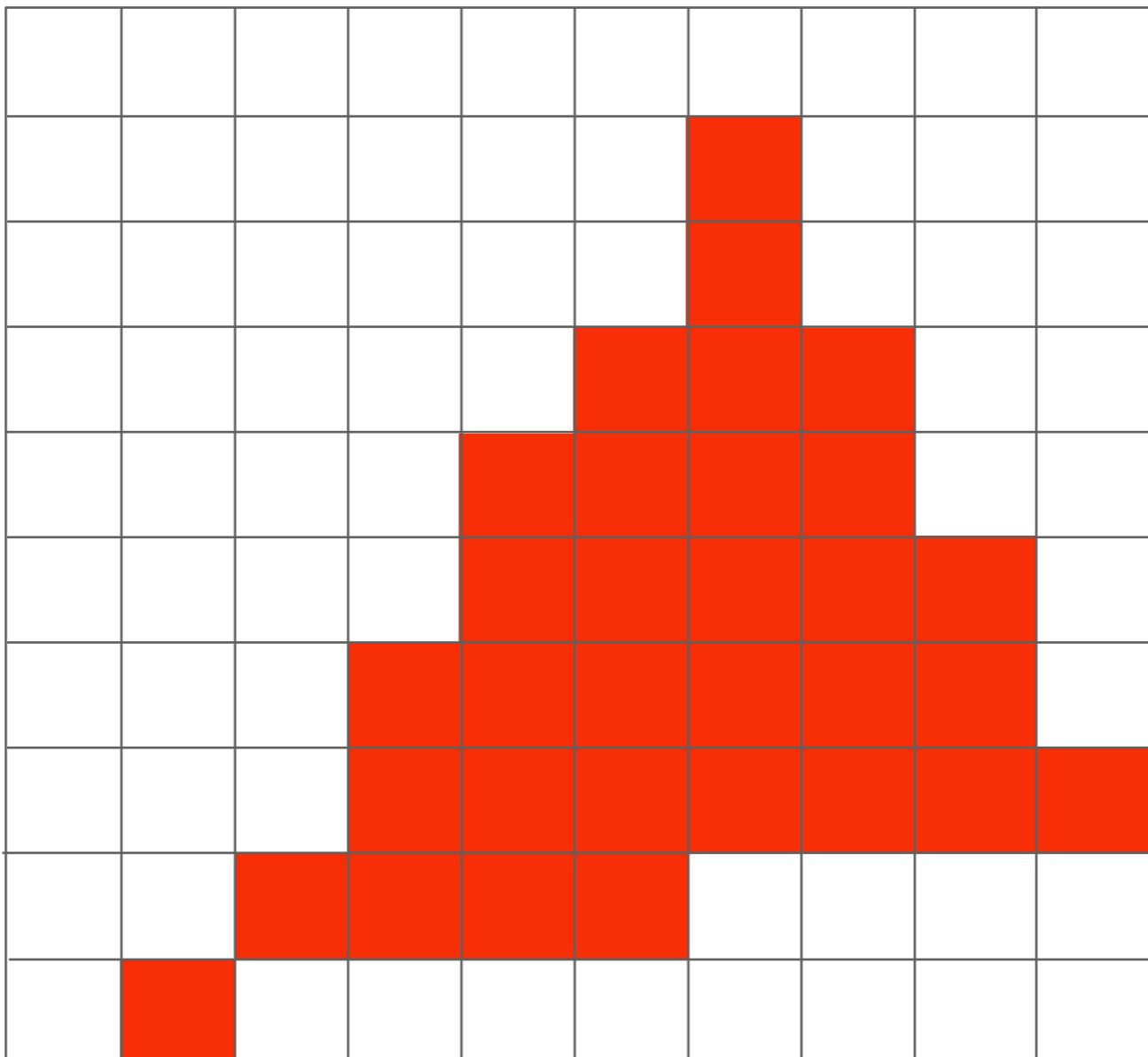


\* LCD pixels do not actually emit light in a square of uniform color, but this approximation suffices for our current discussion

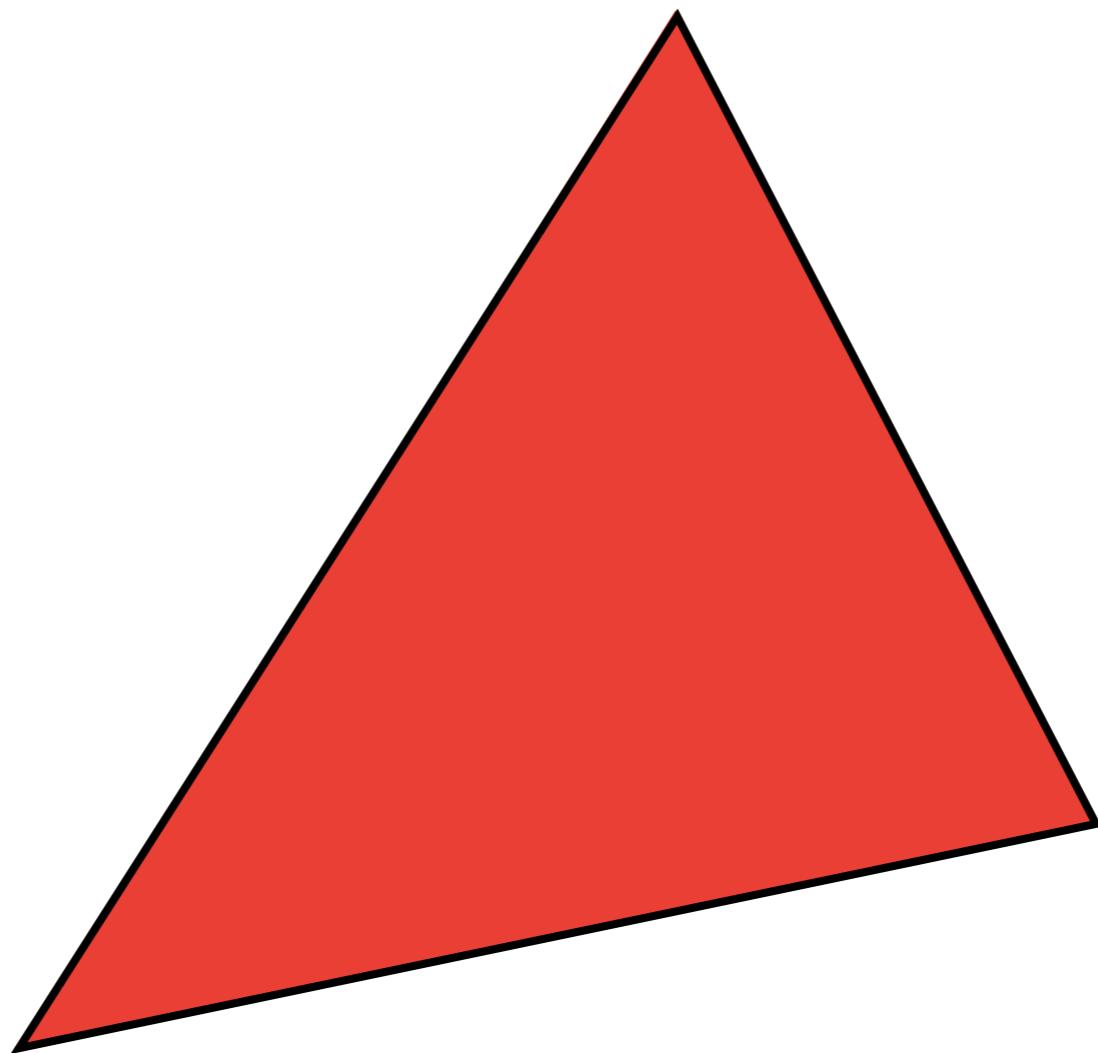
# So, If We Send the Display the Sampled Signal



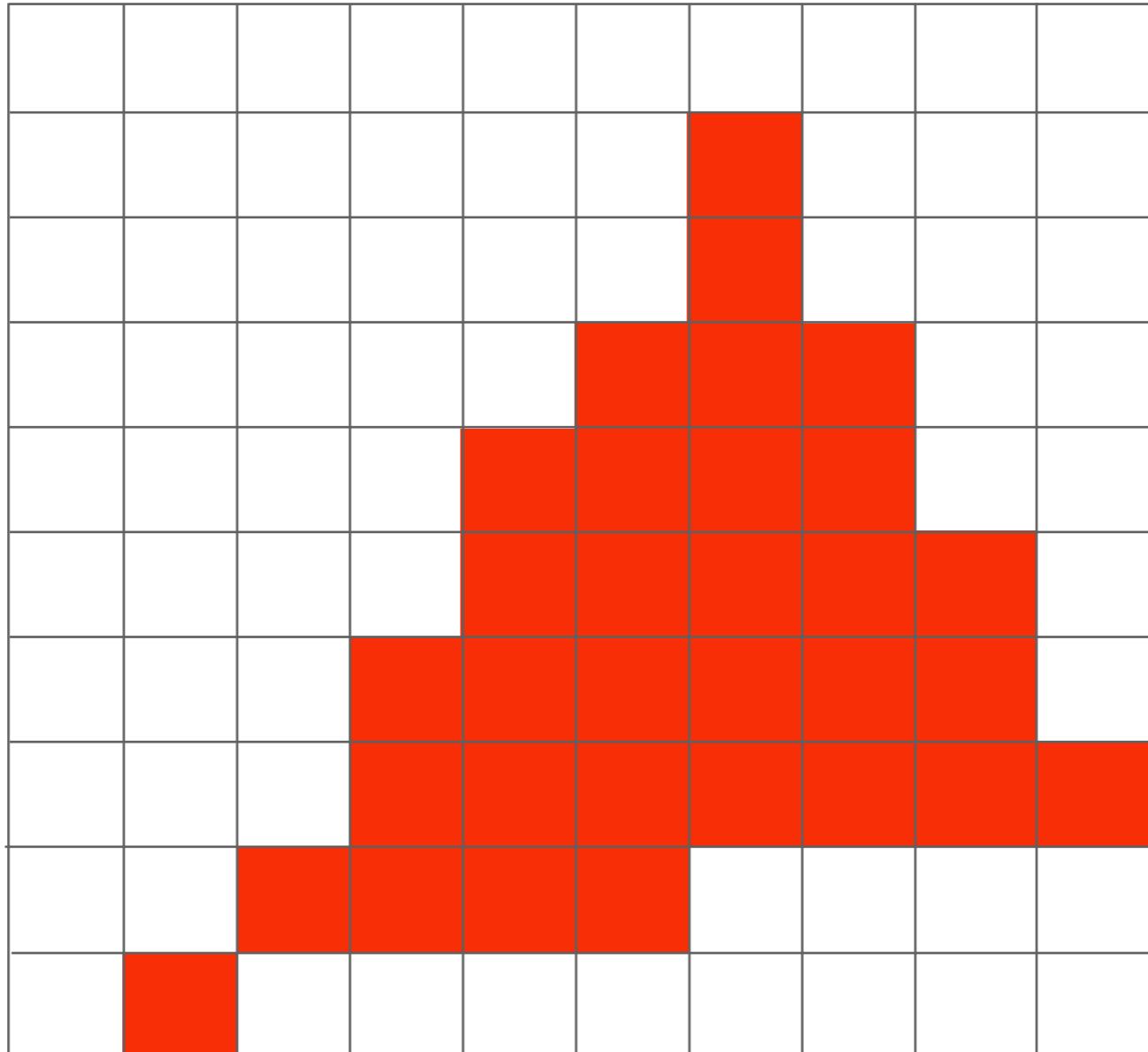
# The Display Physically Emits This Signal



# Compare: The Continuous Triangle Function

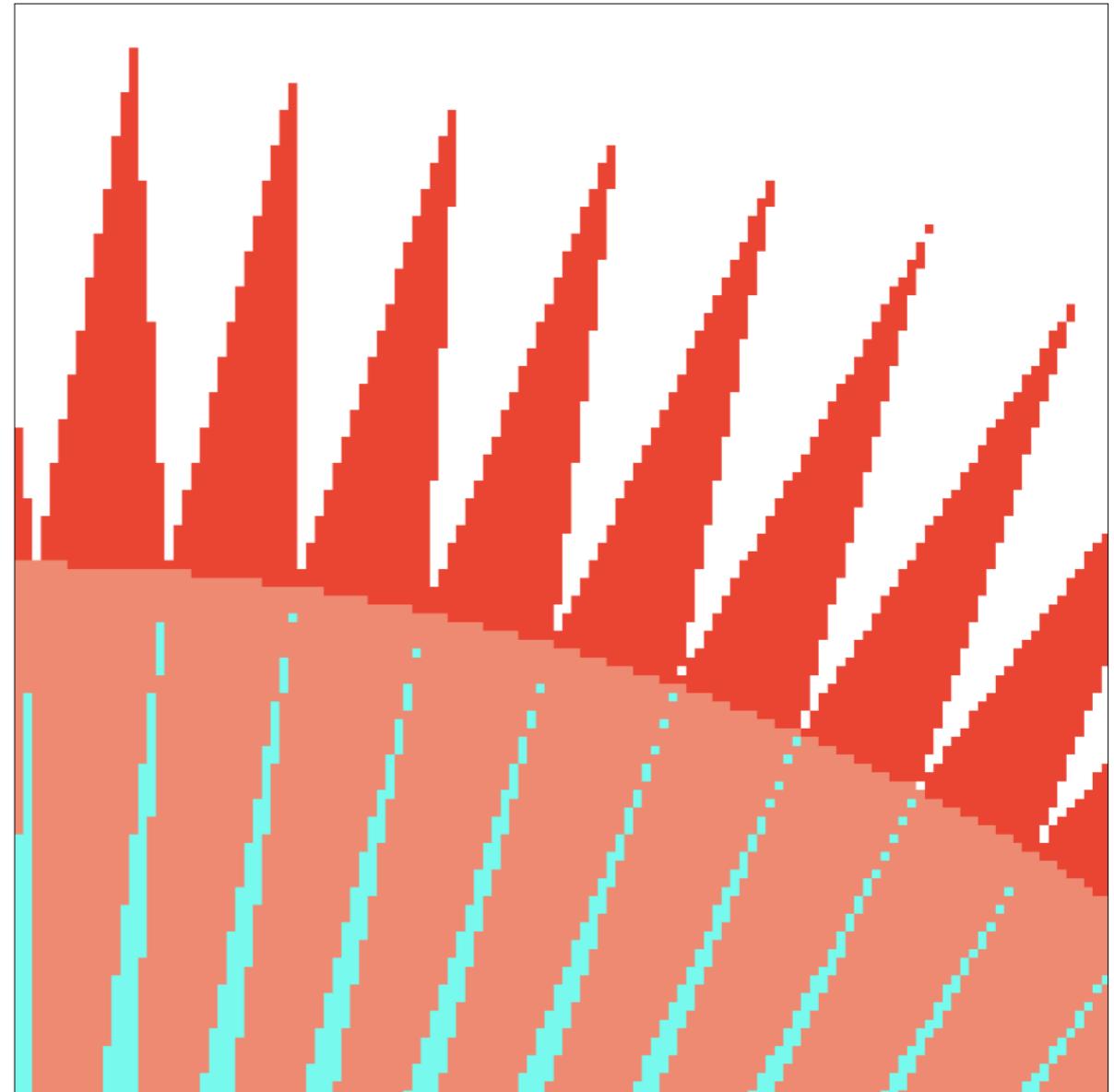
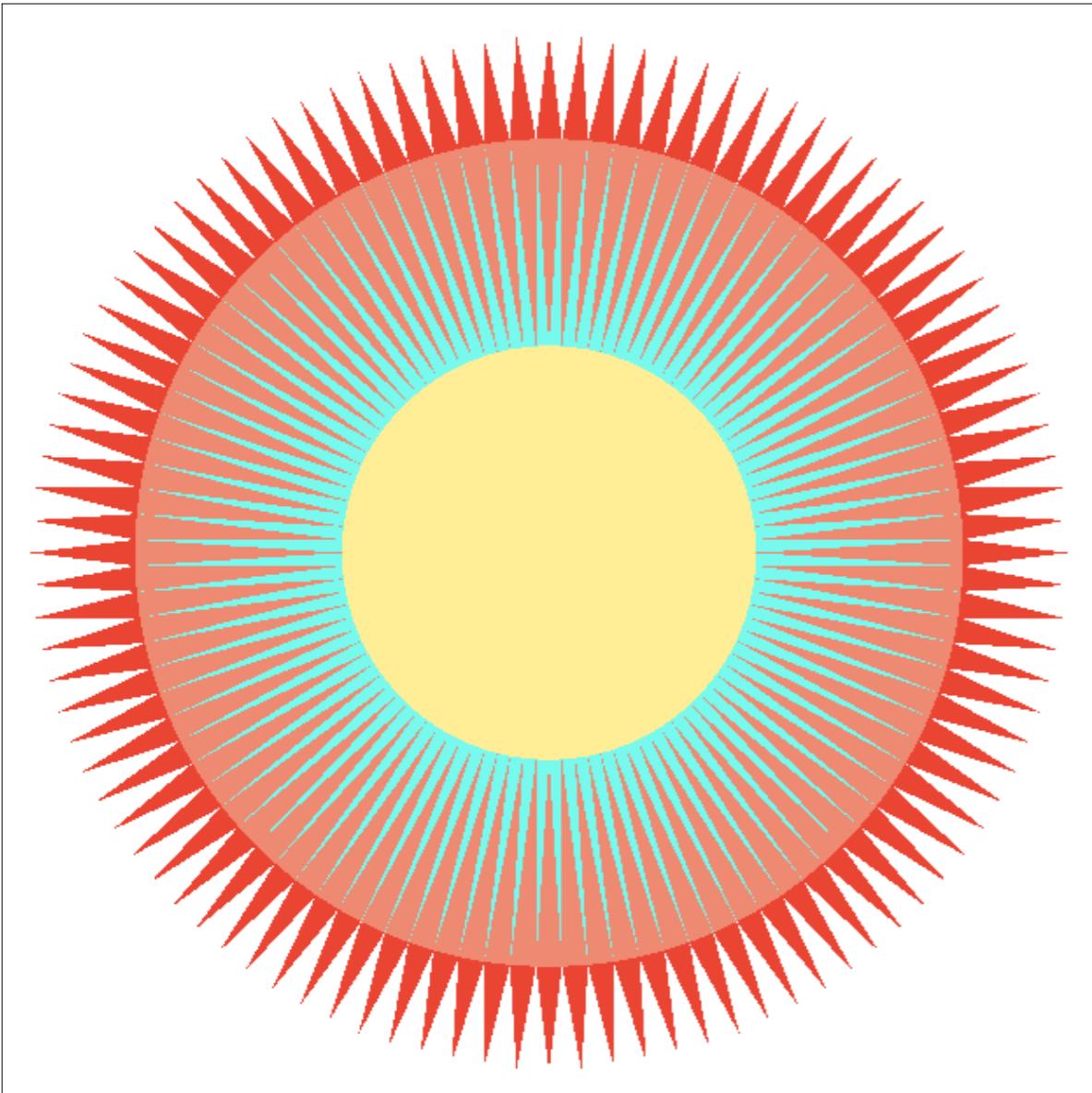


# What's Wrong With This Picture?



Jaggies!

# Aliasing (Jaggies)



Is this the best we can do?

# Thank you!

(And thank Prof. Ravi Ramamoorthi and Prof. Ren Ng for many of the slides!)