

Lecture 1

Intro to Crypto and Cryptocurrencies

Welcome



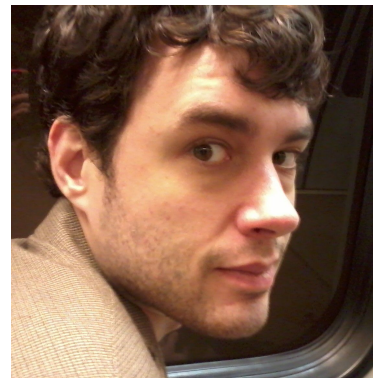
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special guest:
Andrew Miller

This lecture

Crypto background

- hash functions

- digital signatures

- ... and applications

Intro to cryptocurrencies

- basic digital cash

Lecture 1.1:

Cryptographic Hash Functions

Hash function:

- takes any string as input

- fixed-size output (we'll use 256 bits)

- efficiently computable

Security properties:

- collision-free

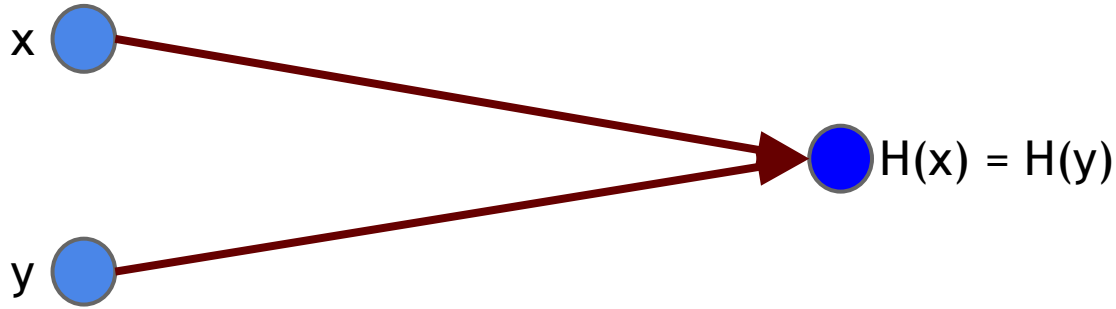
- hiding

- puzzle-friendly

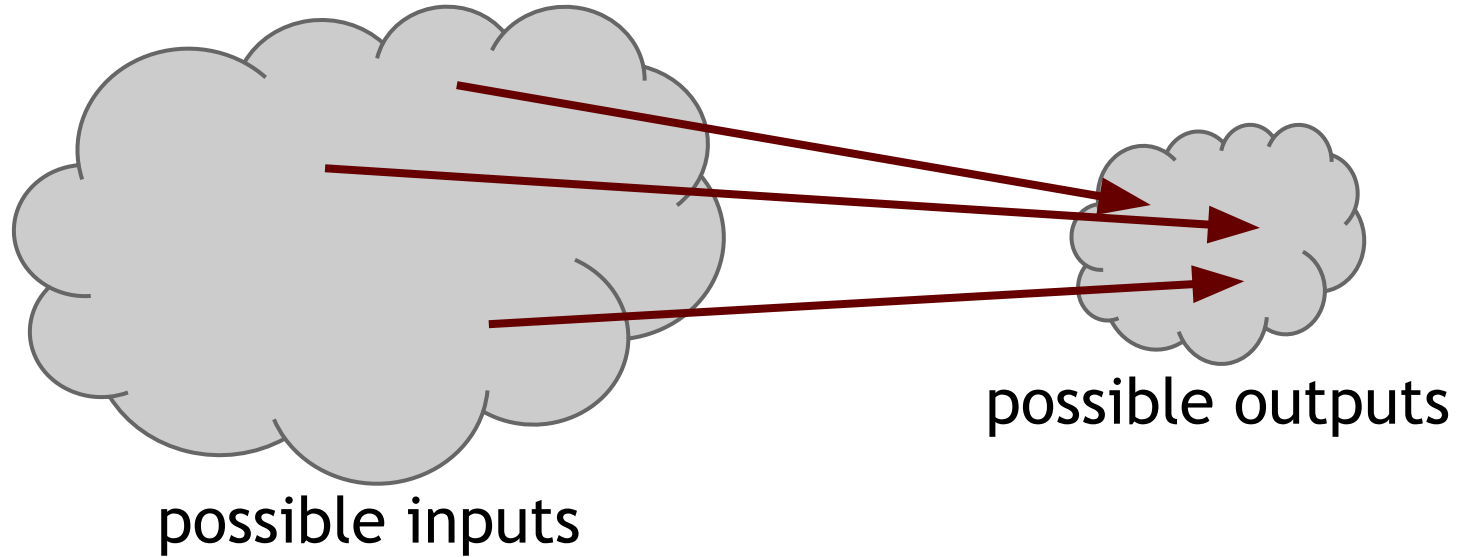
Hash property 1: Collision-free

Nobody can find x and y such that

$$x \neq y \text{ and } H(x) = H(y)$$



Collisions do exist ...



... but can anyone find them?

How to find a collision

try 2^{130} randomly chosen inputs

99.8% chance that two of them will collide

This works no matter what H is ...
... but it takes too long to matter

Is there a faster way to find collisions?

For some possible H 's, yes.

For others, we don't know of one.

No H has been proven collision-free.

Application: Hash as message digest

If we know $H(x) = H(y)$,
it's safe to assume that $x = y$.

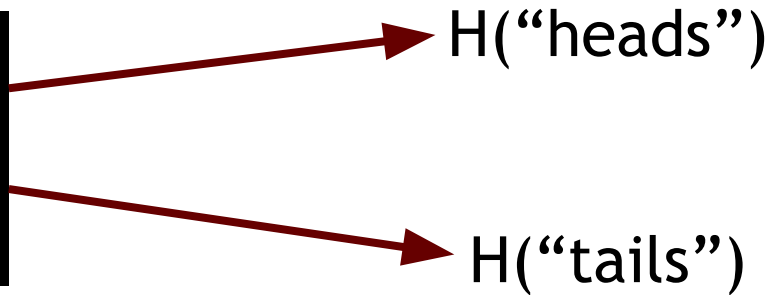
To recognize a file that we saw before,
just remember its hash.

Useful because the hash is small.

Hash property 2: Hiding

We want something like this:

Given $H(x)$, it is infeasible to find x .



easy to find x !

Hash property 2: Hiding

Hiding property:

If r is chosen from a probability distribution that has *high min-entropy*, then given $H(r \parallel x)$, it is infeasible to find x .

High min-entropy means that the distribution is “very spread out”, so that no particular value is chosen with more than negligible probability.

Application: Commitment

Want to “seal a value in an envelope”, and
“open the envelope” later.

Commit to a value, reveal it later.

Commitment API

$(com, key) := \text{commit}(msg)$

$match := \text{verify}(com, key, msg)$

To seal msg in envelope:

$(com, key) := \text{commit}(msg)$ -- then publish com

To open envelope:

publish key, msg

anyone can use $\text{verify}()$ to check validity

Commitment API

$(com, key) := \text{commit}(msg)$

$match := \text{verify}(com, key, msg)$

Security properties:

Hiding: Given com , infeasible to find msg .

Binding: Infeasible to find $msg \neq msg'$ such that
 $\text{verify}(\text{commit}(msg), msg') == \text{true}$

Commitment API

$\text{commit}(msg) := (H(key \mid msg), H(key))$

where key is a random 256-bit value

$\text{verify}(com, key, msg) := (H(key \mid msg) == com)$

Security properties:

Hiding: Given $H(key \mid msg)$, infeasible to find msg .

Binding: Infeasible to find $msg \neq msg'$ such that

$H(key \mid msg) == H(key \mid msg')$

Hash property 3: Puzzle-friendly

Puzzle-friendly:

For every possible output value y ,
if k is chosen from a distribution with high min-entropy,
then it is infeasible to find x such that $H(k \parallel x) = y$.

Application: Search puzzle

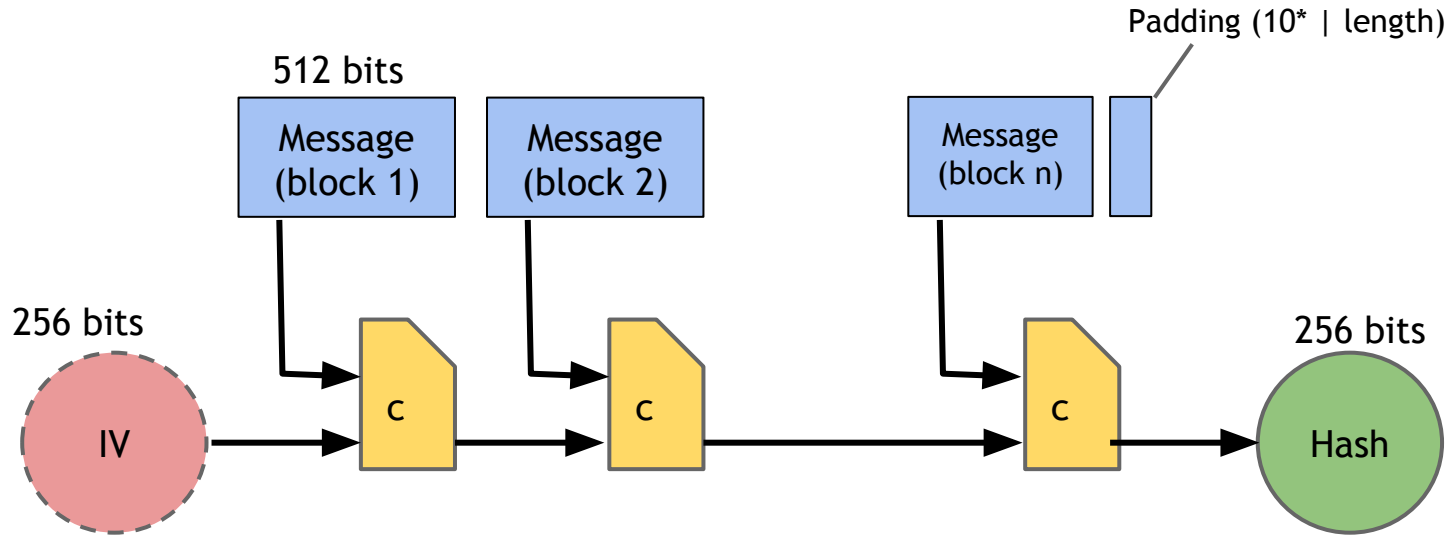
Given a “puzzle ID” id (from high min-entropy distrib.),
and a target set Y :

Try to find a “solution” x such that

$$H(id \mid x) \in Y.$$

Puzzle-friendly property implies that no solving strategy is much better than trying random values of x .

SHA-256 hash function



Theorem: If c is collision-free, then SHA-256 is collision-free.

Lecture 1.2:

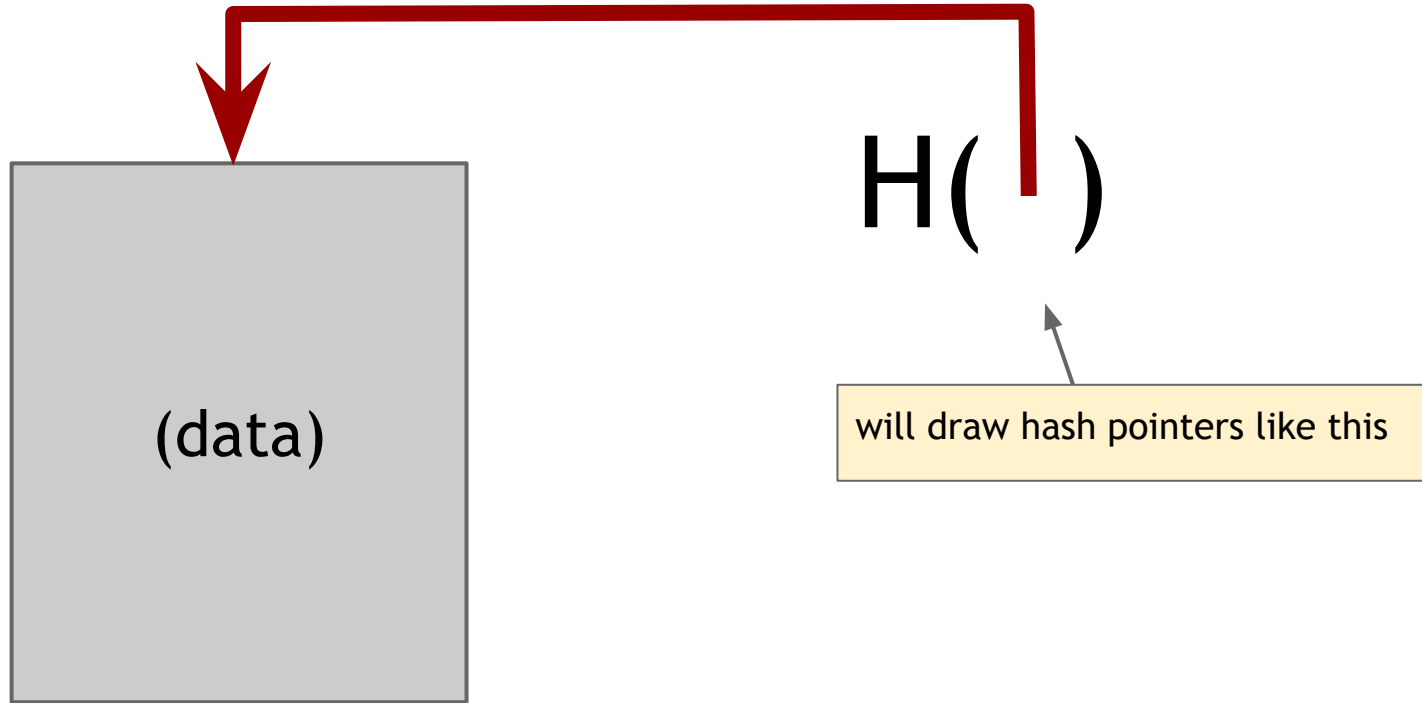
Hash Pointers and Data Structures

hash pointer is:

- * pointer to where some info is stored, and
- * (cryptographic) hash of the info

if we have a hash pointer, we can

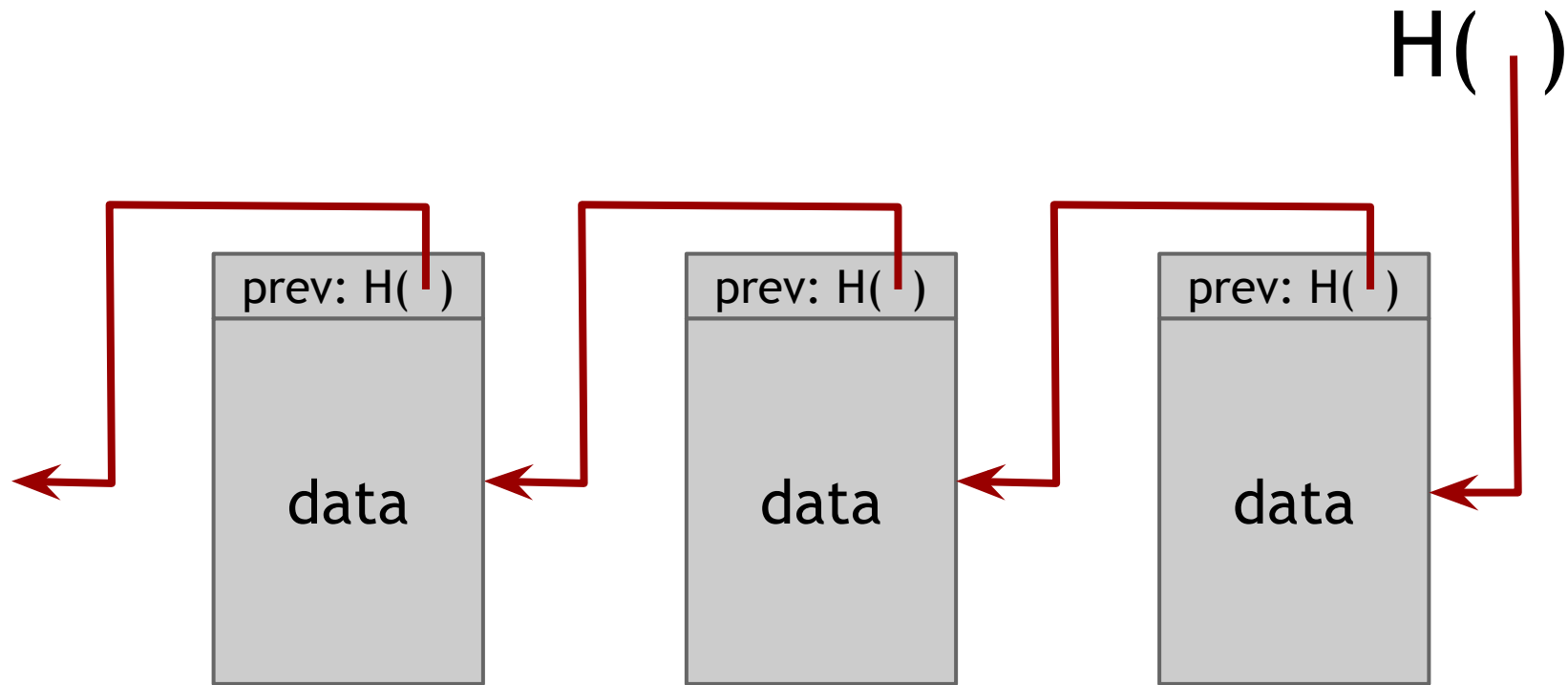
- * ask to get the info back, and
- * verify that it hasn't changed



key idea:

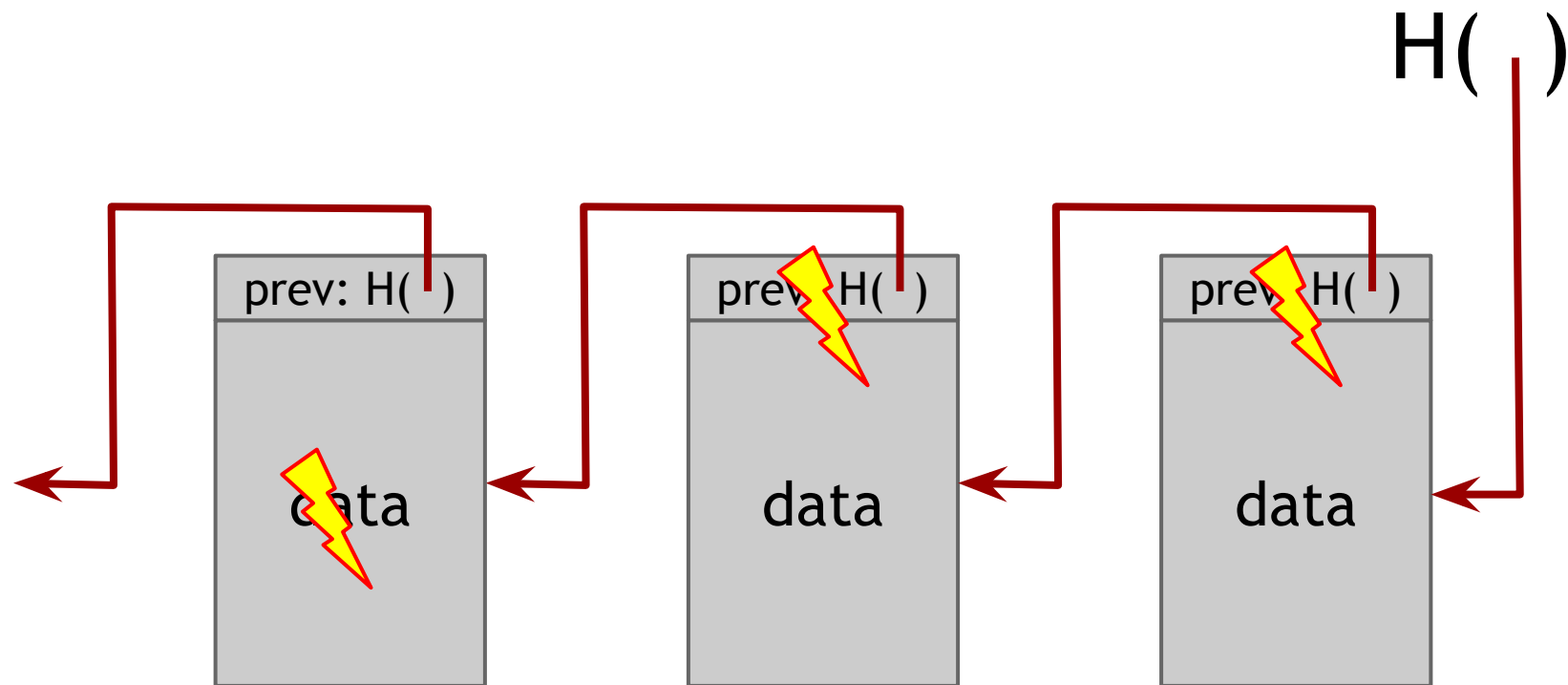
build data structures with hash pointers

linked list with hash pointers = “block chain”



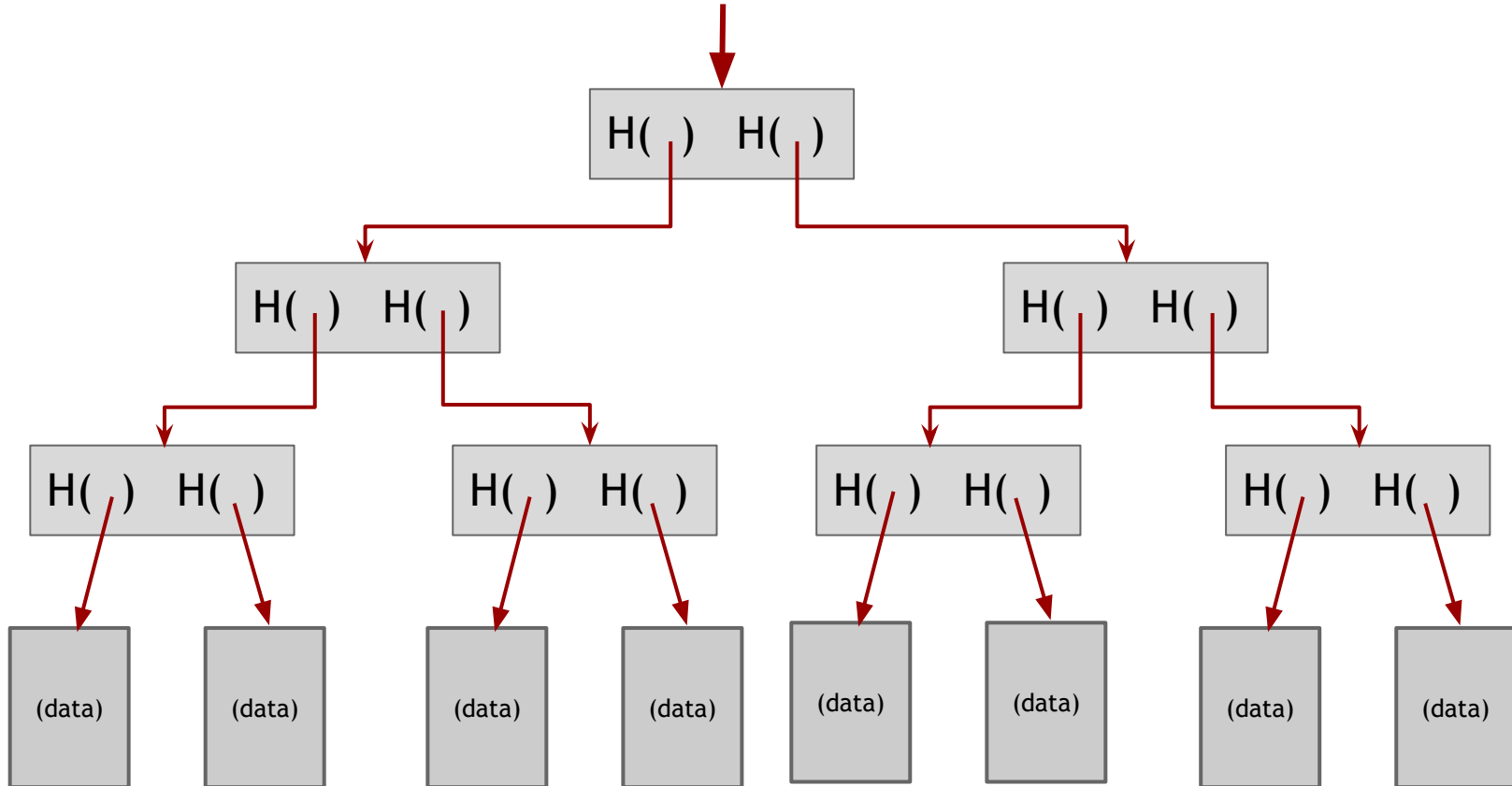
use case: tamper-evident log

detecting tampering

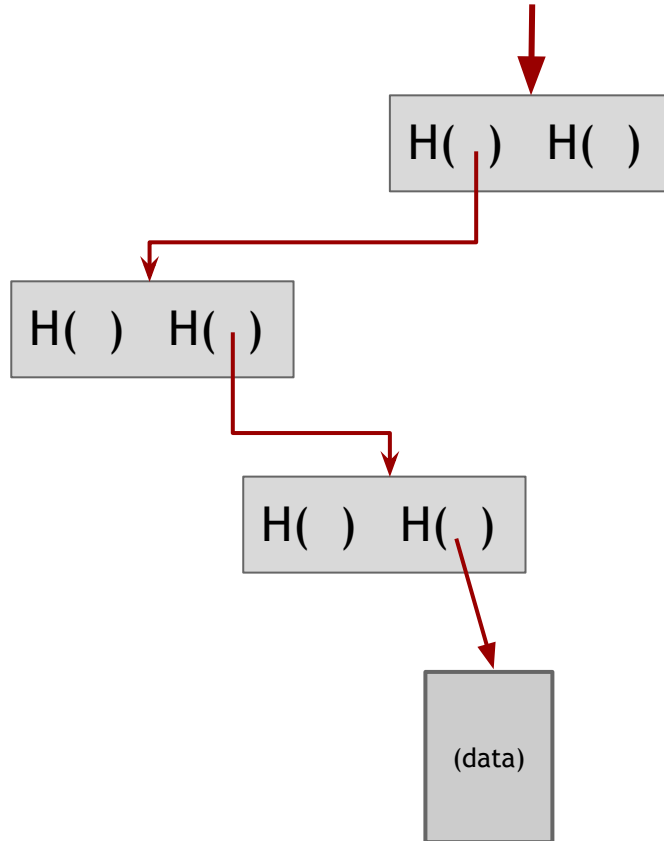


use case: tamper-evident log

binary tree with hash pointers = “Merkle tree”



proving membership in a Merkle tree



show $O(\log n)$ items

Advantages of Merkle trees

Tree holds many items

but just need to remember the root hash

Can verify membership in $O(\log n)$ time/space

Variant: sorted Merkle tree

can verify non-membership in $O(\log n)$

(show items before, after the missing one)

More generally ...

can use hash pointers in any pointer-based
data structure that has no cycles

Lecture 1.3:

Digital Signatures

What we want from signatures

Only you can sign, but anyone can verify

Signature is tied to a particular document
can't be cut-and-pasted to another doc

API for digital signatures


$(sk, pk) := \text{generateKeys}(\text{keysize})$

sk: secret signing key

pk: public verification key

$\text{sig} := \text{sign}(sk, \text{message})$

$\text{isValid} := \text{verify}(pk, \text{message}, \text{sig})$



can be
randomized
algorithms

Requirements for signatures

“valid signatures verify”

`verify(pk, message, sign(sk, message)) == true`

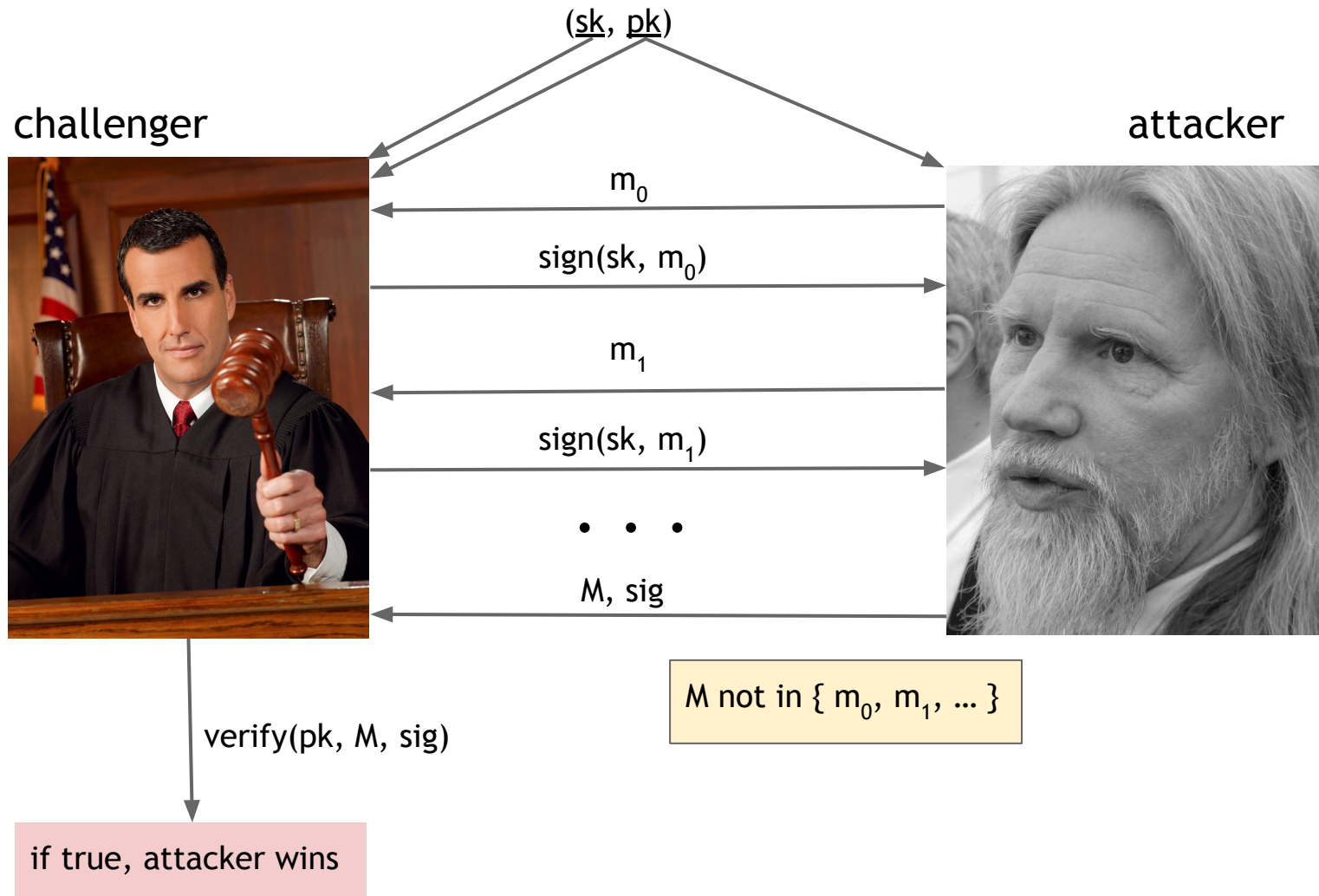
“can’t forge signatures”

adversary who:

knows pk

gets to see signatures on messages of his choice

can’t produce a verifiable signature on another message



Practical stuff...

algorithms are randomized

- need good source of randomness

limit on message size

- fix: use Hash(message) rather than message

fun trick: sign a hash pointer

- signature “covers” the whole structure

Bitcoin uses ECDSA standard

Elliptic Curve Digital Signature Algorithm

relies on hairy math

will skip the details here --- look it up if you care

good randomness is essential

foul this up in generateKeys() or sign() ?

probably leaked your private key

GAME
OVER

Lecture 1.4:

Public Keys as Identities

Useful trick: public key == an identity

if you see sig such that $verify(pk, msg, sig) == true$,
think of it as

pk says, “[msg]”.

to “speak for” pk , you must know matching secret key sk

How to make a new identity

create a new, random key-pair (sk , pk)

pk is the public “name” you can use

[usually better to use $\text{Hash}(pk)$]

sk lets you “speak for” the identity

you control the identity, because only you know sk

if pk “looks random”, nobody needs to know who you are

Decentralized identity management

anybody can make a new identity at any time
make as many as you want!

no central point of coordination

These identities are called “addresses” in Bitcoin.

Privacy

Addresses not directly connected to real-world identity.

But observer can link together an address's activity over time, make inferences.

Later: a whole lecture on privacy in Bitcoin ...

Lecture 1.5:

Simple Cryptocurrencies



GoofyCoin

Goofy can create new coins

signed by pk_{Goofy}

CreateCoin [uniqueCoinID]

New coins belong to me.



A coin's owner can spend it.

signed by pk_{Goofy}
Pay to pk_{Alice} : $H()$

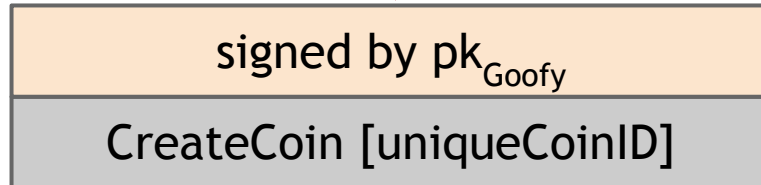
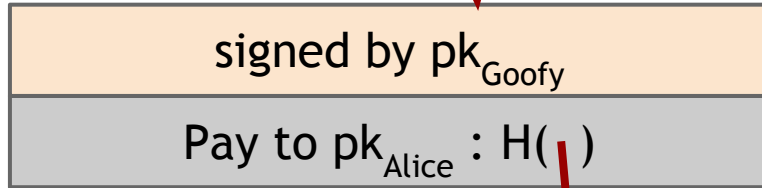
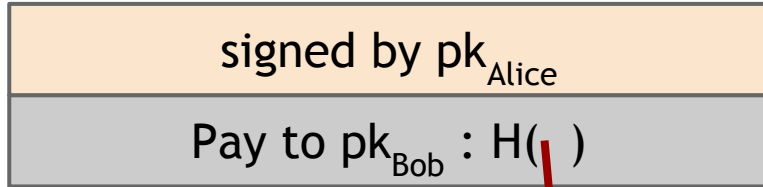


signed by pk_{Goofy}
CreateCoin [uniqueCoinID]

Alice owns it now.



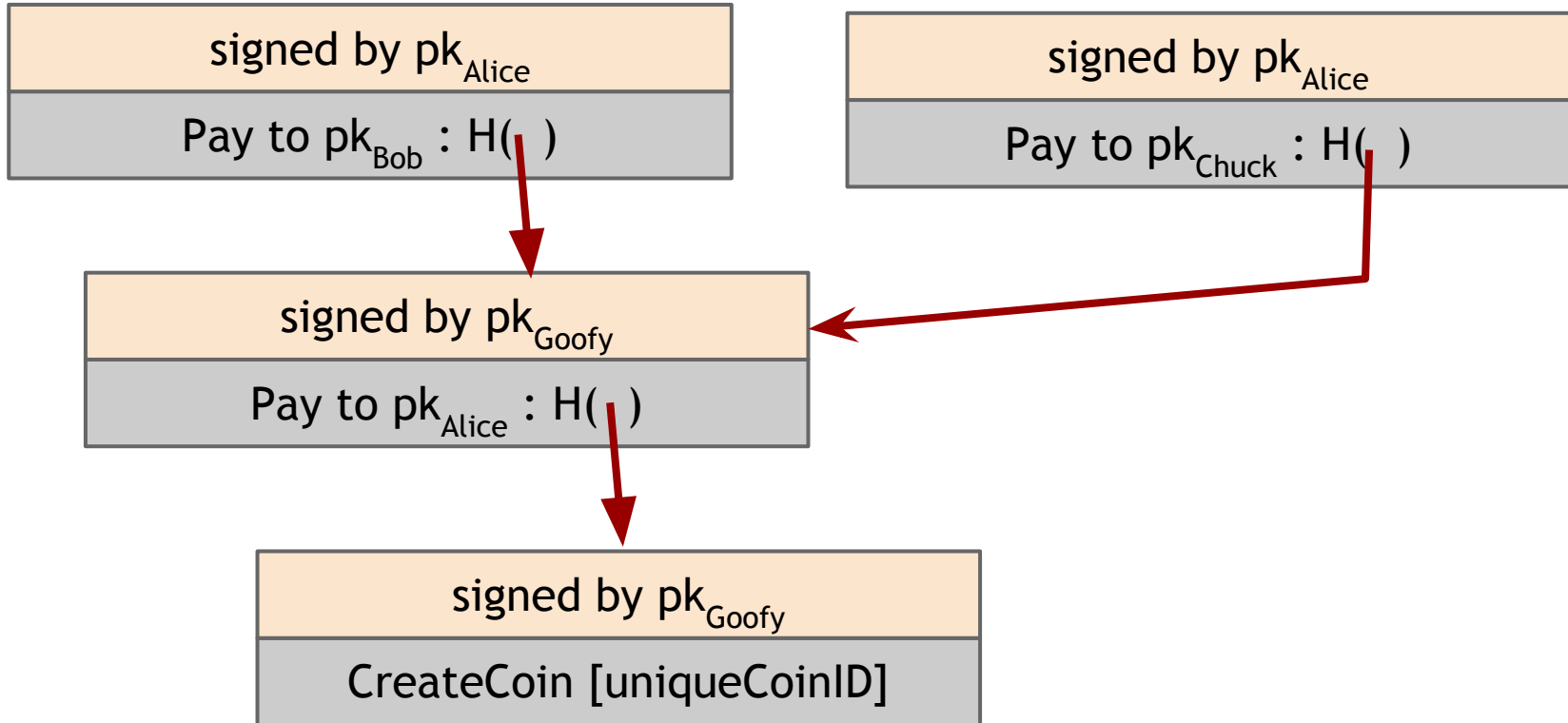
The recipient can pass on the coin again.



Bob owns it now.



double-spending attack



double-spending attack

the main design challenge in digital currency

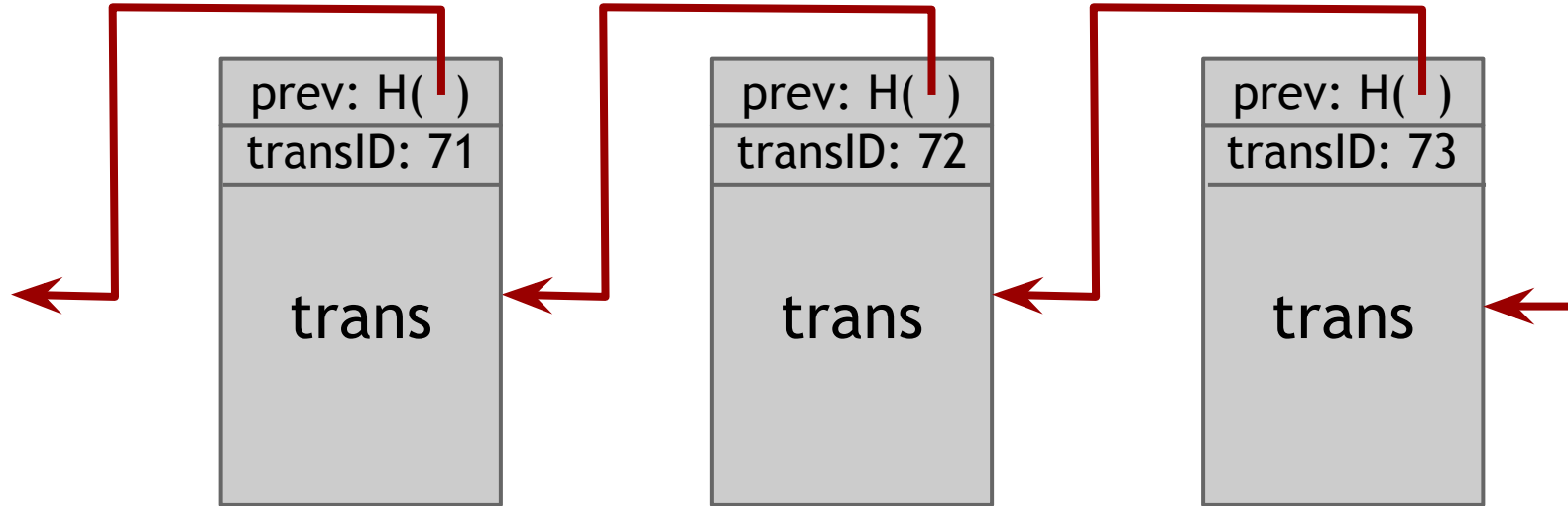


ScroogeCoin

Scrooge publishes a history of all transactions
(a block chain, signed by Scrooge)



$H()$



optimization: put multiple transactions in the same block

CreateCoins transaction creates new coins

transID: 73 type:CreateCoins		
coins created		
<i>num</i>	<i>value</i>	<i>recipient</i>
0	3.2	0x...
1	1.4	0x...
2	7.1	0x...

← coinID 73(0)

← coinID 73(1)

← coinID 73(2)

Valid, because I said so.



PayCoins transaction consumes (and destroys) some coins,
and creates new coins of the same total value

transID: 73 type:PayCoins		
consumed coinIDs: 68(1), 42(0), 72(3)		
coins created		
<i>num</i>	<i>value</i>	<i>recipient</i>
0	3.2	0x...
1	1.4	0x...
2	7.1	0x...
signatures		

Valid if:

- consumed coins valid,
- not already consumed,
- total value out = total value in, and
- signed by owners of all consumed coins

Immutable coins

Coins can't be transferred, subdivided, or combined.

But: you can get the same effect by using transactions
to subdivide: create new trans
consume your coin
pay out two new coins to yourself

Don't worry, I'm honest.



Crucial question:

Can we descroogify the currency,
and operate without any central,
trusted party?