Example: d-regular graph

- Suppose all nodes in $\boldsymbol{G}$ have degree $\boldsymbol{d}$ and $\boldsymbol{G}$ is connected
- **What are some eigenvalues/vectors of $\boldsymbol{G}$?**

$\boldsymbol{A} \cdot \boldsymbol{x} = \boldsymbol{\lambda} \cdot \boldsymbol{x}$ What is λ? What x?

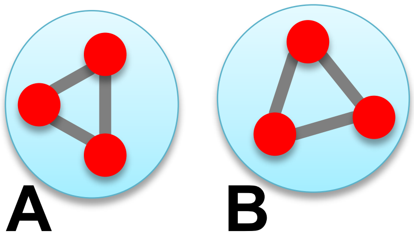
  - **Let's try:** $\boldsymbol{x} = (\mathbf{1}, \mathbf{1}, \dots, \mathbf{1})$
  - **Then:** $\boldsymbol{A} \cdot \boldsymbol{x} = (\boldsymbol{d}, \boldsymbol{d}, \dots, \boldsymbol{d}) = \boldsymbol{\lambda} \cdot \boldsymbol{x}$. **So:** $\boldsymbol{\lambda} = \boldsymbol{d}$
  - **We found eigenpair of $\boldsymbol{G}$:** $\boldsymbol{x} = (\mathbf{1}, \mathbf{1}, \dots, \mathbf{1}), \boldsymbol{\lambda} = \boldsymbol{d}$

Remember the meaning of $\boldsymbol{y} = \boldsymbol{A} \cdot \boldsymbol{x}$:

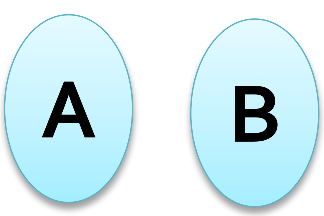
$$y_j = \sum_{i=1}^{n} A_{ij} x_i = \sum_{(j,i) \in E} x_i$$

Example: Graph on 2 components

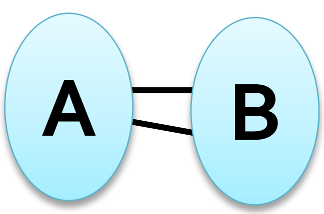
- **What if G is not connected?**
  - G has **2** components, each d-regular



- **What are some eigenvectors?**
  - $x =$ Put all $\mathbf{1}$s on A and $\mathbf{0}$s on B or vice versa
    - $x' = (\underbrace{1, \dots, 1}_{|A|}, \underbrace{0, \dots, 0}_{|B|})$ **then** $\mathrm{A} \cdot x' = (d, \dots, d, 0, \dots, 0)$
    - $x'' = (0, \dots, 0, 1, \dots, 1)$ **then** $A \cdot x'' = (0, \dots, 0, d, \dots, d)$
    - And so in both cases the corresponding $\lambda = d$
- **A bit of intuition:**



$\lambda_1 = \lambda_2$



$\lambda_1 \approx \lambda_2$