PageRank

Random Surfers on the Web
Transition Matrix of the Web
Dead Ends and Spider Traps
Topic-Specific PageRank

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Administrivia – Honor Code

- We’ve had our first HC cases.
- Please, please, please, before you do anything that might violate the HC, talk to me or a TA to make sure it is legitimate.
- It is much easier to get caught than you might think.

Administrivia – Homeworks

- There were a number of people who failed to upload code or HW answers properly and received no credit.
- Also, some people followed general SCPD directions, which you must not do in the future.
- We made some exceptions, e.g., allowing late code uploads.
- But in the future, please do not expect these sorts of exceptions to be made.

Intuition – (1)

- Web pages are important if people visit them a lot.
- But we can't watch everybody using the Web.
- A good surrogate for visiting pages is to assume people follow links randomly.
- Leads to *random surfer* model:
  - Start at a random page and follow random out-links repeatedly, from whatever page you are at.
  - *PageRank* = limiting probability of being at a page.

Intuition – (2)

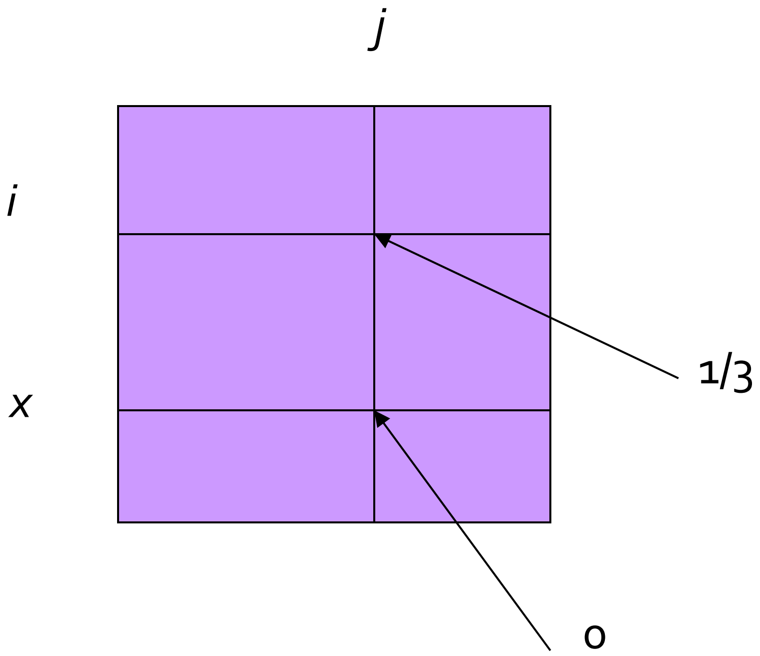
- Solve the recursive equation: “a page is important to the extent that important pages link to it.”
  - Equivalent to the random-surfer definition of PageRank.
- Technically, *importance* = the principal eigenvector of the transition matrix of the Web.
  - A few fixups needed.

Transition Matrix of the Web

- Number the pages 1, 2,… .
  - Page *i* corresponds to row and column *i*.
- *M* [*i*, *j*] = 1/*n* if page *j* links to *n* pages, including page *i* ; 0 if *j* does not link to *i*.
  - *M* [*i, j*] is the probability we'll next be at page *i* if we are now at page *j*.

Example: Transition Matrix

Suppose page *j* links to 3 pages, including *i* but not *x*.



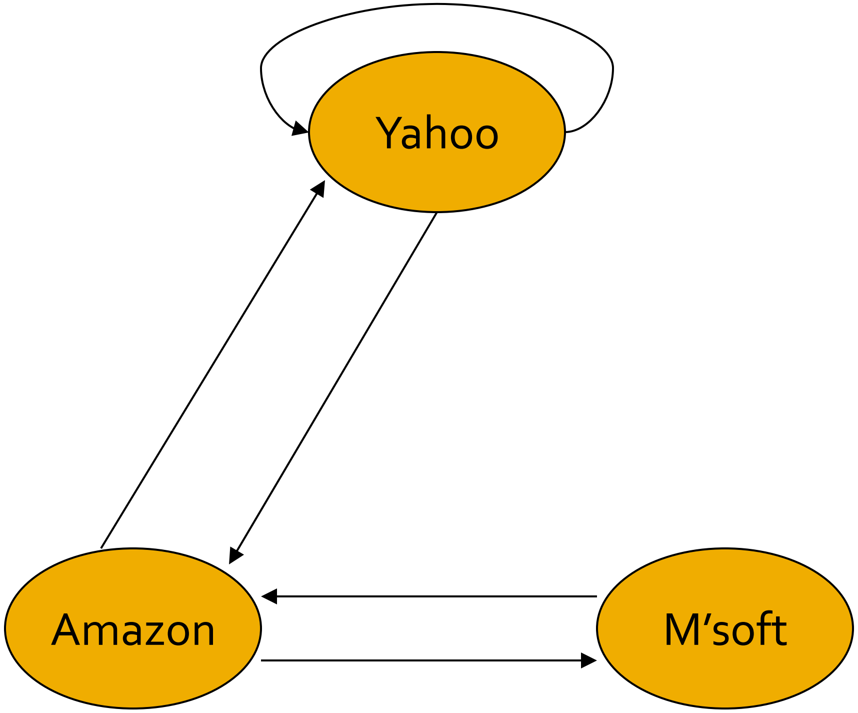
Random Walks on the Web

- Suppose **v** is a vector whose *i* th component is the probability that a random walker is at page *i* at a certain time.
- If a walker follows a link from *i* at random, the probability distribution for walkers is then given by the vector *M***v**.

Random Walks – (2)

- Starting from any vector **u**, the limit $M\ (M\ (\ldots M\ (M\ \mathbf{u}\ )\ \ldots))$ is the long-term distribution of walkers.
- Intuition: pages are important in proportion to how likely a walker is to be there.
- The math: limiting distribution = principal eigenvector of M = PageRank.
  - Note: because M has each column summing to 1, the principal eigenvalue is 1.
    - Why? If **v** is the limit of MM…M**u**, then **v** satisfies the equations **v** = M**v**.

Example: The Web in 1839



$$\begin{array}{c|ccc|}
 & y & a & m \\
\hline
y & 1/2 & 1/2 & 0 \\
a & 1/2 & 0 & 1 \\
m & 0 & 1/2 & 0 \\
\hline
\end{array}$$

Solving The Equations

- Because there are no constant terms, the equations **v** = *M***v** do not have a unique solution.
- In Web-sized examples, we cannot solve by Gaussian elimination anyway; we need to use *relaxation* (= iterative solution).
- Works if you start with any nonzero **u**.

Simulating a Random Walk

- Start with the vector **u** = [1, 1,…, 1] representing the idea that each Web page is given one unit of *importance*.
  - Note: it is more common to start with each vector element = 1/n, where n is the number of Web pages.
- Repeatedly apply the matrix *M* to **u**, allowing the importance to flow like a random walk.
- About 50 iterations is sufficient to estimate the limiting solution.

Example: Iterating Equations

- Equations $\mathbf{v} = M\mathbf{v}$:

$$y = y/2 + a/2$$

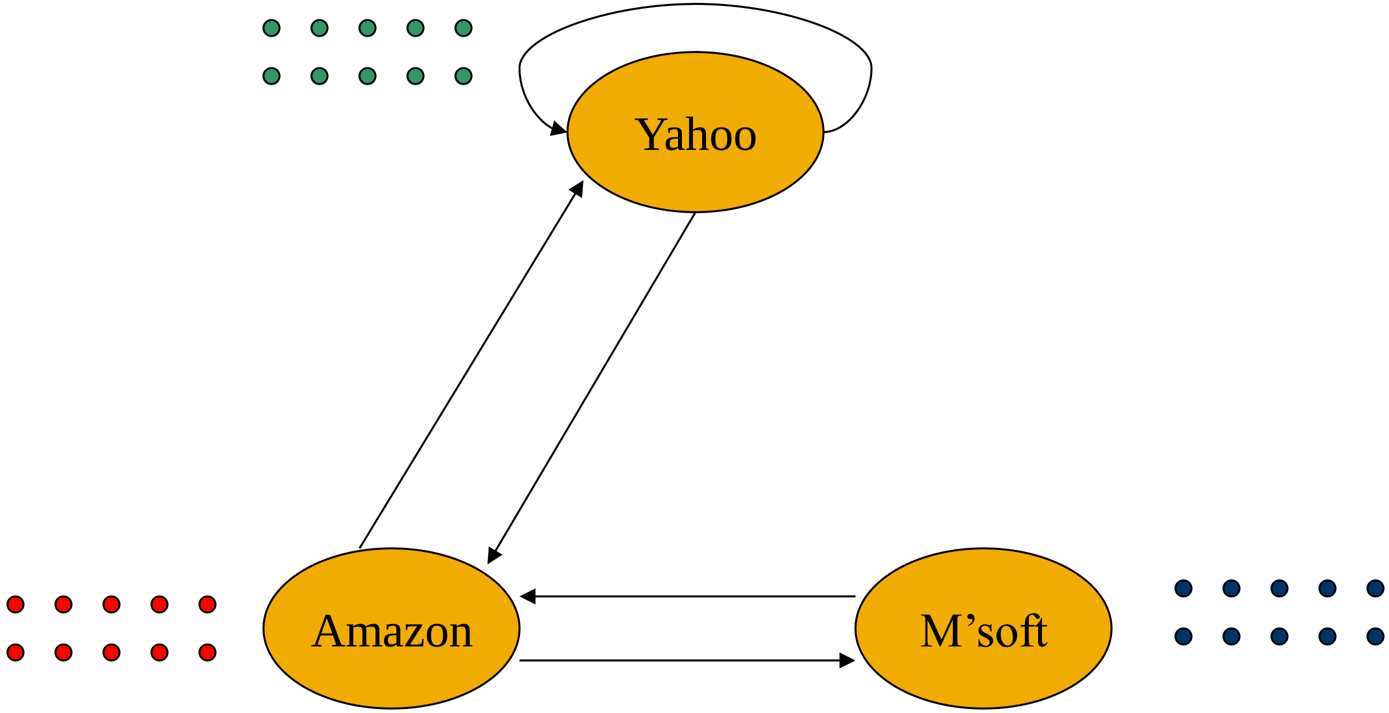
$$a = y/2 + m$$

$$m = a/2$$

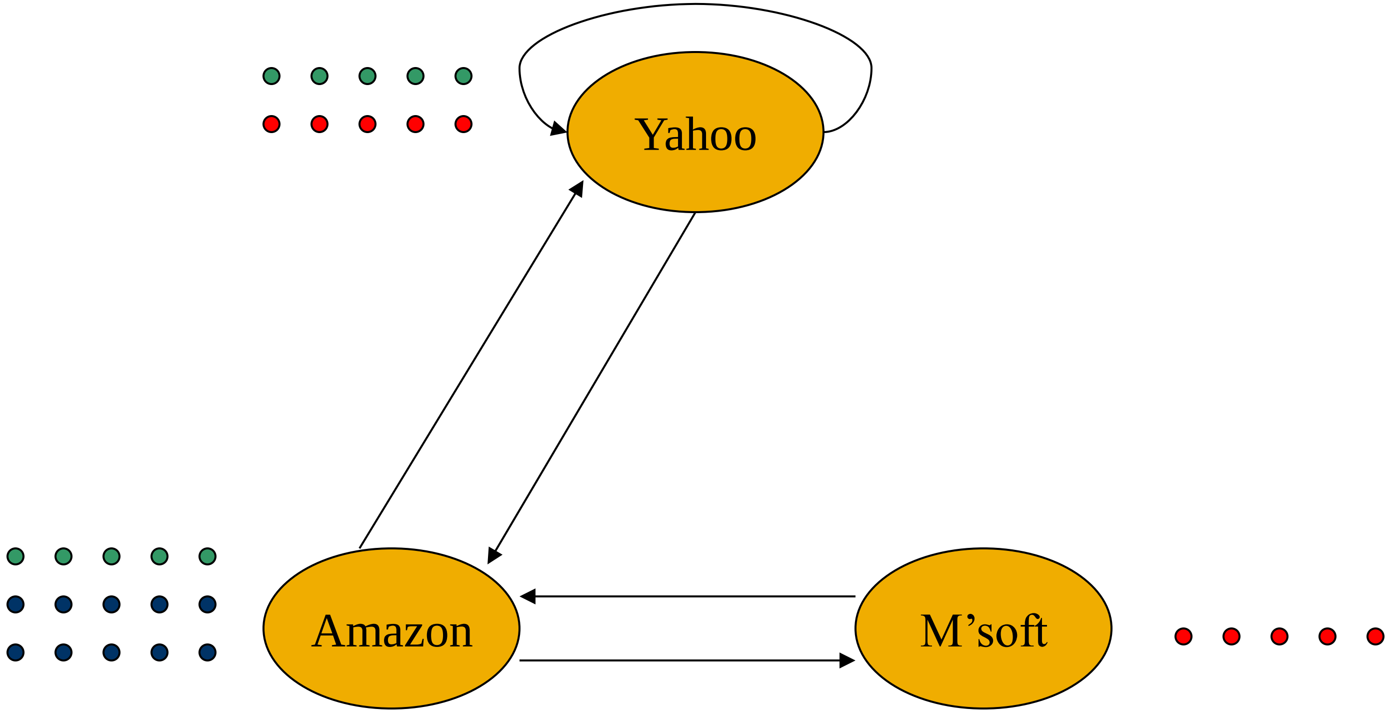
Note: "=" is really "assignment."

y		1	1	5/4	9/8		6/5
a	=	1	3/2	1	11/8	. . .	6/5
m		1	1/2	3/4	1/2		3/5

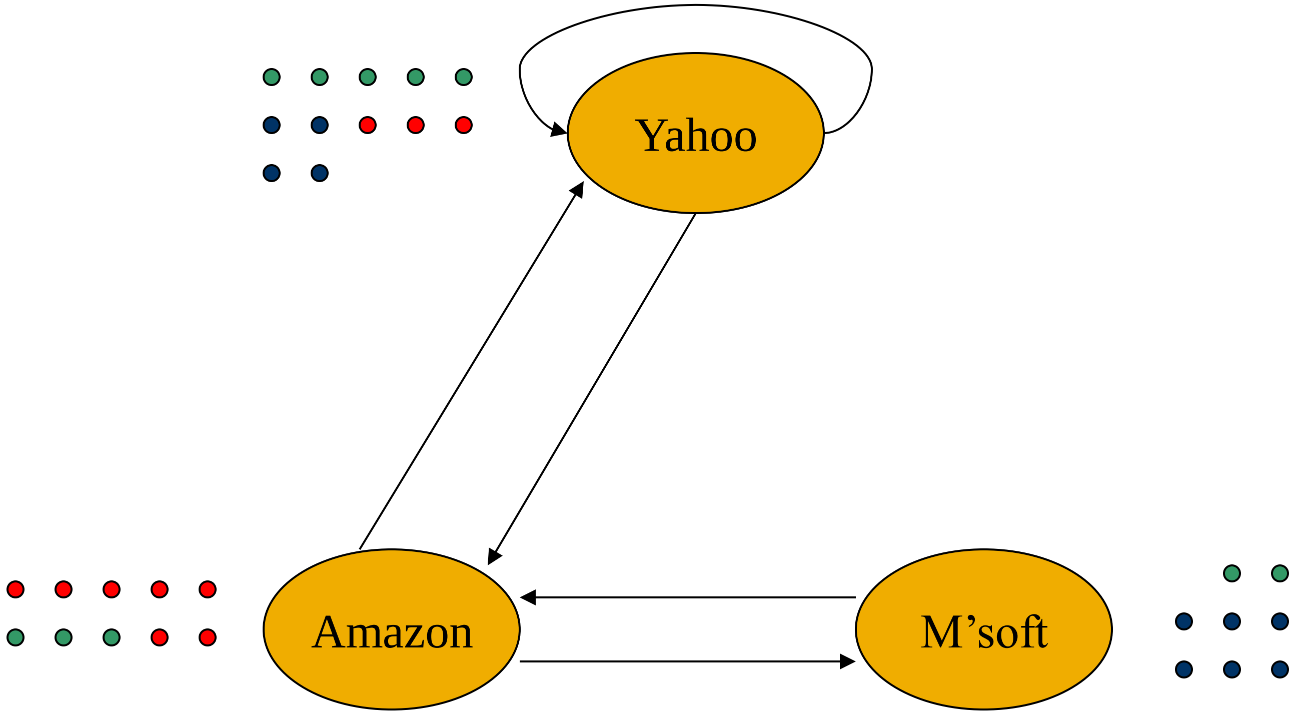
The Walkers



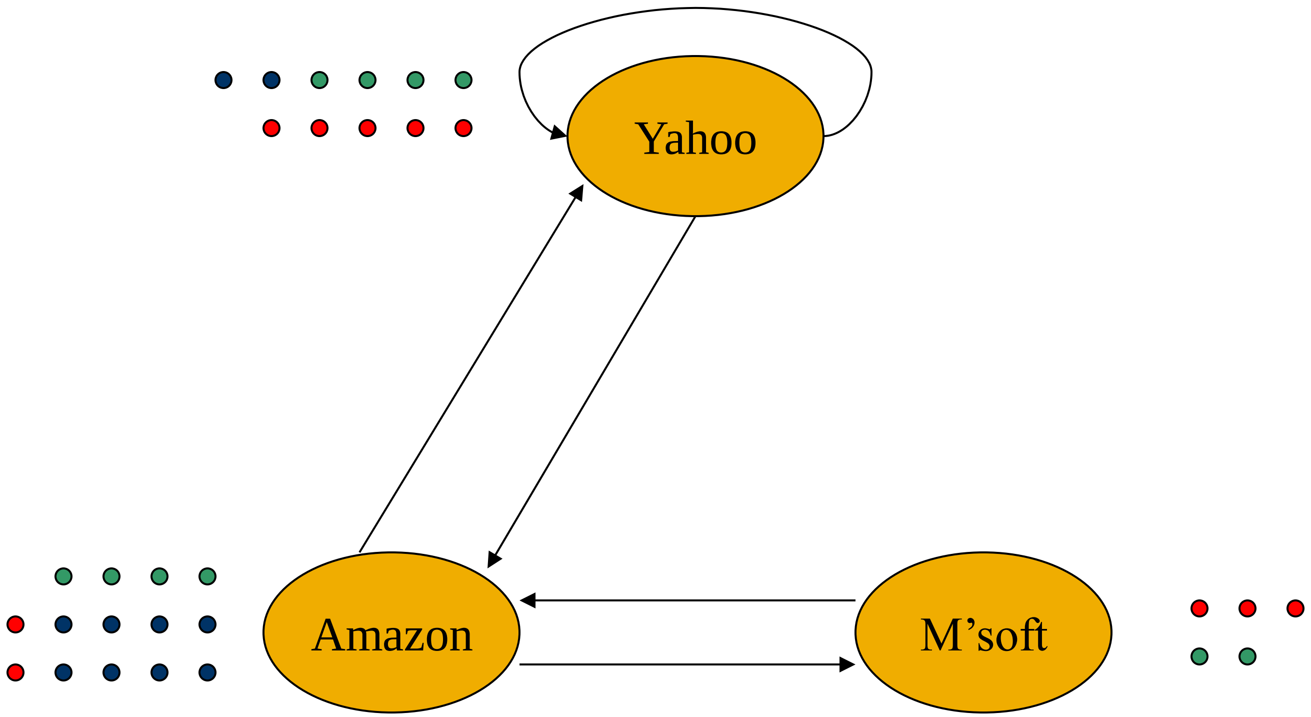
The Walkers



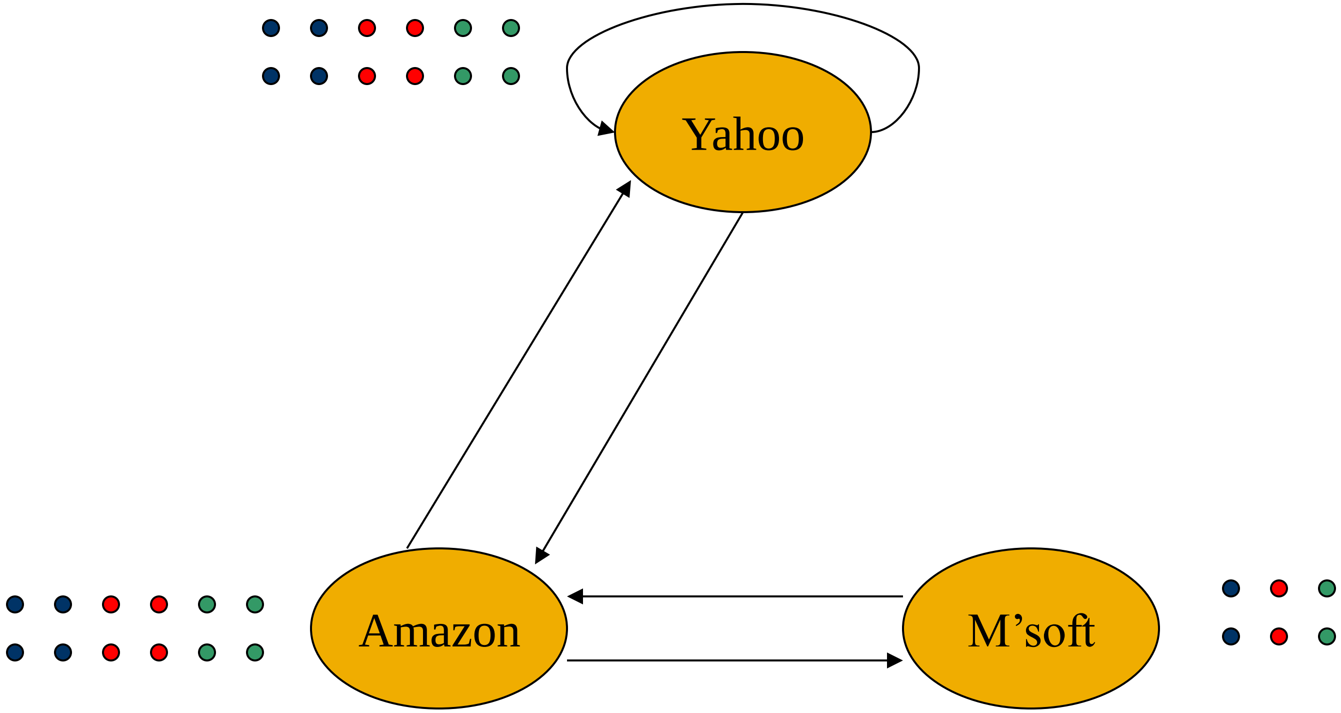
The Walkers



The Walkers



In the Limit …



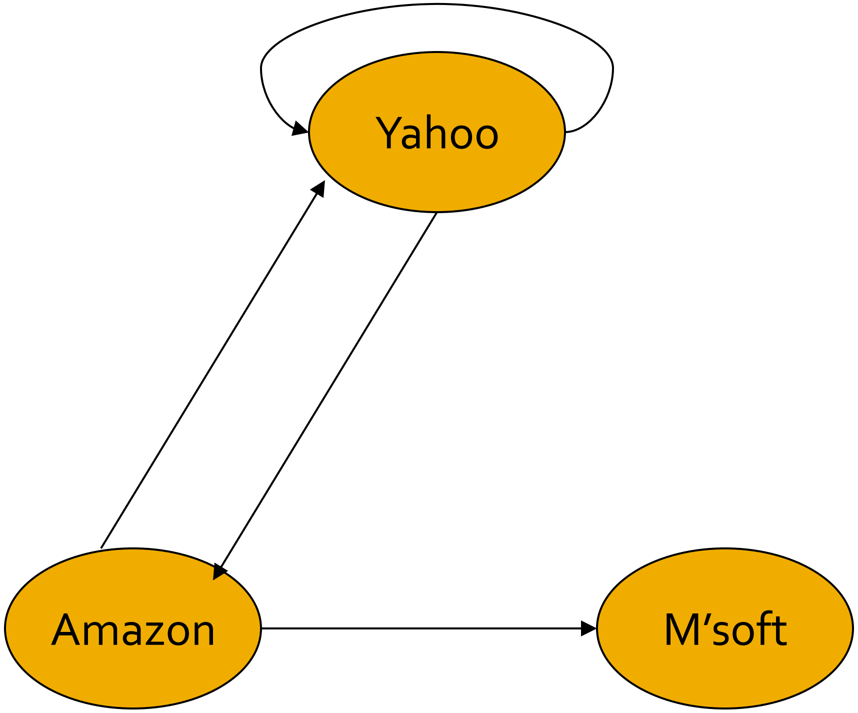
The Web Is More Complex Than That

Dead Ends
Spider Traps
Taxation Policies

Real-World Problems

- Some pages are *dead ends* (have no links out).
  - Such a page causes importance to leak out.
- Other groups of pages are *spider traps* (all out-links are within the group).
  - Eventually spider traps absorb all importance.

Microsoft Becomes Dead End



$$
\begin{array}{c|ccc|}
 & y & a & m \\
\hline
y & 1/2 & 1/2 & 0 \\
a & 1/2 & 0 & 0 \\
m & 0 & 1/2 & 0 \\
\hline
\end{array}
$$

Example: Effect of Dead Ends

- Equations **v** = *M***v**:

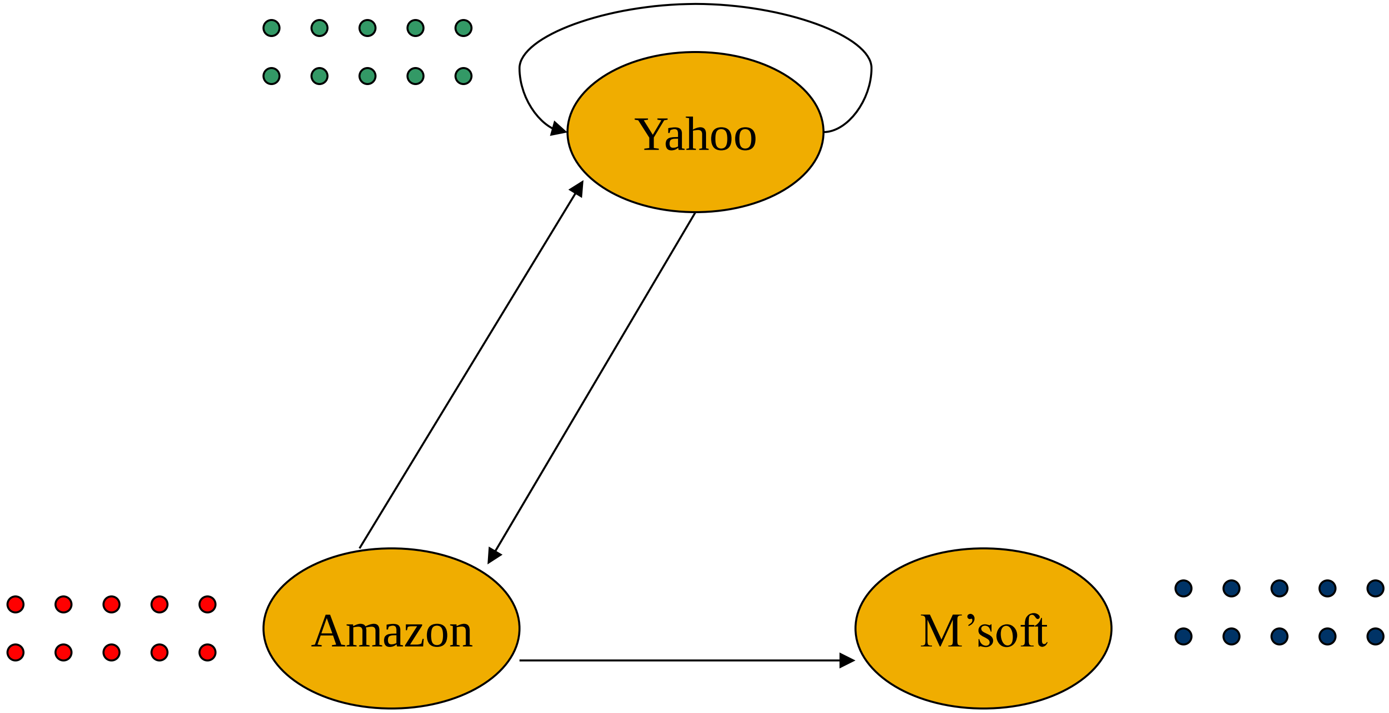
$$y = y/2 + a/2$$

$$a = y/2$$

$$m = a/2$$

$$\begin{array}{c} y \\ a \\ m \end{array} = \quad \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \quad \begin{array}{c} 1 \\ 1/2 \\ 1/2 \end{array} \quad \begin{array}{c} 3/4 \\ 1/2 \\ 1/4 \end{array} \quad \begin{array}{c} 5/8 \\ 3/8 \\ 1/4 \end{array} \quad \ldots \quad \begin{array}{c} 0 \\ 0 \\ 0 \end{array}$$

Microsoft Becomes a Dead End



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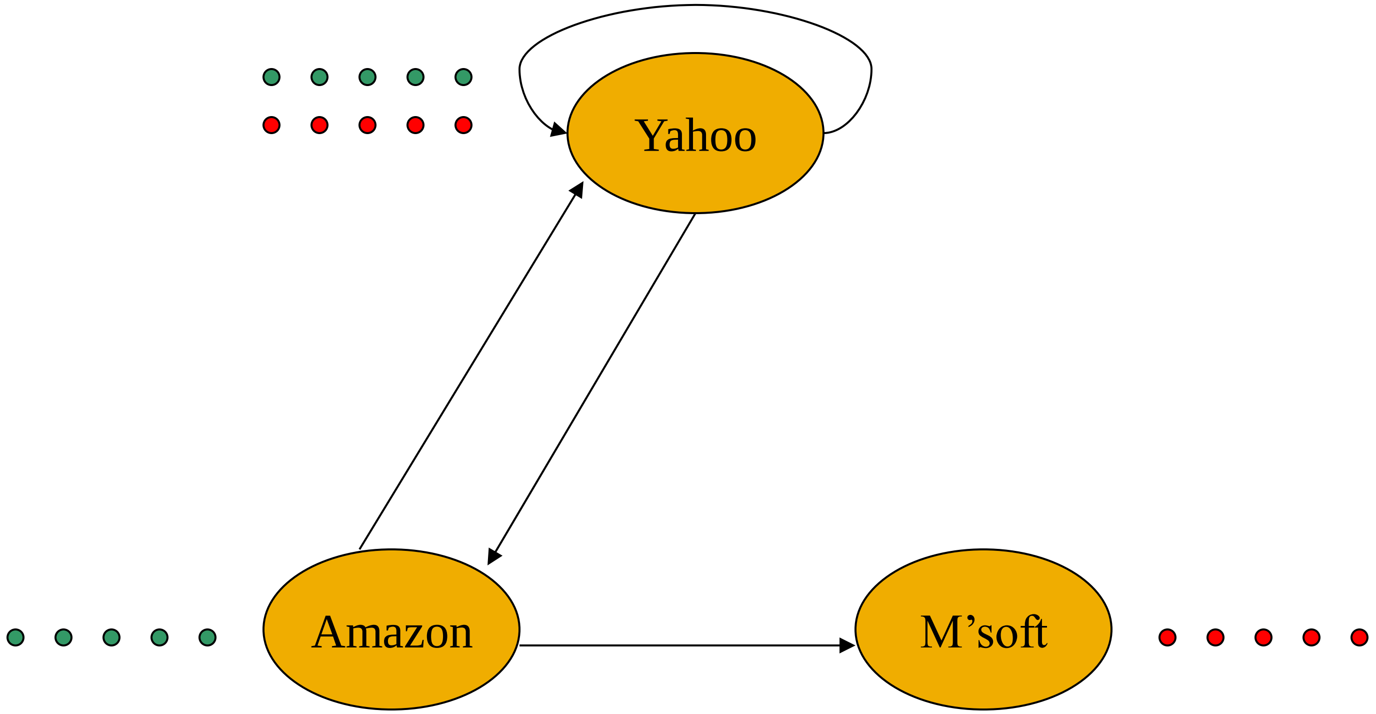

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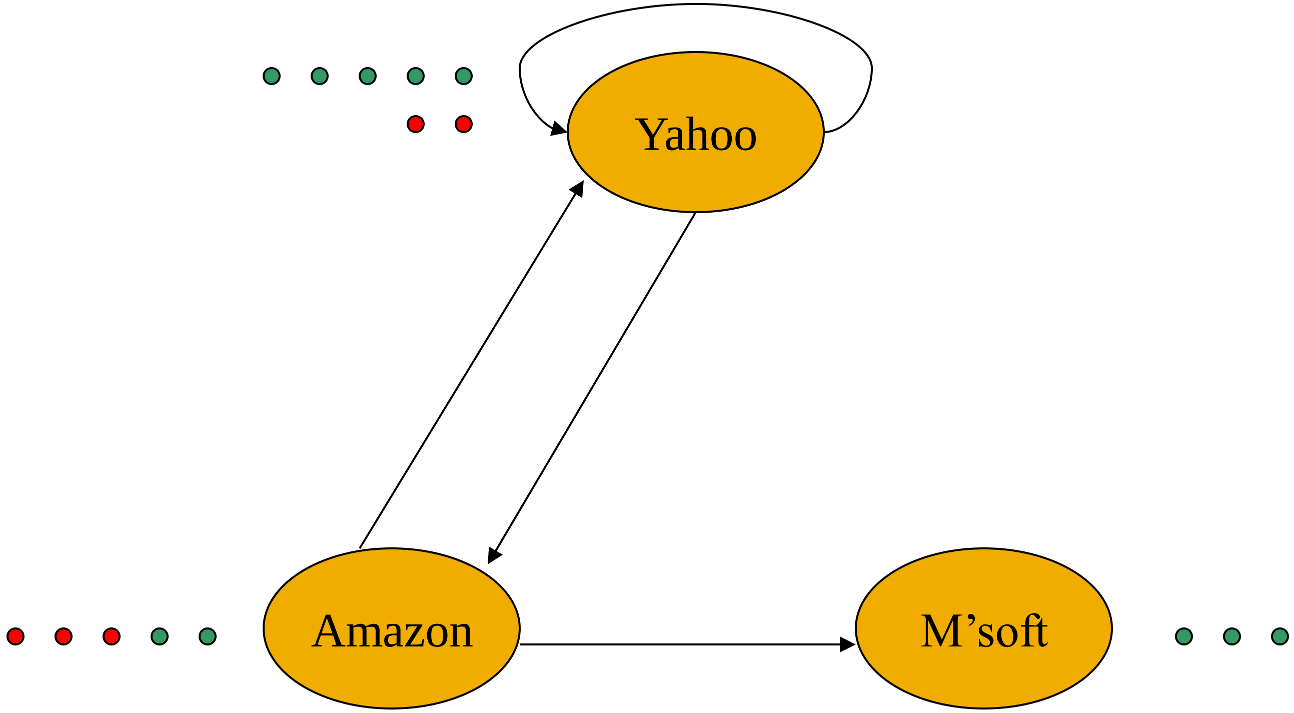

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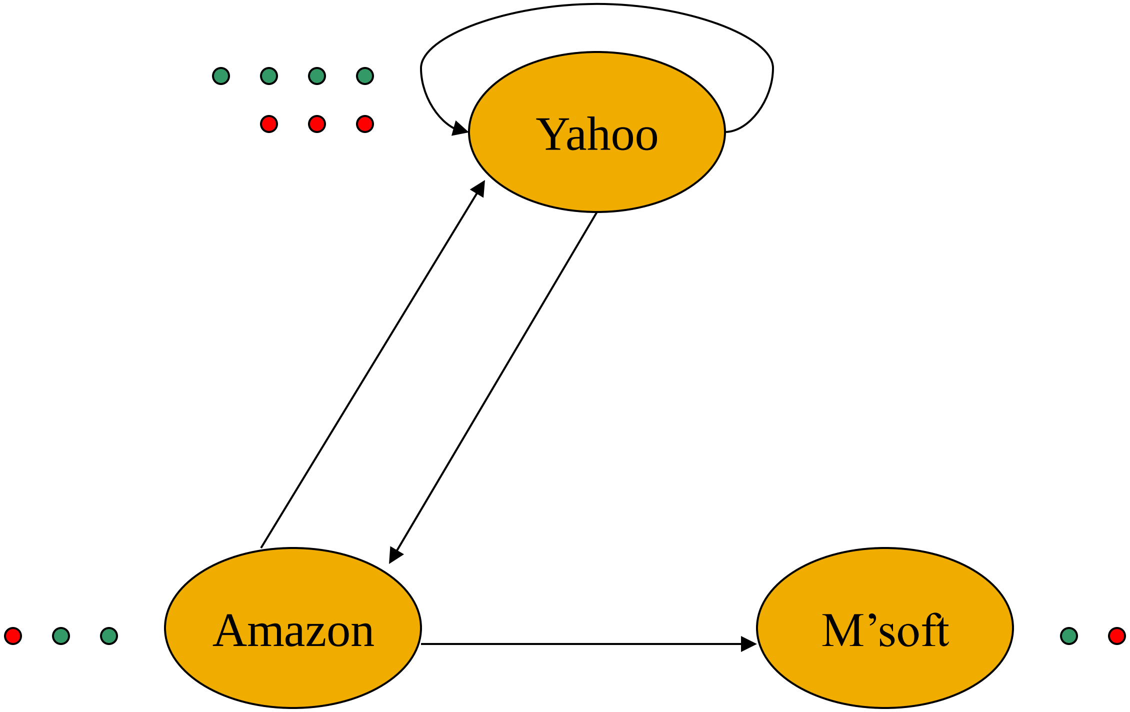

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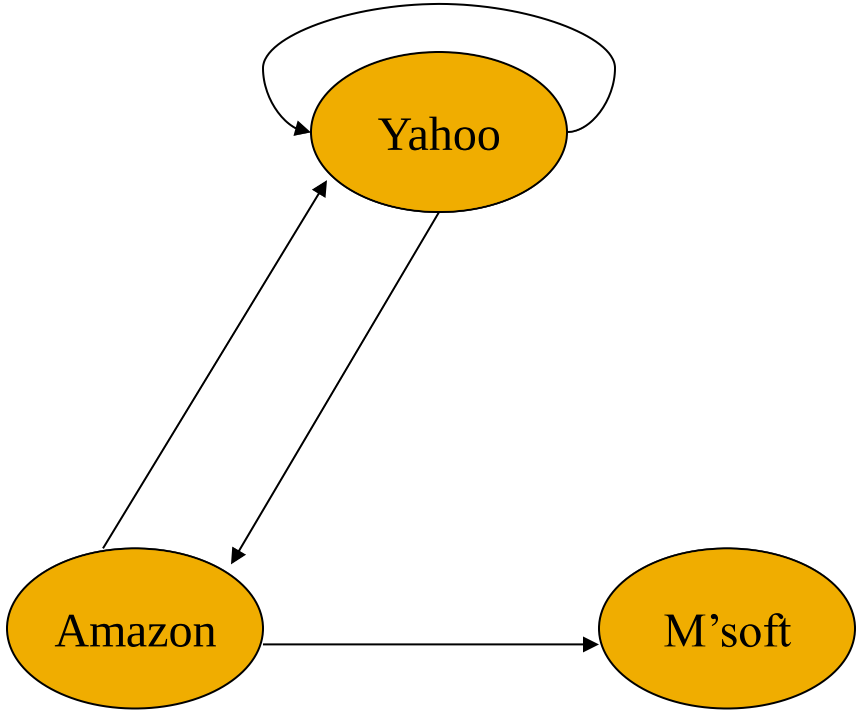
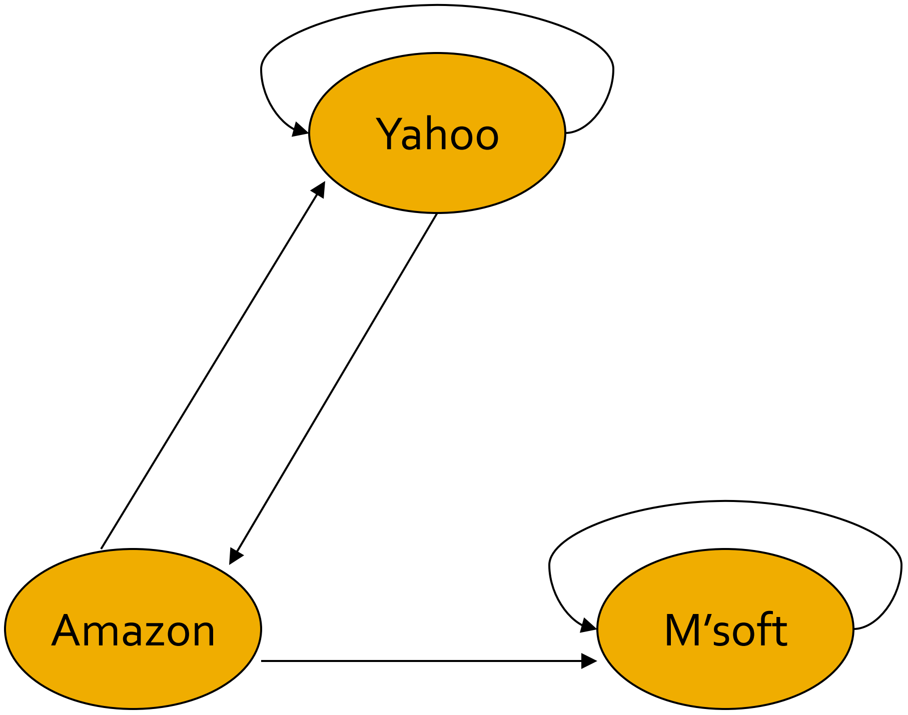


M’soft Becomes Spider Trap



$$
\begin{array}{c|ccc|}
 & y & a & m \\
\hline
y & 1/2 & 1/2 & 0 \\
a & 1/2 & 0 & 0 \\
m & 0 & 1/2 & 1 \\
\hline
\end{array}
$$

Example: Effect of Spider Trap

- Equations **v** = *M***v**:

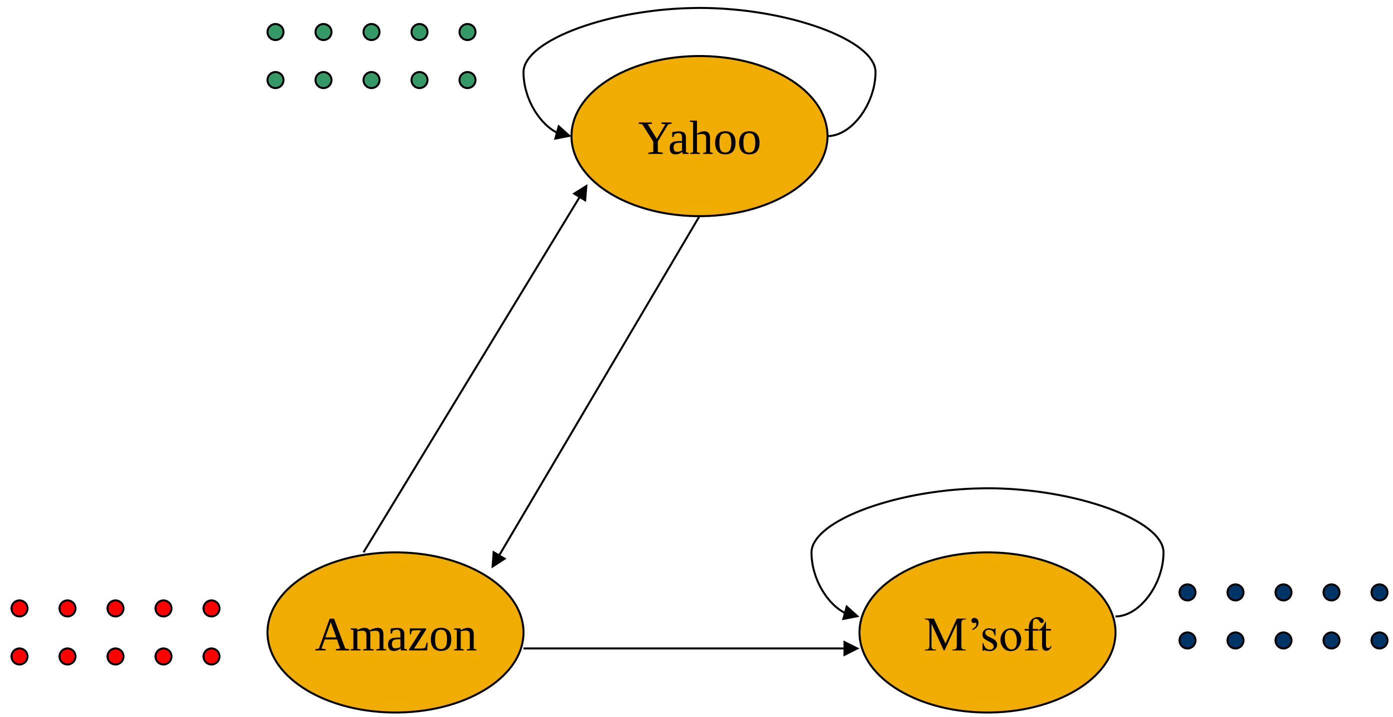
$$y = y/2 + a/2$$

$$a = y/2$$

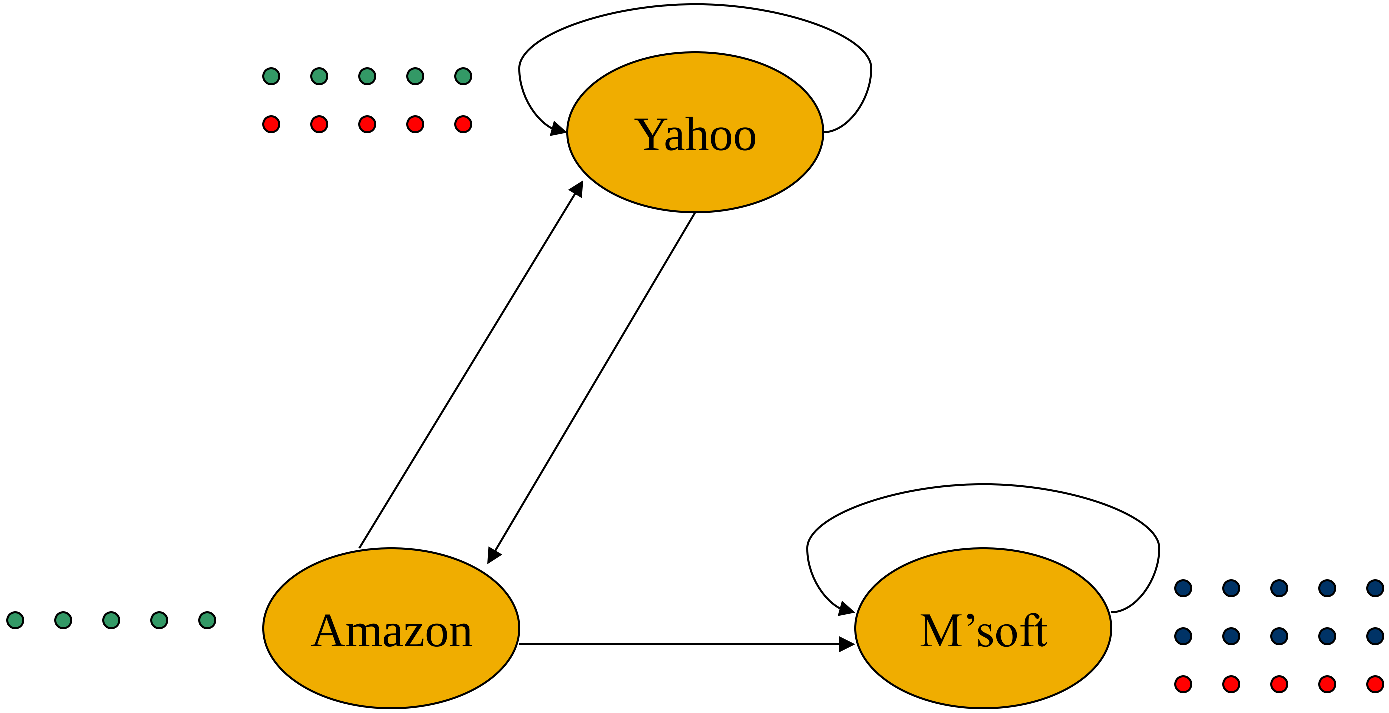
$$m = a/2 + m$$

$$\begin{array}{c} y \\ a \\ m \end{array} = \begin{array}{cccccc} 1 & 1 & 3/4 & 5/8 & & 0 \\ 1 & 1/2 & 1/2 & 3/8 & \cdots & 0 \\ 1 & 3/2 & 7/4 & 2 & & 3 \end{array}$$

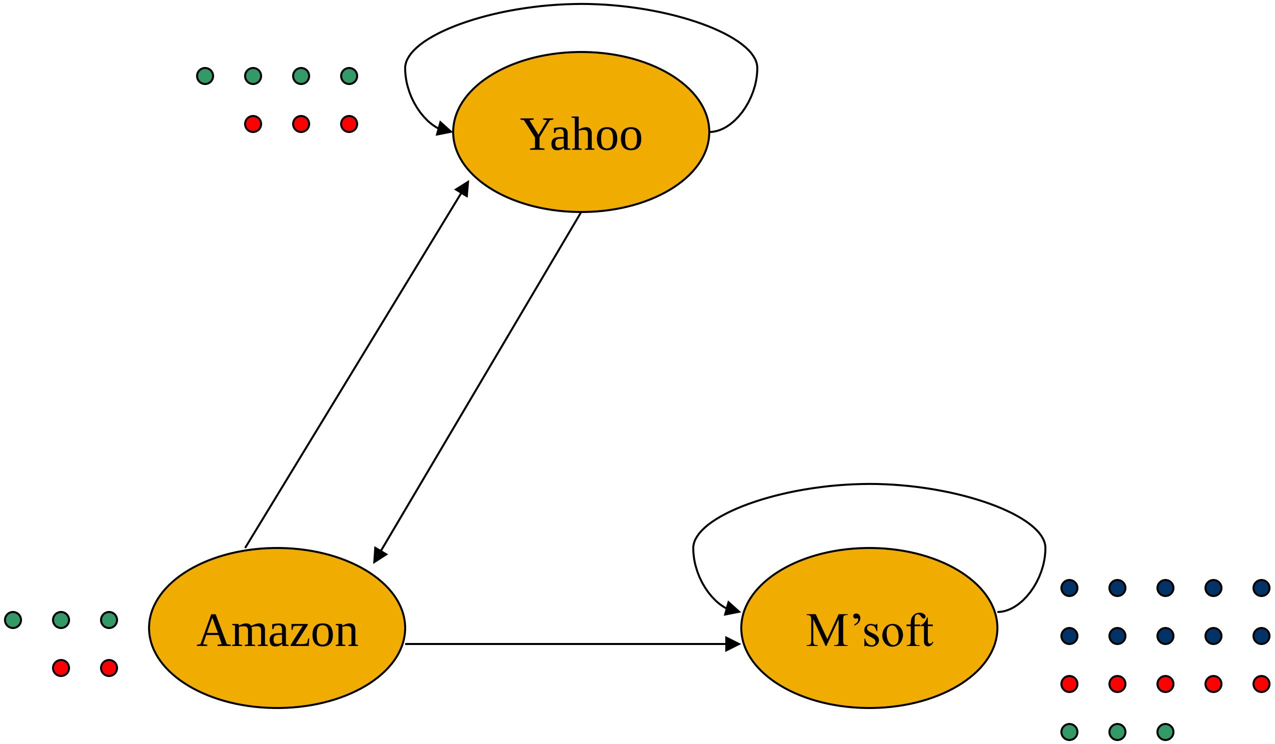
Microsoft Becomes a Spider Trap



Microsoft Becomes a Spider Trap



Microsoft Becomes a Spider Trap



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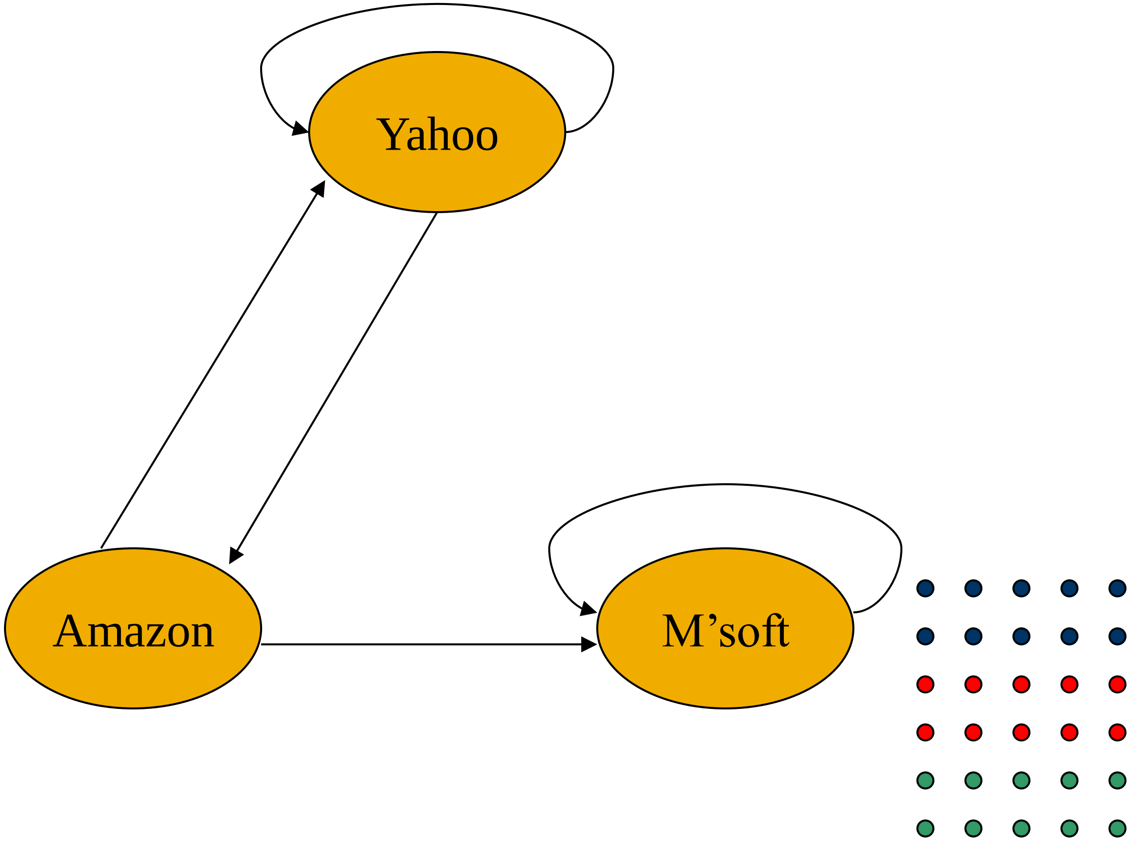


PageRank Solution to Traps, Etc.

- “Tax” each page a fixed percentage at each iteration.
- Add a fixed constant to all pages.
  - Optional but useful: add exactly enough to balance the loss (tax + PageRank of dead ends).
- Models a random walk with a fixed probability of leaving the system, and a fixed number of new walkers injected into the system at each step.
  - Divided equally among all pages.

Example: Microsoft is a Spider Trap; 20% Tax

- Equations $\mathbf{v} = 0.8(M\mathbf{v}) + \mathbf{0.2}$:

$$y = 0.8(y/2 + a/2) + 0.2$$

$$a = 0.8(y/2) + 0.2$$

$$m = 0.8(a/2 + m) + 0.2$$

y		1	1.00	0.84	0.776		7/11
a	=	1	0.60	0.60	0.536	. . .	5/11
m		1	1.40	1.56	1.688		21/11

Topic-Specific PageRank

Focusing on Specific Pages

Teleport Sets

Interpretation as a Random Walk

Topic-Specific Page Rank

- Goal: Evaluate Web pages not just according to their popularity, but also by how relevant they are to a particular topic, e.g. “sports” or “history.”
- Allows search queries to be answered based on interests of the user.
- Example: Search query [jaguar] wants different pages depending on whether you are interested in automobiles, nature, or sports.

Teleport Sets

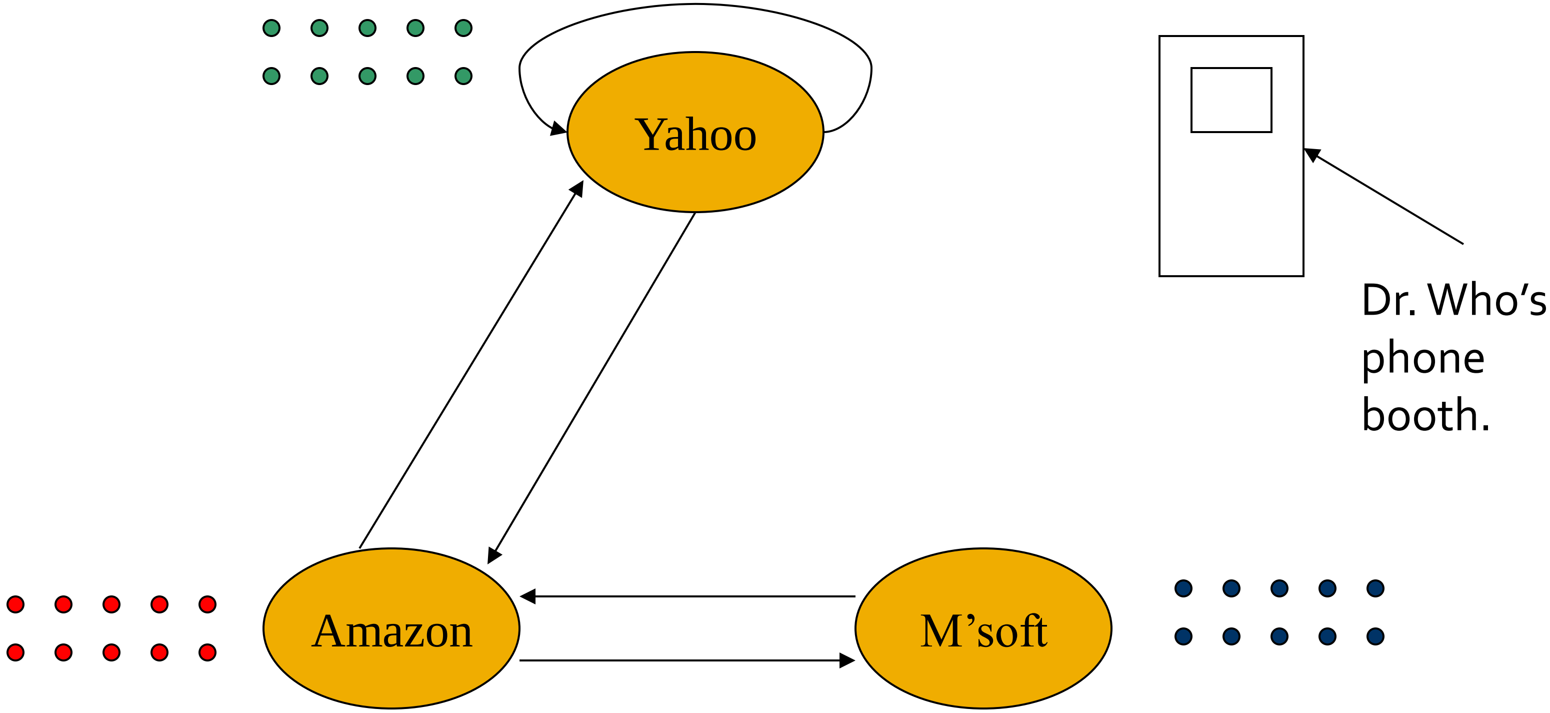
- Assume each walker has a small probability of "teleporting" at any tick.
- Teleport can go to:

1. Any page with equal probability.
   - As in the "taxation" scheme.
2. A set of "relevant" pages (*teleport set*).
   - For *topic-specific* PageRank.

Example: Topic = Software

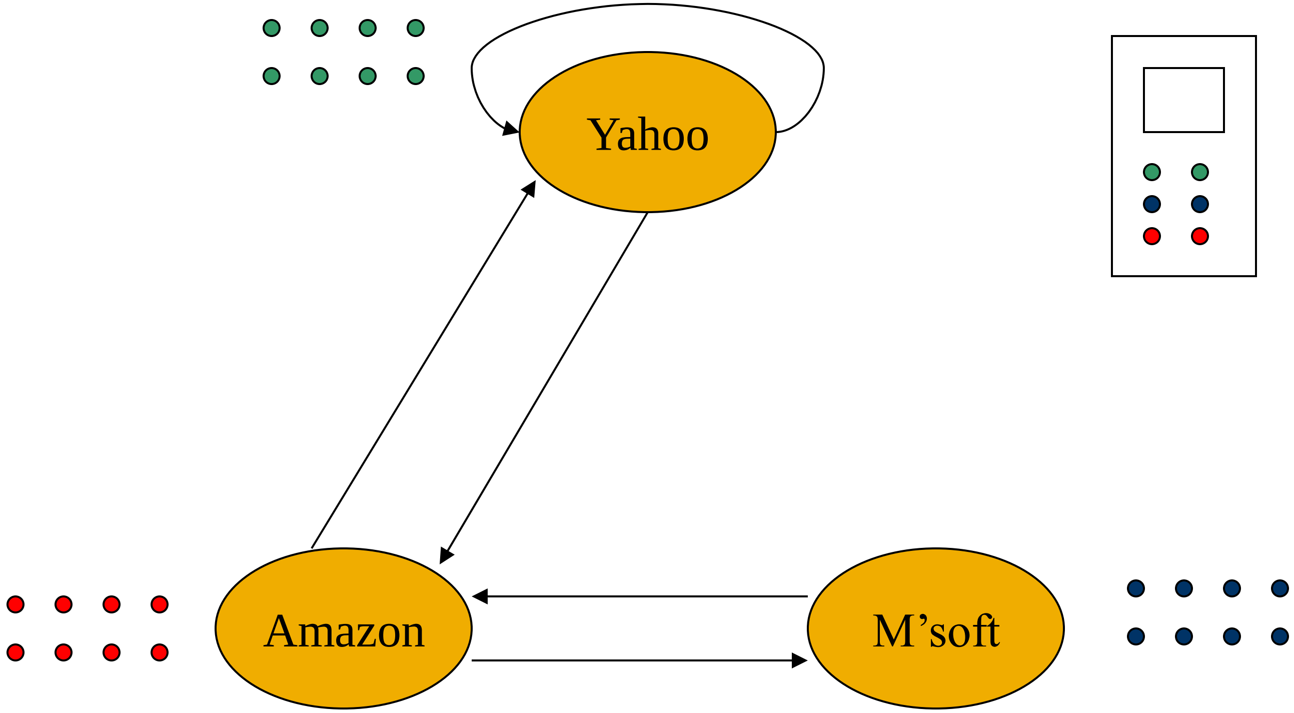
- Only Microsoft is in the teleport set.
- Assume 20% “tax.”
  - I.e., probability of a teleport is 20%.

Only Microsoft in Teleport Set



Dr. Who’s
phone
booth.

Only Microsoft in Teleport Set



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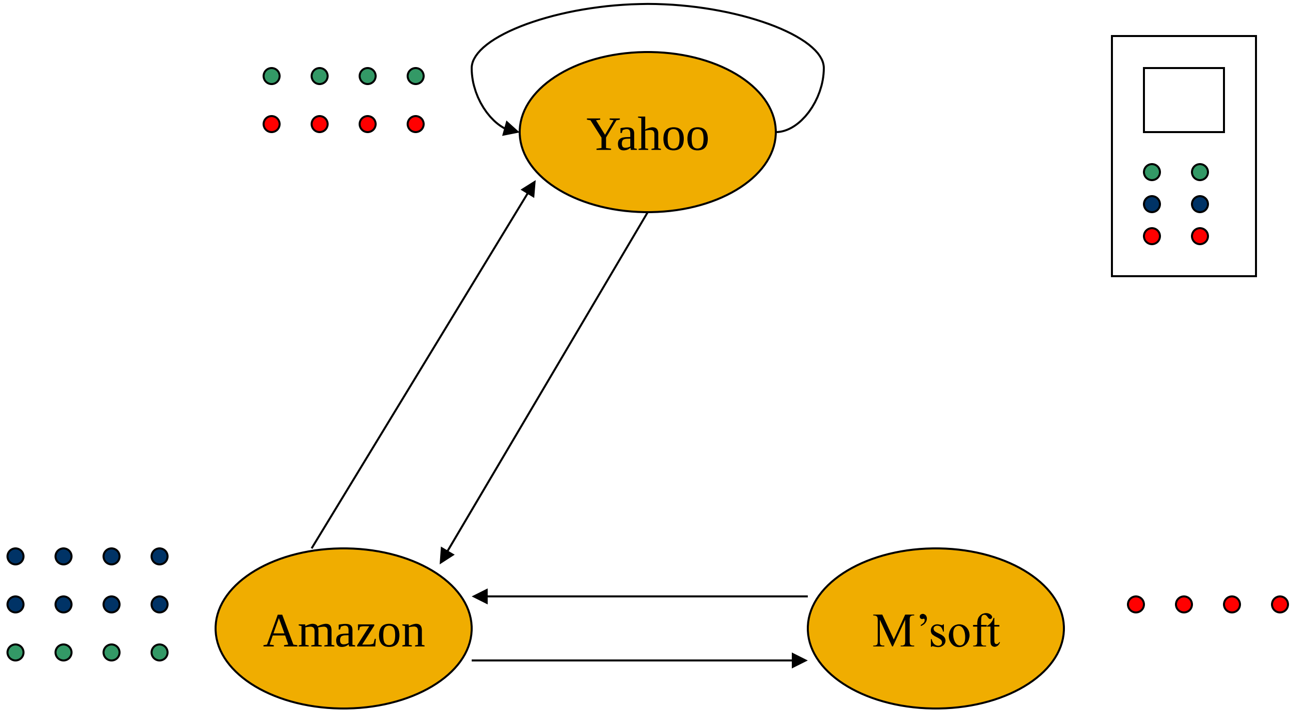

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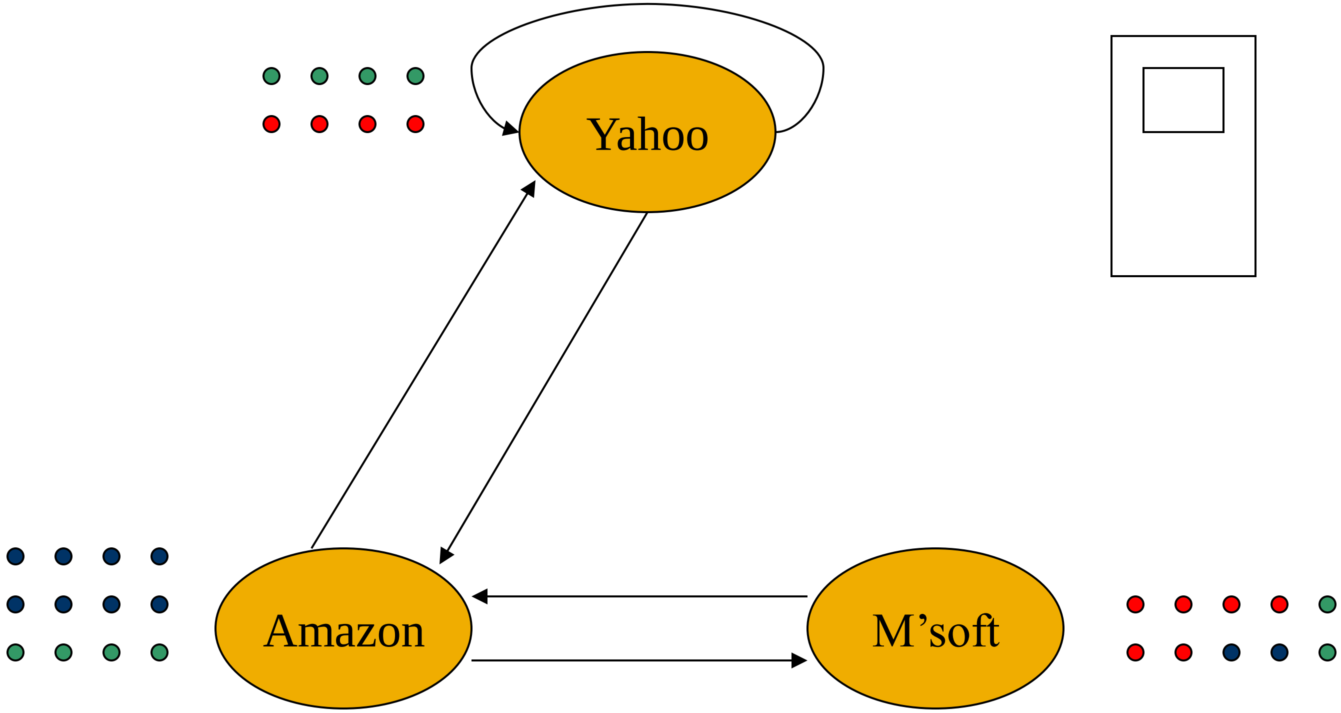

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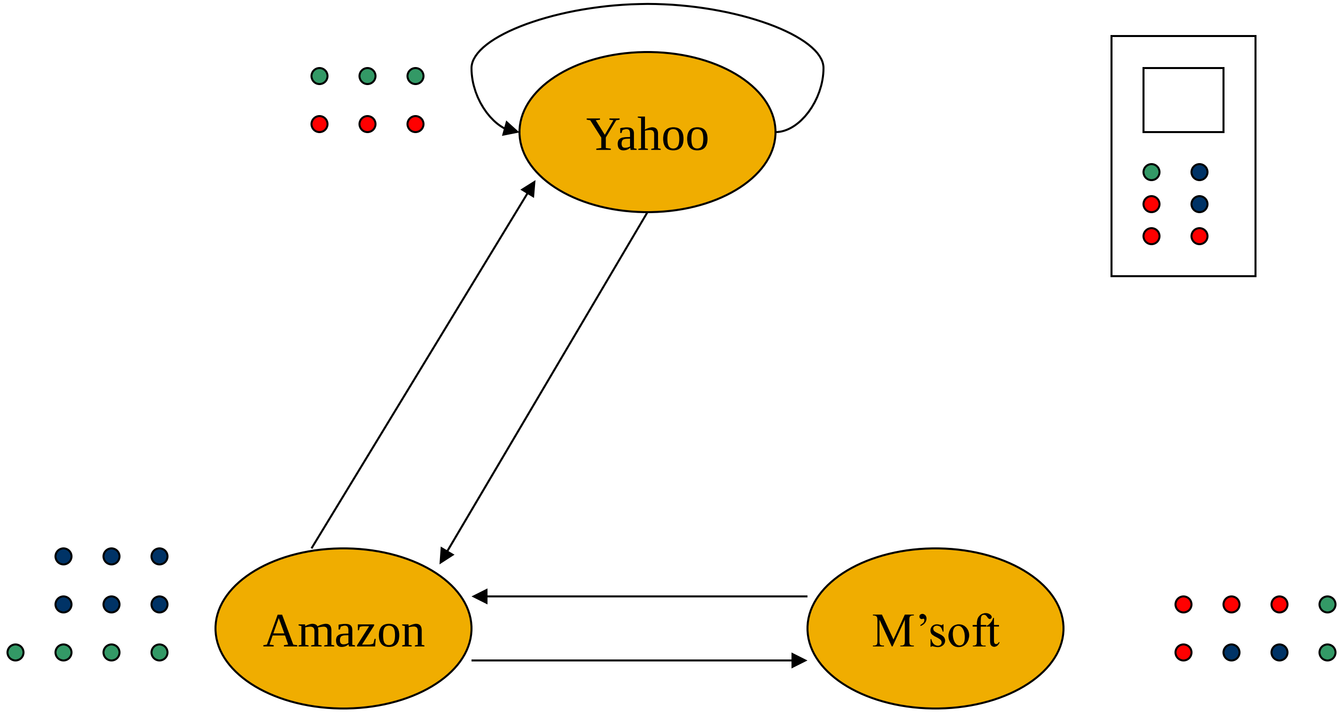

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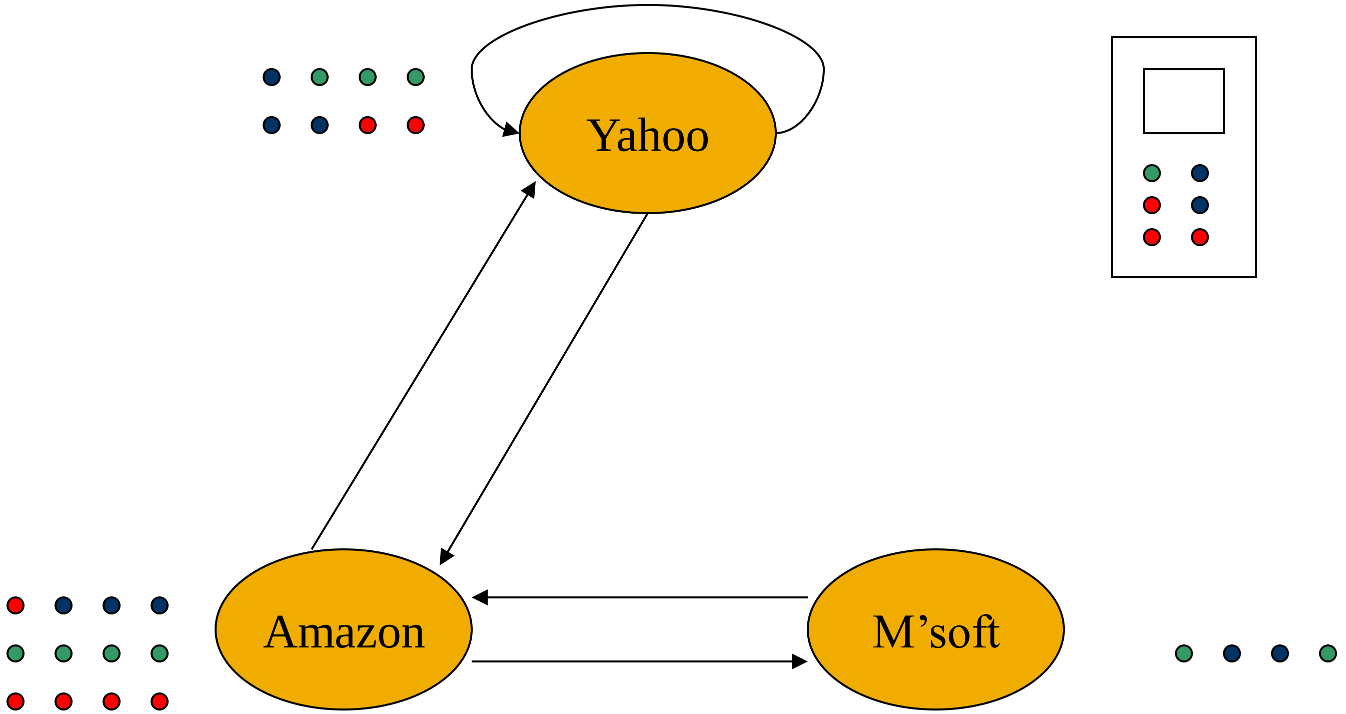

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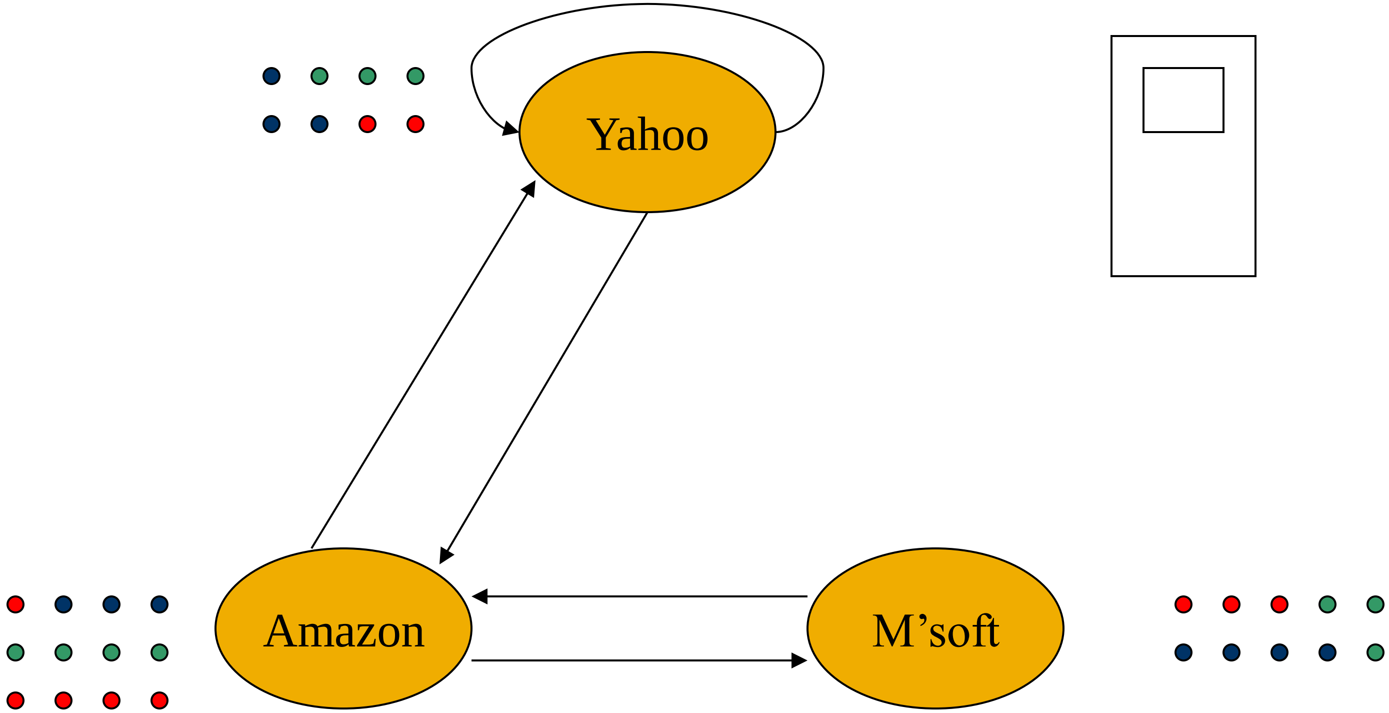

Picking the Teleport Set

1. Choose the pages belonging to the topic in Open Directory.
2. “Learn,” from a training set, the typical words in pages belonging to the topic; use pages heavy in those words as the teleport set.

Application: Link Spam

- Spam farmers create networks of millions of pages designed to focus PageRank on a few undeserving pages.
  - We’ll discuss this technology shortly.
- To minimize their influence, use a teleport set consisting of trusted pages only.
  - Example: home pages of universities.