BigCLAM: How to find F

- **Task:** Given a network $G(V, E)$, estimate F
  - Find F that maximizes the likelihood:

$$arg\ max_F \prod_{(u,v)\in E} p(u,v) \prod_{(u,v)\notin E} (1 - p(u,v))$$

    - where: $P(u,v) = 1 - \exp(-F_u \cdot F_v^T)$
    - Many times we take the logarithm of the likelihood, and call it log-likelihood: $l(F) = \log P(G|F)$

- **Goal:** Find F that maximizes $l(F)$:

$$l(F) = \sum_{(u,v)\in E} \log(1 - \exp(-F_u F_v^T)) - \sum_{(u,v)\notin E} F_u F_v^T$$

BigCLAM: V1.0

$$l(F_u) = \sum_{v \in \mathcal{N}(u)} \log(1 - \exp(-F_u F_v^T)) - \sum_{v \notin \mathcal{N}(u)} F_u F_v^T$$

- **Compute gradient of a single row F_u of F:**

$$\nabla l(F_u) = \sum_{v \in \mathcal{N}(u)} F_v \frac{\exp(-F_u F_v^T)}{1 - \exp(-F_u F_v^T)} - \sum_{v \notin \mathcal{N}(u)} F_v$$

- **Coordinate gradient ascent:**

$\mathcal{N}(u)$.. Set out outgoing neighbors

  - **Iterate over the rows of F:**
    - Compute gradient $\nabla l(F_u)$ of row u (while keeping others fixed)
    - Update the row F_u: $F_u \leftarrow F_u + \eta \nabla l(F_u)$
    - Project F_u back to a non-negative vector: If $F_{uC} < 0$: $F_{uC} = 0$
- **This is slow!** Computing $\nabla l(F_u)$ takes linear time!

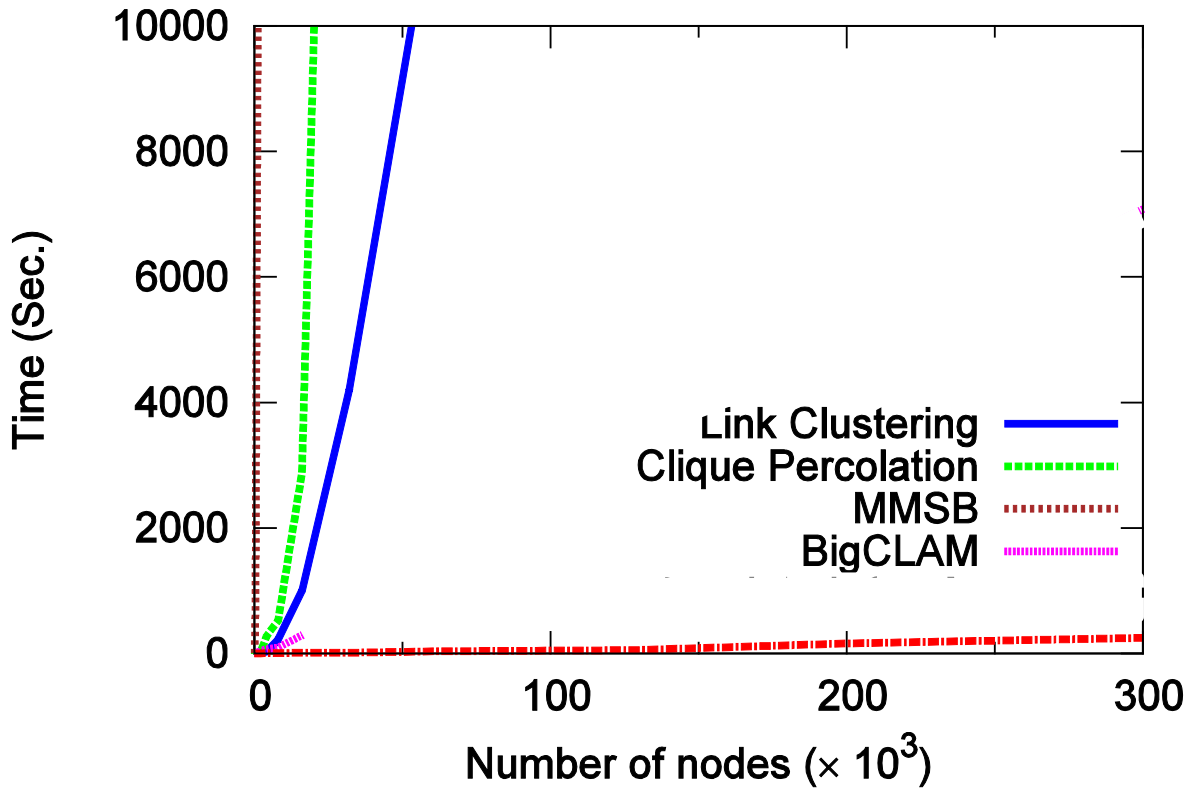
BigCLAM: V2.0

- **However, we notice:**

$$\sum_{v \notin \mathcal{N}(u)} F_v = (\sum_{v} F_v - F_u - \sum_{v \in \mathcal{N}(u)} F_v)$$

  - We cache $\sum_v \boldsymbol{F_v}$
  - So, computing $\sum_{v \notin \mathcal{N}(u)} \boldsymbol{F_v}$ now takes **linear time** in the degree $|\mathcal{N}(\boldsymbol{u})|$ of $\boldsymbol{u}$
    - In networks degree of a node is much smaller to the total number of nodes in the network, so this is a significant speedup!

BigClam: Scalability



- **BigCLAM takes 5 minutes for 300k node nets**
  - **Other methods take 10 days**
- **Can process networks with 100M edges!**

More details at…

- Overlapping Community Detection at Scale: A Nonnegative Matrix Factorization Approach by J. Yang, J. Leskovec. *ACM International Conference on Web Search and Data Mining (WSDM)*, 2013.
- Detecting Cohesive and 2-mode Communities in Directed and Undirected Networks by J. Yang, J. McAuley, J. Leskovec. *ACM International Conference on Web Search and Data Mining (WSDM)*, 2014.
- Community Detection in Networks with Node Attributes by J. Yang, J. McAuley, J. Leskovec. *IEEE International Conference On Data Mining (ICDM)*, 2013.