Computational Advertising

Greedy Algorithms
Competitive Algorithms
Picking the Best Ad
The Balance Algorithm

Jeffrey D. Ullman
Stanford University

Slides primarily taken from Anand Rajaraman

Administrivia

- Final Exam
- Last Lectures

Final Exam

- To be held in Dinkelspiel Auditorium, 8:30-11:30AM, Wednesday March 16.
  - Do not go to Nvidia Aud.
- Exam will cover entire course.
  - No programming.
  - No proofs, but you may need to prove something to yourself to be sure of getting the right answer.
  - Open book/notes.
  - Internet allowed, but not communication with others.

Words of Advice Re Final

- Although you can access any on-line source of information, e.g., the text or Wikipedia, you will find that there are a lot of questions to be answered in very little time.
- If you have to read a section of the book for every question, you will waste too much time.

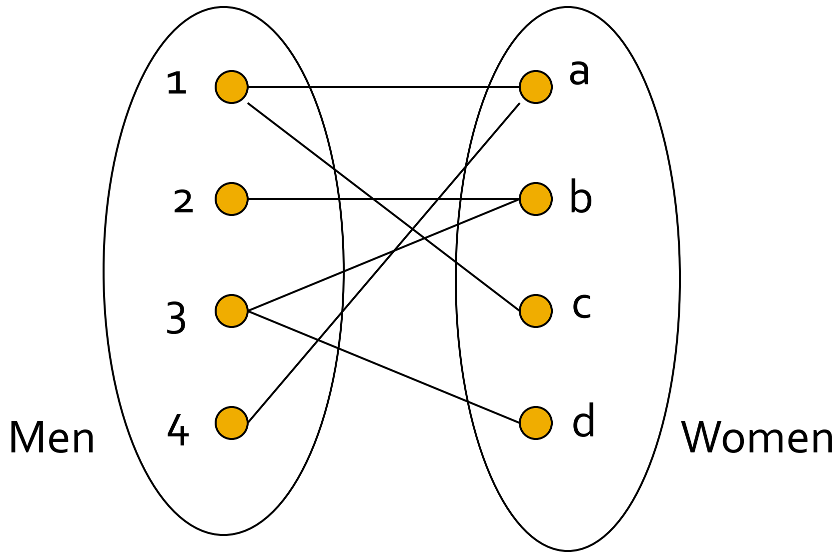
Last Lectures

- March 3: Jeff finishes up Computational Advertising + comparison between MapReduce-like systems and bulk-synchronous systems.
- March 8: Hima and Tim talk about submodular optimization.
- March 10: Caroline on multi-arm bandits + Jeff on the design of good MapReduce algorithms.

Online Algorithms

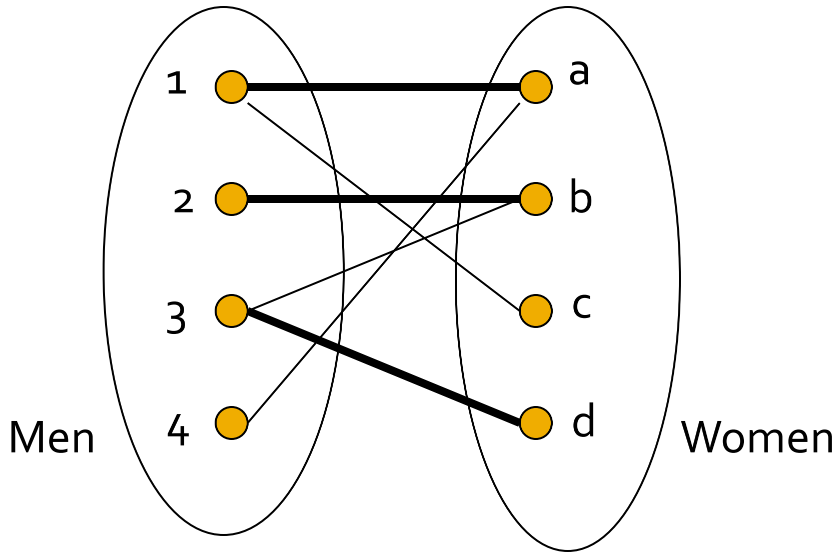
- Classic model of (*offline*) algorithms:
  - You get to see the entire input, then compute some function of it.
- *Online algorithm*:
  - You get to see the input one piece at a time, and need to make irrevocable decisions along the way.
  - Similar to data stream models.

Example: Bipartite Matching



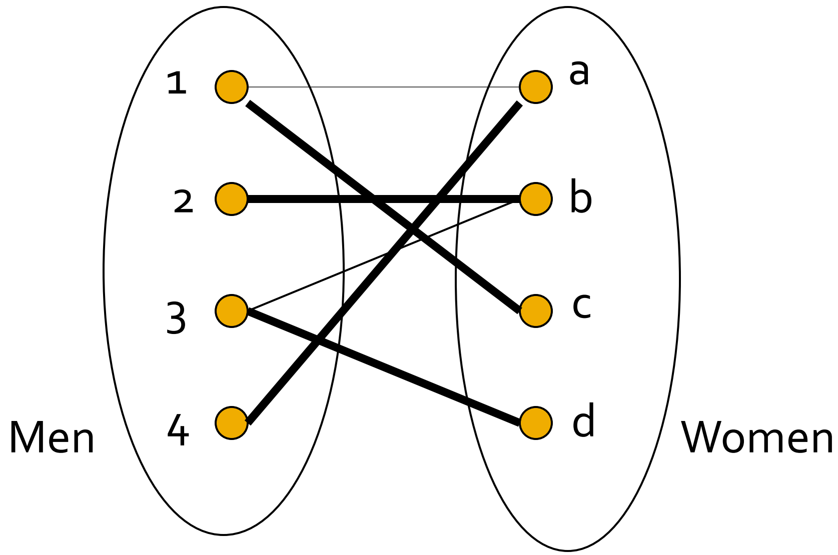
- Two sets of nodes.
- Some edges between them.
- Maximize the number of nodes paired 1-1 by edges.

Bipartite Matching – (2)



M = {(1,a),(2,b),(3,d)} is a *matching* of cardinality |M| = 3.

Bipartite Matching – (3)



M = {(1,c),(2,b),(3,d),(4,a)} is a *perfect matching* (all nodes matched).

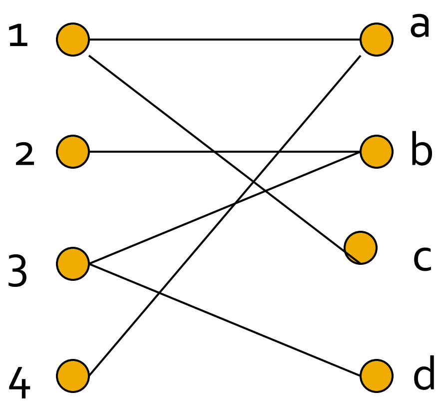
Matching Algorithm

- Problem: Find a maximum-cardinality matching for a given bipartite graph.
  - A perfect one if it exists.
- There is a polynomial-time offline algorithm (Hopcroft and Karp 1973).
- But what if we don't have the entire graph initially?

Online Matching

- Initially, we are given the set of men.
- In each round, one woman's set of choices is revealed.
- At that time, we have to decide either to:
  - Pair the woman with a man.
  - Don't pair the woman with any man.
- Example applications: assigning tasks to servers or Web requests to threads.

Online Matching – (2)



(1,a)

(2,b)

(3,d)

Greedy Algorithm

- Pair the new woman with any eligible man.
  - If there is none, don’t pair the woman.
- How good is the algorithm?

Competitive Ratio

- For input I, suppose greedy produces matching M_{greedy} while an optimal matching is M_{opt}.

Competitive ratio =

$$\min_{\text{all possible inputs } I} (|M_{greedy}|/|M_{opt}|).$$

Greedy Has Competitive Ratio 1/2

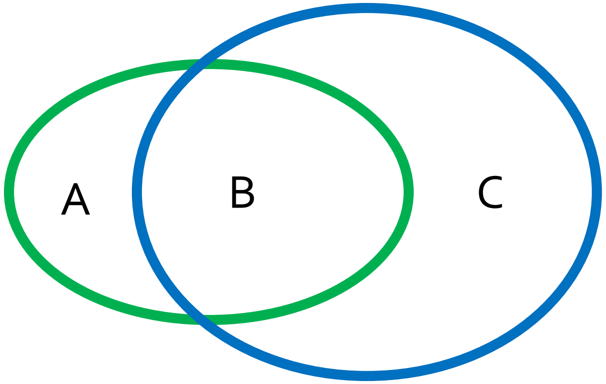
- Let O be the optimal matching, and G the matches produced by a run of the greedy algorithm.
- Consider the sets of women:

  A: Matched in G, not in O.

  B: Matched in both.

  C: Matched in O, not in G.

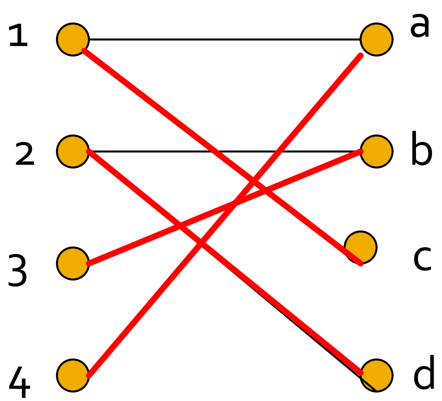
Proof of Competitive Ratio 1/2



- During the greedy matching, every woman in C found her optimal match taken by another woman.
- Thus, $|A| + |B| \geq |C|$.
- Surely, $|A| + |B| \geq |B|$.
- Thus, $|G| = |A| + |B| \geq (|B| + |C|)/2 = |O|/2$.

If you're greater than each of two things, you are greater than their average.

Worst-Case Scenario



(1,a)

(2,b)

|Greedy| = 2;

|Opt| = 4.

History of Web Advertising

- Banner ads (1995-2001).
  - Initial form of web advertising.
  - Popular websites charged X$ for every 1000 “impressions” of ad.
    - Called “CPM” rate.
    - Modeled on TV, magazine ads.
  - Untargeted to demographically targeted.
  - Low clickthrough rates.
    - low ROI for advertisers.

Performance-Based Advertising

- Introduced by Overture around 2000.
  - Advertisers “bid” on search keywords.
  - When someone searches for that keyword, the highest bidder’s ad is shown.
  - Advertiser is charged only if the ad is clicked on.
- Similar model adopted by Google with some changes around 2002.
  - Called “Adwords.”

Web 2.0

- Performance-based advertising works!
  - Multi-billion-dollar industry.
- Interesting problems:
  - What ads to show for a search?
  - If I'm an advertiser, which search terms should I bid on and how much should I bid?

Adwords Problem

- A stream of queries arrives at the search engine
  - q1, q2,…
- Several advertisers bid on each query.
- When query q_i arrives, search engine must pick a subset of advertisers whose ads are shown.
- Goal: maximize search engine's revenues.
- Clearly we need an online algorithm!
- Simplest online algorithm is Greedy.

Complications – (1)

- Each ad has a different likelihood of being clicked.
- Example:
  - Advertiser 1 bids $2, click probability = 0.1.
  - Advertiser 2 bids $1, click probability = 0.5.
    - Click-through rate measured by historical performance.
- Simple solution:
  - Instead of raw bids, use the "expected revenue per click."

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
A	$1.00	1%	1 cent
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
A	$1.00	1%	1 cent

Complications – (2)

- Each advertiser has a limited budget
  - Search engine guarantees that the advertiser will not be charged more than their daily budget.

Simplified Model (For Now)

- Assume all bids are 0 or 1.
- Each advertiser has the same budget B.
- One advertiser is chosen per query.
- Let's try the greedy algorithm:
  - Arbitrarily pick an eligible advertiser for each keyword.

Bad Scenario For Greedy

- Two advertisers A and B.
- A bids on query x, B bids on x and y.
- Both have budgets of $4.
- Query stream: x x x x y y y y.
- Possible greedy choice: B B B B _ _ _ _.
- Optimal: A A A A B B B B.
- Competitive ratio = 1/2.
  - This is actually the worst case.

Balance Algorithm [MSVV]

- [Mehta, Saberi, Vazirani, and Vazirani].
- For each query, pick the advertiser with the largest unspent budget who bid on this query.
  - Break ties arbitrarily.

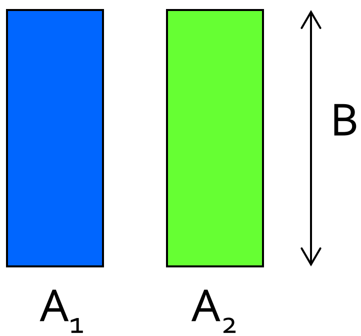
Example: Balance

- Two advertisers A and B.
- A bids on query x, B bids on x and y.
- Both have budgets of $4.
- Query stream: x x x x y y y y.
- Balance choice: B A B A B B _ _.
- Optimal: A A A A B B B B.
- Competitive ratio = 3/4.

Analyzing Balance

- Consider simple case: two advertisers, A_1 and A_2, each with budget B > 1, an even number.
- We'll consider the case where the optimal solution exhausts both advertisers' budgets.
  - I.e., optimal revenue to search engine = 2B.
- Balance must exhaust at least one advertiser's budget.
  - If not, we can allocate more queries.
  - Assume Balance exhausts A_2's budget.

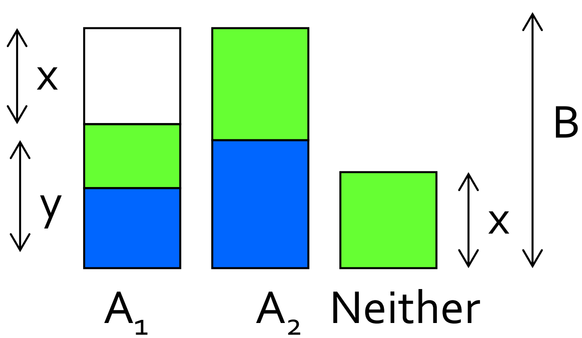
Analyzing Balance



Queries allocated to A_1 in optimal solution

Queries allocated to A_2 in optimal solution

Opt revenue = 2B
Balance revenue = 2B-x = B+y

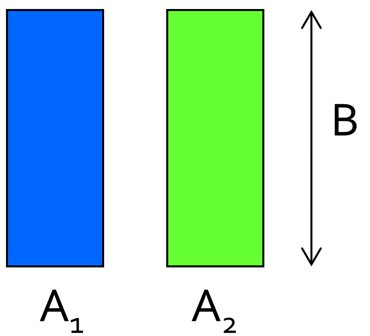


Balance allocation

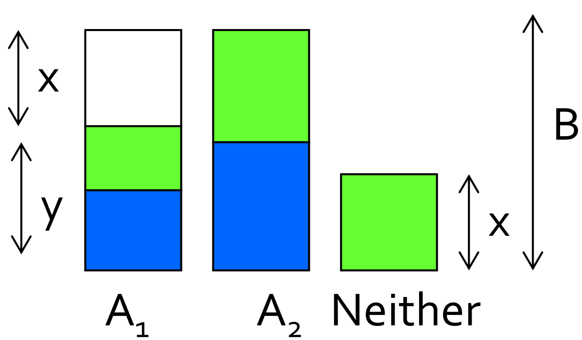
Note: only green queries can be assigned to neither.
A blue query could have been assigned to A_1.

We claim $y \geq x$ (next slide).
Balance revenue is minimum for x=y=B/2.
Minimum Balance revenue = 3B/2.
Competitive Ratio = 3/4.

Analyzing Balance: Two Cases

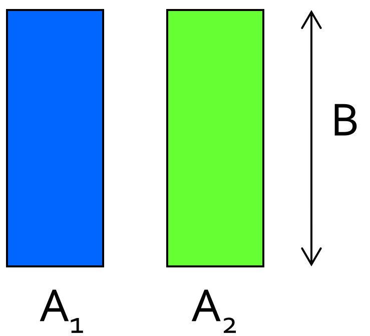


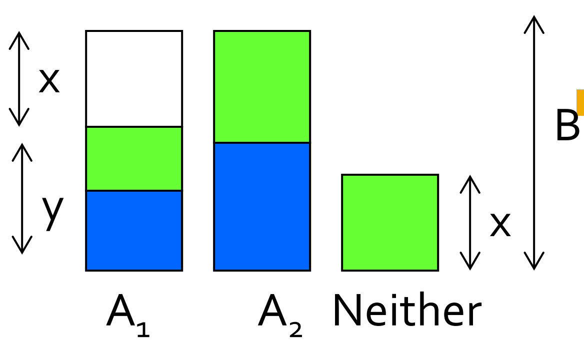
- Case 1: At least half the blue queries are assigned to A_1 by Balance.
  - Then $y \geq B/2$, since the blues alone are $\geq B/2$.
- Case 2: Fewer than half the blue queries are assigned to A_1 by Balance.
  - Let q be the last blue query assigned by Balance to A_2.



Balance allocation

Analyzing Balance – (3)





Balance allocation

- Since A_1 obviously bid on q, at that time, the budget of A_2 must have been at least as great as that of A_1.
- Since more than half the blue queries are assigned to A_2, at the time of q, A_2's remaining budget was at most B/2.
- Therefore so was A_1's, which implies $x \leq B/2$, and therefore $y \geq B/2$ and $y \geq x$.
- Thus Balance uses $\geq 3B/2$.

General Result

- In the general case, competitive ratio of Balance is 1–1/e = approx. 0.63.
- Interestingly, no online algorithm has a better competitive ratio.
- Won’t go through the details here, but let’s see the worst case that gives this ratio.

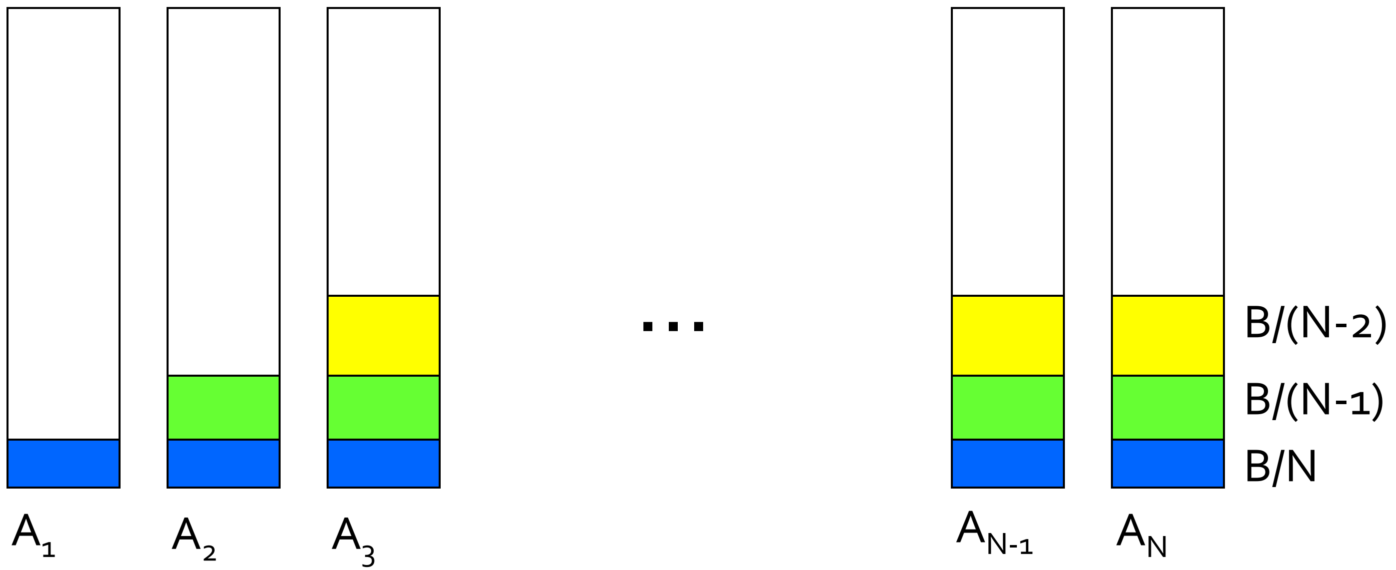
Worst Case for Balance

- N advertisers, each with budget B >> N >> 1.
- N*B queries appear in N rounds.
- Each round consists of a single query repeated B times.
- Round 1 queries: bidders $A_1, A_2, \ldots, A_N$.
- Round 2 queries: bidders $A_2, A_3, \ldots, A_N, \ldots$
- Round i queries: bidders $A_i, \ldots, A_N, \ldots$
- Round N queries: only A_N bids.
- Optimum allocation: round i queries to A_i.
  - Optimum revenue N*B.

Pattern to Remember

- After i rounds, the first i advertisers have dropped out of the bidding.
  - Why? All subsequent queries are ones they do not bid on.
- Thus, they never get any more queries, even though they have budget left.

Balance Allocation



After k rounds, sum of allocations to each of $A_k, \ldots, A_N$ is
$S_k = S_{k+1} = \ldots = S_N = \sum_{1 \leq i \leq k} B/(N-i+1)$.

If we find the smallest k such that $S_k \geq B$, then after k rounds we cannot allocate any queries to any advertiser.

BALANCE Analysis

B/1 B/2 B/3 … B/(N-k+1) … B/(N-1) B/N

S_1

S_2

Each width represents the amount of budget spent after k rounds.

$S_k = B$

Or in terms of fractions (dividing by B):

1/1 1/2 1/3 … 1/(N-k+1) … 1/(N-1) 1/N

S_1

S_2

$S_k = 1$

BALANCE analysis

- Fact: $H_n = \sum_{1 \le i \le n} 1/i \sim= \log_e(n)$ for large n.
  - Result due to Euler.

$1/1 \quad 1/2 \quad 1/3 \quad \ldots \quad 1/(N-k+1) \quad \ldots \quad 1/(N-1) \quad 1/N$

log(N)

log(N) - 1 $\qquad S_k = 1$

$S_k = 1$ implies $H_{N-k} = \log(N) - 1 = \log(N/e)$.

$N-k = N/e$ (Why? $\log(N-k) = H_{N-k} = \log(N/e)$).

$k = N(1-1/e) \sim= 0.63N$.

Balance analysis

- So after the first N(1-1/e) rounds, we cannot allocate a query to any advertiser.
- Revenue = BN(1-1/e).
- Competitive ratio = 1-1/e.

General Version of Problem

- Arbitrary bids, budgets.
- Balance can be terrible.
- Example: Consider two advertisers A_1 and A_2, each bidding on query q.
  - A_1: $x_1 = 1$, $b_1 = 110$.
  - A_2: $x_2 = 10$, $b_2 = 100$.
- First 10 occurrences of q all go to A_1, and A_1 then gets 10 q's for every one that A_2 gets.
  - What if there are only 10 occurrences of q?
    - Opt yields \$100; Balance yields \$10.

Generalized Balance

- Arbitrary bids; consider query q, bidder i.
  - Bid = x_i.
  - Budget = b_i.
  - Amount spent so far = m_i.
  - Fraction of budget remaining $f_i = 1-m_i/b_i$.
- Define $\psi_i(q) = x_i(1-e^{-f_i})$.
- Allocate query q to bidder i with largest value of $\psi_i(q)$.
- Same competitive ratio (1-1/e).