IE 529 Fall 2016 Final Project

Jifu Zhao Date: 10/31/2016

I. PCA

Algorithm 1 PCA in Feature Spaces

procedure PCA(X)
  given input: $X_{n\times m} \leftarrow \left[\mathbf{x}_1; \mathbf{x}_2; \cdots; \mathbf{x}_n\right]^T$
  de-mean (or standardize): $x_{ij} \leftarrow x_{ij} - \bar{x}_j \ or \ x_{ij} \leftarrow \frac{x_{ij} - \bar{x}_j}{s_j}$
  calculate covariance matrix: $Cov \leftarrow \frac{1}{n} X^T X$
  singular value decomposition (SVD): $[U, S, V] \leftarrow svd(Cov)$
  choose the first k eigenvectors: $E_{m\times k} \leftarrow \left[\mathbf{u}_1; \mathbf{u}_2; \cdots; \mathbf{u}_k\right]$
  project the test data $\mathbf{x}$: $\mathbf{p} \leftarrow E^T \mathbf{x}$
finish

II. Kernel PCA

Algorithm 2 Kernel PCA

procedure K-PCA(X)
  given input: $X_{n\times m} \leftarrow \left[\mathbf{x}_1; \mathbf{x}_2; \cdots; \mathbf{x}_n\right]^T$
  calculate kernel matrix $K_{n\times n}$: $k_{ij} \leftarrow k(\mathbf{x}_i, \mathbf{x}_j)$
  centralize $K : K' \leftarrow K - \mathbb{I}_n K/n - K\mathbb{I}_n/n + \mathbb{I}_n K \mathbb{I}_n/n^2$
  calculate eigenvector $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \cdots, \boldsymbol{\alpha}_d$ according to: $n\lambda\boldsymbol{\alpha} = K'\boldsymbol{\alpha}$
  normalize eigenvector according to: $n\lambda_i \boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_i = 1$
  project the test data $\mathbf{x}$: $p_i(\mathbf{x}) \leftarrow \sum_{j=1}^{n} \alpha_{ij} k(\mathbf{x}, \mathbf{x}_j)$
finish

Note: $\mathbb{I}_n$ stands for $n \times n$ matrix with all values equal to 1.