

## **I. Lloyd's (K-means) Algorithm**

1. In this part, the polynomial regression is conducted on the given dataset. The derivation is relatively simple, here we only give one simple version.

## **II. Greedy K-centers Algorithm**

## **III. Single-Swap Algorithm**

## **IV. Spectral Clustering Algorithm**

## **V. Expectation Maximization (EM) Algorithm**

```

1
2 def polynomial_regression(x, y, degree, x_range):
3     """ function to perform polynomial regression and compute least suares error
4
5     Parameters:
6     -----
7     x: input array, should have one dimension
8     y: fit goal, shoule have one dimension
9     degree: polynomial degress
10
11     return:
12     -----
13     w: weight matrix, the first value is intercept
14     prediction: predicted values
15     error: least squares error
16     """
17
18     n = len(x)
19     # map x into multiple columns
20     X = np.zeros((n, degree + 1))
21     X[:, 0] = 1
22     for i in range(1, degree + 1):
23         X[:, i] = x ** i
24
25     # reshape y into n by 1 format
26     Y = np.array([y]).T
27
28     # compute w, prediction and error
29     w = np.dot(np.dot(np.linalg.inv(np.dot(X.T, X)), X.T), Y)
30     error = np.sum((np.dot(X, w) - Y) ** 2)
31
32     # compute the prediction
33     new_X = np.zeros((len(x_range), degree + 1))
34     new_X[:, 0] = 1
35     for i in range(1, degree + 1):
36         new_X[:, i] = x_range ** i
37     prediction = np.dot(new_X, w)
38
39     return w, error, prediction

```

Listing 1: Python example