Week11

IN0003

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TUM

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What about the last week?

- The first stage to Ocaml is gone.
- Now we would introduce a proving tool **Big Step**
- We also use the Ocaml syntax for Big Step. So recrusion.
- Do you have the lectures? They are helpful for this topic.

Homework

- Some of you receive **0 points** with ocaml Homework because compilation, termination.
- Try to write email following the guide in Piazza, a pined post
- https://piazza.com/class/jnadp6tfu067ov?cid=312
- You have chance until 20.01.2019.

Statement to this Slide

- This slide is aimed at the presentation in my tutorial.
- Can be some style, which not 100% same of the orginal soltion and the lecture slide.
- Because the big step is too too long. It can't fit the beamer or the white board or the exam sheet.
- **If there are some mismatch with the orginal solution, please following the style in orginal solution**
- And tell me where is wrong.

Big step

- Lecture part **verification**.
- A tool to test the program, which value it has, to see it the reliability.
- A tool to test some program, to see if they can have a same result.
- Unless specified otherwise, all rules used in a big-step proof tree must be annotated and all axioms ($v \implies v$) must be written down.

Assignment 11.1.1 Big Steps

We define these functions:

```
let rec f = fun l ->
        match l with [] -> 1 | x::xs -> x + g xs
and g = fun l ->
        match l with [] -> 0 | x::xs -> x * f xs
```

Consider the following expressions. Find the values they evaluate to and construct a big-step proof for that claim.

1. `let f = fun a -> (a+1,a-1)::[] in f 7`

Assignment 11.1.1 Big Steps

1 [(7+1,7-1)] as warm up

Big step tree:

$$\pi_0 = \mathrm{LI}\ \frac{}{\texttt{[(7+1,7-1)]} \Rightarrow}$$

Assignment 11.1.1 Big Steps

1 [(7+1,7-1)] as warm up

Big step tree:

$$\pi_0 = \mathsf{LI}\ \dfrac{\mathsf{TU}\ \dfrac{}{\texttt{(7+1,7-1)} \Rightarrow}}{\texttt{[(7+1,7-1)]} \Rightarrow}$$

Assignment 11.1.1 Big Steps

1 [(7+1,7-1)] as warm up

Big step tree:

$$\pi_0 = \mathrm{LI}\ \dfrac{\mathrm{TU}\ \dfrac{\mathrm{OP}\ \dfrac{7 \Rightarrow 7 \quad 1 \Rightarrow 1 \quad 7+1 \Rightarrow}{7+1 \Rightarrow} \qquad \mathrm{OP}}{(7+1,7-1) \Rightarrow}}{[(7+1,7-1)] \Rightarrow}$$

Assignment 11.1.1 Big Steps

1 [(7+1,7-1)] as warm up

Big step tree:

$$\pi_0 = \mathrm{LI}\ \dfrac{\mathrm{TU}\ \dfrac{\mathrm{OP}\ \dfrac{7 \Rightarrow 7 \quad 1 \Rightarrow 1 \quad 7+1 \Rightarrow 8}{7+1 \Rightarrow 8} \qquad \mathrm{OP}}{(7+1,7-1) \Rightarrow}}{[(7+1,7-1)] \Rightarrow}$$

Assignment 11.1.1 Big Steps

1 [(7+1,7-1)] as warm up

Big step tree:

$$\pi_0 = \text{LI}\ \dfrac{\text{TU}\ \dfrac{\text{OP}\ \dfrac{7 \Rightarrow 7 \quad 1 \Rightarrow 1 \quad 7+1 \Rightarrow 8}{7+1 \Rightarrow 8} \quad \text{OP}\ \dfrac{7 \Rightarrow 7 \quad 1 \Rightarrow 1 \quad 7-1 \Rightarrow 6}{7-1 \Rightarrow 6}}{(7+1,7-1) \Rightarrow}}{[(7+1,7-1)] \Rightarrow}$$

Assignment 11.1.1 Big Steps

1. [(7+1,7-1)] as warm up
2. result will be applied later

Big step tree:

$$\pi_0 = \dfrac{\text{LI}\ \dfrac{\text{TU}\ \dfrac{\text{OP}\ \dfrac{7 \Rightarrow 7 \quad 1 \Rightarrow 1 \quad 7+1 \Rightarrow 8}{\texttt{7+1} \Rightarrow \texttt{8}} \quad \text{OP}\ \dfrac{7 \Rightarrow 7 \quad 1 \Rightarrow 1 \quad 7-1 \Rightarrow 6}{\texttt{7-1} \Rightarrow \texttt{6}}}{\texttt{(7+1,7-1)} \Rightarrow \texttt{(8,6)}}}{\texttt{[(7+1,7-1)]} \Rightarrow}}$$

Assignment 11.1.1 Big Steps

1. [(7+1,7-1)] as warm up
2. result will be applied later

Big step tree:

$$\pi_0 = \text{LI}\ \frac{\text{TU}\ \dfrac{\text{OP}\ \dfrac{7 \Rightarrow 7 \quad 1 \Rightarrow 1 \quad 7+1 \Rightarrow 8}{\texttt{7+1} \Rightarrow \texttt{8}} \quad \text{OP}\ \dfrac{7 \Rightarrow 7 \quad 1 \Rightarrow 1 \quad 7-1 \Rightarrow 6}{\texttt{7-1} \Rightarrow \texttt{6}}}{\texttt{(7+1,7-1)} \Rightarrow \texttt{(8,6)}}}{\texttt{[(7+1,7-1)]} \Rightarrow \texttt{[(8,6)]}}$$

Assignment 11.1.1 Big Steps

```
let f = fun a -> (a+1,a-1)::[] in f 7
```

$\pi_1 = \text{fun a} \rightarrow [(a+1, a-1)] \Longrightarrow \text{fun a} \rightarrow [(a+1, a-1)]$

(A very useful helper)

$$\text{LD}\ \frac{\pi_1 \quad (fun\ a{-}{>}[(a+1, a-1)])\ 7 \Rightarrow}{let\ f = fun\ a{-}{>}[(a+1, a-1)]in\ f\ 7 \Rightarrow}$$

Assignment 11.1.1 Big Steps

```
let f = fun a -> (a+1,a-1)::[] in f 7
```

$\pi_1 = \text{fun a} \to [(a+1, a-1)] \Longrightarrow \text{fun a} \to [(a+1, a-1)]$

(A very useful helper)

$$\text{LD}\ \frac{\pi_1 \qquad \text{APP'}\ \dfrac{\pi_1 \quad 8 \Rightarrow 8\ ?}{(\mathit{fun}\ a{-}{>}\ [(a+1, a-1)])\ 7 \Rightarrow}}{\mathit{let}\ f = \mathit{fun}\ a{-}{>}\ [(a+1, a-1)]\ \mathit{in}\ f\ 7 \Rightarrow}$$

Assignment 11.1.1 Big Steps

```
let f = fun a -> (a+1,a-1)::[] in f 7
```

$\pi_1 = \text{fun a} \to [(a+1, a-1)] \Longrightarrow \text{fun a} \to [(a+1, a-1)]$

(A very useful helper)

$$\text{LD}\ \frac{\pi_1 \qquad \text{APP'}\ \dfrac{\pi_1 \quad 8 \Rightarrow 8 \quad \textit{we had this}}{(\textit{fun } a- > [(a+1, a-1)])\ 7 \Rightarrow}}{\textit{let } f = \textit{fun } a- > [(a+1, a-1)] \textit{in } f\ 7 \Rightarrow}$$

Assignment 11.1.1 Big Steps

```
let f = fun a -> (a+1,a-1)::[] in f 7
```

$\pi_1 = \text{fun a} \to [(a+1, a-1)] \Longrightarrow \text{fun a} \to [(a+1, a-1)]$

(A very useful helper)

$$\text{LD}\ \frac{\pi_1 \qquad \text{APP'}\ \dfrac{\pi_1 \quad 8 \Rightarrow 8 \quad \pi_0}{(fun\ a{-}{>}\ [(a+1, a-1)])\ 7 \Rightarrow}}{let\ f = fun\ a{-}{>}\ [(a+1, a-1)] in\ f\ 7 \Rightarrow}$$

Assignment 11.1.1 Big Steps

```
let f = fun a -> (a+1,a-1)::[] in f 7
```

π_1 = fun a $\rightarrow [(a+1, a-1)] \Longrightarrow$ fun a $\rightarrow [(a+1, a-1)]$
(A very useful helper)

$$\text{LD}\ \frac{\pi_1 \quad \text{APP'}\ \dfrac{\pi_1 \quad 8 \Rightarrow 8 \quad \pi_0}{(\mathit{fun}\ a{-}{>}\ [(a+1, a-1)])\ 7 \Rightarrow \texttt{[(8,6)]}}}{\mathit{let}\ f = \mathit{fun}\ a{-}{>}\ [(a+1, a-1)]\ \mathit{in}\ f\ 7 \Rightarrow}$$

Assignment 11.1.1 Big Steps

```
let f = fun a -> (a+1,a-1)::[] in f 7
```

π_1 = fun a $\rightarrow [(a+1, a-1)] \Longrightarrow$ fun a $\rightarrow [(a+1, a-1)]$
(A very useful helper)

$$\text{LD}\ \frac{\pi_1 \quad \text{APP'}\ \dfrac{\pi_1 \quad 8 \Rightarrow 8 \quad \pi_0}{(\mathit{fun}\ a{-}{>}\ [(a+1, a-1)])\ 7 \Rightarrow \texttt{[(8,6)]}}}{\mathit{let}\ f = \mathit{fun}\ a{-}{>}\ [(a+1, a-1)]\ \mathit{in}\ f\ 7 \Rightarrow \texttt{[(8,6)]}}$$

Assignment 11.1.1 Big Steps

- OP: math operation: +, -, *, /
- TU: Tupel
- LI: List
- GD: global definition: functions
- LD: local defintion: let ... in ...
- APP: Function calling
- PM: pattern matching
- More? Refer to the sheet 11 from last year.

Assignment 11.1.1 Big Steps

- Get how to perform the Big Step?
- We start with the bottom, try to get the first thing we could calculate at the top.
- The try to give back the result step by step to the bottom.
- If we get where start. Then finish!
- (A very necessary)Hint: Try to use the π_0, π_1 to save time

Assignment 11.1.2 Big Steps

```
f [3;6]
```

$$\pi_f = \text{GD}\ \frac{\begin{array}{l}\texttt{f} = \texttt{fun l -> match l with [] -> 1 | x::xs -> x+g xs} \\ \quad \texttt{fun l -> match l with [] -> 1 | x::xs -> x+g xs} \\ \Rightarrow \texttt{fun l -> match l with [] -> 1 | x::xs -> x+g xs}\end{array}}{\texttt{f} \Rightarrow \texttt{fun l -> match l with [] -> 1 | x::xs -> x+g xs}}$$

$$\pi_g = \text{GD}\ \frac{\begin{array}{l}\texttt{g} = \texttt{fun l -> match l with [] -> 0 | x::xs -> x*f xs} \\ \quad \texttt{fun l -> match l with [] -> 0 | x::xs -> x*f xs} \\ \Rightarrow \texttt{fun l -> match l with [] -> 0 | x::xs -> x*f xs}\end{array}}{\texttt{g} \Rightarrow \texttt{fun l -> match l with [] -> 0 | x::xs -> x*f xs}}$$

Assignment 11.1.2 Big Steps

$$\text{APP'}\ \frac{\texttt{f}\ \texttt{[3;6]}\ \quad \pi_f\ \ \texttt{[3;6]} \Rightarrow \texttt{[3;6]}}{\texttt{f}\ \texttt{[3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

$$\text{APP'}\ \frac{\texttt{f [3;6]}\ \pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6] match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

`f [3;6]`

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6] 3+g [6]} \Rightarrow}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

`f [3;6]`

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{\texttt{3} \Rightarrow \texttt{3}\ \ \texttt{g}\ \ \texttt{[6]} \Rightarrow}{\texttt{3+g}\ \ \texttt{[6]} \Rightarrow}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

`f [3;6]`

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{\texttt{3} \Rightarrow \texttt{3}\ \ \texttt{g}\ \ \texttt{[6]} \Rightarrow}{\texttt{3+g}\ \ \texttt{[6]} \Rightarrow}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{3 \Rightarrow 3 \qquad \text{APP'}\ \dfrac{\pi_g\ \texttt{[6]} \Rightarrow \texttt{[6]}}{\texttt{g [6]} \Rightarrow}}{\texttt{3+g [6]} \Rightarrow}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

`f [3;6]`

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{3 \Rightarrow 3 \qquad \text{APP'}\ \dfrac{\pi_g\ \texttt{[6]} \Rightarrow \texttt{[6]}}{\texttt{g [6]} \Rightarrow}}{\texttt{3+g [6]} \Rightarrow}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

No space, we need a π_0

Assignment 11.1.2 Big Steps

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{3 \Rightarrow 3 \qquad \text{APP'}\ \dfrac{\pi_g\ \texttt{[6]} \Rightarrow \texttt{[6]}}{\texttt{g [6]} \Rightarrow}}{\texttt{3+g [6]} \Rightarrow}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

No space, we need a π_0

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

Your time to exercise. 10 min

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \text{PM}\ \frac{[6] \Rightarrow [6]\ \ 6 * f\ [] \Rightarrow}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \text{PM}\ \frac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \quad f\ [] \Rightarrow}{6 * f\ [] \Rightarrow}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \text{PM}\ \frac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \quad f\ [] \Rightarrow}{6 * f\ [] \Rightarrow}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \text{PM}\ \frac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \qquad \text{APP'}\ \dfrac{\pi_f \quad [] \Rightarrow []}{f\ [] \Rightarrow}}{6 * f\ [] \Rightarrow}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \text{PM}\ \frac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \qquad \text{APP'}\ \dfrac{\pi_f \quad [] \Rightarrow [] \qquad \text{PM}\ \dfrac{}{match[]with[] -> 1|x :: xs -> x + g\ xs \Rightarrow}}{f\ [] \Rightarrow}}{6 * f\ [] \Rightarrow}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \dfrac{\text{PM}\ \dfrac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \qquad \text{APP'}\ \dfrac{\pi_f \quad [] \Rightarrow [] \qquad \text{PM}\ \dfrac{[] \Rightarrow [] \quad 1 \Rightarrow 1}{match[]with[]- > 1|x :: xs- > x + g\ xs \Rightarrow}}{f\ [] \Rightarrow}}{6 * f\ [] \Rightarrow}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \dfrac{\text{PM}\ \dfrac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \qquad \text{APP'}\ \dfrac{\pi_f \quad [] \Rightarrow [] \qquad \text{PM}\ \dfrac{[] \Rightarrow [] \quad 1 \Rightarrow 1}{match[]with[] -> 1|x :: xs -> x + g\ xs \Rightarrow 1}}{f\ [] \Rightarrow 1}}{6 * f\ [] \Rightarrow}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \dfrac{\text{PM}\ \dfrac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \qquad \text{APP'}\ \dfrac{\pi_f \quad [] \Rightarrow [] \qquad \text{PM}\ \dfrac{[] \Rightarrow [] \quad 1 \Rightarrow 1}{match[]with[] -> 1|x :: xs -> x + g\ xs \Rightarrow 1}}{f\ [] \Rightarrow 1} \qquad 6*1 \Rightarrow}{6 * f\ [] \Rightarrow}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \dfrac{\text{PM}\ \dfrac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \qquad \text{APP'}\ \dfrac{\pi_f \quad [] \Rightarrow [] \qquad \text{PM}\ \dfrac{[] \Rightarrow [] \quad 1 \Rightarrow 1}{match[]with[] -> 1|x :: xs -> x + g\ xs \Rightarrow 1}}{f\ [] \Rightarrow 1} \qquad 6*1 \Rightarrow 6}{6 * f\ [] \Rightarrow}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \dfrac{\text{PM}\ \dfrac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \qquad \text{APP'}\ \dfrac{\pi_f \quad [] \Rightarrow [] \qquad \text{PM}\ \dfrac{[] \Rightarrow [] \quad 1 \Rightarrow 1}{match[]with[] -> 1|x :: xs -> x + g\ xs \Rightarrow 1}}{f\ [] \Rightarrow 1} \qquad 6*1 \Rightarrow 6}{6 * f\ [] \Rightarrow 6}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow}$$

Assignment 11.1.2 Big Steps

```
match [6] with [] -> 0 | x::xs -> x*f xs
```

$$\pi_0 = \text{PM}\ \frac{[6] \Rightarrow [6] \qquad \text{OP}\ \dfrac{6 \Rightarrow 6 \qquad \text{APP'}\ \dfrac{\pi_f \quad [] \Rightarrow [] \qquad \text{PM}\ \dfrac{[] \Rightarrow [] \quad 1 \Rightarrow 1}{match[]with[] -> 1|x :: xs -> x + g\ xs \Rightarrow 1}}{f\ [] \Rightarrow 1} \qquad 6*1 \Rightarrow 6}{6 * f\ [] \Rightarrow 6}}{\texttt{match [6] with [] -> 0 | x::xs -> x*f xs} \Rightarrow 6}$$

Assignment 11.1.2 Big Steps

`f [3;6]`

We are back!

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{3 \Rightarrow 3 \qquad \text{APP'}\ \dfrac{\pi_g\ \texttt{[6]} \Rightarrow \texttt{[6]} \quad \pi_0}{\texttt{g [6]} \Rightarrow 6}}{\texttt{3+g [6]} \Rightarrow}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

`f [3;6]`

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{3 \Rightarrow 3 \qquad \text{APP'}\ \dfrac{\pi_g\ \texttt{[6]} \Rightarrow \texttt{[6]} \quad \pi_0}{\texttt{g [6]} \Rightarrow 6} \qquad 3+6 \Rightarrow}{\texttt{3+g [6]} \Rightarrow}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

`f [3;6]`

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{3 \Rightarrow 3 \qquad \text{APP'}\ \dfrac{\pi_g\ \texttt{[6]} \Rightarrow \texttt{[6]} \quad \pi_0}{\texttt{g [6]} \Rightarrow 6} \qquad 3+6 \Rightarrow 9}{\texttt{3+g [6]} \Rightarrow}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

`f [3;6]`

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{3 \Rightarrow 3 \qquad \text{APP'}\ \dfrac{\pi_g\ \texttt{[6]} \Rightarrow \texttt{[6]} \quad \pi_0}{\texttt{g [6]} \Rightarrow 6} \qquad 3+6 \Rightarrow 9}{\texttt{3+g [6]} \Rightarrow 9}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow}}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{3 \Rightarrow 3 \qquad \text{APP'}\ \dfrac{\pi_g\ \texttt{[6]} \Rightarrow \texttt{[6]} \quad \pi_0}{\texttt{g [6]} \Rightarrow 6} \qquad 3+6 \Rightarrow 9}{\texttt{3+g [6]} \Rightarrow 9}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow 9}}{\texttt{f [3;6]} \Rightarrow}$$

Assignment 11.1.2 Big Steps

`f [3;6]`

$$\text{APP'}\ \frac{\pi_f\ \texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{PM}\ \dfrac{\texttt{[3;6]} \Rightarrow \texttt{[3;6]} \qquad \text{OP}\ \dfrac{3 \Rightarrow 3 \qquad \text{APP'}\ \dfrac{\pi_g\ \texttt{[6]} \Rightarrow \texttt{[6]} \quad \pi_0}{\texttt{g [6]} \Rightarrow 6} \qquad 3+6 \Rightarrow 9}{\texttt{3+g [6]} \Rightarrow 9}}{\texttt{match [3;6] with [] -> 1 | x::xs -> x+g xs} \Rightarrow 9}}{\texttt{f [3;6]} \Rightarrow 9}$$

Assignment 11.1.3 Big Steps

- Any Questions until now?

Assignment 11.1.3 Big Steps

- Any Questions until now?

```
(fun x -> x 3) (fun y z -> z y) (fun w -> w + w)
```

- Your time to exercise. 15min

Assignment 11.1.3 Big Steps

```
(fun x -> x 3) (fun y z -> z y) (fun w -> w + w)
```

$\mathsf{f}_1 = \mathit{fun}\ x \rightarrow x\ 3$

$f_2 = \mathit{fun}\ y\ z \rightarrow z\ y$

$f_3 = \mathit{fun}\ w \rightarrow w + w$

$\pi_1 = \mathsf{f}_1 \implies f_1$

$\pi_2 = \mathsf{f}_2 \implies f_2$

$\pi_3 = \mathsf{f}_3 \implies f_3$

Summary

- The Big Step workaround:
- Remember the keyword like: **LI, GD, LD, APP, ...**
- We start from bottom, also end at bottom.
- Remember to use f_1, π_1 annotations to save the unnecessary writing in EXAM!
- More to Big Step: refer to the exercise 11 from last year.

Summary

- The Big Step is just basic work around as a proving tool.
- **Time** is ultimate factor in the exam.
- If you master the time with Big Step, then this part is 100% easy for you.
- In exam, then you save time to consider more with Ocaml.
- Paper Size Problem in exam: ask for more papers!
- Any Questions?