QO - Week 10

January 29, 2021

Exercise 1

`Unrank(8, 64)`

- Init: $\pi := [0; 1; 2; 3; 4; 5; 6; 7]$
- *iteration* $i = n = 8$
  - `swap(`$\pi[i-1] = \pi[7]$`,` $\pi[r \ mod \ i] = \pi[0]$`)`
  - $r = \lfloor r/i \rfloor = 8$
  - After swapping: $\pi := [7; 1; 2; 3; 4; 5; 6; 0]$
- *iteration* $i = 7$
  - `swap(`$\pi[i-1] = \pi[6]$`,` $\pi[r \ mod \ i] = \pi[1]$`)`
  - $r = \lfloor r/i \rfloor = 1$
  - After swapping: $\pi := [7; 6; 2; 3; 4; 5; 1; 0]$
- *iteration* $i = 6$
  - `swap(`$\pi[i-1] = \pi[5]$`,` $\pi[r \ mod \ i] = \pi[1]$`)`
  - $r = \lfloor r/i \rfloor = 0$
  - After swapping: $\pi := [7; 5; 2; 3; 4; 6; 1; 0]$
- *iteration* $i = 5$
  - `swap(`$\pi[i-1] = \pi[4]$`,` $\pi[r \ mod \ i] = \pi[0]$`)`
  - $r = \lfloor r/i \rfloor = 0$
  - After swapping: $\pi := [4; 5; 2; 3; 7; 6; 1; 0]$
- *iteration* $i = 4$
  - `swap(`$\pi[i-1] = \pi[3]$`,` $\pi[r \ mod \ i] = \pi[0]$`)`
  - $r = \lfloor r/i \rfloor = 0$

- After swapping: $\pi := [3; 5; 2; 4; 7; 6; 1; 0]$

- *iteration* $i = 3$
  - `swap`$(\pi[i-1] = \pi[2],\ \pi[r\ mod\ i] = \pi[0])$
  - $r = \lfloor r/i \rfloor = 0$
  - After swapping: $\pi := [2; 5; 3; 4; 7; 6; 1; 0]$

- *iteration* $i = 2$
  - `swap`$(\pi[i-1] = \pi[1],\ \pi[r\ mod\ i] = \pi[0])$
  - $r = \lfloor r/i \rfloor = 0$
  - After swapping: $\pi := [5; 2; 3; 4; 7; 6; 1; 0]$

- *iteration* $i = 1$
  - `swap`$(\pi[i-1] = \pi[0],\ \pi[r\ mod\ i] = \pi[0])$
  - $r = \lfloor r/i \rfloor = 0$
  - After swapping: $\pi := [5; 2; 3; 4; 7; 6; 1; 0]$

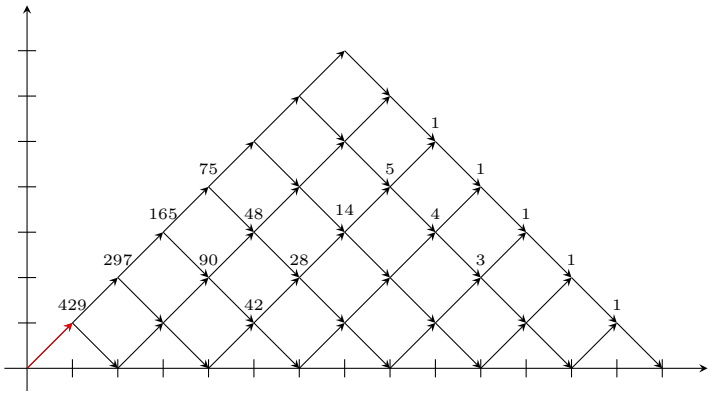
$$\texttt{Unrank(8, 64)} = [5; 2; 3; 4; 7; 6; 1; 0]$$

Exercise 2

`Unrank(8, 125)`

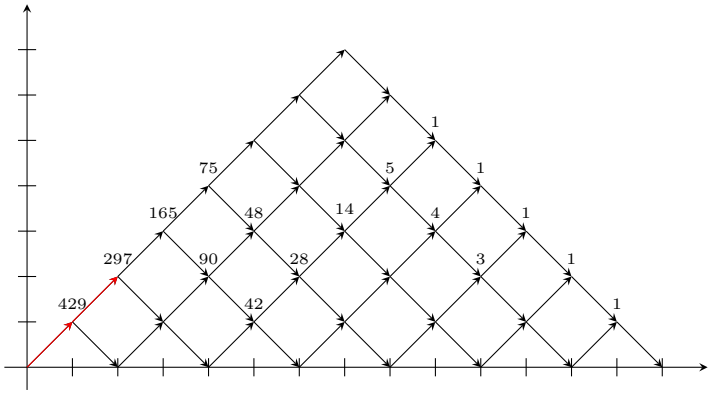
Init:

- rank = 125
- Building Dyck Word: (



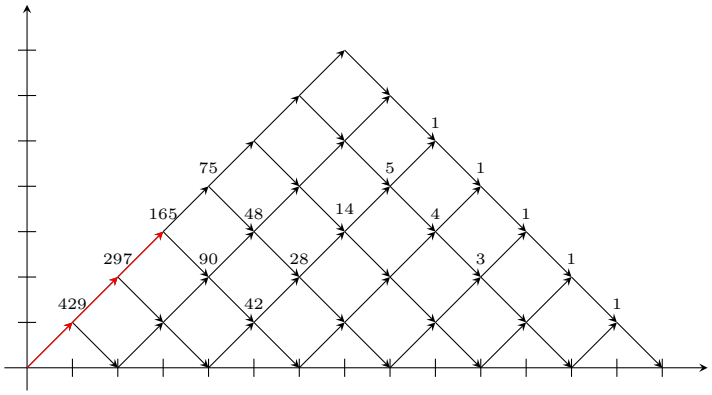
Step 1:

- rank = 125 < 297
- Go Up
- Building Dyck Word: ((



Step 2:

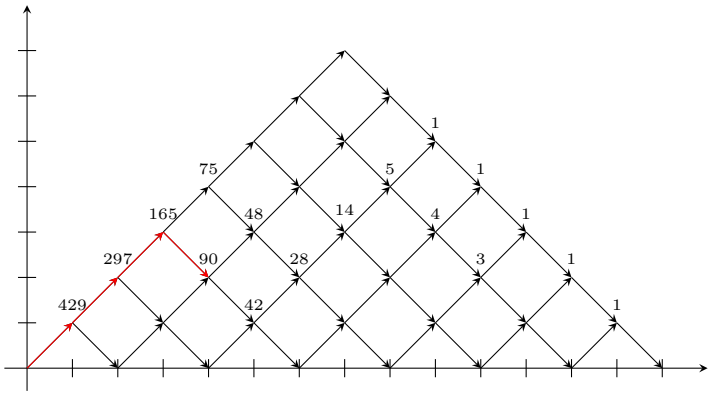
- rank = 125 < 165
- Go Up
- Building Dyck Word: (((



Step 3:

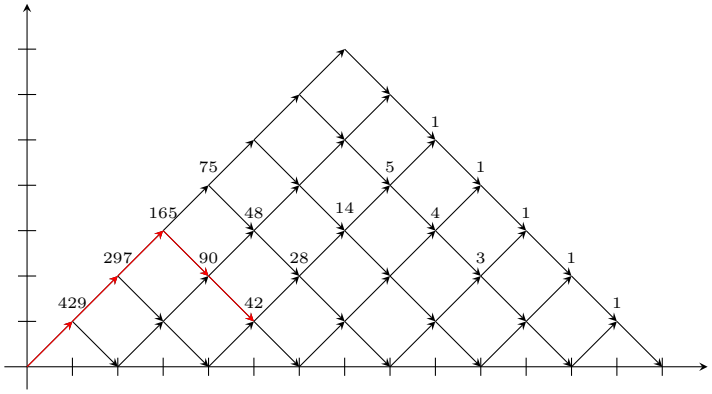
- rank = 125 ≥ 75
- rank = 125 - 75 = 50
- Go down

- Building Dyck Word: (((



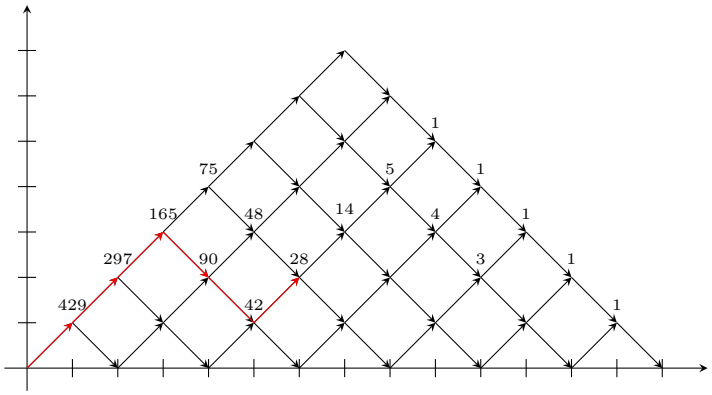
Step 4:

- rank = 50 $\geq$ 48
- rank = 50 - 48 = 2
- Go down
- Building Dyck Word: ((())



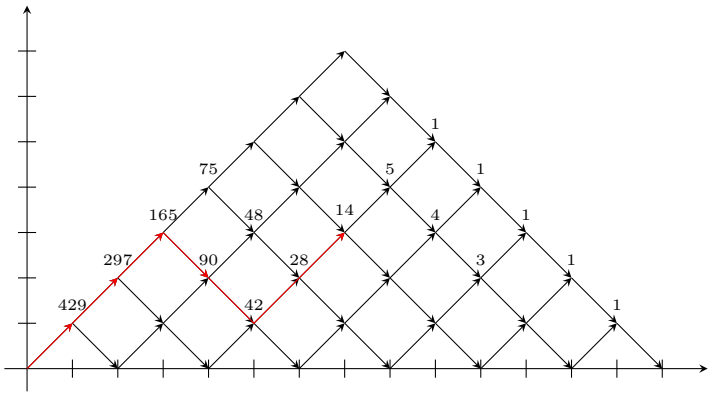
Step 5:

- rank = 2 $<$ 28
- Go Up
- Building Dyck Word: ((()))(



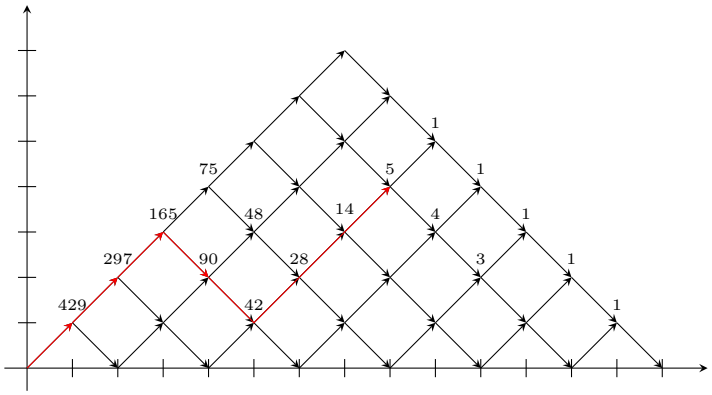
Step 6:

- rank $= 2 < 14$
- Go Up
- Building Dyck Word: (((())((



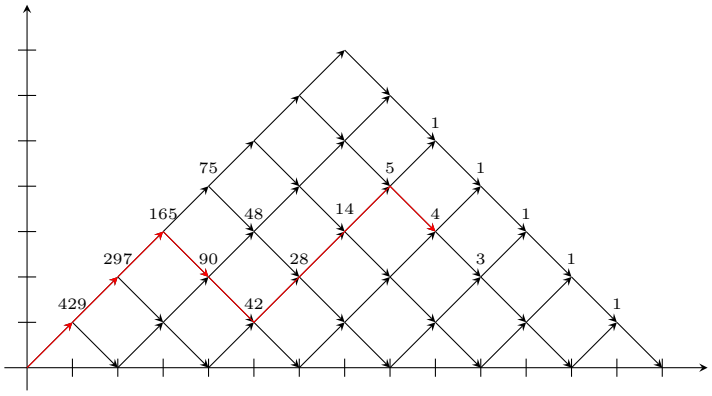
Step 7:

- rank $= 2 < 5$
- Go Up
- Building Dyck Word: (((()(((



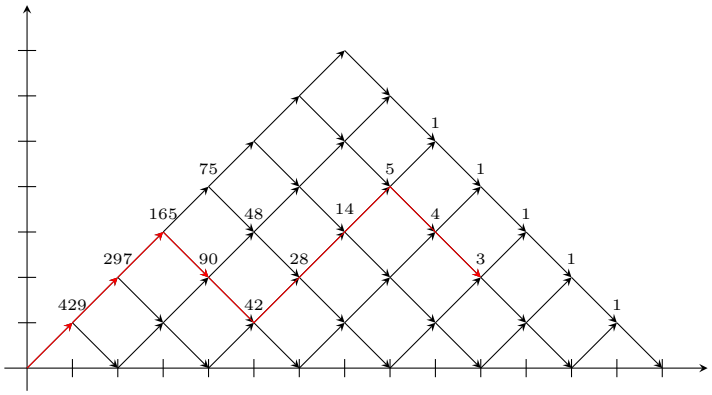
Step 8:

- rank = $2 \geq 1$
- rank = 2 - 1 = 1
- Go down
- Building Dyck Word: (())(((



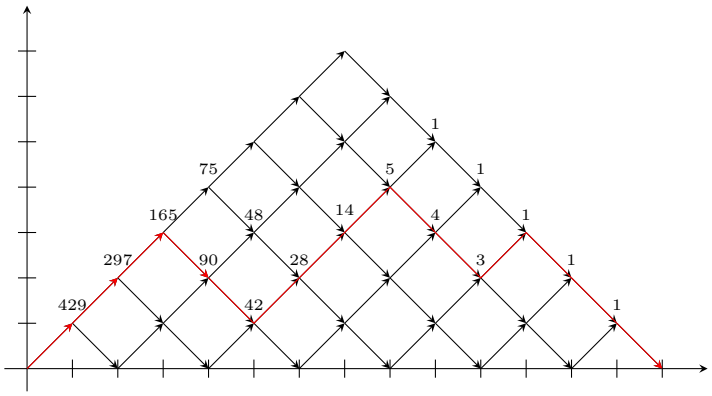
Step 9:

- rank = $1 \geq 1$
- rank = 1 - 1 = 0
- Go down
- Building Dyck Word: (())((())

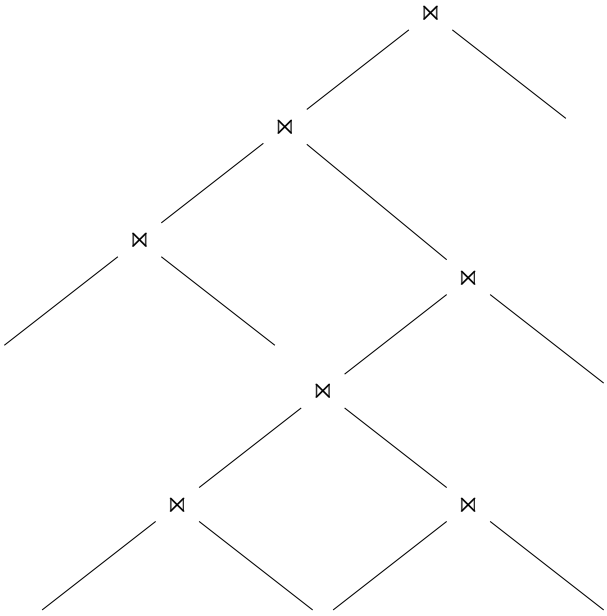


Step 9:

- rank $= 0 < 1$
- Go Up and finish due to rank $= 0$
- Building Dyck Word: ((())((())()))



`Unrank(8, 125) := Dyck Word:  ((())((())()))`



The final result

The final result from Exercise 1 and Exercise 2: