Problem1

November 17, 2024

[1]:
```
import numpy as np
from torchvision import datasets
from torchvision.transforms import ToTensor

# Load the USPS dataset
usps_train = datasets.USPS(root='usps', download=True, transform=ToTensor(),␣
 ↪train=True)

# Extract the first 100 samples for labels 1 and 8
images, labels = [], []
for img, label in usps_train:
    if label == 1 and len([l for l in labels if l == 1]) < 100:
        images.append(img.numpy().squeeze())
        labels.append(1)
    elif label == 8 and len([l for l in labels if l == 8]) < 100:
        images.append(img.numpy().squeeze())
        labels.append(0)
    if len(labels) == 200:
        break

# Convert lists to NumPy arrays
images = np.array(images)
labels = np.array(labels)

print(f"Dataset shape: {images.shape}")
print(f"Labels shape: {labels.shape}")
```

```
Dataset shape: (200, 16, 16)
Labels shape: (200,)
```

[2]:
```
conv_kernel = np.random.randn(5, 5) * 0.1  # Random 5x5 kernel
fc_weight = np.random.randn(12 * 12) * 0.1  # Fully connected weights (144␣
 ↪elements)
fc_bias = 0.0  # Fully connected bias

def relu(x):
    """ReLU activation function."""
    return np.maximum(0, x)
```

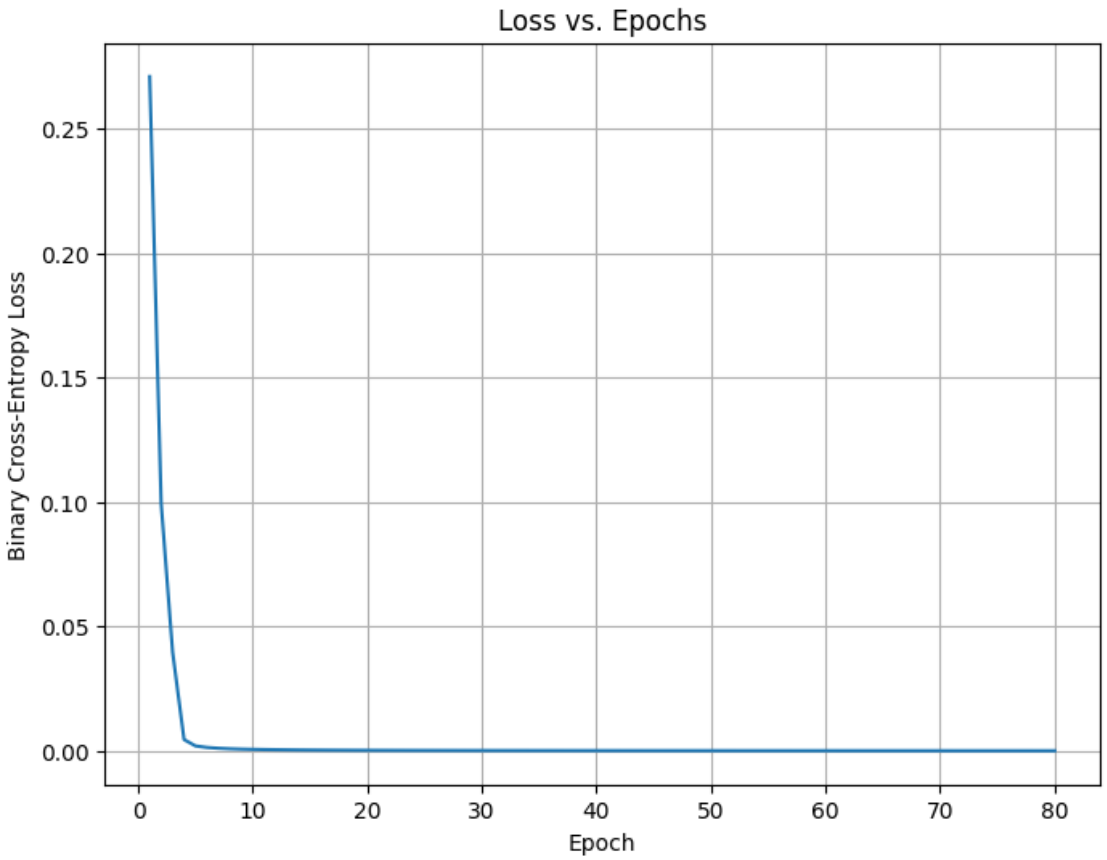
```
def sigmoid(x):
    """Sigmoid activation function."""
    return 1 / (1 + np.exp(-x))

def forward(image):
    """Forward pass through the CNN."""
    # Convolutional layer
    conv_out = np.zeros((12, 12))
    for i in range(12):
        for j in range(12):
            conv_out[i, j] = np.sum(image[i:i+5, j:j+5] * conv_kernel)

    # ReLU activation
    relu_out = relu(conv_out)

    # Flatten layer
    flatten_out = relu_out.flatten()

    # Fully connected layer
    logits = np.dot(flatten_out, fc_weight) + fc_bias
    prediction = sigmoid(logits)  # Output probability

    return prediction, relu_out, conv_out, flatten_out
```

[3]:
```
def backward(image, label, prediction, relu_out, conv_out, flatten_out):
    """Backward pass using simplified gradient computation."""
    d_loss = prediction - label

    # Gradients for fully connected layer
    d_fc_weight = d_loss * flatten_out
    d_fc_bias = d_loss

    # Gradients for ReLU output
    d_relu = d_loss * fc_weight.reshape(12, 12)
    d_relu[conv_out <= 0] = 0

    # Gradients for convolutional kernel
    d_kernel = np.zeros_like(conv_kernel)
    for i in range(12):
        for j in range(12):
            d_kernel += image[i:i+5, j:j+5] * d_relu[i, j]

    return d_fc_weight, d_fc_bias, d_kernel
```

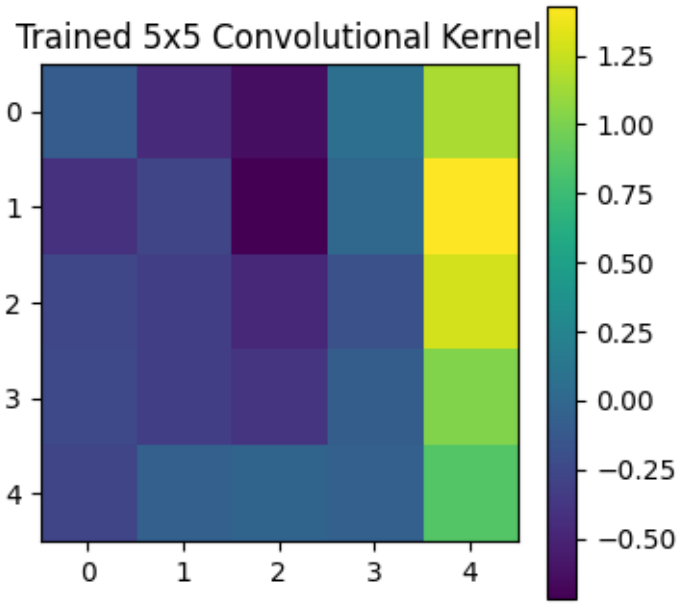
[4]:
```
# Training parameters
learning_rate = 0.1
```

```
epochs = 80
losses = []

# Training loop
for epoch in range(epochs):
    epoch_loss = 0.0

    for img, label in zip(images, labels):
        # Forward pass
        prediction, relu_out, conv_out, flatten_out = forward(img)

        # Compute binary cross-entropy loss
        loss = -label * np.log(prediction + 1e-7) - (1 - label) * np.log(1 -␣
 ↪prediction + 1e-7)
        epoch_loss += loss

        # Backward pass
        d_fc_weight, d_fc_bias, d_kernel = backward(img, label, prediction,␣
 ↪relu_out, conv_out, flatten_out)

        # Update parameters
        global fc_weight, fc_bias, conv_kernel
        fc_weight -= learning_rate * d_fc_weight
        fc_bias -= learning_rate * d_fc_bias
        conv_kernel -= learning_rate * d_kernel

    # Compute and store average loss for the epoch
    avg_loss = epoch_loss / len(images)
    losses.append(avg_loss)
```

[5]:
```
import matplotlib.pyplot as plt

# Plot the loss curve
plt.figure(figsize=(8, 6))
plt.plot(range(1, epochs + 1), losses)
plt.title('Loss vs. Epochs')
plt.xlabel('Epoch')
plt.ylabel('Binary Cross-Entropy Loss')
plt.grid()
plt.show()
```



[6]:
```
# Display the trained convolutional kernel
plt.figure(figsize=(4, 4))
plt.imshow(conv_kernel, cmap='viridis', interpolation='nearest')
plt.colorbar()
plt.title('Trained 5x5 Convolutional Kernel')
plt.show()
```


Trained 5x5 Convolutional Kernel
0
1
2
3
4
0
1
2
3
4
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50

Problem2

November 12, 2024

[1]:
```
#Import Libraries
import torch
from torchvision import datasets, transforms
from torch.utils.data import DataLoader
from torch import nn, optim
import matplotlib.pyplot as plt
from torchvision.transforms import ToTensor
```

[2]:
```
data_train = datasets.USPS(root='usps', train=True, download=True,␣
 ↪transform=ToTensor())
data_test = datasets.USPS(root='usps', train=False, download=True,␣
 ↪transform=ToTensor())

train = DataLoader(data_train, batch_size=512, shuffle=True)
test = DataLoader(data_test)
```

[3]:
```
img, label = data_train[0]
img.shape
```

[3]:
```
torch.Size([1, 16, 16])
```

[4]:
```
class CNN(nn.Module):
    def __init__(self):
        super().__init__()
        self.conv1 = nn.Conv2d(in_channels=1, out_channels=4, kernel_size=5)
        self.conv2 = nn.Conv2d(in_channels=4, out_channels=4, kernel_size=3)
        self.pool = nn.MaxPool2d(kernel_size=2)
        self.a = nn.ReLU()
        self.output = nn.Linear(16, 10)
        self.flatten = nn.Flatten()
    def forward(self, x):
        x = self.pool(self.a(self.conv1(x)))
        x = self.pool(self.a(self.conv2(x)))
        x = self.flatten(x)
        x = self.output(x)
        return x
```

[5]:
```
model = CNN().to('cuda')
loss_fn = nn.CrossEntropyLoss()
optimizer = optim.SGD(model.parameters(), lr=0.5)
```

[6]:
```
epochs = 50
train_loss = torch.zeros(epochs)
train_acc = torch.zeros(epochs)
test_acc = torch.zeros(epochs)

for epoch in range(epochs):
    cumalative = 0
    total = 0
    for X, Y in train:
        X, Y = X.to('cuda'), Y.to('cuda')
        out = model(X)
        loss = loss_fn(out, Y)
        optimizer.zero_grad()
        loss.backward()
        optimizer.step()

        cumalative += (out.argmax(axis=1)==Y).sum().item()
        total += loss.item()
    train_loss[epoch] = total /len(train)
    train_acc[epoch] = cumalative/len(data_train)
    correct = 0
    with torch.no_grad():
        for Xt, Yt in test:
            Xt, Yt = Xt.to('cuda'), Yt.to('cuda')
            out_test = model(Xt)
            correct += (out_test.argmax(axis=1) == Yt).sum().item()
    test_acc[epoch] = correct / len(data_test)
    print(f"Epoch {epoch+1}/{epochs}, Loss: {train_loss[epoch]:.4f}, "f"Train␣
 ↪Acc: {train_acc[epoch]*100:.2f}%, Test Acc: {test_acc[epoch]*100:.2f}%")
```

```
Epoch 1/50, Loss: 2.1015, Train Acc: 24.10%, Test Acc: 40.81%
Epoch 2/50, Loss: 1.6419, Train Acc: 42.93%, Test Acc: 54.66%
Epoch 3/50, Loss: 1.0607, Train Acc: 64.24%, Test Acc: 64.42%
Epoch 4/50, Loss: 0.7890, Train Acc: 74.79%, Test Acc: 68.51%
Epoch 5/50, Loss: 0.7845, Train Acc: 73.79%, Test Acc: 70.00%
Epoch 6/50, Loss: 0.6231, Train Acc: 79.37%, Test Acc: 75.29%
Epoch 7/50, Loss: 0.5376, Train Acc: 82.54%, Test Acc: 78.82%
Epoch 8/50, Loss: 0.5443, Train Acc: 81.54%, Test Acc: 81.76%
Epoch 9/50, Loss: 0.4414, Train Acc: 85.98%, Test Acc: 76.33%
Epoch 10/50, Loss: 0.4174, Train Acc: 86.52%, Test Acc: 79.07%
Epoch 11/50, Loss: 0.4227, Train Acc: 87.09%, Test Acc: 81.91%
Epoch 12/50, Loss: 0.3694, Train Acc: 88.34%, Test Acc: 84.45%
Epoch 13/50, Loss: 0.3043, Train Acc: 90.91%, Test Acc: 83.91%
Epoch 14/50, Loss: 0.3025, Train Acc: 91.13%, Test Acc: 81.07%
```

```
Epoch 15/50, Loss: 0.6125, Train Acc: 80.19%, Test Acc: 86.15%
Epoch 16/50, Loss: 0.2853, Train Acc: 91.70%, Test Acc: 86.25%
Epoch 17/50, Loss: 0.2697, Train Acc: 92.26%, Test Acc: 80.67%
Epoch 18/50, Loss: 0.3180, Train Acc: 90.73%, Test Acc: 84.16%
Epoch 19/50, Loss: 0.2820, Train Acc: 91.35%, Test Acc: 86.10%
Epoch 20/50, Loss: 0.2554, Train Acc: 92.46%, Test Acc: 82.56%
Epoch 21/50, Loss: 0.2518, Train Acc: 92.44%, Test Acc: 87.89%
Epoch 22/50, Loss: 0.2314, Train Acc: 93.39%, Test Acc: 87.14%
Epoch 23/50, Loss: 0.3355, Train Acc: 89.12%, Test Acc: 89.24%
Epoch 24/50, Loss: 0.2152, Train Acc: 93.54%, Test Acc: 88.99%
Epoch 25/50, Loss: 0.2345, Train Acc: 92.77%, Test Acc: 88.99%
Epoch 26/50, Loss: 0.2036, Train Acc: 93.87%, Test Acc: 88.64%
Epoch 27/50, Loss: 0.2886, Train Acc: 91.11%, Test Acc: 88.59%
Epoch 28/50, Loss: 0.1938, Train Acc: 94.39%, Test Acc: 89.59%
Epoch 29/50, Loss: 0.2021, Train Acc: 94.01%, Test Acc: 89.69%
Epoch 30/50, Loss: 0.2017, Train Acc: 94.05%, Test Acc: 86.35%
Epoch 31/50, Loss: 0.2279, Train Acc: 92.87%, Test Acc: 89.89%
Epoch 32/50, Loss: 0.1875, Train Acc: 94.42%, Test Acc: 90.03%
Epoch 33/50, Loss: 0.2190, Train Acc: 93.49%, Test Acc: 90.33%
Epoch 34/50, Loss: 0.2209, Train Acc: 93.27%, Test Acc: 83.16%
Epoch 35/50, Loss: 0.3914, Train Acc: 88.62%, Test Acc: 88.24%
Epoch 36/50, Loss: 0.1800, Train Acc: 94.60%, Test Acc: 89.39%
Epoch 37/50, Loss: 0.1828, Train Acc: 94.55%, Test Acc: 89.69%
Epoch 38/50, Loss: 0.1862, Train Acc: 94.71%, Test Acc: 86.45%
Epoch 39/50, Loss: 0.5612, Train Acc: 83.86%, Test Acc: 89.59%
Epoch 40/50, Loss: 0.1863, Train Acc: 94.60%, Test Acc: 89.24%
Epoch 41/50, Loss: 0.1653, Train Acc: 95.17%, Test Acc: 89.99%
Epoch 42/50, Loss: 0.1571, Train Acc: 95.45%, Test Acc: 88.64%
Epoch 43/50, Loss: 0.1579, Train Acc: 95.24%, Test Acc: 91.13%
Epoch 44/50, Loss: 0.1466, Train Acc: 95.83%, Test Acc: 91.38%
Epoch 45/50, Loss: 0.1510, Train Acc: 95.65%, Test Acc: 90.58%
Epoch 46/50, Loss: 0.1376, Train Acc: 95.93%, Test Acc: 91.43%
Epoch 47/50, Loss: 0.1332, Train Acc: 96.02%, Test Acc: 91.13%
Epoch 48/50, Loss: 0.1674, Train Acc: 94.82%, Test Acc: 90.78%
Epoch 49/50, Loss: 0.1327, Train Acc: 96.24%, Test Acc: 89.39%
Epoch 50/50, Loss: 0.1397, Train Acc: 95.84%, Test Acc: 90.98%
```

[7]:
```python
# Plot Cross-Entropy Loss
plt.figure(figsize=(10, 5))
plt.plot(range(epochs), train_loss, label='Training Loss')
plt.xlabel('Epoch')
plt.ylabel('Cross-Entropy Loss')
plt.title('Training Loss over Epochs')
plt.legend()
plt.show()

# Plot Training and Testing Accuracy on the same graph
```

```
plt.figure(figsize=(10, 5))
plt.plot(range(epochs), train_acc * 100, label='Training Accuracy')
plt.plot(range(epochs), test_acc * 100, label='Testing Accuracy',␣
  ↪color='orange')
plt.xlabel('Epoch')
plt.ylabel('Accuracy (%)')
plt.title('Training and Testing Accuracy over Epochs')
plt.legend()
plt.show()
```