A1

Simplify the following formulas:

a. $p \wedge (p \wedge q)$
b. $\overline{\overline{p} \vee q}$
c. $\overline{p \Rightarrow \overline{q}}$

Answer

(a) $p \wedge q$
(b) $p \wedge \overline{q}$
(c) $p \wedge q$

A2

Show that the argument

"If p and q, then r. Therefore, if not r, then not p or not q."

is valid. In other words, show that the logic used in the argument is correct.

Answer

Symbolically, the argument says

$$[(p \wedge q) \Rightarrow r] \Rightarrow [\overline{r} \Rightarrow (\overline{p} \vee \overline{q})]. \tag{2.5.3}$$

We want to show that it is a tautology. It is easy to verify with a truth table. We can also argue that this compound statement is always true by showing that it can never be false.

Suppose, on the contrary, that ([eqn:tautology]) is false for some choices of p, q, and r. Then

$$(p \wedge q) \Rightarrow r \quad \text{must be true,} \qquad \text{and} \qquad \overline{r} \Rightarrow (\overline{p} \vee \overline{q}) \quad \text{must be false.} \tag{2.5.4}$$

For the second implication to be false, we need

$$\overline{r} \quad \text{to be true,} \qquad \text{and} \qquad \overline{p} \vee \overline{q} \quad \text{to be false.} \tag{2.5.5}$$

They in turn imply that r is false, and both $\overline{p}$ and $\overline{q}$ are false; hence both p and q are true. This would make $(p \wedge q) \Rightarrow r$ false, contradicting the assumption that it is true. Thus, ([eqn:tautology]) cannot be false, it must be a tautology.

A3

Use the truth tables method to determine whether $(\neg p \vee q) \wedge (q \rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$ *(denoted with* φ*) is satisfiable.*

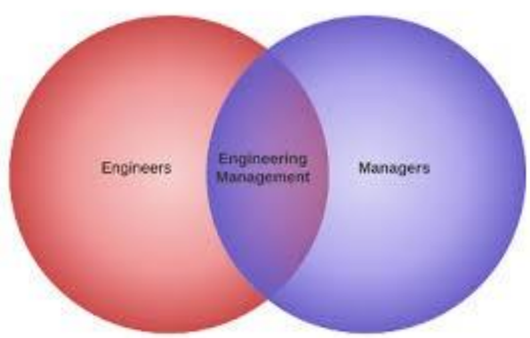
Solution.

p	q	r	$\neg p \vee q$	$\neg r \wedge \neg p$	$q \rightarrow \neg r \wedge \neg p$	$(p \vee r)$	φ
T	T	T	T	F	F	T	F
T	T	F	T	F	F	T	F
T	F	T	F	F	T	T	F
T	F	F	F	F	T	T	F
F	T	T	T	F	F	T	F
F	T	F	T	T	T	F	F
F	F	T	T	F	T	T	**T**
F	F	F	T	T	T	F	F

There exists an interpretation satisfying φ*, thus* φ *is satisfiable.*

B1

In a school there are 25 teachers who teach engineering or management. Of these, 15 teach engineering and 6 teach both engineering and management. How many teach management?



Solution

Let M be the set of teachers who teach management and E be the set of teachers who teach engineering. Here ‘or’ means union and ‘and’ means intersection. So, we have

$$n (E \cup M) = 25 , n (E) = 15 \text{ and } n (E \cap M) = 6$$

We have to calculate $n (M)$.

According to formula

$$n (E \cup M) = n (E) + n (M) - n (E \cap M),$$

We obtain

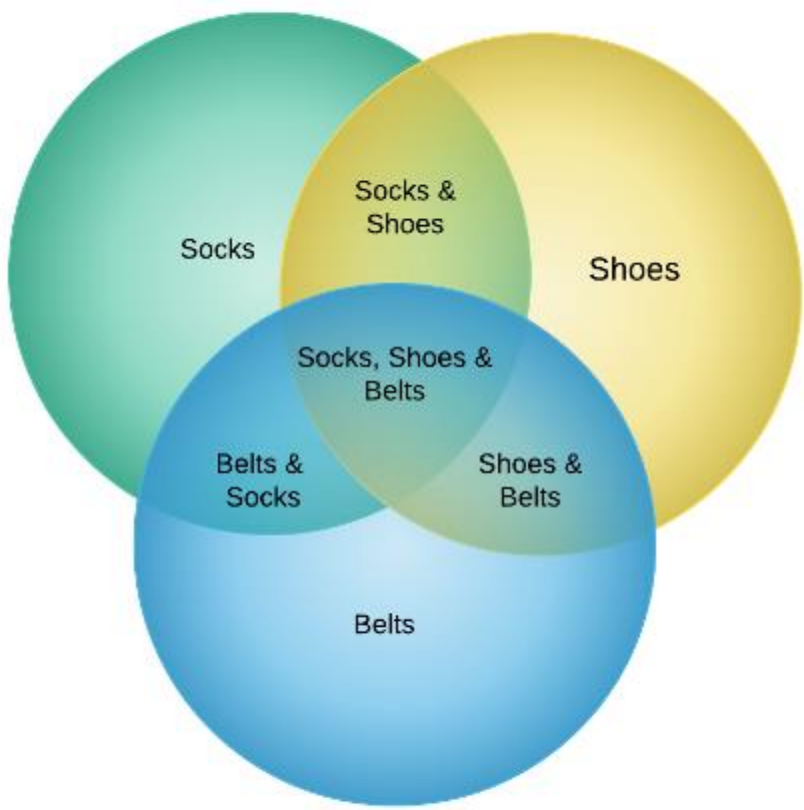
$$25 = 15 + n (M) - 6$$

Thus $n (M) = 16$

Hence 16 teachers teach management.

B2

In a shop, 380 people buy socks, 150 people buy shoes and 200 people buy belt. If there are total 580 people who bought either socks or shoes or belt and only 30 people bought all the three things? So how many people bought exactly two things.



Solution

Let S, H and B represent the set of number of people bought socks, shoes and belt respectively.

So, $n(S) = 380$, $n(H) = 150$, $n(B) = 200$

$n(S \cup H \cup B) = 580$, $n(S \cap H \cap B) = 30$

Therefore, $n(S \cup H \cup B) = n(S) + n(H) + n(B) - n(S \cap H) - n(H \cap B) - n(B \cap S) + n(S \cap H \cap B)$,

Now, we will put values given in the formula,

$$580 = 380 + 150 + 200 - n(S \cap H) - n(H \cap B) - n(B \cap S) + 30$$

This gives that,

$$n(S \cap H) + n(H \cap B) + n(B \cap S) = 180$$

But this includes the number of people who bought all the three items also. So we have to deduct these numbers of people from it.

Let, $n(S \cap H \cap B) = a$

As we can see from the Venn diagram,

$n(S \cap H) - a + n(H \cap B) - a + n(B \cap S) - a =$ the required number

$n(S \cap H) + n(H \cap B) + n(B \cap S) - 3a$

$180 - 90 = 90$

Hence, 90 people are there who bought exactly two things.

B3 There are 500 students in a school, 220 like science subject, 180 like math and 40 like both science and math. Find the number of students who like

- Science but not math
- Math but not science
- Either math or science

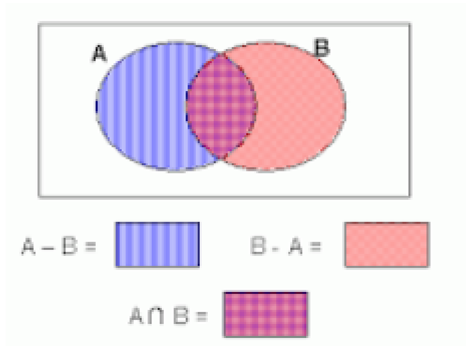
Solution

Let the total number of students be U that is, the universal set.

Let A is the set of number of students who like science.

Let B is the set of number of students who like math.

Here, $n(U) = 500$ $n(A) = 220$ $n(B) = 180$ $n(A \cap B) = 40$



- Here we have to find the number of students who like science but not math, so symbolically we have to find A-B.

As it is showing in the Venn diagram,

$A = (A-B) \cup (A \cap B)$

$n(A) = n(A-B) + n(A \cap B)$

$n(A-B) = n(A) - n(A \cap B)$

$= 220 – 40 = 180$

Hence, the numbers of students who like science only not math are 180.

- Here we have to find the number of students who like math but not science, so symbolically we have to find B-A.

$B = (B-A) \cup (A \cap B)$

$n(B) = n(B-A) + n(A \cap B)$

$n(B-A) = n(B) - n(A \cap B)$

$=180-40=140$

Hence, the numbers of students who like math only not science are 140.

- Here we have to find the number of students who like either math or science.

$n(A \cup B) = n(A) + n(B) – n(A \cap B)$

$= 220 + 180 – 40$

$= 360$