Bairstow's Method - Description of The Method

Description of The Method

Bairstow's approach is to use Newton's method to adjust the coefficients *u* and *v* in the quadratic until its roots are also roots of the polynomial being solved. The roots of the quadratic may then be determined, and the polynomial may be divided by the quadratic to eliminate those roots. This process is then iterated until the polynomial becomes quadratic or linear, and all the roots have been determined.

Long division of the polynomial to be solved

by yields a quotient and a remainder such that

A second division of by is performed to yield a quotient and remainder with

The variables, and the are functions of and . They can be found recursively as follows.

$$\begin{aligned} b_n &= b_{n-1} = 0, & f_n &= f_{n-1} = 0, \\ b_i &= a_{i+2} - ub_{i+1} - vb_{i+2} & f_i &= b_{i+2} - uf_{i+1} - vf_{i+2} \qquad (i = n-2, \ldots, 0), \\ c &= a_1 - ub_0 - vb_1, & g &= b_1 - uf_0 - vf_1, \\ d &= a_0 - vb_0, & h &= b_0 - vf_0. \end{aligned}$$

The quadratic evenly divides the polynomial when

Values of and for which this occurs can be discovered by picking starting values and iterating Newton's method in two dimensions

$$\begin{bmatrix} u \\ v \end{bmatrix} := \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} \frac{\partial c}{\partial u} & \frac{\partial c}{\partial v} \\ \frac{\partial d}{\partial u} & \frac{\partial d}{\partial v} \end{bmatrix}^{-1} \begin{bmatrix} c \\ d \end{bmatrix} := \begin{bmatrix} u \\ v \end{bmatrix} - \frac{1}{vg^2 + h(h - ug)} \begin{bmatrix} -h & g \\ -gv & gu - h \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

until convergence occurs. This method to find the zeroes of polynomials can thus be easily implemented with a programming language or even a spreadsheet.

Read more about this topic: Bairstow's Method

Famous quotes containing the words *description of the, description of, description* and/or *method*:

> "Do not require a *description of the* countries towards which you sail. The description does not describe them to you, and to- morrow you arrive there, and know them by inhabiting them."
> —Ralph Waldo Emerson (1803–1882)

> "Once a child has demonstrated his capacity for independent functioning in any area, his lapses into dependent behavior, even though temporary, make the mother feel that she is being taken advantage of....What only yesterday was a *description of* the child's stage in life has become an indictment, a judgment."
> —Elaine Heffner (20th century)

> "The next Augustan age will dawn on the other side of the Atlantic. There will, perhaps, be a Thucydides at Boston, a Xenophon at New York, and, in time, a Virgil at Mexico, and a Newton at Peru. At last, some curious traveller from Lima will visit England and give a *description* of the ruins of St. Paul's, like the editions of Balbec and Palmyra."
> —Horace Walpole (1717–1797)

> ""I have usually found that there was *method* in his madness."

"Some folk might say there was madness in his *method*."
—Sir Arthur Conan Doyle (1859–1930)

Source(s):

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