Comparing the Results of Different Voting Schemes

Arik Rundquist, Jay Upadhyaya, Jihan Song

ajr4322, jau464, js97924

The University of Texas at Austin

M 362K – Probability I

Dr. Mark Maxwell

28 April 2022

Table of Contents

Abstract

The majority opinion of a group of people is most easily obtained through the use of voting schemes. However, there are many different voting methods to choose from, and each method has its benefits and drawbacks. In this project, a survey about chocolate bars and methods of transportation in Austin was distributed to a group of students at the University of Texas at Austin. The survey asked for the voters' preference through three different voting schemes: Ranking (Relative), Approval (Like / Dislike), and Scaled Approval (0 to 10). Each scheme introduced the same scenario, but with a different way of recording the result. These results were then summarized into a series of probability models to compare the effect of the scheme on the winning candidates. The analysis of these models show that a change in voting schemes can have subtle, yet significant impacts on the results of the survey.

Keywords: voting schemes, fairness, probability distribution function, cumulative distribution function, discrete random variable, binomial random variable.

Introduction

In recent years, the integrity of the underlying election systems has been called into question. Whether it be from accusations of voter fraud or claims of foreign interference, it is clear that there is a growing concern about the fairness of the systems in place to facilitate the collecting and measuring votes. While particular issues surrounding interference or fraud can only be mitigated by increased security surrounding the infrastructure involved in vote collection, the issue of fairness is an interesting one. In a traditional voting scheme, each voter may vote for only one candidate; to vote for an option is equivalent to voting against all the others, meaning it is a zero-sum game. This is what is currently employed in American elections for positions in government as part of the electoral college (Ray, n.d.) . It is simple to implement and to interpret. However, it is not the only voting scheme; other methodologies exist to determine which options a population prefer, with each system having their own unique strengths and failings.

In this work, we examine several voting systems, asking participants to vote on the same candidates in multiple ways. We analyze the results, looking specifically for instances where the scheme used appears to alter the results of the vote. We conclude with a discussion of fairness and how to "fairly" determine which scheme should be used in a particular circumstance.

Schemes

We collected votes using three unique schemes. First, we asked for candidates to be ranked; second, we asked for voters to vote for or against each candidate independently; third, we asked for voters to rank how strongly they favored each candidate on a scale from zero to ten (*Alternative Voting Systems,* 2020).

1. Ranking

   This scheme allows for a top k analysis of candidates. For the case when $k = 1$, this is equivalent to a traditional voting scheme. For $k = 2$, we determine how frequently each candidate was either the first or second choice of the voter, etc. Lower values of k better represent the strongest feelings of the voters, while higher values may better capture overall population sentiments; however, results tend to become less significant as k increases. Finding an optimal value of k requires a significant amount of judgment and justification. A top k analysis should consider how results shift across values of k.

2. Approval

   The motivation for this scheme is best explained by a hypothetical example. Suppose three individuals are running for office. Candidate A and Candidate B are each staunchly supported by 40% of the population. In a traditional vote, Candidate C would be left with only 20% of the total votes. Suppose, however, that half of the individuals who voted for Candidate A or Candidate B would also be okay with Candidate C winning the vote; in this case, we would find that 40% of the population supported Candidate A, 40% supported Candidate B, and 60% supported Candidate C. This example demonstrates a case where the candidate with the fewest votes in a traditional system might actually satisfy the most people.

3. Scaled Approval

This scheme allows voters to indicate how strongly they support a particular option. It is similar to approval voting in that it allows voters to cast multiple votes, but whereas approval voting only presents voters with a binary decision, this scheme captures additional information as voters can weight some of their votes more than others.

Methodology

We first developed a survey through Google Forms with two scenarios and multiple options for each. To compare the various voting schemes, we gave the voters three different systems to select their preferred options: ranked, approval, and scaled. In the first section, we gathered the voters' opinions on candy bars by utilizing the voting schemes mentioned above. The second section had the same format, with the voters reporting their most frequently used methods of transportation in Austin. To disperse these surveys, we sent the link in various UT group chats, including class and social group chats. We obtained 32 unique data points from this approach and moved on to data compilation.

The results from the Google form were placed into a spreadsheet, which we used to analyze the voters' opinions and how they were impacted by the voting scheme at hand. We then converted some of the results from words into numbers to convert the results into valid probability distributions (such as converting a response "most frequent" to $X = 1$, "second most frequent" to $X = 2$, etc… where X is the random variable representing the rank of the option, see Fig. 1.6-1.11, 2.6-2.10). For each scheme, we calculated the measures of central tendency and spread.

Analysis

1. Chocolate Bars

In a traditional voting scheme (top k where $k = 1$), Reese's Cups are the clear winner (see Fig. 1.5), and at $k = 2$, Reese's continues to dominate the votes. This indicates that many people prefer Reese's over any other chocolate bar in the study. However, at $k = 3$, Snickers overtakes Reese's, and by $k = 5$, Reese's has fallen behind Snickers, Twix, and Milky Way. This indicates that while there is a large group that prefers Reese's over other chocolates, for the remainder of the voters it ranks poorly. In contrast, it seems that although few people would choose Snickers or Twix as their favorite candy bar, many people still like them more than some other chocolate bars, causing these two bars to rank towards the upper-middle.

In particular, examination of Fig. 1.3 shows that more people like Twix than like any other chocolate. Unlike the ranking system, the approval system does not track individual's preferences of one option over another; instead, it shows general population approbation. As Twix received the maximum number of votes under this system, we can conclude that the majority of people would be content with Twix, although, considering the results from the Top K experiment, few would receive their favorite chocolate bar.

These observations are supported by the data presented in Fig. 1.4. The scores for Reese's are heavily skewed, with the majority of scores being very high but featuring a large spread. In contrast, the scores for Twix were less skewed and thus tended to be slightly under the scores for Reese's. Of all the chocolates, Twix had the least variation, with the smallest range and standard deviation (Fig. 1.1).

We note that while the results for the $k = 1$ experiment and the approval voting experiment differ in their indication of the most popular chocolate bar (Reese's and Twix,

respectively), choosing $k = 3$ and comparing with the approval votes and scaled votes indicates that Reese's, Snickers, and Twix are consistently the top choices of the sample population, although their relative ordering can differ.

Interestingly, 3 Musketeers and Almond Joys each had large standard deviations, with Almond Joys also having a large interquartile range. This suggests that whereas other chocolates receive fairly consistent scores, these two have much more variability in people's perception of them, with some people strongly liking them and others disliking them.

2. Modes of Transportation

For the purposes of examining how voting schemes can affect the outcome of the vote, this experiment was not as useful as the chocolate bar experiment. Walking is overwhelmingly the most common mode of transportation, with driving a clear second. Given that our sample drew heavily from other students on campus, this result is not surprising. While the method used in this experiment did not affect the two most common modes of transportation, it does affect the ordering of the lowest three – biking, taking the bus, and scootering. As such, we will consider only this subset of data to determine how the voting scheme could affect the outcome of a poll to determine the most frequently used mode of public transportation.

More people responded that they frequently biked compared to those that responded that they frequently took the bus or rode a scooter (see Fig. 2.3). However, the expected value of the score for the frequency of taking the bus is significantly larger than that of biking or scootering (3.29 compared to biking, which has an expected score of 2.48, see Fig. 2.1). This corresponds to taking the bus receiving significantly more votes than biking or scootering under the ranking system at $k = 3$ (note that since the vast majority of votes went to walking and driving at $k = 1$

and $k = 2$, using $k = 3$ approximates using $k = 1$ with those two modes of transportation removed, see Fig. 2.5). Thus, the method of voting used does impact the results of this poll.

Connection to Course Content

1. The Measures of Central Tendency and Dispersion

We allocated a basic concept of the measures of central tendency - mean, median, and mode - to rank preferred chocolate bars and methods of transportation in Austin based on each voting scheme. With the measures of central tendency, we used dispersion, mainly standard deviation, to add the additional factor of measuring popularity. This is due to the fact that even though the rank is slightly higher, if it was the result of high variability, it would likely not be the best option to represent the population.

2. Discrete Random Variable

Both the ranked and scaled voting schemes can be represented by a discrete random variable. The interval of the options given to the voters in the ranked system were [1,6] for chocolates and [1,5] for the transportation method (one rank per candidate). For the scaled system, the interval of the discrete random variable was [0,10], with 0 being the least favorable choice and 10 being the most favorable choice.

3. Binomial Random Variable

The approval scheme can be represented by a binomial random variable. For every option, it could be either 0 (dislike / infrequent) or 1 (like / frequent). Scaled approval could be represented by a binomial random variable by dividing the interval [0,10] into two sections: [0,4] being 0, [6,10] being 1, and discounting 5 as a neutral value.

"Fairness"

It is difficult to make a general statement about the fairness of each of the tested voting systems. Fairness is a subjective area; however, using the measure of central tendency and dispersion, we tried to suggest “generally liked” options with compelling reasons and explanations. The point of this project is not to come up with the best voting scheme but rather to suggest that 1) each voting scheme might result in different candidates winning, and 2) each voting scheme has different advantages.

We propose that the voting method used should depend upon the objectives of those running the poll:

1. If the purpose is to satisfy as many people as possible, the approval system should be used. Some examples include:
    a. Maxwell wants to know which chocolate bar he could bring to class as a prize that most students would want.
    b. Deciding which day of the week works best to meet up and work on an inquiry project.
2. If the purpose is to prioritize various decisions, the rank system should be used. Options can then be ordered by average rank, or a couple values of k can be considered. Some examples include:
    a. Determining the order in which features should be added to a software product based on customer preference data.
    b. You and your friends are rewatching your favorite episodes of The Office and need to pick the order to watch them in.

3. If the purpose is to decide what will provide the most value to a population, scaled voting should be used. Some examples where this scheme would be appropriate are:
    a. Deciding whether increasing the availability of bikes, buses, or electric scooters around campus would provide the most value to students.
    b. UT wants to decide which courses from the course catalog to offer next semester from the results of a student poll.

Limitations

After we conducted a survey, we realized that asking people to rank candidates first and then evaluate each independently could easily affect their evaluations. After building a mental model of how the various candidates are related, it is hard to think of them separately. We believe this could lead to more extreme (both positive and negative) evaluations in the scaled approval section. Another limitation was the relatively small and non-diverse sample, as we only polled students at UT. As a result, these findings may not be generalizable to the entire population other than students.

Applications and Future Works

These findings have numerous practical applications at both local and national levels. As concerns over the fairness of the elections and electoral college are rising, future research done on this very subject could lead to a shift in the way votes are accounted for in elections for government officials. Even minute differences in the results from the change in voting schemes used can have drastic impacts in the results of a wide-scale application such as an election, which

further highlights the importance of selecting the right scheme that leads to the most fair outcome.

The schemes presented in this work are not comprehensive. Many schemes could be constructed. In the future, we would like to examine a system in which voters distribute k votes among the candidates. This is similar to the scaled system, but imposes additional constraints on the voter, which might result in greater numerical stability of the results of such a vote. We also could test a party-list system ("Party List", 2017), in which the voters choose nominees for each party, and then choose a winner from the list of all party candidates. This is somewhat similar to a tournament-style bracket system. If this study was to be continued, then testing these different voting schemes can help show how voting between subgroups affect the results of the overall vote.

Reflection

Throughout the process of this project, we were able to create our own statistical model in a context with real-world connections. While collecting data, analyzing the differences in voting schemes with probability concepts, and determining the fairness of each scheme, we had to continuously ask ourselves if the ideas and data points on the paper were valid for the purposes of the study. One example of this was the removal of the walking and driving options from the results of the transportation study, as we knew that this data would likely skew the results. Asking these questions helped us to refine our goal with the research and ultimately allow us to develop a better overall study and analysis.

Data Book

References

"Alternative Voting Systems." NCSL, National Conference of State Legislatures, 25 June 2020, https://www.ncsl.org/research/elections-and-campaigns/alternative-voting-systems.aspx.

Cuff, Paul, et al. "Voting Research - Voting Theory." Princeton University, The Trustees of Princeton University, 6 Aug. 2021, https://www.princeton.edu/~cuff/voting/theory.html.

"Party List Proportional Representation." Electoral Reform Society ERS, 2017, https://www.electoral-reform.org.uk/voting-systems/types-of-voting-system/party-list-pr/.

Ray, Michael. "How Does the Electoral College Work?" *Encyclopædia Britannica*, Encyclopædia Britannica, Inc., https://www.britannica.com/story/how-does-the-electoral-college-work.