Indexing and Query Processing (Inverted Index)

CS4422/7263 Information Retrieval
Lecture 05

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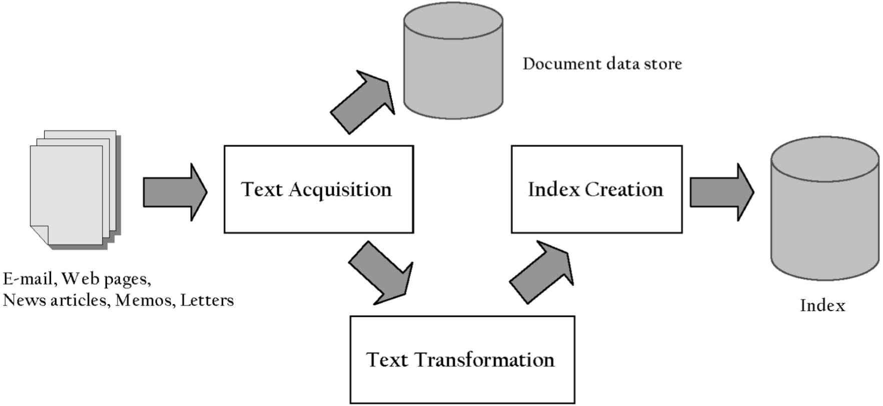
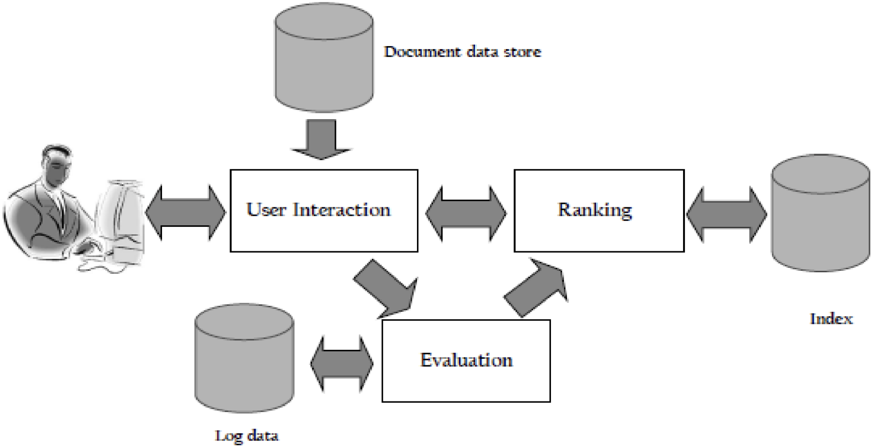
1. **Indexing Process**
    - Motivation
    - Inverted Index
    - Index Compression
    - Index Construction
2. **Query Processing**

The Indexing Process

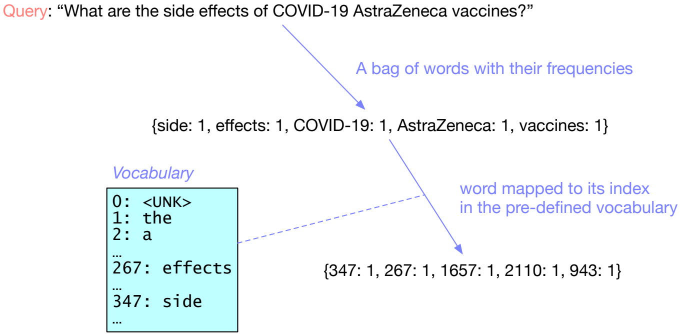


The Querying Process



Where are we now?

- We **crawled** webpages from the Web
- We **extracted** documents (body text) from the webpages
- We **tokenized and normalized** the documents
- Now, we can **represent** a document as a Bag-of-Words



Query: “What are the side effects of COVID-19 AstraZeneca vaccines?”

A bag of words with their frequencies

{side: 1, effects: 1, COVID-19: 1, AstraZeneca: 1, vaccines: 1}

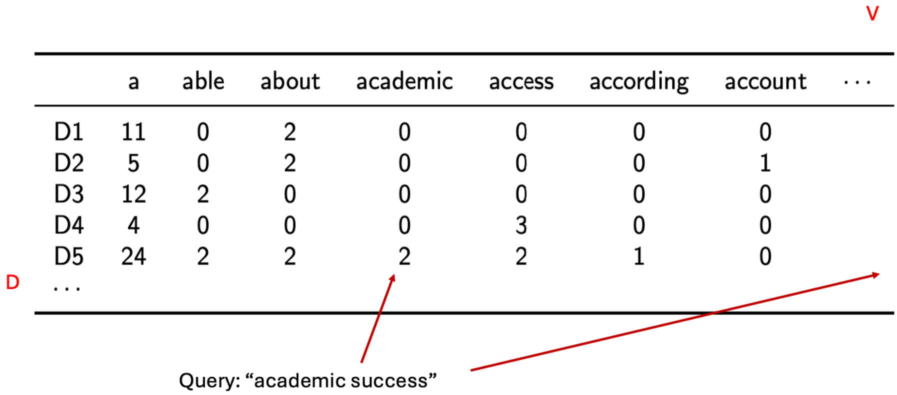
Vocabulary

```
0: <UNK>
1: the
2: a
…
267: effects
…
347: side
…
```

word mapped to its index
in the pre-defined vocabulary

{347: 1, 267: 1, 1657: 1, 2110: 1, 943: 1}

Document-Term Matrix



	a	able	about	academic	access	according	account	· · ·
D1	11	0	2	0	0	0	0	
D2	5	0	2	0	0	0	1	
D3	12	2	0	0	0	0	0	
D4	4	0	0	0	3	0	0	
D5	24	2	2	2	2	1	0	
· · ·								

Query: “academic success”

Vectore Representations for Documents

- A document is represented as a vector of term frequencies

$$d = (4, 0, 0, 0, 3, 0, \ldots, tf_{|V|})$$

- A query can also be represented as a vector of term frequencies

$$q = (0, 0, 0, 1, 0, 0, \ldots, tf_{|V|})$$

- The matching terms between the query and the documents are used to compute a similarity score
- *dot product* is a common similarity measure

$$sim(q, d) = q \cdot d = \sum_{i=1}^{|V|} q_i \cdot d_i$$

Complexity Analysis (Time)

```
1  rel_docs = list()
2  for qt in q:
3      for d in C:
4          for w in d:
5              if qt == w:
6                  rel_docs.append(d)
7  return rel_docs
```

- $O(q \times |D| \times l)$
  - where q is the average length of queries, and
  - l is the average length of documents
- line 3 is the bottleneck, where $|D|$ can be huge

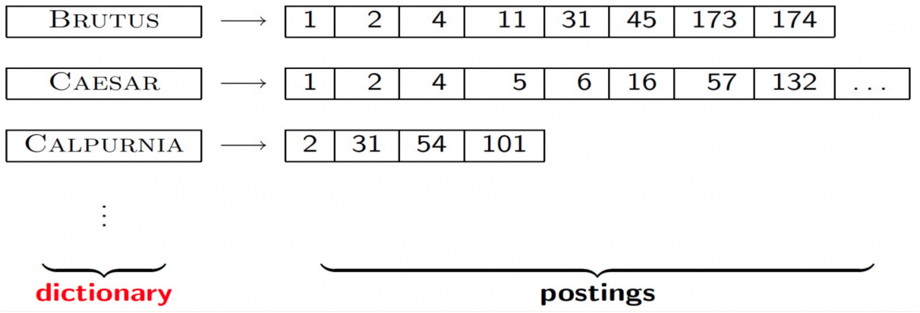
Complexity Analysis (Space)

- $O(|D| \times |V|)$
  - where V is the vocabulary size
- Zipf's law: Each document contains only a small fraction of the vocabulary
- Space efficiency can be greatly improved by storing only the non-zero entries of the document-term matrix
- Any Solution??

1. **Indexing Process**
   - Motivation
   - Inverted Index
   - Index Compression
   - Index Construction

2. **Query Processing**

Solution: Use Inverted Index



Brutus ⟶ 1 | 2 | 4 | 11 | 31 | 45 | 173 | 174

Caesar ⟶ 1 | 2 | 4 | 5 | 6 | 16 | 57 | 132 | . . .

Calpurnia ⟶ 2 | 31 | 54 | 101

⋮

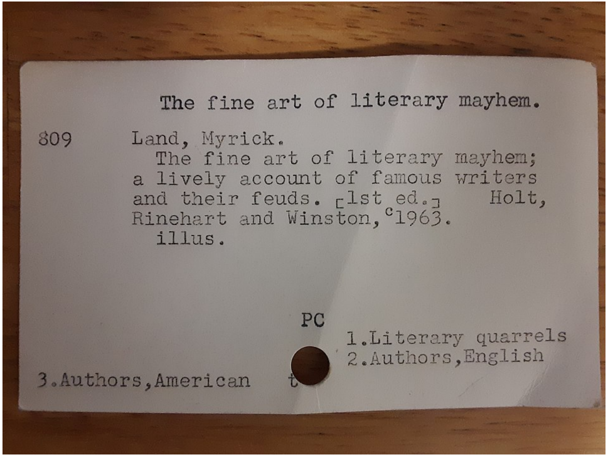
- **Inverted Index** is the term-to-documents mapping
- **Dictionary** is the data structure for storing the term vocabulary
- **Posting** is a list that records which documents the term occur in

Library Catalog

How to fill in a stack application form
Book number
Name and Address of reader
Classified Catalogue

Figure: The card catalog in a library

Library Catalog



The fine art of literary mayhem.

809 Land, Myrick.
The fine art of literary mayhem; a lively account of famous writers and their feuds. [1st ed.] Holt, Rinehart and Winston, ^{c}1963.
illus.

PC

1.Literary quarrels
2.Authors,English
3.Authors,American t

Figure: An title index card

Example Document Collection

S_1 Tropical fish include fish found in tropical environments around the world, including both freshwater and salt water species.

S_2 Fishkeepers often use the term tropical fish to refer only those requiring fresh water, with saltwater tropical fish referred to as marine fish.

S_3 Tropical fish are popular aquarium fish, due to their often bright coloration.

S_4 In freshwater fish, the coloration typically derives from iridescence, while salt water fish are generally pigmented.

Four sentences from the Wikipedia entry for *tropical fish*

Forward Index

S_1 tropical, fish, include, found, in, environments, around, the, world, ...

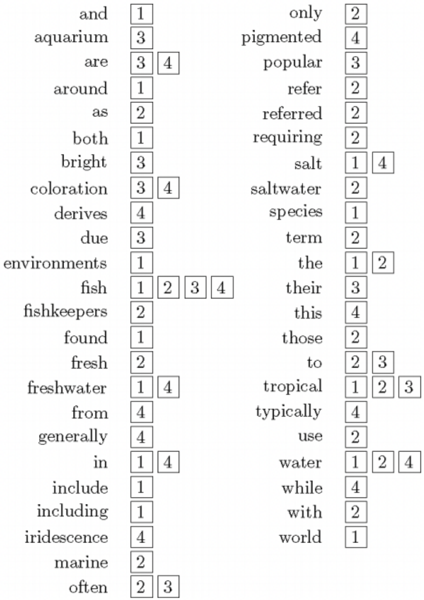
S_2 fishkeepers, often, use, the, term, tropical, to, fish, refer, only, ...

S_3 tropical, fish, are, popular, aquarium, fish, due, to, their, often, ...

S_4 in, freshwater, fish, the, coloration, typically, derives, from, iridescence, ...

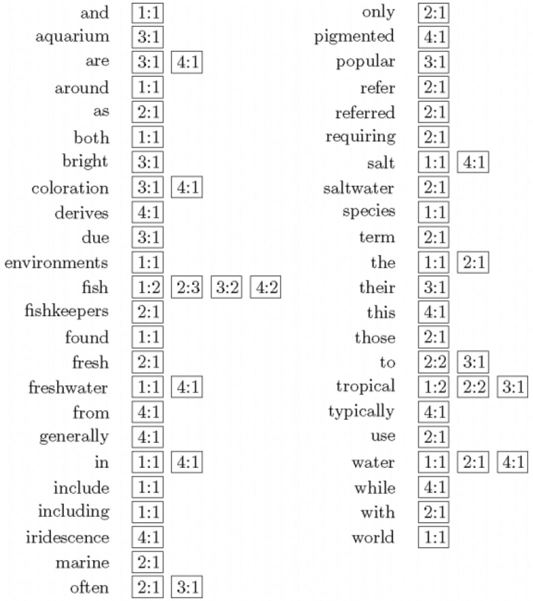
documents (sentences) and their terms

- **Inverted-Index**, an inversion of the forward index
- Simple inverted index for the documents (sentences)



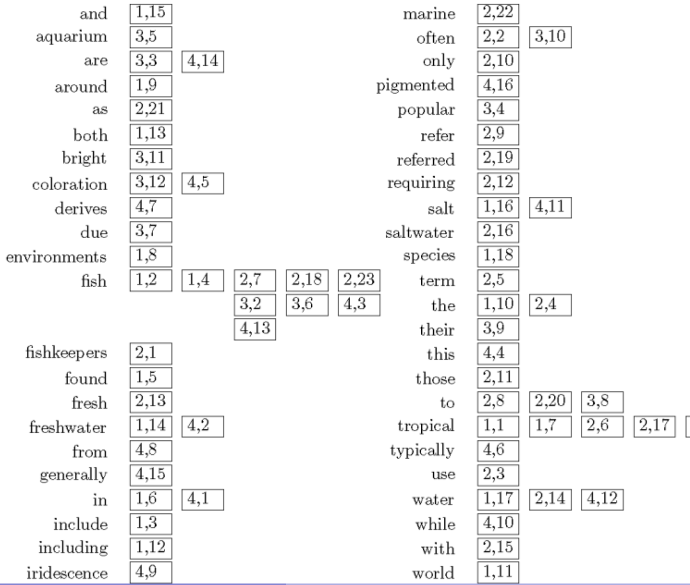
and	1
aquarium	3
are	3 4
around	1
as	2
both	1
bright	3
coloration	3 4
derives	4
due	3
environments	1
fish	1 2 3 4
fishkeepers	2
found	1
fresh	2
freshwater	1 4
from	4
generally	4
in	1 4
include	1
including	1
iridescence	4
marine	2
often	2 3
only	2
pigmented	4
popular	3
refer	2
referred	2
requiring	2
salt	1 4
saltwater	2
species	1
term	2
the	1 2
their	3
this	4
those	2
to	2 3
tropical	1 2 3
typically	4
use	2
water	1 2 4
while	4
with	2
world	1

- Inverted index with counts (**term frequencies**) supports better ranking algorithms



and	1:1			
aquarium	3:1			
are	3:1	4:1		
around	1:1			
as	2:1			
both	1:1			
bright	3:1			
coloration	3:1	4:1		
derives	4:1			
due	3:1			
environments	1:1			
fish	1:2	2:3	3:2	4:2
fishkeepers	2:1			
found	1:1			
fresh	2:1			
freshwater	1:1	4:1		
from	4:1			
generally	4:1			
in	1:1	4:1		
include	1:1			
including	1:1			
iridescence	4:1			
marine	2:1			
often	2:1	3:1		
only	2:1			
pigmented	4:1			
popular	3:1			
refer	2:1			
referred	2:1			
requiring	2:1			
salt	1:1	4:1		
saltwater	2:1			
species	1:1			
term	2:1			
the	1:1	2:1		
their	3:1			
this	4:1			
those	2:1			
to	2:2	3:1		
tropical	1:2	2:2	3:1	
typically	4:1			
use	2:1			
water	1:1	2:1	4:1	
while	4:1			
with	2:1			
world	1:1			

- term/document frequencies can be inferred
- more features for better ranking



and	1,15
aquarium	3,5
are	3,3 4,14
around	1,9
as	2,21
both	1,13
bright	3,11
coloration	3,12 4,5
derives	4,7
due	3,7
environments	1,8
fish	1,2 1,4 2,7 2,18 2,23 3,2 3,6 4,3 4,13
fishkeepers	2,1
found	1,5
fresh	2,13
freshwater	1,14 4,2
from	4,8
generally	4,15
in	1,6 4,1
include	1,3
including	1,12
iridescence	4,9
marine	2,22
often	2,2 3,10
only	2,10
pigmented	4,16
popular	3,4
refer	2,9
referred	2,19
requiring	2,12
salt	1,16 4,11
saltwater	2,16
species	1,18
term	2,5
the	1,10 2,4
their	3,9
this	4,4
those	2,11
to	2,8 2,20 3,8
tropical	1,1 1,7 2,6 2,17
typically	4,6
use	2,3
water	1,17 2,14 4,12
while	4,10
with	2,15
world	1,11

Data Structure — Dictionary

- Modest size
- Stay in memory
- Choices of data structure: **Hash, B-Tree, B+Tree...**
- Some IR systems use hashes, some trees
- Criteria in choosing *hash* or *search tree*
  - Fixed number of terms or keep growing
  - Relative frequencies with which various keys are accessed
  - How many terms

Data Structure — Postings

- Sequential access is expected
- Stay on disk
- Information to store: docID, term frequency, positions, ...
- Likely to be huge: Compression is needed

Hashes

- Each vocabulary term is hashed to a unique integer
- **Pros**:
  - Lookup is fast: $\mathcal{O}(1)$
- **Cons**:
  - No easy way to find all terms starting with prefix (e.g., 'automat')
  - Need to rehash when the dictionary grows

Search Trees — Binary Tree

- Binary tree (simple), B-tree, B+tree (more usual)
- **Pros**:
  - Solves the prefix problem (finding all terms starting with ‘automat’)
- **Cons**:
  - Slower: $\mathcal{O}(\log M)$ [and this requires balanced tree]
    - ★ M is called the order of the tree, that is the number of allowed children per node
  - Rebalancing binary trees is expensive
  - But B+trees mitigate the rebalancing problem

B+tree[1]

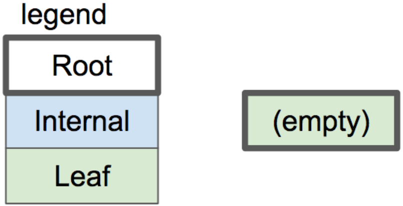
- *B+tree* is a good data structure for indexes:
  - Searching for a particular value is fast (logarithmic time)
  - Inserting / deleting a value you’ve already found is fast
  - Traversing a range of values is fast (unlike a hash map)
- Unlike a binary tree, each node in a B+Tree can have more than 2 children.
- Each node can have up to M children

B+Tree Properties

- Every node has at most M children.
- Every (internal) node, except for the root and the leaves, has at least $\lceil M/2 \rceil$ children.
- The root node has at least two children unless it is a leaf.
- All leaves appear on the same level.
- A non-leaf node with k children contains $k - 1$ keys.

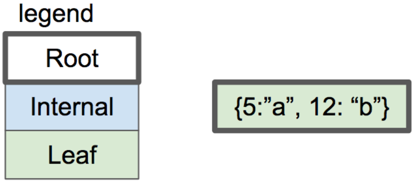
B+tree example

- Let's take an example to see how a B-tree grows. To keep things simple, the tree will be order 3 ($M = 3$). That means:
- An empty B+tree has a single node: the root node.
- The root node starts as a leaf node with zero key/value pairs:



B+tree example

- If we insert a couple key/value pairs, they are stored in the leaf node in sorted order.
- Two items are added. Let’s insert a new item (1: “c”), where 1 is the key and “c” is the value.



legend

Root

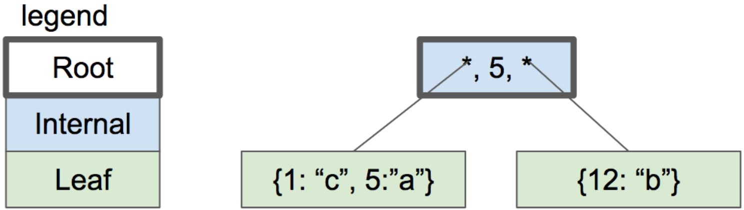
Internal

Leaf

{5:”a”, 12: “b”}

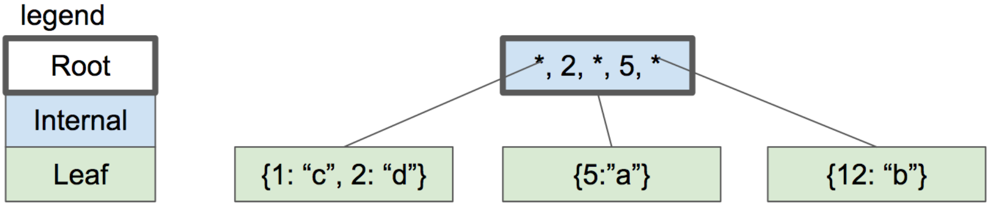
B+tree example

- When we insert another, we have to split the leaf node and put half the pairs in each node.
- Let's insert a new item (2: "d")



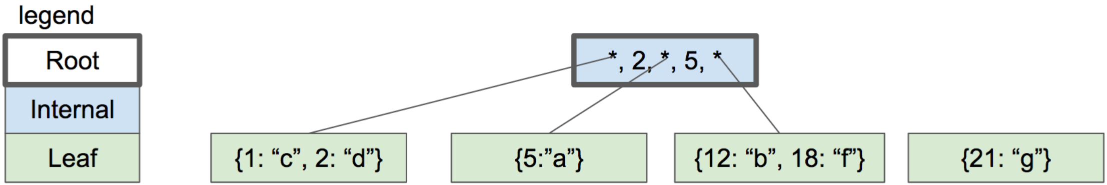
B+tree example

- Now let's insert the key 2. First we look up which leaf node it would be in if it was present, and we arrive at the left leaf node. The node is full, so we split the leaf node and create a new entry in the parent node.
- Now, we are adding items with keys 18 and 21.



B+tree example

- We get to the point where we have to split again, but there’s no room in the parent node for another key/pointer pair.



legend

Root

Internal

Leaf

*, 2, *, 5, *

{1: “c”, 2: “d”}

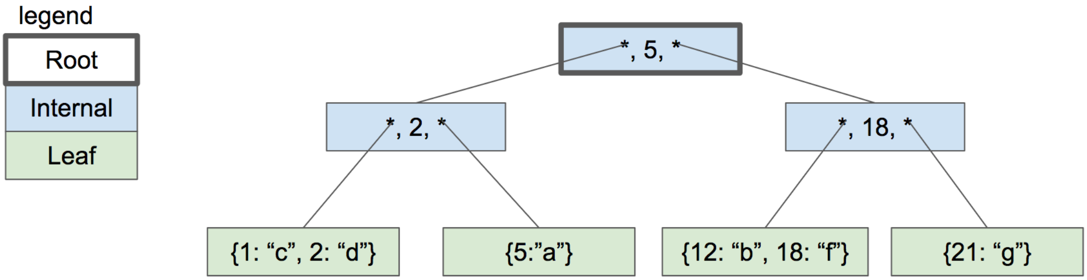
{5:”a”}

{12: “b”, 18: “f”}

{21: “g”}

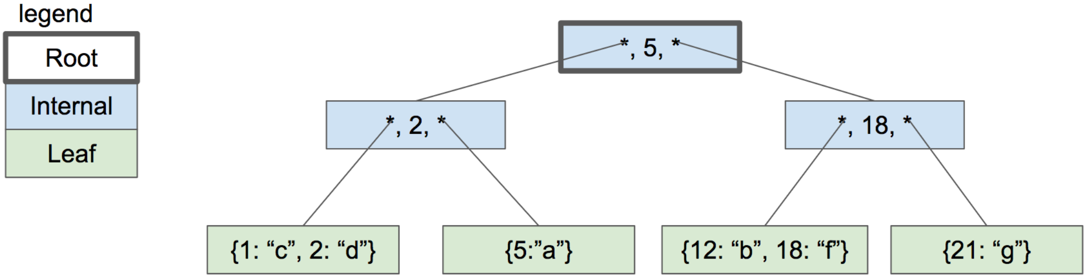
B+tree example

- The solution is to split the root node into two internal nodes, then create new root node to be their parent.



B+tree example

- The depth of the tree only increases when we split the root node.
- Every leaf node has the same depth and close to the same number of key/value pairs, so the tree remains balanced and quick to search.



1. **Indexing Process**
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2. **Query Processing**

Reuters RCV1 Statistics

Statistics	Value
documents (N)	800,000
avg. # tokens per doc (L)	200
vocabulary size (M) (# terms)	400,000
avg. # bytes per token (incl. spaces/punct.)	6
avg. # bytes per token (without spaces/punct.)	4.5
avg. # bytes per term	7.5
non-positional postings	100,000,000

Reuters RCV1 Statistics (Cont.)

	Dictionary	Non-positional Index	Positional Index
Unfiltered	484,494	109,971,179	197,879,290
Numbers removed	473,723	100,680,242	179,158,204
Case folding	39,523	96,969,056	179,158,204
Stopwords removed (30)	391,493	83,390,443	121,857,825
Stopwords removed (50)	391,373	67,001,847	94,516,599
Stemming	322,383	63,812,300	94,516,599

Size in tokens (= number of position entries in postings)

Index Compression

- The Postings file is LARGE
  - For the most common term ‘the’, the posting likely to have $|D|$ entries
- We need to store each posting compactly
- For Reuters ($|D| = 800,000$), we would use 32 bits per docID when using 4-byte integers
- Alternatively, we can use $\log_2 800,000 \approx 20$ bits per docID.
- *Our goal is to use far fewer than 20 bits per docID*

Posting Compression — Intuition

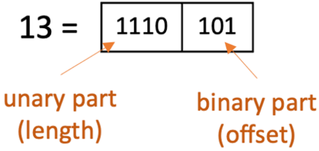
- Posting is a docID list in ascending order
- Instead of using docID, we can store the gaps between docIDs
- Zipf's law:
  - More frequent word: a long list with smaller gaps
  - Less frequent word: a short list with longer gaps
- Use fewer bits for encoding small (high frequency) integers
  - **Variable-length coding**
    - ★ Unary code
    - ★ Gamma γ-code
    - ★ Delta δ-code

Unary Code

- Represent n as n 1s with a final 0.
- Unary code for 3 is 1110.
- Unary code for 40 is 11111111111111111111111111111111111111110.
- This doesn't look promising, but...
  - Optimal if $P(n) = 2^{-n}$

Gamma Code

- Bit-level codes
- Represent a gap G as a pair of length and offset
- Offset is G in binary, with the leading bit cut off
  - For example, 13 →1101 →101
- Length is the length of offset
  - For 13 (offset is 101), the length is 3
- *Why do we need a length code?*
- *Why is it okay to remove the leading bit?*



13 = 1110 | 101

unary part (length)

binary part (offset)

Gamma Code (Cont.)

- How many bits are needed to encode a gap G?
  - To encode the offset: $\lfloor \log_2 G \rfloor$ bits
  - To encode the length: $\lfloor \log_2 G \rfloor + 1$ bits
  - Total: $2 \times \lfloor \log_2 G \rfloor + 1$ bits
- That implies, the coding scheme is optimal for $P(n) \approx \frac{1}{2n^2}$
- Cons:
  - Bit-level; machines have word boundaries
  - Schemes using byte or word aligned code are favorable

Variable Byte (VB) Code

- Begin with one byte to store G and dedicate 1 bit in it to be a continuation bit c (*Similar to the Unicode coding scheme*)
- If $G \leq 127$, binary-encode it in the 7 available bits and set $c = 1$
- Else encode G's lower-order 7 bits and then use additional bytes to encode the higher order bits using the same algorithm
- At the end, set the continuation bit of the last byte to 1

docIDs	**824**	**829**	**215406**
gaps	-	5	214577
VB code (binary)	00000101 00000000	10000101	00001101 00001100 10110001
VB code (hexadecimal)	06 B8	85	0D 0C D1

Reuters RCV1 Statstics

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avg. # bytes per token (incl. spaces/punct.)	6
avg. # bytes per token (without spaces/punct.)	4.5
avg. # bytes per term	7.5
non-positional postings	100,000,000
postings, uncompressed (32-bit words)	400 MB
postings, uncompressed (20-bits $= \log_2 1M$)	250 MB
postings, Gamma encoded	101 MB
postings, variable byte encoded	116 MB

Delta Encoding

- Delta encoding is a way of storing or transmitting data in the form of **differences (deltas)** between sequential data.
- **Assuming that the docids, positions, etc. in our lists are *small and repetitive,***
- We can utilize the encoding schemes with differences between the numbers to reduce the index size.

Byte-aligned Code

- In production systems, inverted lists are stored using byte-aligned codes for delta-encoded integer sequences.
- For better computing performance, our index lists can be encoded using byte-aligned code schemes (e.g., VB).

Putting it All Together

- Let's compress an inverted index postings with delta encoding.
- The raw postings contain a sequence of tuples as `(docid, tf, [pos1, pos2, ...])`

  `(1,2,[1,7]), (2,3,[6,17,197]), (3,1,[1])`
- We **delta-encode** the docid and position sequences independently.

  `(1,2,[1,6]), (1,3,[6,11,180]), (1,1,[1])`
- Finally, we encode the integers using VByte.

  `81 82 81 86 81 83 86 8B 01 B4 81 81 81`

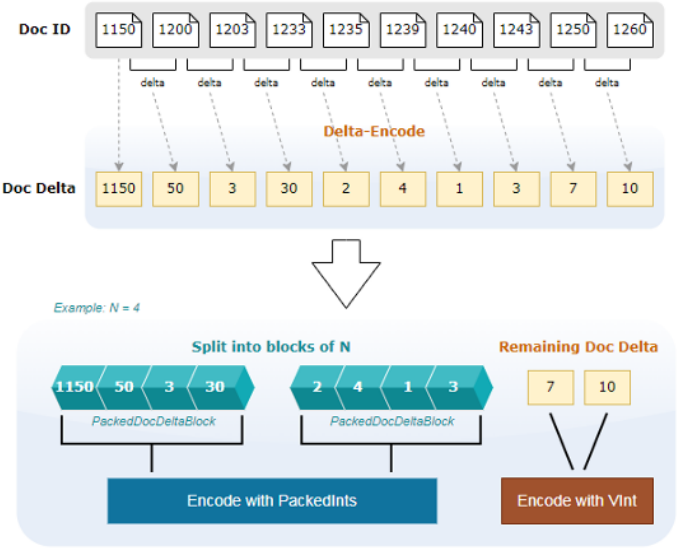

Figure: Lucene's compression method utilizing PackedDocDeltaBlock of 128 deltas. In this figure N=4 for simplicity

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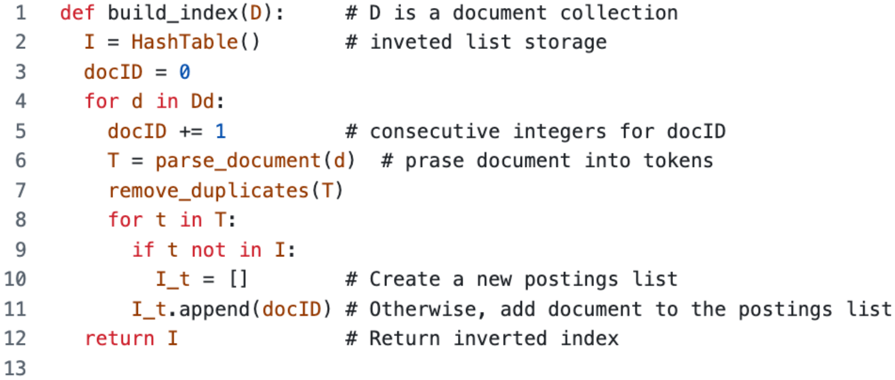
Challenges of Indexing

Index construction (or indexing) is the process of building an inverted index from a corpus.

1. Document (postings) size exceeds the memory limit
   - Say, one posting entry (termID, docID) requires 8 bytes; $100,000,000 \times 8/1024^2$ is near 800 Mb
   - Typical document collections are much larger
2. While indexing, we parse docs one at a time; final postings for a term are incomplete until the end.

We need to store intermediate results on disk.

Simple Indexer



```
def build_index(D):     # D is a document collection
  I = HashTable()       # inveted list storage
  docID = 0
  for d in Dd:
    docID += 1          # consecutive integers for docID
    T = parse_document(d)  # prase document into tokens
    remove_duplicates(T)
    for t in T:
      if t not in I:
        I_t = []        # Create a new postings list
      I_t.append(docID) # Otherwise, add document to the postings list
  return I              # Return inverted index

```

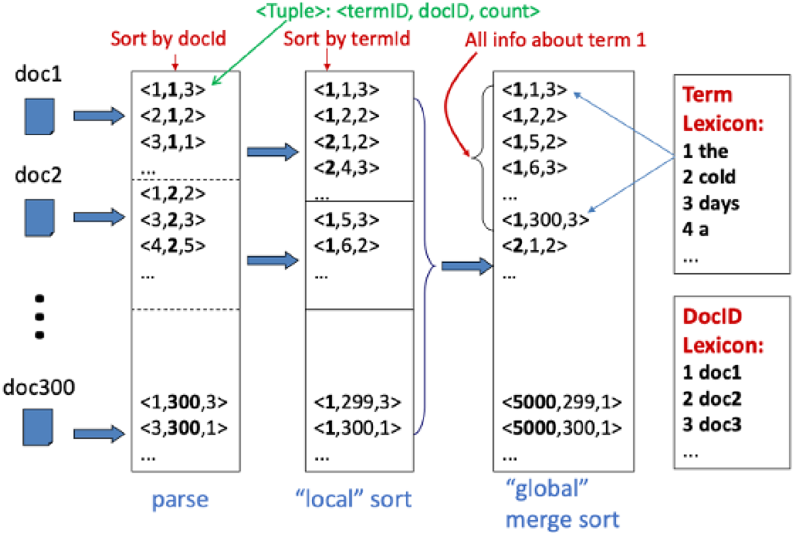
Sort using disk as “memory”?

- Can we use the same index construction algorithm for larger collections, *but by using disk instead of memory?*
  - No: Sorting T = 100,000,000 records on disk is too slow - too many disk seeks.
- We need an external sorting algorithm.

External Sorting?

External Sorting is a class of sorting algorithms that can handle massive amounts of data. In the sorting phase, chunks of data small enough to fit in main memory are read, sorted, and written out to a temporary file. In the merge phase, the sorted subfiles are combined into a single larger file.

Merge Sort



<Tuple>: <termID, docID, count>

Sort by docId

Sort by termId

All info about term 1

doc1

doc2

doc300

<1,1,3>
<2,1,2>
<3,1,1>
...
<1,2,2>
<3,2,3>
<4,2,5>
...
<1,300,3>
<3,300,1>
...

parse

<1,1,3>
<1,2,2>
<2,1,2>
<2,4,3>
...
<1,5,3>
<1,6,2>
...
<1,299,3>
<1,300,1>
...

"local" sort

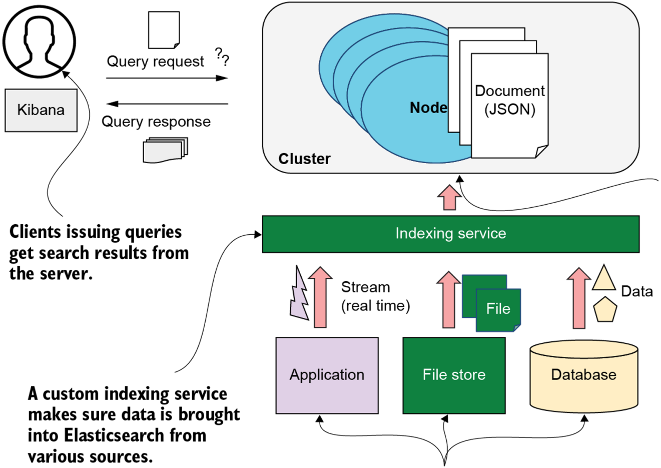
<1,1,3>
<1,2,2>
<1,5,2>
<1,6,3>
...
<1,300,3>
<2,1,2>
...
<5000,299,1>
<5000,300,1>
...

"global" merge sort

Term Lexicon:
1 the
2 cold
3 days
4 a
...

DocID Lexicon:
1 doc1
2 doc2
3 doc3
...

ElasticSearch (ES) Indexing/Querying Pipeline



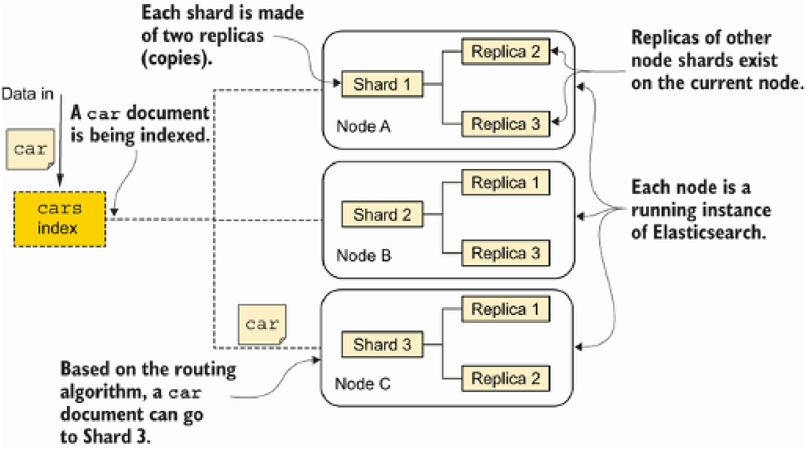
Clients issuing queries get search results from the server.

Data is analyzed and stored by the engine during indexing.

A custom indexing service makes sure data is brought into Elasticsearch from various sources.

Data flows into Elasticsearch from multiple sources.

ElasticSearch (ES) Indexes



Each shard is made of two replicas (copies).

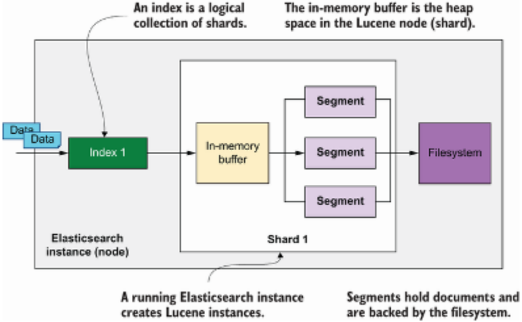
A car document is being indexed.

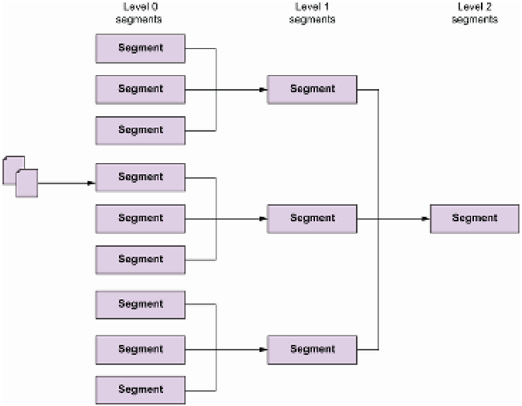
Replicas of other node shards exist on the current node.

Each node is a running instance of Elasticsearch.

Based on the routing algorithm, a car document can go to Shard 3.

Mechanics of Indexing


An index is a logical collection of shards.
The in-memory buffer is the heap space in the Lucene node (shard).
Data
Data
Index 1
In-memory buffer
Segment
Segment
Segment
Filesystem
Elasticsearch instance (node)
Shard 1
A running Elasticsearch instance creates Lucene instances.
Segments hold documents and are backed by the filesystem.



Lucene's Index

- Lucene index file formats

1. Indexing Process
   - Motivation
   - Inverted Index
   - Index Compression
   - Index Construction

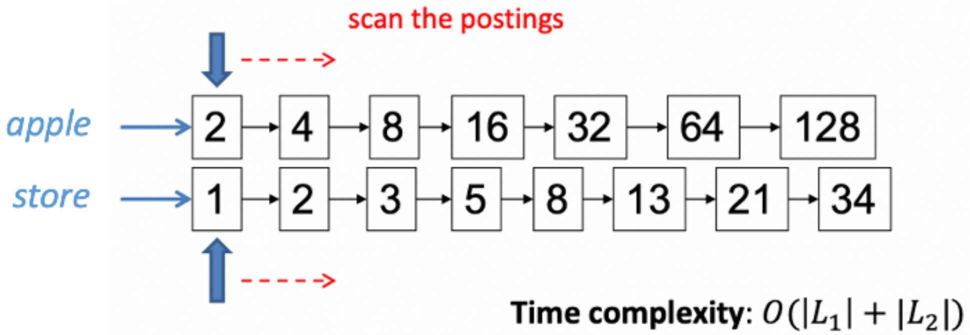
2. **Query Processing**

Search with an inverted index

- Parse query syntax
  - e.g., `Apple AND store, orange OR apple`
- Perform the same text processing procedures as on documents
  - **tokenization** (e.g., John's →John + 's)
  - **normalization** (e.g., U.S.A. →usa)
  - **stemming** (e.g., beautiful →→ beauti)
  - **stopwords** (e.g., remove the, a, to, ...)

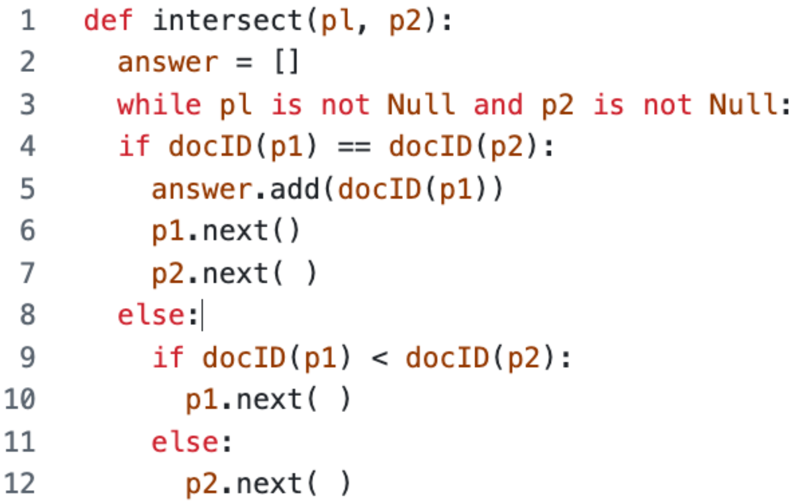
Query, Conjunction

AND example: *apple AND store*



Time complexity: $O(|L_1| + |L_2|)$

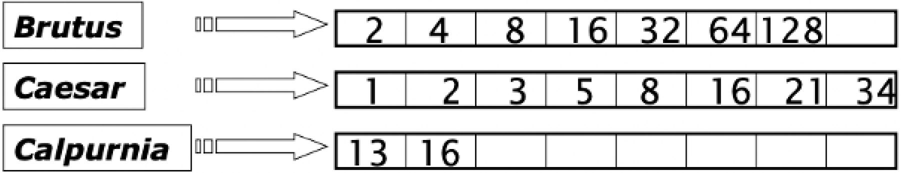
Query, Conjunction (pseudo code)



```
def intersect(pl, p2):
  answer = []
  while pl is not Null and p2 is not Null:
  if docID(p1) == docID(p2):
    answer.add(docID(p1))
    p1.next()
    p2.next( )
  else:
    if docID(p1) < docID(p2):
      p1.next( )
    else:
      p2.next( )
```

Query Optimization

- Consider a query that is an “AND” of n terms, for example,
- **q:** `(Brutus AND Calpurnia AND Caesar)`



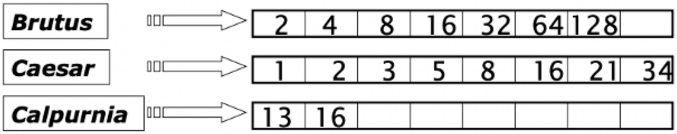
Brutus → 2 4 8 16 32 64 128

Caesar → 1 2 3 5 8 16 21 34

Calpurnia → 13 16

- For each of the n terms, get its postings, then “AND” them together
- **What is the best order for query processing?**

Query Optimization (Cont.)



Brutus 2 4 8 16 32 64 128

Caesar 1 2 3 5 8 16 21 34

Calpurnia 13 16

- Process in order of increasing term frequency (i.e., posting list lengths) can help increase the speed of merging posting lists.
- Start with the smallest set, then keep cutting further; This is why we keep document frequencies in our dictionary.

(**Brutus** AND **Calpurnia** AND **Caesar**)

optimize → (**Calpurnia** AND **Brutus**) AND **Caesar**

Query Optimization — General Rules

- Query: `(london OR beijing) AND (train OR plane)`
- Get document frequencies for all terms
- Estimate the size of each OR by the sum of its doc. freq.'s (conservative)
- Process in increasing order of OR sizes.

Phrase Queries

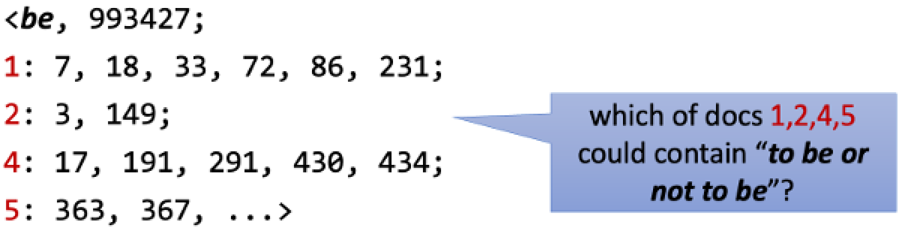
- For example, we want to be able to answer queries such as “`Apple Store`” as a phrase.
- Thus, the sentence “Which **store** should I go to buy an **apple**?” is not supposed to be a match.
- n-grams generally do not work for this.
- Large dictionary size, how to break long phrase into n-grams?
- For this, we need to store term positions in the postings lists.

(proof of concept) Positional Indexes

In the postings, store, for each term the position(s) in which tokens of it appear:

```
<term, document frequency;
doc1: position1, position2, ... ;
doc2: position1, position2, ...>
```

Positional indexes — example



```
<be, 993427;
1: 7, 18, 33, 72, 86, 231;
2: 3, 149;
4: 17, 191, 291, 430, 434;
5: 363, 367, ...>
```

which of docs 1,2,4,5 could contain “***to be or not to be***”?

- For phrase queries, we use a merge algorithm *recursively at the document level* (not the term level).
- But we now need to deal with more than just equality.

Processing a Phrase Query

- Extract inverted index entries (i.e., postings) for each distinct term: **to, be, or, not.**
- Merge their *doc:position* lists to enumerate all positions with "to be or not to be".
  - to: 2:1,17,74,222,551; 4:8,16,190,429,433; 7:13,23,191; ...
  - be: 1:17,19; 4:17,191,291,430,434; 5:14,19,101; ...
  - {to, be, or, not}: ... 4:8,16,17,190,191,291,429,430,431,432,433,434; ...
- If doc4 contains the phrase, you'll see a sequence like 429, 430, 431, 432, 433, 434 from the merged *doc:position* list.
- Generally, the same strategy can be used for proximity search.

Processing a Phrase Query (Cont.)

- `(employment place)~3`
  - Here, ~k means "within k words of (on either side)"
  - Clearly, positional indexes can be used for such queries
- Equality condition: $|t_2.\text{pos} - t_1.\text{pos}| = 1$
  - t_1 must be immediately before or after t_2 in any matched document
- Inequality condition: $|t_2.\text{pos} - t_1.\text{pos}| \leq k$
  - t_1 and t_2 must be positioned within k words

Processing a Phrase Query (Cont.)

- A positional index expands postings storage substantially.
  - Even though indices can be compressed.
- Nevertheless, a *positional index is now standardly used because of the power and usefulness of phrase and proximity queries*, whether used explicitly or implicitly in a ranking retrieval system.
- A positional index is 2–4 times as large as a non-positional index.
- Positional index size is 35–50% of the volume of the original text.

Summary

- Questions?