

Text Classification

CS4422/7263 Information Retrieval

Lecture 08

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- 1 **Supervised learning**
 - Applications of Text Classification
 - Classification Methods
- 2 Text Representations
- 3 Rocchio Classification
- 4 k Nearest Neighbor Classification
- 5 Decision Tree Algorithm

Text classification

- **Classification** (also called categorization) is a ubiquitous enabling technology in data science; studied within pattern recognition, statistics, and machine learning.
- Definition: The activity of assigning a predefined class (or category) to a data item belongs to.
- Formulated as the task of **generating a hypothesis** (or “classifier” or “model”)

$$h : D \rightarrow C,$$

where $D = x_1, x_2, \dots$ is a domain of data items and $C = c_1, \dots, c_n$ is a finite set of classes (the **classification scheme**, or codeframe).

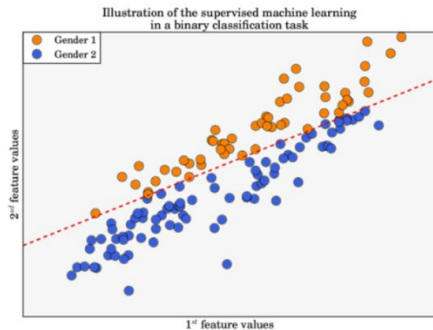
Text classification — what is and is not

- Different from clustering, where the groups ("clusters") and the number of which are not known in advance
- Classification always involves a **subjective judgment**; the membership of a data item into a class must not be determinable with certainty.
 - ▶ E.g., predicting whether a natural number belongs to Prime or NonPrime is not a classification task.
- In text classification, data items are **textual** (e.g., news articles, emails, tweets, product reviews, sentences, questions, queries, etc.) or partly textual (e.g., Web pages).

Main types of classification in machine learning

Binary classification

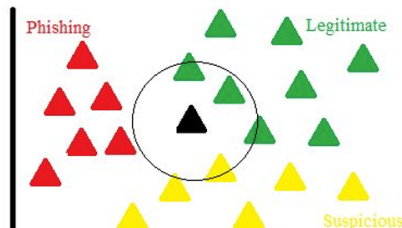
- Tasks that have two classes: {True, False}, {Positive, Negative}, etc.
 - ▶ E.g., Assigning emails to one of {Spam, Legitimate}
- Suitable algorithms:
 - ▶ Logistic regression
 - ▶ Support vector machine (SVM)



Main types of classification in machine learning

Single-label Multi-class classification

- $h : D \rightarrow C$ (each item belongs to exactly one class) and $C = \{c_1, \dots, c_n\}$ with $n > 2$
 - ▶ E.g., Assigning news articles to one of {Home, News, International, Entertainment, Lifestyles, Sports}
- The number of classes can be very large on some problems. (e.g., biomedical entity classification, $|C| > 30,000$)
- Suitable algorithms:
 - ▶ k-Nearest Neighbors
 - ▶ Decision Trees
 - ▶ Naïve Bayes
 - ▶ Random Forest



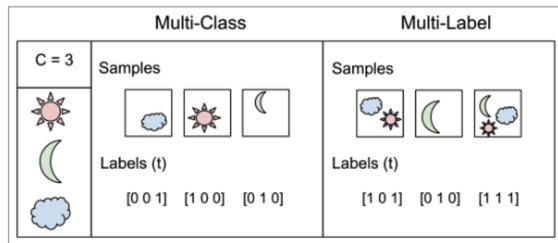
Main types of classification in machine learning

Multi-label Multi-class classification

- $h : D \rightarrow 2^C$ (each item may belong to zero, one, or several classes) and $C = \{c_1, \dots, c_n\}$ with $n > 2$
 - ▶ E.g., Assigning computer science articles to the classes in the ACM classification system

- Suitable algorithms:

- ▶ Decision Trees
- ▶ Random Forests
- ▶ Gradient Boosting



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Application 1: Knowledge Organization

- Long tradition in both science and the humanities; goal was organizing knowledge, i.e., conferring structure to an otherwise unstructured body of knowledge
- The rationale is that using a **structured body of knowledge** is easier / more effective than if this knowledge is unstructured
- **Automated classification** tries to automate the tedious task of assigning data items based on their content, a task otherwise performed by human annotators (a.k.a. “assessors”, or “coders”)

Application 1: Knowledge Organization (cont'd)

- Examples;
 - ▶ Classifying news articles for selective dissemination.
 - ▶ Classifying scientific papers into specialized taxonomies .
 - ▶ Classifying patents.
 - ▶ Classifying topic-related tweets by sentiment.
 - ▶ ...
- Retrieval (as in search engines) could also be viewed as (binary) classification into *Relevant vs. NonRelevant*.

Application 2: Filtering

- In IR, **Filtering** refers to the activity of blocking a set of NonRelevant items from a dynamic stream, thereby leaving only the Relevant ones.
 - ▶ E.g., **Spam filtering**, attempting to tell legitimate messages from Spam messages.
 - ▶ Detecting unsuitable content (e.g., porn, violent content, racist content, cyberbullying, fake news) is also an important application.
- Filtering is an example of binary classification.
- Collaborative filtering in recommendation systems.

Application 3: Empowering other IR tasks

- Functional to improving the effectiveness of other tasks in IR or NLP; e.g.,
 - ▶ Classifying queries by intent within search engines
 - ▶ Classifying questions by type in question-answering systems
 - ▶ Classifying named entities
 - ▶ Word sense disambiguation in NLP systems
 - ▶ Sentiment analysis
 - ▶ ...
- Many of these tasks involve classifying very small texts (e.g., queries, questions, sentences), and stretch the notion of “text” classification quite a bit.

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Classification Methods 1

Manual Classification

- Used by the original Yahoo! Directory
- Looksmart, About.com, OpenDataPlane (ODP), PubMed
- Accurate when job is done by experts
- Consistent when the problem size and team is small
- Difficult and expensive to scale
 - ▶ We need automatic classification methods for big problems.

Classification Methods 2

Feature Engineering

- Hand-coded rule-based classifiers.
- One technique used by news agencies, intelligence agencies, etc..
- Widely deployed in government and enterprise.
- Vendors provide "IDE" for writing such rules.
- Commercial systems have complex query languages.
- Accuracy can be high if a rule has been carefully refined over time by a subject expert.
- Building and maintaining these rules is expensive.
- E.g., ('longer' AND 'harder' AND 'stronger') → Spam

Classification Methods 3: Supervised learning

Supervised Learning Approach

- A generic (task-independent) learning algorithm is used to train a classifier from a set of manually classified examples
- From the training examples, the model learns the textual characteristics which belongs to a class
- Advantages:
 - ▶ Annotating training examples is cheaper than writing classification rules
 - ▶ Easy update to changing conditions (e.g., addition of new classes, deletion of existing classes, shifted meaning of existing classes, etc.)

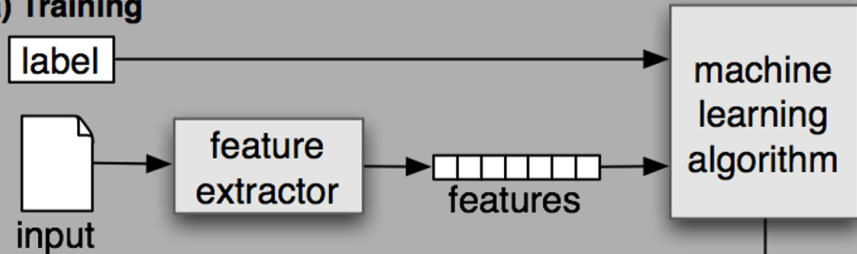
Classification Methods 3: Supervised learning

Supervised Learning Approach

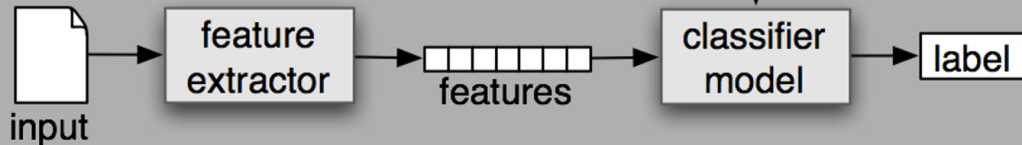
- Methods:
 - ▶ Naïve Bayes (simple, common)
 - ▶ k-Nearest Neighbors (simple, powerful)
 - ▶ Support-vector machines (newer, generally more powerful)
 - ▶ Decision trees and Random forests
 - ▶ Neural networks
- No free lunch: need hand-classified training data
- Many commercial systems use a mix of methods

Supervised learning for classification

(a) Training



(b) Prediction



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Representing text for classification purposes

- In order to be input to a learning algorithm (or a classifier), all training (or unlabeled) documents are converted into **vectors** in a vector space model.
- The dimensions of the vector space are called **features** (or terms, or covariates), and **the number K of features** used is called the dimensionality of the vector space.
- In order to generate a vector-based representation for a set of documents D , the following steps need to be taken.
 - 1 Feature Design and Extraction
 - 2 (Feature Selection or Feature Synthesis)
 - 3 Feature Weighting

Representing text for classification purposes

- For topic classification, a typical choice is to make the set of features coincide with the **set of words** that occur in the training set
 - ▶ Unigram model (i.e., bag-of-words), bigram model, character n-grams
 - ▶ With a unigram model, the dimensionality K is the number of words
- Depends on the task, other features can be utilized:
 - ▶ Author, URL, email address, punctuation, document length ...
- The choice of features for a classification task (feature design) is dictated by the distinctions we want to capture, and is left to the designer

Feature selection

- Number of features can easily exceed 10^5 , esp. if word n-grams are used, consequently causing "overfitting" and high computational cost.
- **Feature selection** is to identify the most discriminative features, so that other non-informative features can be discarded
- We can use **Mutual Information (MI)** to filter out the less contributing features.
- MI is very important notion in information theory.
- It measures **the expected "amount of information" held in a random variable**; MI can be thought of as the reduction in uncertainty about one random variable given knowledge of another.

Mutual Information (MI)

- Intuitively, MI measures the information that X and Y share;
- High MI indicates a large reduction in uncertainty;
- Low MI indicates a small reduction;
- and zero mutual information between two random variables means the variables are independent.

$$MI(t_k, c_i) = \sum_{c \in \{c_i, \bar{c}_i\}} \sum_{t \in \{t_i, \bar{t}_i\}} P(t, c) \log_2 \frac{P(t, c)}{P(t)P(c)}$$

Mutual Information (MI) (Cont.)

- In [search engine technology](#), we want to select the features that maximizes the discriminative ability.
- E.g., Pointwise Mutual Information (PMI) score for bigrams to find the keywords in a context

$$PMI(w_1, w_2) = \log_2 \frac{P(w_1, w_2)}{P(w_1)P(w_2)}$$

Feature Synthesis

- **Matrix decomposition techniques** (e.g., PCA, SVD, LSA) can be used to synthesize new features
- These techniques are based on the *principles of distributional semantics*, which states that the semantics of a word can be defined by the words that co-occur within the context
 - ▶ *"You shall know a word by the company it keeps"*
 - ▶ **Word embeddings** is an example of dimension reduction using the distributional semantics via a "deep learning" approach

Simple

a document $d \rightarrow x$

Naïve Bayesian Method for Text Classification

$$P(y|x) = \frac{P(x|y)P(y)}{P(x)}$$

a class $c \rightarrow y$

Naïve Bayes Classifier

$$C_{MAP} = \arg \max_{c \in C} P(c|d)$$

MAP is “Maximum a Posteriori” = most likely class

$$= \arg \max_{c \in C} \frac{P(d|c)P(c)}{P(d)}$$

Bayes Rule

$$= \arg \max_{c \in C} \frac{P(d|c)P(c)}{P(d)}$$

Given a document, $P(d)$ is the same over all classes

$$= \arg \max_{c \in C} P(x_1, x_2, \dots, x_n | c) P(c)$$

Document d represented as features x_1, x_2, \dots, x_n

Naïve Bayes Classifier

$$C_{MAP} = \arg \max_{c \in C} P(x_1, x_2, \dots, x_n | c) P(c)$$

- Assumptions:

- ▶ Bag of Words: Assume position doesn't matter
- ▶ Conditional Independence: Assume the feature probabilities

$$P(x_1, x_2, \dots, x_n | c) = P(x_1 | c) P(x_2 | c) \cdots P(x_n | c)$$

$$C_{NB} = \arg \max_{c \in C} P(c) \prod_{x \in d} P(x | c)$$

MLE in Naïve Bayes Classifier

$$C_{NB} = \arg \max_{c \in C} P(c) \prod_{x \in d} P(x|c)$$

- For estimating the factors, we simply use the frequencies in the data.

- $\hat{P}(c_j) = \frac{\text{doc_count}(C = c_j)}{N}$

- $\hat{P}(w_i|c_j) = \frac{\text{count}(w_i, c_j)}{\sum_{w \in V} \text{count}(w, c_j)}$

- ▶ $\hat{P}(w_i|c_j)$ is the fraction of times the word w_i appears among all words in documents of topic c_j

Smoothing

- Zero probabilities must be smoothed.
- $\forall i, \hat{P}(w_i|c_j) \neq 0$
- **Laplace (add-1) smoothing**

$$\begin{aligned}\hat{P}(w_i|c_j) &= \frac{\text{count}(w_i, c_j)}{\sum_{w \in V} \text{count}(w, c_j)} \\ &= \frac{\text{count}(w_i, c_j) + 1}{\sum_{w \in V} (\text{count}(w, c_j) + 1)} \\ &= \frac{\text{count}(w_i, c_j) + 1}{\sum_{w \in V} \text{count}(w, c_j) + |V|}\end{aligned}$$

Naïve Bayes: Learning

- ① From training corpus, build a vocabulary.
- ② Calculate $\hat{P}(c_j)$ terms:
 - ▶ For each c_j in C Do
 - ★ $D_j \leftarrow$ all docs with class c_j
 - ★ $\hat{P}(c_j) \leftarrow |D_j|/N$
- ③ Calculate $\hat{P}(w_i|c_j)$ terms:
 - ▶ $T_j \leftarrow$ single doc containing all docs in D_j
 - ▶ For each word w_i in the vocabulary,
 - ★ $n_i \leftarrow$ # of occurrences of w_i in T_j
 - ★ $\hat{P}(w_i|c_j) \leftarrow (N_i + \alpha) / (n + \alpha|V|)$

NB Classifier — Example

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w|c) = \frac{\text{count}(w,c)+1}{\text{count}(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Priors:

$$P(c) = \frac{3}{4}$$

$$P(j) = \frac{1}{4}$$

Choosing a class:

$$P(c|d5) \propto \frac{3}{4} * \left(\frac{3}{7}\right)^3 * \frac{1}{14} * \frac{1}{14} \approx 0.0003$$

Conditional Probabilities:

$$P(\text{Chinese}|c) = (5+1) / (8+6) = 6/14 = 3/7$$

$$P(\text{Tokyo}|c) = (0+1) / (8+6) = 1/14$$

$$P(\text{Japan}|c) = (0+1) / (8+6) = 1/14$$

$$P(\text{Chinese}|j) = (1+1) / (3+6) = 2/9$$

$$P(j|d5) \propto \frac{1}{4} * \left(\frac{2}{9}\right)^3 * \frac{2}{9} * \frac{2}{9} \approx 0.0001$$

Naïve Bayes Classifier – Analysis

- **Advantages**

- ▶ fast and low storage requirements
- ▶ works well with multi-class prediction problems
- ▶ If the independence assumption holds, works better than other models with less training data
- ▶ With many equally important features (e.g., categorical input variables), performs better in comparison to numerical variables

- **Disadvantages**

- ▶ For "zero frequency" cases, a smoothing technique is required
- ▶ Even though, it's a probability estimate, the outputs can't be directly used for the prediction probability.
- ▶ It assumes that all the features are independent; conditional independence assumption is violated by real-world data.

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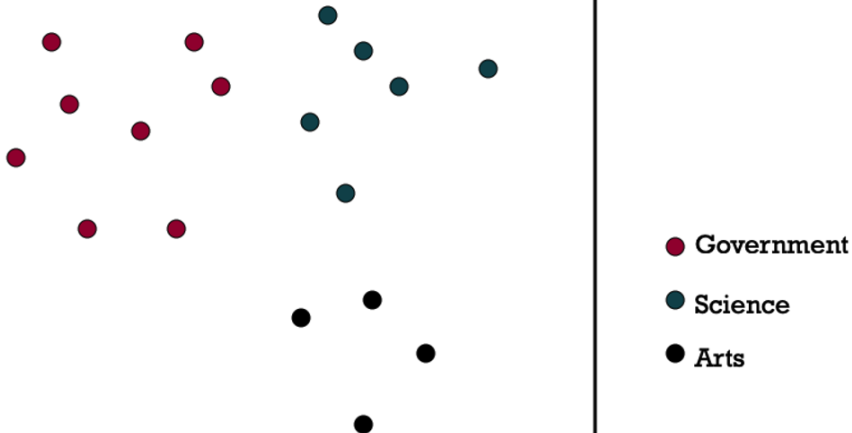
Remember: Vector Space Representation

- Each document is a vector, one component for each term (=word)
- Normally, normalize vectors to unit length.
- High-dimensional vector space:
 - ▶ Terms are axes
 - ▶ 10,000+ dimensions, or even 100,000+
 - ▶ Docs are vectors in this space
- How can we do classification in this space?

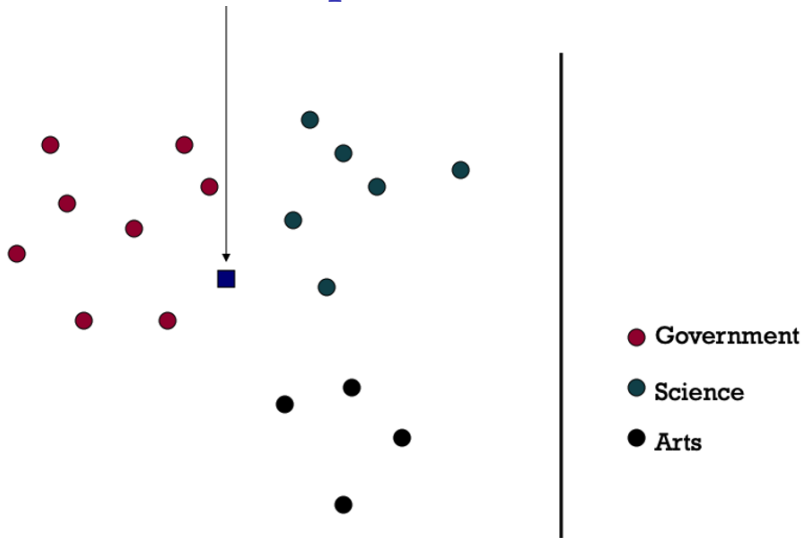
Classification using Vector Spaces

- In vector space classification,
 - ▶ training set corresponds to a labeled set of points (equivalently, vectors)
 - ▶ **Premise 1:** Documents in the same class form a contiguous region of space (i.e., a cluster)
 - ▶ **Premise 2:** Documents from different classes don't overlap (much)
- Learning a classifier is to **build surfaces to delineate classes in the space**

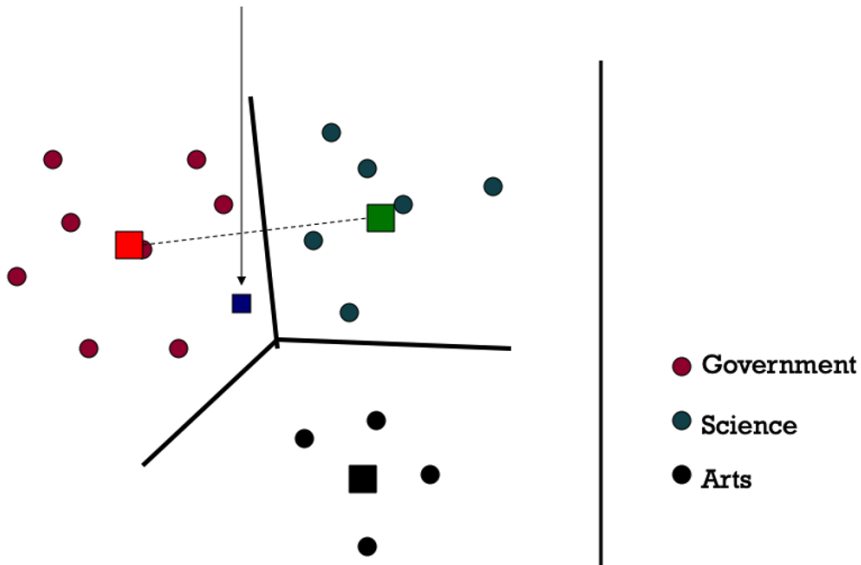
Documents in a Vector Space



Documents in a Vector Space



Documents in a Vector Space



Definition of centroid

$$\vec{\mu}(c) = \frac{1}{|D_c|} \sum_{d \in D_c} \vec{v}(d)$$

- Where D_c is the set of all documents that belong to class c and $v(d)$ is the vector space representation of d .
- Note that centroid will in general not be a unit vector even when the inputs are unit vectors.

- The boundary between two classes in Rocchio classification is **the set of points with equal distance from the two centroids**.

- $|a_1| = |a_2|$
- $|b_1| = |b_2|$
- $|c_1| = |c_2|$

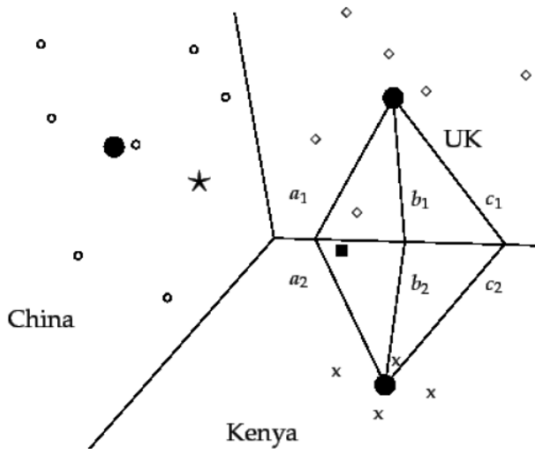


Figure 14.3: Rocchio classification.

- The boundary line (or hyperplane) in M-dimensional space is the set of points that satisfy:

Boundary between two classes

- The boundary line (or hyperplane) in M-dimensional space is the set of points that satisfy:

$$\vec{w}^T \vec{x} = b$$

- \vec{w} is the M-dimensional normal vector of the hyperplane and b is a constant, such that
 - ▶ $\vec{w} = \vec{\mu}(c_1) - \vec{\mu}(c_2)$
 - ▶ $b = \frac{1}{2} (|\vec{\mu}(c_1)|^2 - |\vec{\mu}(c_2)|^2)$
 - ▶ how?
- A line divides a plane in two, a plane divides 3-dimensional space in two, and hyperplanes divide higher-dimensional spaces in two.
- Basically, the Rocchio classifier is to determine $\vec{\mu}(c)$ that the point is closest to and then assign it to c .

Rocchio classification – Pseudocode

```
def train_rocchio(C, D):  
    mu = []  
    for c in C:  
        n = 0  
        for d in c:  
            n += 1  
            mu_c += vec(d)  
        mu_c = mu_c / n  
        mu.append(mu_c)  
  
def apply_rocchio(mu, d):  
    return argmin(dist(mu, d)) # or argmax(cos(mu, d))
```

Rocchio classification — Example

docs	term weights (tf.idf)					
	Chinese	Japan	Tokyo	Macao	Beijing	Shanghai
d1	0	0	0	0	1.0	0
d2	0	0	0	0	0	1.0
d3	0	0	0	1.0	0	0
d4	0	0.71	0.71	0	0	0
d5	0	0.71	0.71	0	0	0
mu_c	0	0	0	0.33	0.33	0.33
mu_j	0	0.71	0.71	0	0	0

$$(1 + \log_{10} tf_{t,d}) \log_{10}(4/df_t)$$

• The separating hyperplane has the following parameters:

Multi-class classification

How can we perform multi-class classification using a linear classifier, when $C = \{C_1, C_2, \dots, C_k\}$ and $k > 2$? There are two main solutions:

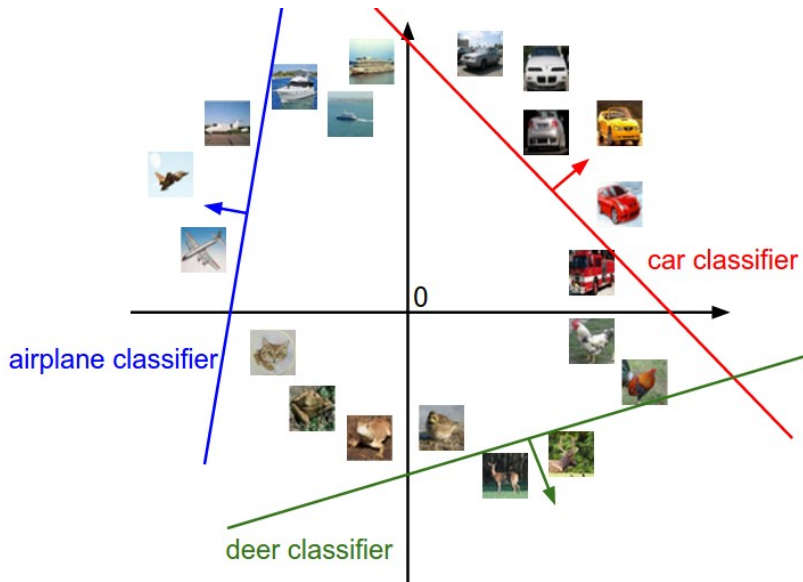
- **One-vs-All**

- ▶ Train a binary (linear) classifier for each class.
- ▶ For example,
 - ★ C_1 vs. C_2, \dots, C_k
 - ★ C_2 vs. C_1, C_3, \dots, C_k
 - ★ C_k vs. C_1, C_2, \dots, C_{k-1}
- ▶ If multiple classes are predicted for a single example, choose the one with highest confidence level.

- **One-vs-One**

- ▶ Train a classifier for each pair of classes:
- ▶ For example,
 - ★ C_1 vs. C_2
 - ★ C_1 vs. C_3 ,
 - ★ ...
- ▶ A majority vote is then performed to find the correct class.

Multi-class Classification



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k Nearest Neighbor Classification

- kNN
- To classify a document d :
 - ▶ Define k -neighborhood as the k nearest neighbors of d
 - ▶ Pick the majority class label in the k -neighborhood
 - ▶ For larger k can roughly estimate $P(c|d)$ as $\#(c)/k$

k Nearest Neighbor Learning

- Learning: just store the labeled training examples D
- Testing instance x (under 1NN)
 - ▶ Compute similarity between x and all examples in D .
 - ▶ Assign x the category of the most similar example in D .
- Does not compute anything beyond storing the examples
- Also called:
 - ▶ Case-based learning
 - ▶ Memory-based learning
 - ▶ Lazy learning
- Rationale of kNN: contiguity hypothesis
 - ▶ “Documents in the same class form a contiguous region, and regions of different classes do not overlap.”

k Nearest Neighbor Learning

- Using only the closest example (1NN) is subject to errors due to:
 - ▶ A single atypical example
 - ▶ Noise (i.e., an error) in the category label of a single training example
- More robust: find the k examples and return the majority category of these k .
- k is typically odd to avoid ties; 3 and 5 are most common

Nearest Neighbor with Inverted Index

- Naively finding nearest neighbors requires a linear search through $|D|$ documents in collection.
- But determining k nearest neighbors is the same as determining the k **best retrievals** using the test document as a query to a database of training documents.
- Use standard vector space **inverted index** methods to find the k nearest neighbors
- Testing Time: $O(B|V_t|)$ where B is the average number of training documents in which a test-document word appears.
- Typically $B \ll |D|$.

kNN: Discussion

- No feature selection necessary
- No training necessary
- Scales well with large number of classes
 - ▶ Don't need to train n classifiers for n classes
- Classes can influence each other
 - ▶ Small changes to one class can have ripple effect
- Done naively, very expensive at test time
- In most cases, it's more accurate than NB or Rocchio
 - ▶ As the amount of data goes to infinity, it has to be a great classifier! — it's "Bayes optimal"

Bias vs. Capacity

- Consider asking a botanist: Is an object a tree?
 - ▶ High capacity, Low bias
 - ★ Botanist who memorizes
 - ★ Will always say "no" to new object (e.g., different # of leaves)
 - ▶ Low capacity, high bias
 - ★ Lazy botanist
 - ★ Says "yes" if the object is green
 - ▶ We want the middle ground

kNN vs. Naïve Bayes

- Bias/Variance tradeoff (Variance \sim Capacity)
- kNN has **high variance** and **low bias**
 - ▶ Infinite memory
- Rocchio/NB has **low variance** and **high bias**
 - ▶ Linear decision surface between classes

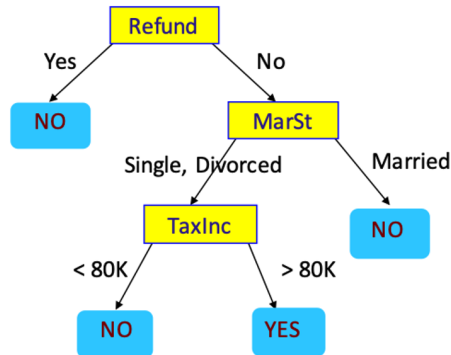
Summary: Representation of Text Categorization Attributes

- Representations of text are usually very high dimensional
 - ▶ "The curse of dimensionality"
- High-bias algorithms should generally work best in high-dimensional space
 - ▶ They prevent overfitting
 - ▶ They generalize more
- For most text categorization tasks, there are many relevant features & many irrelevant ones

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Decision Tree — Representation

- A tree structure
 - ▶ Each internal node: test one feature X_i
 - ▶ Each branch from a node: selects some value for X_i
 - ▶ Each leaf node: prediction for Y
- **Question 1: What function does a decision tree represent?**
 - ▶ C.f., In linear regression, we use a linear function of the input to predict the output
- **Question 2: Given a decision tree, how do we assign a label to a test point?**

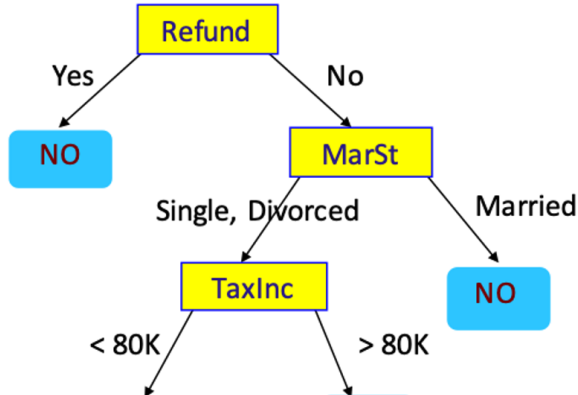


cheating? under declared income?

Decision Tree algorithm — Tax Evasion Detection

Query Data

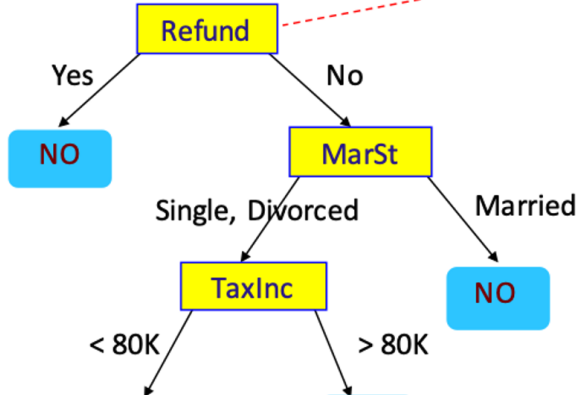
X_1	X_2	X_3	Y
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Decision Tree algorithm — Tax Evasion Detection

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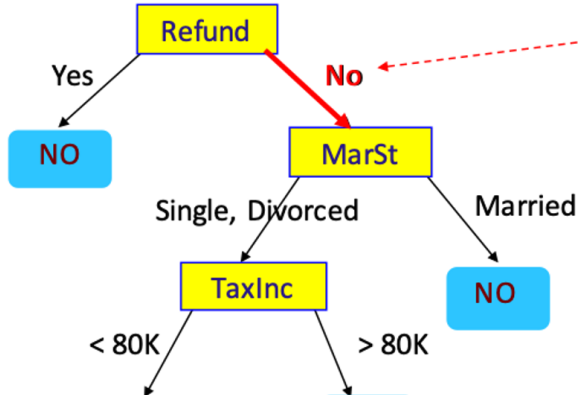
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Decision Tree algorithm — Tax Evasion Detection

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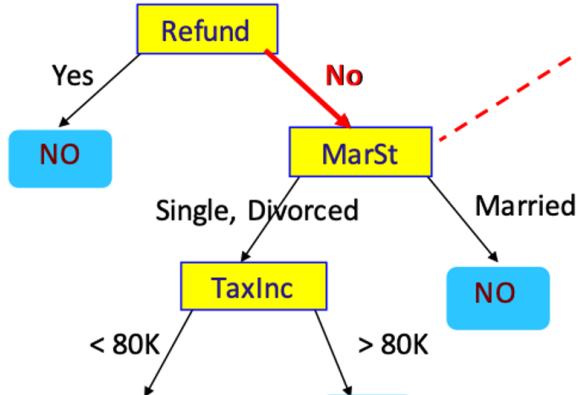
X_1	X_2	X_3	Y
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Decision Tree algorithm — Tax Evasion Detection

Query Data

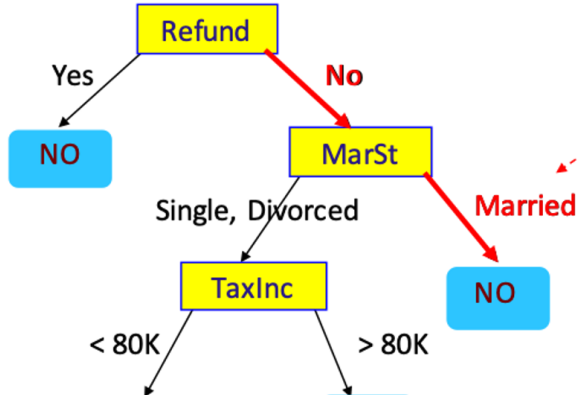
X_1	X_2	X_3	Y
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



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Query Data

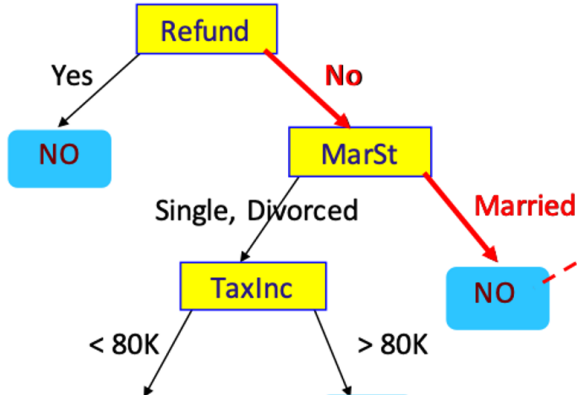
X_1	X_2	X_3	Y
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No	Married	80K	?



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Query Data

X_1	X_2	X_3	Y
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Assign Cheat to "No"

Decision Tree

- **So far...**

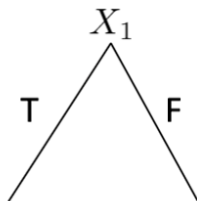
- ▶ What function does a decision tree represent
- ▶ Given a decision tree, how do we assign label to a test point

- **Now ...**

- ▶ How do we learn a decision tree from training data?

Which feature is better?

X_1	X_2	Y
T	T	T
T	F	T
T	T	T
T	F	T
F	T	T
F	F	F
F	T	F
F	F	F

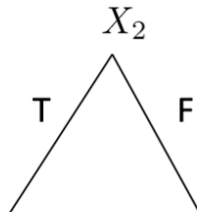


Y: 4 Ts
0 Fs

Absolutely
sure

Y: 1 Ts
3 Fs

Kind of
sure



Y: 3 Ts
1 Fs

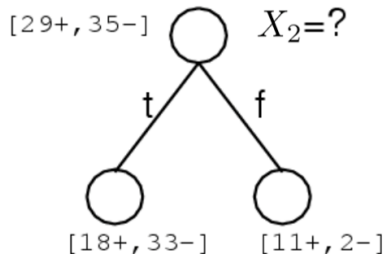
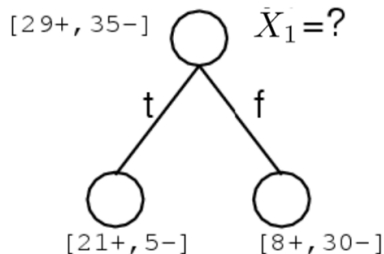
Kind of
sure

Y: 2 Ts
2 Fs

Absolutely
unsure

- Good split if we are more certain about classification after split.

Which feature is better, mathematically?



Pick the attribute/feature which yields **maximum information gain**:

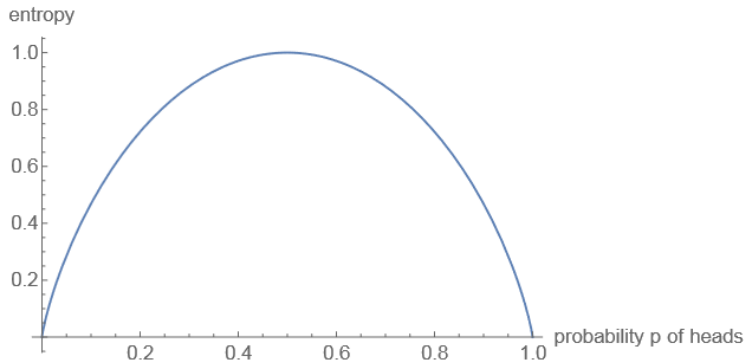
$$\arg \max_i I(Y, X_i) = \arg \max_i [H(Y) - H(Y|X_i)],$$

where $H(Y)$ is the entropy of Y and $H(Y|X_i)$ is the conditional entropy of Y given X .

Entropy

- Entropy of a random variable Y

$$H(X) = - \sum_{x \in X} P(x) \log_2 P(x)$$



Information Theory-based interpretation: $H(Y)$ is the expected number of bits

Information Gain

- Advantage of an attribute means **decrease in uncertainty**.

- ▶ Entropy of Y before split:

$$H(Y) = - \sum_y P(Y = y) \log_2 P(Y = y)$$

- ▶ Entropy of Y after splitting based on X_i :

$$\begin{aligned} H(Y|X) &= \sum_{x \in X} P(x) H(Y|X = x) \\ &= - \sum_x P(x) P(Y|X = x) \log_2 P(Y|X = x) \end{aligned}$$

Maximum Information Gain

- Information Gain (I) measures the reduction in entropy (or surprise) by observing a feature to a given value of a random variable

$$I(Y, X_i) = H(Y) - H(Y|X_i)$$

- Maximum Information Gain = Minimum Conditional Entropy**

$$\begin{aligned}\arg \max_i I(Y, X_i) &= \arg \max_i [H(Y) - H(Y|X_i)] \\ &= \arg \min_i H(Y|X_i) \\ &= \arg \min_i P(Y = y|X_i = x) \log_2 P(Y = y|X_i = x)\end{aligned}$$

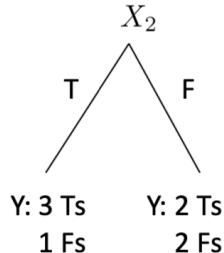
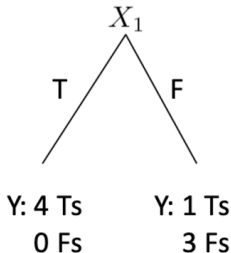
Which feature is best to split?

- Pick the attribute/feature which yields maximum information gain, which provides maximum information about Y .

Maximum Information Gain

$$H(Y | X_i) = - \sum_x P(X_i = x) \sum_y P(Y = y | X_i = x) \log_2 P(Y = y | X_i = x)$$

X_1	X_2	Y
T	T	T
T	F	T
T	T	T
T	F	T
F	T	T
F	F	F
F	T	F
F	F	F



$$\hat{H}(Y|X_1) = -\frac{1}{2}[1 \log_2 1 + 0 \log_2 0] - \frac{1}{2}\left[\frac{1}{4} \log_2 \frac{1}{4} + \frac{3}{4} \log_2 \frac{3}{4}\right]$$

$$\hat{H}(Y|X_2) = -\frac{1}{2}\left[\frac{3}{4} \log_2 \frac{3}{4} + \frac{1}{4} \log_2 \frac{1}{4}\right] - \frac{1}{2}\left[\frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{2} \log_2 \frac{1}{2}\right]$$

> 0

How to learn a decision tree

Recursive and greedy way to build a decision tree

- 1 Pick an attribute with the highest IG at an internal node.
- 2 Categorize data items based on the attribute values.
- 3 For each group:
 - ▶ if no examples – return majority from parent,
 - ▶ else if all examples in the same class – return the class,
 - ▶ otherwise loop to step 1 after removing the current feature.

ID3

Top-down induction (ID3, iterative Dichotomiser 3) ID3 is one of the DT methods (e.g., C4.5, C5, ...) Information gain is used to select the attributes.

- 1 $X \leftarrow$ the "best" decision feature for next node
- 2 Assign X as decision feature for node
- 3 For each value of X , create new descendant of node (Discrete features)
- 4 Sort training examples to leaf nodes
- 5 If training examples perfectly classified, then stop, else iterate over new leaf nodes.
- 6 Repeat (steps 1-5) after removing current feature
- 7 When all features exhausted, assign majority label to the leaf node.

Decision Tree — Analysis

Decision Trees

• Advantages

- ▶ Easy to understand (interpretable)
- ▶ Easy to generate rules (intuitive)
- ▶ Reduce problem complexity
- ▶ Good with discrete attributes
- ▶ Easily deals with missing values (just treat as another value)
- ▶ Fast at test time

• Disadvantages

- ▶ Few hyperparameters. (this can be an advantage too)
- ▶ A document is only connected with one branch (hard clustering)
- ▶ Once a mistake is made at a higher level, any subtree is wrong
- ▶ Does not handle continuous variable well
- ▶ Too big of a tree may suffer from overfitting.

Decision Tree – Summary

- Can be used for classification, regression and density estimation too.
- The overfitting problem:
 - ▶ must use tricks to find “simple trees”, e.g.,
 - ★ Pre-pruning: fixed depth/fixed number of leaves
 - ★ post-pruning: Chi-square test of independence
 - ★ Complexity penalized / MDL (minimum description length) model selection
- Decision trees \rightarrow Random Forests \rightarrow Gradient-boosted Decision Trees $\rightarrow \dots$
- In practice, an ensemble model is used.

Wrap Up

- Naïve Bayes
 - ▶ fast and robust to irrelevant features
 - ▶ very good in domains with many equally important features
 - ▶ A good dependable baseline for text classification
- Decision trees
 - ▶ Simple non-linear, discriminative classifier
 - ▶ Easy to interpret
- In real-world
 - ▶ You should exploit domain specific structure!!

Summary

- and discussion

Reference