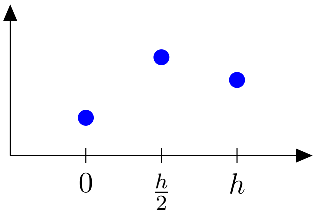
F. Bornemann, C. Ludwig

Assignment 8

The interpolation polynomial p (of degree ≤ 2) is interpolating a function f at the points 0, $h/2$ and h. Give q and r for the error:

$$E(h) := \max_{x\in[0,h]} |f(x) - p(x)| = c|f^{(q)}(\xi)|h^r.$$



Solution

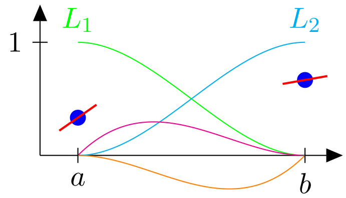
$q = r = 3$.

Assignment 9 (Hermite interpolation)

To construct a interpolation polynomial p (of degree ≤ 3) with the properties

$$p(a) = y_a, \quad p(b) = y_b, \quad p'(a) = w_a \quad \text{and} \quad p'(b) = w_b$$

for given data (y_a, y_b, w_a, w_b) we use the Lagrange idea:



(a) Give the formulas for the four polynomials L_j $(j = 1 : 4)$ such that L_1 is interpolating the data $(y_a, y_b, w_a, w_b) = (1, 0, 0, 0)$, L_2 is interpolating $(0, 1, 0, 0)$, L_3 is interpolating $(0, 0, 1, 0)$ and L_4 is interpolating $(0, 0, 0, 1)$.

(b) Give a simple formula (using the L_j) for p that is interpolating the data (y_a, y_b, w_a, w_b).

(c) Write a Matlab function `hermiteinterpol(a,b,y,w,xx)` that evaluates the interpolation polynomial p at the points `xx`.

Solution

(a)

$$L_1 = \frac{(b-x)^2}{(a-b)^3}(3a - b - 2x) \qquad L_3 = \frac{(x-a)(b-x)^2}{(a-b)^2}$$

$$L_2 = \frac{(a-x)^2}{(a-b)^3}(-3b + a + 2x) \qquad L_4 = \frac{(x-b)(a-x)^2}{(a-b)^2}$$

(b)

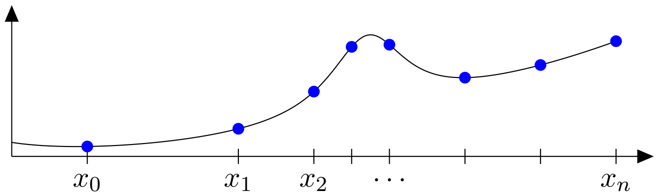
$$p(x) = y_a L_1(x) + y_b L_2(x) + w_a L_3(x) + w_b L_4(x)$$

(c)

```
function yy=hermiteinterpol(a,b,y,w,xx)
% evaluate Hermite polynomial p on interval [a,b]
% function yy=hermiteinterpol(a,b,y,w,xx)
%  p(a)  = y(1),  p(b)  = y(2)
%  p'(a) = w(1),  p'(b) = w(2)
% yy = p(xx)
L = {...
    @(x) (b-x).^2 .* (3*a-b-2*x) ./(a-b).^3 ,....
    @(x) (a-x).^2 .* (a-3*b+2*x)./(a-b).^3, ....
    @(x) (b-x).^2 .* (x-a) ./ (a-b).^2 , ...
    @(x) (a-x).^2 .* (x-b) ./ (a-b).^2 };

yy = y(1)*L{1}(xx) + w(1)*L{3}(xx) + ...
     y(2)*L{2}(xx) + w(2)*L{4}(xx);
end
```

Assignment 10 (Splines vs. piecewise Hermite interpolation)
For a function f the values $y_j = f(x_j)$ and the derivatives $w_j = f'(x_j)$ are given.



We compare the spline (using not-a-knot end conditions) with a piecewise Hermite interpolation, i. e. on every interval $[x_j, x_{j+1}]$ a Hermite polynomial (see assignment 9) is constructed.

(a) Write a Matlab function for evaluating the spline and the piecewise Hermite interpolant.

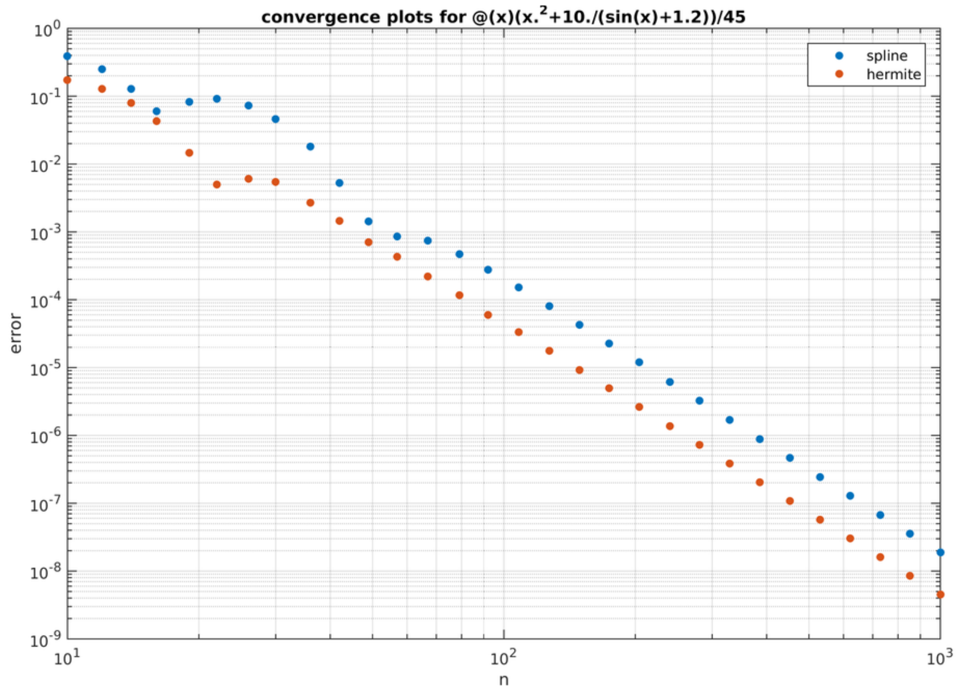
(b) Produce for some f (e. g. $f(x) = \sin(x)$) convergence plots. Use equidistant nodes with $h := x_{j+1} - x_j$ in an interval $[a, b]$ and plot the error vs. h.

(c) Give the complexity to evaluate the spline and the Hermite interpolation for one x ($x \neq x_j$).

Solution

(a) & (b)

```
function [yy_spline,yy_hermite] = A10a(x,y,w,xx)
% evaluate spline and hermite interpolation for xx.
%   y=f(x) are the function values and
%   w=f'(x) are the values of the derivatives
yy_spline = interp1(x,y,xx,'spline');
yy_hermite = zeros(size(xx));
for j=1:numel(x)-1
    in_interval = xx>=x(j) & xx<=x(j+1);
    yy_hermite(in_interval) = ...
        hermiteinterpol(x(j),x(j+1),y(j:j+1),w(j:j+1),xx(in_interval));
end
end
```



(c) For the spline: Even if only one evaluation is needed, all 2nd derivatives of the spline have to be calculated (by solving the corresponding linear system). Complexity: $O(n)$.
For the hermite interpolation: One can first check for which interval $x \in [x_j, x_{j+1}]$. Then one only needs calculate the hermite interpolation polynomial there. Complexity: $O(1)$.