

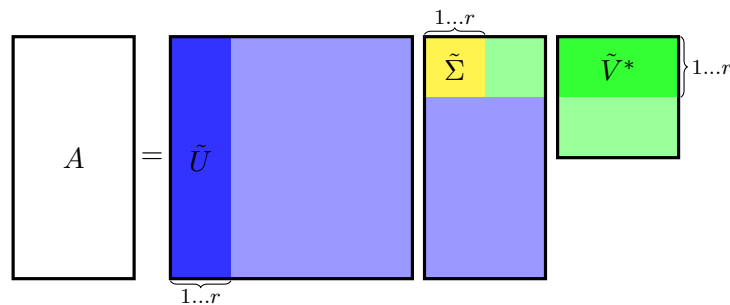
Assignment 28 (basis for nullspace and image)

 Use the complete singular value decomposition $A = U\Sigma V'$ of $A \in \mathbb{R}^{m \times n}$ to compute

- (a) an orthogonal basis of the nullspace of A and
- (b) an orthogonal basis of the image of A .

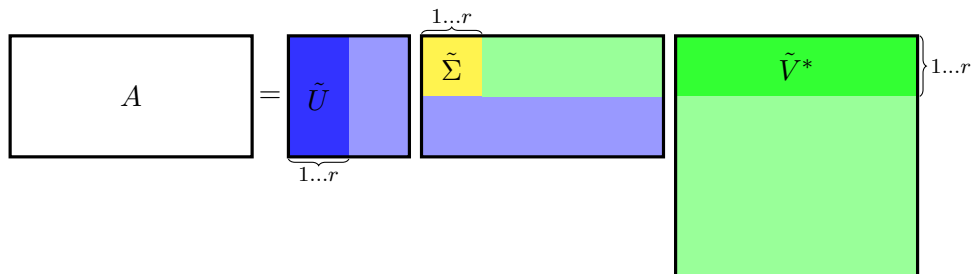
Solution

For $A \in \mathbb{R}^{m \times n}$ the complete singular value decomposition is given by $A = U\Sigma V'$ with $U \in \mathbb{R}^{m \times m}$, $V \in \mathbb{R}^{n \times n}$ and $\Sigma \in \mathbb{R}^{m \times n}$. If $\sigma_1, \dots, \sigma_r$ are all singular values that are non-zero, then we do the following partitioning for $m > n$



The diagram shows the matrix equation $A = U \Sigma V'$ for the case $m > n$. The matrix A is represented by a white rectangle. It is equal to the product of three matrices: U , Σ , and V' . The matrix U is a tall rectangle with a blue vertical strip on the left, labeled \tilde{U} and $1 \dots r$ below it. The matrix Σ is a tall rectangle with a yellow top strip labeled $\tilde{\Sigma}$ and $1 \dots r$ above it, and a light blue bottom strip. The matrix V' is a tall rectangle with a green top strip labeled \tilde{V}^* and $1 \dots r$ to its right, and a light green bottom strip.

 and for $m < n$:



The diagram shows the matrix equation $A = U \Sigma V'$ for the case $m < n$. The matrix A is represented by a wide rectangle. It is equal to the product of three matrices: U , Σ , and V' . The matrix U is a wide rectangle with a blue vertical strip on the left, labeled \tilde{U} and $1 \dots r$ below it. The matrix Σ is a wide rectangle with a yellow top strip labeled $\tilde{\Sigma}$ and $1 \dots r$ above it, and a light blue bottom strip. The matrix V' is a wide rectangle with a green top strip labeled \tilde{V}^* and $1 \dots r$ to its right, and a light green bottom strip.

In both cases we have u_1, \dots, u_r is an orthogonal basis for the image of A and v_{r+1}, \dots, v_n is an orthogonal basis for the nullspace of A .

Matlab code, see zip-file.

Assignment 29

In the (x, y) plane a cat is hunting a mouse: The speed of the cat is always $v_C = 2$ and it is always heading in the direction of the mouse. At time $t = 0$ the mouse is starting at point $(0, 0)$ and it is running straight in the direction $(0, 1)$ with speed $v_M = 1$. The cat is starting at point $(1, 0)$.

- (a) Write down the differential equations for the path of the cat and the mouse, respectively.
- (b) Use Matlab's `ode45` to calculate, when the distance between the cat and the mouse is 10^{-5} . (Hint: see the help of the `Events` option in `odeset` for the event detection.)

Solution

(a) For the mouse we have

$$\begin{pmatrix} x'_M \\ y'_M \end{pmatrix} = \begin{pmatrix} 0 \\ v_M \end{pmatrix}, \quad \begin{pmatrix} x_M(0) \\ y_M(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

The direction of the cat is given by

$$r := \begin{pmatrix} x_M \\ y_M \end{pmatrix} - \begin{pmatrix} x_C \\ y_C \end{pmatrix}.$$

Hence we obtain for the cat

$$\begin{pmatrix} x'_C \\ y'_C \end{pmatrix} = v_C \cdot r / \|r\|_2.$$