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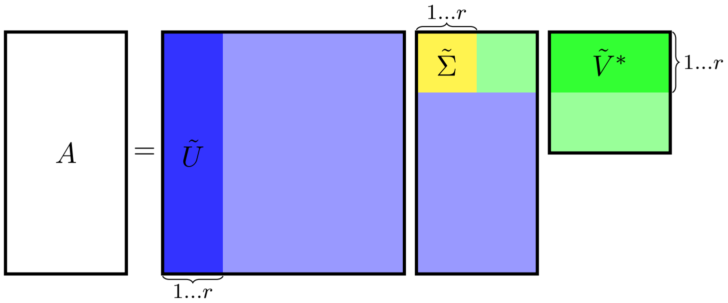
Assignment 28 (basis for nullspace and image)
Use the complete singular value decomposition $A = U\Sigma V'$ of $A \in \mathbb{R}^{m\times n}$ to compute

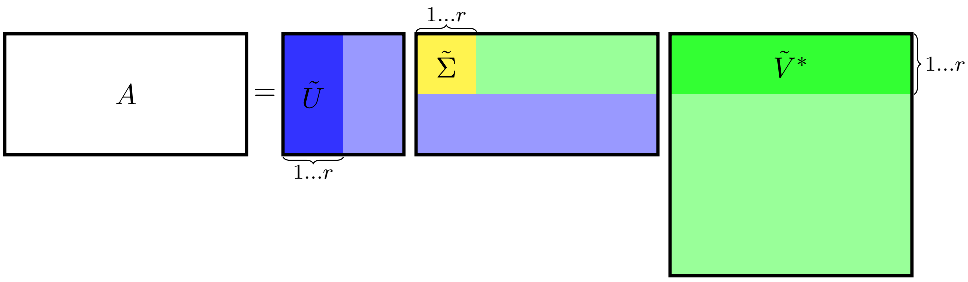
(a) an orthogonal basis of the nullspace of A and

(b) an orthogonal basis of the image of A.

Solution
For $A \in \mathbb{R}^{m\times n}$ the complete singular value decomposition is given by $A = U\Sigma V'$ with $U \in \mathbb{R}^{m\times m}$, $V \in \mathbb{R}^{n\times n}$ and $\Sigma \in \mathbb{R}^{m\times n}$. If $\sigma_1, \dots, \sigma_r$ are all singular values that are non-zero, then we do the following partitioning for $m > n$



and for $m < n$:



In both cases we have $u_1, \dots, u_r$ is an orthogonal basis for the image of A and $v_{r+1}, \dots, v_n$ is an orthogonal basis for the nullspace of A.
Matlab code, see zip-file.

Assignment 29
In the (x, y) plane a cat is hunting a mouse: The speed of the cat is always $v_C = 2$ and it is always heading in the direction of the mouse. At time $t = 0$ the mouse is starting at point $(0, 0)$ and it is running straight in the direction $(0, 1)$ with speed $v_M = 1$. The cat is starting at point $(1, 0)$.

(a) Write down the differential equations for the path of the cat and the mouse, respectively.

(b) Use Matlab's `ode45` to calculate, when the distance between the cat and the mouse is 10^{-5}. (Hint: see the help of the `Events` option in `odeset` for the event detection.)

Solution

(a) For the mouse we have

$$\begin{pmatrix} x_M' \\ y_M' \end{pmatrix} = \begin{pmatrix} 0 \\ v_M \end{pmatrix}, \qquad \begin{pmatrix} x_M(0) \\ y_M(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

The direction of the cat is given by

$$r := \begin{pmatrix} x_M \\ y_M \end{pmatrix} - \begin{pmatrix} x_C \\ y_C \end{pmatrix}.$$

Hence we obtain for the cat

$$\begin{pmatrix} x_C' \\ y_C' \end{pmatrix} = v_C \cdot r / \|r\|_2.$$