Institut für
Informatik

Informatik V
Chair for
Scientific Computing

Scientific Computing Lab

Results Worksheet 3

Worksheet 3 – Solution

$$N_x = N_y = 3$$

$$A = \begin{pmatrix}
-64 & 16 & & 16 & & & & & \\
16 & -64 & 16 & & 16 & & & & \\
 & 16 & -64 & & & 16 & & & \\
16 & & & -64 & 16 & & 16 & & \\
 & 16 & & 16 & -64 & 16 & & 16 & \\
 & & 16 & & 16 & -64 & & & 16 \\
 & & & 16 & & & -64 & 16 & \\
 & & & & 16 & & 16 & -64 & 16 \\
 & & & & & 16 & & 16 & -64
\end{pmatrix}$$

Worksheet 3 – Solution

1) m-file create_matrix.m

```
for j=1:N_y
   for i=1:N_x
      r = (j-1)*N_x+i;
      A(r,r) = -2*(N_x+1)^2-2*(N_y+1)^2;
      if ~(i==1)
         A(r,r-1) = (N_x+1)^2;
      end  …
      if ~(j==1)
         A(r,r-N_x) = (N_y+1)^2;
      end …
   end
end
```

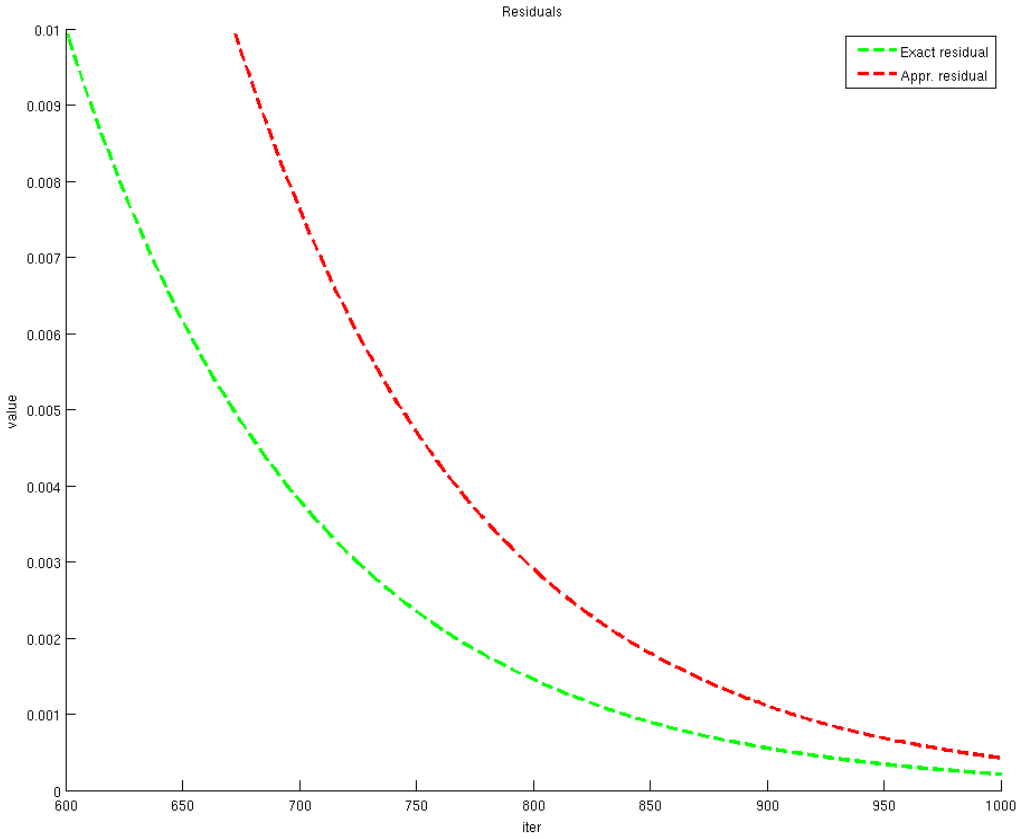
Worksheet 3 – Solution

2) m-file GaussSeidel.m

```
while (res>0.0001)
    res=0.0;
    for j=2:N_y+1
        for i=2:N_x+1
            x_m(i,j)=(d_1*(x_m(i-1,j)+x_m(i+1,j))+…
        end
    end
    for j=2:N_y+1
        for i=2:N_x+1
            res=res+(b((j-2)*N_x+i-1)+a_ii*x_m(i,j)-…
        end
    end
    res=sqrt(res/(N_x*N_y));
end
```

Worksheet 3 – Solution

Comparison of real residual (extra loops) and approx. residual (31x31):



Worksheet 3 – Solution

3) m-file worksheet3.m

```
tic; x=A\b; time(1)=toc;
memory(1)=numel(A)+numel(b)+numel(x);
%transform x to a matrix for visualisation
   x_m=zeros(N_x+2,N_y+2);
   for i=2:N_x+1
   for j=2:N_y+1
      x_m(i,j)=x((j-2)*N_x+i-1);
   end
end …
subplot(2,1,1); mesh(coord1,coord2,x_m);
subplot(2,1,2);contour(coord1,coord2,x_m);
```

Worksheet 3 – Solution

3) m-file worksheet3.m

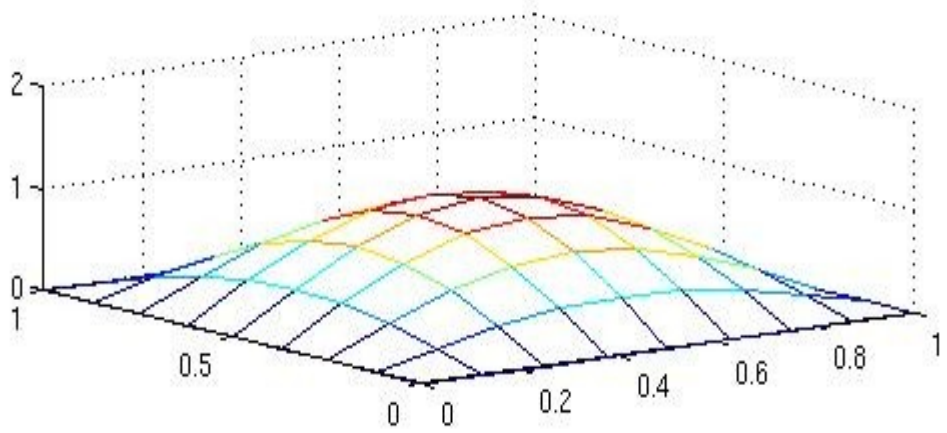
```
S=sparse(A);
tic; x=S\b; time(2)=toc;
memory(2)=nnz(S)+numel(b)+numel(x);
```

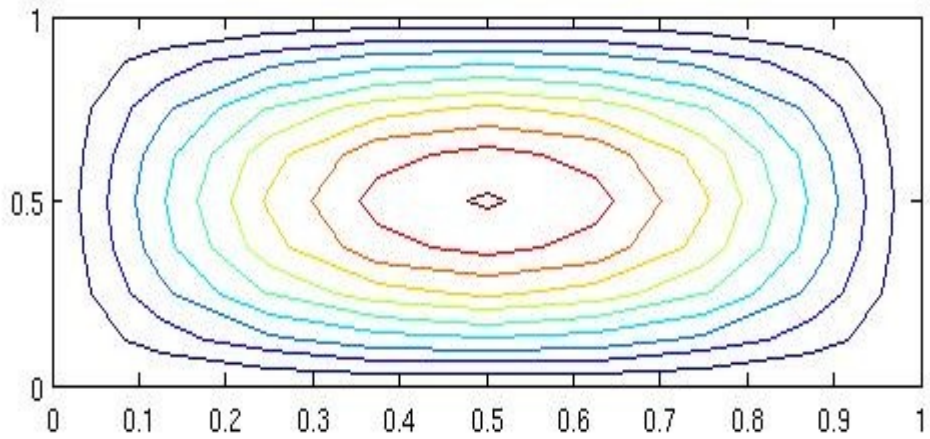
Worksheet 3 – Solution

3) m-file worksheet3.m

```
tic; x_m=GaussSeidel(b,N_x,N_y); time(3)=toc
memory(3)=numel(b)+numel(x_m)
```

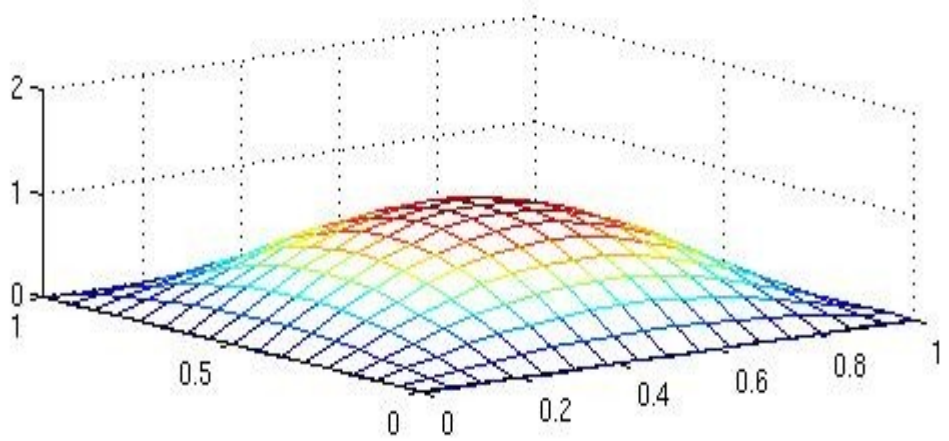
Worksheet 3 – Solution

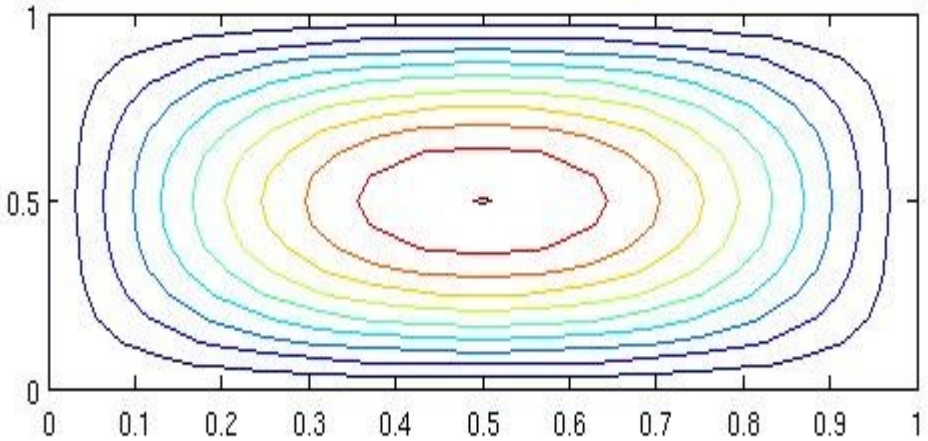




$N_x = N_y = 7$

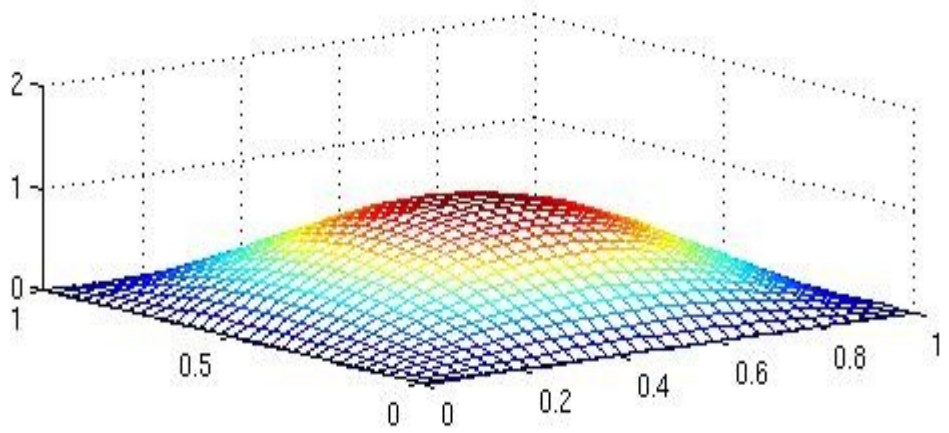
Worksheet 3 – Solution

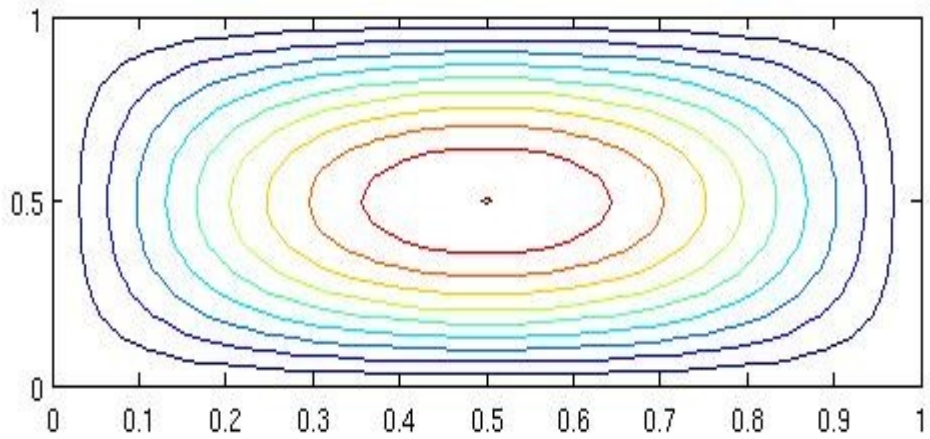




$N_x = N_y = 15$

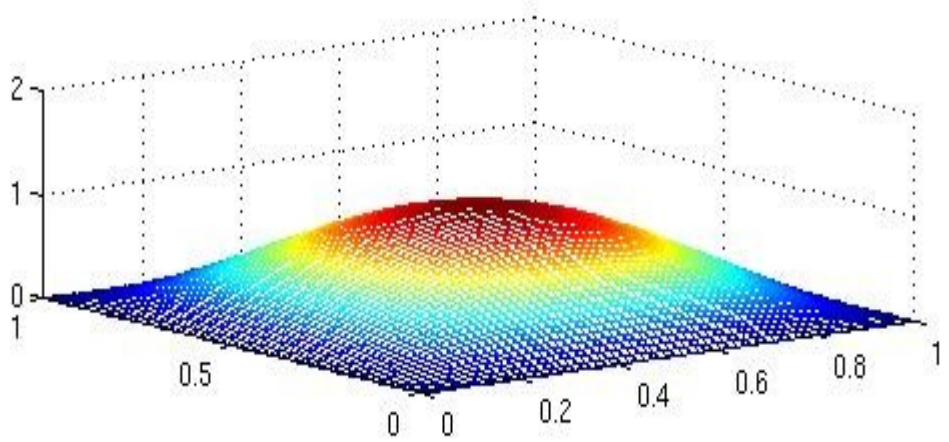
Worksheet 3 – Solution

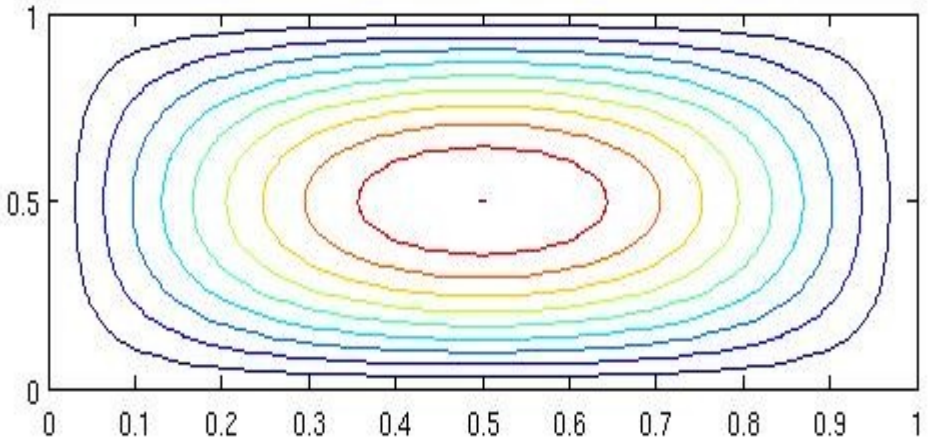




$N_x = N_y = 31$

Worksheet 3 – Solution





$N_x = N_y = 63$

Worksheet 3 – Solution

direct solution with full matrix

	N_x, N_y	7	15	31	63
Dense	runtime	0.0002	0.0044	0.1936	10.4078
	storage	2,499	51,075	925,443	15,760,899

Worksheet 3 – Solution

direct solution with sparse matrix

	N_x, N_y	7	15	31	63
Dense	runtime	0.0002	0.0044	0.1936	10.4078
	storage	2,499	51,075	925,443	15,760,899
Sparse	runtime	0.0014	0.0045	0.0182	0.0826
	storage	315	1,515	6,603	27,531

Worksheet 3 – Solution

Gauss-Seidel without explicit matrix

	N_x, N_y	7	15	31	63
Dense	runtime	0.0002	0.0044	0.1936	10.4078
	storage	2,499	51,075	925,443	15,760,899
Sparse	runtime	0.0014	0.0045	0.0182	0.0826
	storage	315	1,515	6,603	27,531
Gauss-Seidel	runtime	0.0017	0.0148	0.1831	2.7531
	storage	130	514	2,050	8,194

Worksheet 3 – Solution

Gauss-Seidel without explicit matrix

	N_x, N_y	7	15	31	63
Dense	runtime	0.0002	0.0044	0.1936	10.4078
	storage	2,499	51,075	925,443	15,760,899
Sparse	runtime	0.0014	0.0045	0.0182	0.0826
	storage	315	1,515	6,603	27,531
Gauss-Seidel	runtime	0.0017	0.0148	0.1831	2.7531
	storage	130	514	2,050	8,194
	iterations	69	271	1078	4305

Worksheet 3 – Solution

FD discretisation (5-P-Stencil) – convergence order

N_x, N_y	15	31	63	127
error	0.0017	$4.10*10^{-4}$	$9.70*10^{-5}$	$2.02*10^{-5}$
error red.	4.35	4.14	4.23	4.83

Second order!