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Assignment 11 (calculation of maximal order)
Calculate c, q and r in the error term

$$e(h) := \left|\int_0^h f(x)\,dx - Q(f)\right| = cf^{(q)}(0)h^r + O(h^{r+1})$$

for the following quadrature formulas. Give the orders of the quadrature formulas.

(a) $Q(f) = h\dfrac{f(0) + 3f(h/3) + 3f(2h/3) + f(h)}{8}$

(b) $Q(f) = h\dfrac{5f((\frac{1}{2} - \frac{\sqrt{15}}{10})h) + 8f(\frac{h}{2}) + 5f((\frac{1}{2} + \frac{\sqrt{15}}{10})h)}{18}$

Hint: *Please* use computer algebra software tools for the symbolic calculations!

Solution
We use the Taylor formula:

```
warning off symbolic:sym:int:notFound

syms x h;
f=symfun(sym('f(x)'),x);

q1=@(h) h*( f(0)+3*f(h/3)+3*f(2*h/3)+f(h) )/8;
E1=@(h) int(f(x),x,0,h)-q1(h);
disp('Fehlerterm für F1');
pretty(simplify(taylor(E1(h),h,'Order',6)))

alpha=0*h+sqrt(15)/10;
q2=@(h) (h/18)*(5*f((-alpha+1/2)*h)+8*f(h/2)+5*f((alpha+1/2)*h));
E2=@(h) int(f(x),x,0,h)-q2(h);
disp('Fehlerterm für F2');
pretty(simplify(taylor(E2(h),h,'Order',8)))
```

We obtain the error terms:

$$\text{(a)}\ \frac{-f^{(4)}(0)}{6480}h^5 \qquad \text{(b)}\ \frac{f^{(6)}(0)}{201600}h^7.$$

So (a) has order $p = 4 = s$ and (b) has order $p = 6 = 2s$.

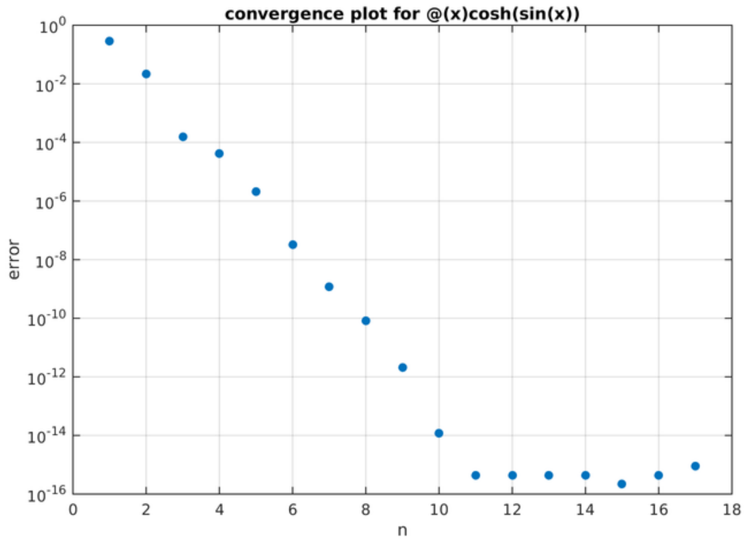
Assignment 12 (Gauss-Legendre quadrature)
Produce convergence plots using Gauss-Legendre quadrature to approximate the following integrals

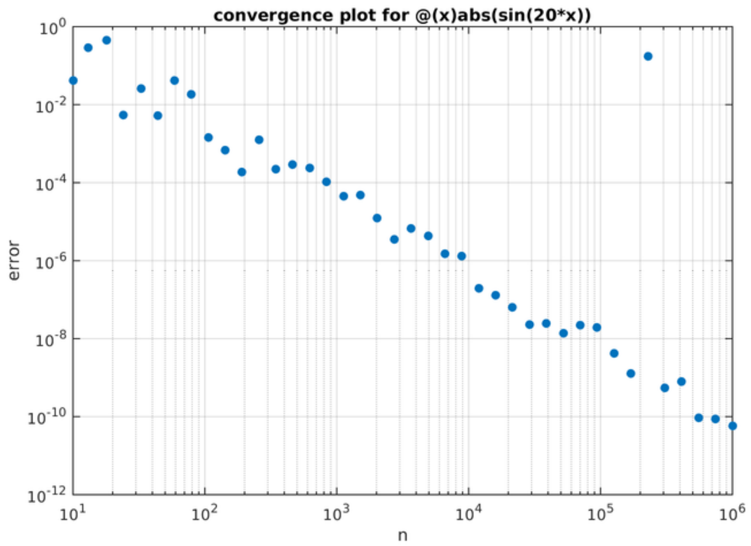
$$\int_{-1}^{1} \cosh(\sin x)\,dx \quad \text{and} \quad \int_{-1}^{1} |\sin(20x)|\,dx.$$

Explain the results.
Hint: Use legpts of the Chebfun toolbox to get the nodes and weights for a Gauss-Legendre quadrature formula.

Solution

Code see zip-file.





Assignment 13 (composite Simpson quadrature)

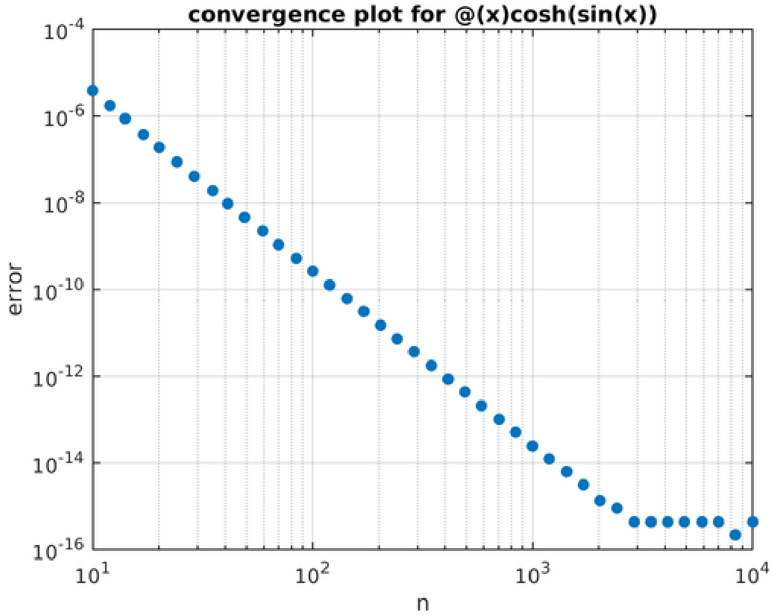
Implement the composite Simpson quadrature for equidistant subdivision nodes. Don't use any loops (like `for`, `while`, etc.).

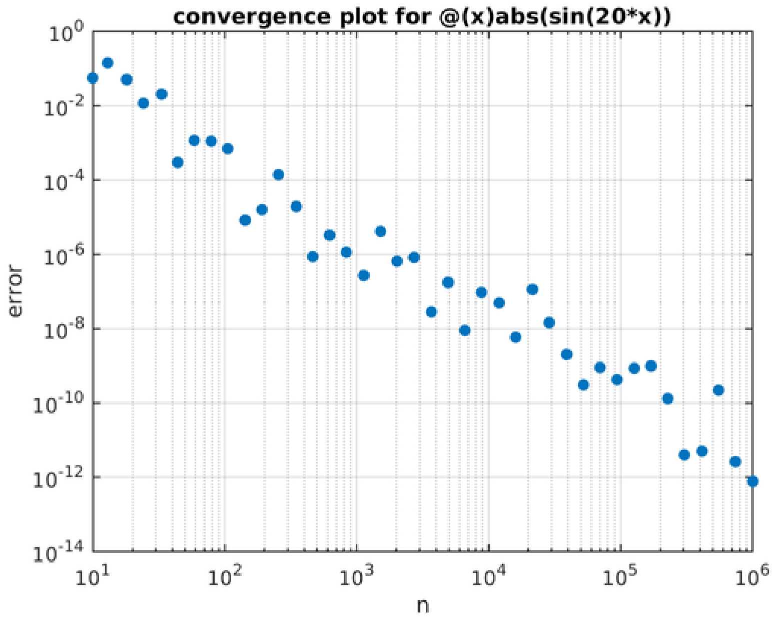
Produce convergence plots for the integrals given in assignment 12.

Solution

```
function result = A13_Simpson(a,b,f,n)
assert(n>1);
x=linspace(a,b,2*n-1)';    % simpson nodes;
                            %x_j, (x_j+x_(j+1))/2, x_(j+1)

w=repmat([2,4],1,n);  % weights: [1,4,2,4,2,4,...,4,1]*h/6
w(1)=1;w(2*n-1)=1; w(2*n:end)=[];
w=w*(b-a)/(6*(n-1));
result = w*f(x);
end
```


convergence plot for @(x)cosh(sin(x))
error
n
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{1}
10^{2}
10^{3}
10^{4}


convergence plot for @(x)abs(sin(20*x))
error
n
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}