Scientific Computing Lab I: Worksheet 2 Answer Sheet

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1. Question 1

1.1. a)

By looking at the relationship between h_x (or h_y) and the maximum δt required for the solution to be stable, we can deduce that for $\mathrm{O(h_x^2)}, O(\delta \mathrm{t}^2)$.

This is due to the first order discritisation of time when implementing implicit Euler method. For instance, Adams-Moulton method (which is second order in time as well) order of h_x and maximum δt required for the solution to be stable will be the same (both of them in first order).

2. Question 2

Considering that the discritsation errors arising from time and space are independent, we can express the total discritsation error in the following way:

$\epsilon_{total} = \epsilon_{space} + \epsilon_{time}$ (1)

Hence,

$\mathrm{O}(\epsilon_{total})$ =$\mathrm{O}(\epsilon)$ + $O(h^2)$ (2)

3. Question 3

This question really brings down to a sensitive study between y_n and y_{n+1}. Assume the relationship between y_n and y_{n+1} is y_{n+1} = c + y_n, where c is a real number. Ideally, for infinitesimally small time step c would be 0, making $y_{n+1} = y_n$. By inserting this relationship into the linearised equations:

Second part inside the bracket of linearisation 1 becomes:

$$\Big(1 - \frac{c + y_n}{10}\Big) y_n \tag{3}$$

Hence,

$$\left(1 - \frac{y_n}{10}\right) y_n - \frac{c}{10} y_n \tag{4}$$

Similarly, second part inside the bracket of linearisation 2 becomes:

$$\left(1 - \frac{y_n}{10}\right)(c + y_n) \tag{5}$$

Hence,

$$\left(1 - \frac{y_n}{10}\right) y_n + \left(1 - \frac{y_n}{10}\right) c \tag{6}$$

The last terms in equations (2) and (3) are the error terms arising from linearisation. Such deviation is smaller from the linearisation 1 than linearisation 2 (try with varying y_n values). Therefore, linearisation 1 works better than linearisation 2.

4. Question 4

4.1. 1)

For the IVP from worksheet 1, explicit scheme is preferred due to its fast convergence. As previously discussed, implicit schemes does not make solution more accurate but only make improvements in stability. Considering no divergence is observed when using explicit scheme for the ODE from the worksheet 1, there is no need to use implicit schemes which would lead to slower convergence.

4.2. 2)

For the IVP from worksheet 2, the question really brings down to the accuracy required. For cases where a high accuracy is desired, explicit schemes would be the right choice since it's faster than implicit schemes, whilst the accuracy for a given time step is the same for both (provided the same order method is used). On the other hand, if accuracy is not as emphasised as the computation time, implicit schemes would be preferred since big time steps with explicit schemes will explode the solution.