Strangeness Enhancement and Canonical Suppression

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We demonstrate the essential role of canonical suppression in strangeness enhancement. The pattern of enhancement of strange and multistrange baryons observed by the WA97 collaboration can be understood on this basis. Besides, it is shown that in canonical approach strangeness enhancement is a decreasing function of collision energy. It is the largest at $\sqrt{s} = 8.7$ GeV where the enhancement of Ω, Ξ and Λ is of the order of 100, 20 and 3 respectively.

I. INTRODUCTION

Ultra-relativistic heavy ion collisions provide a unique opportunity to study the properties of highly excited hadronic matter under extreme conditions of high density and temperature [1–5]. From the analysis of rapidity distribution of protons and of their transverse energy measured in 158 A GeV/c Pb+Pb collisions an estimate of the initial condition leads to energy density of 2-3 GeV/fm^3 and a baryon density of the order of 0.7/fm^3. Lattice QCD at vanishing baryon density suggests that the phase transition from confined to the quark-gluon plasma (QGP) phase appears at the temperature $T_c = 173 \pm 8$ MeV which corresponds to the critical energy density [6] $\epsilon_c \sim 0.6 \pm 0.3$ GeV/fm^3. One could thus conclude that the required initial conditions for quark deconfinement are already reached in heavy ion collisions at top SPS energy and RHIC energies. Thus, of particular relevance was to find experimental probes to check whether the produced medium in its early stage was indeed in the QGP phase. Different probes have been theoretically proposed and studied in relation with CERN-SPS and more recently with BNL-RHIC experiments. We will concentrate on strange hadrons and particularly on multistrange baryons. More precisely we will examine the issue of enhancement of strange and multistrange baryons as a possible signature of QGP formation.

II. STRANGENESS ENHANCEMENT AND QGP

It was long ago argued that enhanced production of strange particles in nucleus-nucleus (AA) collisions, relatively to proton-proton (pp) or to proton-nucleus (pA) collisions, could be a signal of QGP formation [7]. In QGP the production and equilibration of strangeness is very efficient due to a large gluon density and a low energy threshold for dominant QCD processes of $s\bar{s}$ production [7,8]:

$$GG \rightarrow s\bar{s} \tag{1}$$

In contrast, in a hadronic system, e.g., in pp, the higher threshold for strangeness production was argued to make the strangeness yield considerably smaller and the equilibration time much longer than in QGP. From these strangeness QGP characteristics one expects a *global strangeness enhancement*, which should increase from pp, pA to AA collisions, as well as *enhancement of multistrange (anti)baryons*. The global strangeness content in the collision fireball is measured by $< s\bar{s} > /N_{part}$, the total number of strange quarks per participant nucleon or by $< s\bar{s} > / < u\bar{u} + d\bar{d} >$, the total number of strange quarks per produced light quark. Furthermore, this (anti)hyperon enhancement was predicted to depend on the strangeness content of the (anti)hyperons and to appear in a typical hierarchy:

$$E_\Lambda < E_\Xi < E_\Omega$$

This hierarchy is observed by the WA97 and NA57 collaborations [9,10] which measure the yield per participant in Pb+Pb relative to p+Pb and p+Be collisions. In particular the WA97 data show a pattern with this hierarchy and a saturation of enhancement for a number of participant nucleons $N_{part} > 100$. Recent results of the NA57 collaboration [10] are showing in addition an abrupt change of anti-cascade enhancement for lower N_{part}. Similar behavior was previously seen on the K^+ yield measured by the NA52 experiment in Pb-Pb collisions [11]. These results are very interesting as they might be interpreted as an indication of the onset of new dynamics. However, a more detailed experimental study and theoretical understanding are still required here. It is, e.g., not clear what is the origin of different centrality dependence of Ξ and $\bar{\Xi}$. Nevertheless, this abrupt change could be possibly accounted for in canonical approach by assuming a very particular increase of volume parameter with centrality [12].

Anyway, strangeness enhancement is also seen at low energies, and found to be a decreasing function of collision energy in a compilation of the data on K^+/π^+ ratio in A+A relative to p+p collisions, where the double ratio $(K^+/\pi^+)_{(AA)}/(K^+/\pi^+)_{pp}$ can be considered as an enhancement measure [13,14]. Such a behaviour with collision energy could also be expected for multistrange baryons. It was indeed shown [15,16] that a statistical model (SM) implementing canonical strangeness conservation explains the WA97 pattern and predicts that enhancement is a decreasing function of collision energy. This, we summarize in the following sections.

III. STRANGENESS ENHANCEMENT AND CANONICAL SUPPRESSION

The enhancement E measured in experiments is the ratio of the yield of a given (anti)baryon per participant nucleon in the large AA system to the yield of the same (anti)baryon in a the small pp or pA system:

$$E = \frac{(Yield)_{|AA}}{(Yield)_{|pA}} \tag{2}$$

In a large system with a large number of produced particles, the conservation law of a quantum number, e.g., strangeness, can be implemented on the average by using the corresponding chemical potential. This is the Grand Canonical formulation (GC). In a small system, however, with small particles multiplicities, conservation laws must be implemented locally on an event-by-event basis [17,18]. This is the Canonical formulation (C). The (C) conservation of quantum numbers is known to severely reduce the phase space available for particle production [17]. This the canonical suppression (CS). If in Eq. (2) the denominator is reduced by CS then E is increased. That is, in our approach, the essence of strangeness enhancement from pp, pA to AA collisions.

To better understand CS, consider a gas of particles with strangeness $s = +1, 0, -1$ and with *total* strangeness $S = 0$ (this condition is imposed by the fact that in heavy ion collisions the initial state of the system is strangeness neutral). Group theory projection techniques [19,20] allow to construct the partition function of the gas, from which all thermal physical quantities are derived. One obtains for the thermal kaon density in the (C) formulation

$$n_K^C = \frac{Z_K^1}{V} \frac{S_1}{\sqrt{S_1 S_{-1}}} \frac{I_1(x_1)}{I_0(x_1)} \tag{3}$$

where

$$Z_K^1 = V \frac{g_K}{2\pi^2} m_k^2 T K_2 \left(\frac{m_K}{T}\right) \tag{4}$$

$$S_1 = Z_K^1 + Z_\Lambda^1 + Z_{K^\star}^1 + \ldots \tag{5}$$

$$x_1 \equiv 2\sqrt{S_1 S_{-1}} \propto V \tag{6}$$

V is the volume parameter which is assumed, as usual, to be linear in N_{part}:

$$V = \frac{V_0}{2} N_{part} \tag{7}$$

where $V_0 \approx 7\,\mathrm{fm}^3$ is taken as the volume of the nucleon. S_1 and S_{-1} are the sum over one-particle partition functions for particles carrying strangenes 1 and -1 respectively. I_1, I_0 and K_2 are modified Bessel and Hankel functions. The density n_K^C in Eq. (3) depends on the volume, but for $x_1 \to \infty$, $I_1(x_1)/I_0(x_1) \to 1$ and n_K^C reaches its GC limit, n_K^{GC}, independent of the volume. For $x_1 \to 0$ $I_1(x_1)/I_0(x_1) \to x_1/2$ and the density n_K^C is linearly dependent on the volume. One clearly sees on Eq. (3) that the factor

$$F_{CS1} = \frac{I_1(x_1)}{I_0(x_1)} \tag{8}$$

called canonical suppression factor, measures the deviation of the kaon density from its GC value n_K^{GC}.

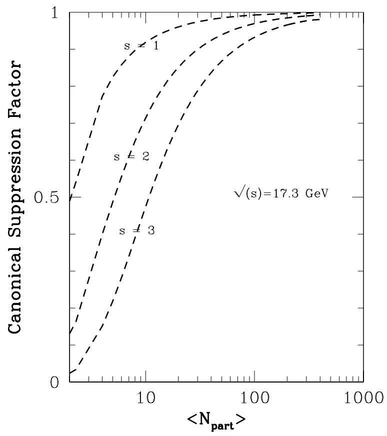

FIG. 1. Canonical suppression factor for three values of particle strangeness: $s = 1, 2, 3$, at top CERN-SPS energy.

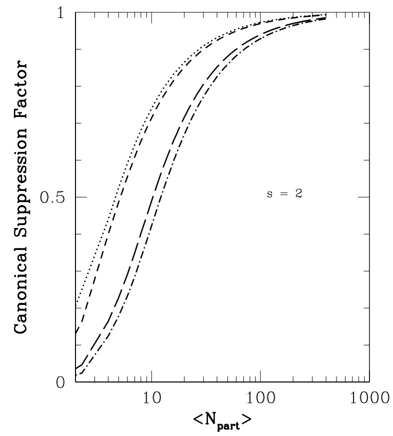

FIG. 2. dotted line: $\sqrt{s} = 130$GeV; short dashed line: $\sqrt{s} = 17.3$GeV; long dashed line: $\sqrt{s} = 12.3$GeV; dot-short dashed line: $\sqrt{s} = 8.3$GeV

The previous considerations can naturally be extended to a gas of particles with strangeness $s = 0, \pm1, \pm2, \pm3$, i.e., to a gas composed of all particles (antiparticles) and their resonances. With the condition that the gas has total strangeness $S = 0$, the canonical particle density of a particle i having strangeness s reads [21]

$$n_i^C = \frac{1}{V}\frac{Z_i^1}{Z_{S=0}^C}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} a_3^p a_2^n a_1^{-2n-3p-s} I_n(x_2) I_p(x_3) I_{-2n-3p-s}(x_1) \tag{9}$$

where

$$a_i = \sqrt{S_i/S_{-i}} \tag{10}$$
$$x_i = 2\sqrt{S_i S_{-i}} \propto V \tag{11}$$
$$Z_{S=0}^C = \sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} a_3^p a_2^n a_1^{-2n-3p} I_n(x_2) I_p(x_3) I_{-2n-3p}(x_1) \tag{12}$$

In Eq. (10), $S_i = \sum_k Z_k^1$ where the sum is over all particles and resonances carrying strangeness i. For a particle of mass m_k, with spin-isospin degeneracy factor g_k, carrying baryon number B_k and electric charge Q_K, with corresponding chemical potential μ_B and μ_Q, the one-particle partition function is given, in the Boltzmann, by

$$Z_k^1 \equiv V\frac{g_k}{2\pi^2} m_k^2 T K_2\left(\frac{m_k}{T}\right)\exp(B_k\mu_B + Q_k\mu_Q) \tag{13}$$

Here too one can show [15] that for small x_1

$$n_i^C \simeq \frac{Z_i^1}{V}\frac{(S_1)^s}{(S_{+1}S_{-1})^{s/2}}\;\frac{I_s(x_1)}{I_0(x_1)} \tag{14}$$

and the canonical suppression factor is now

$$F_{CSs} = \frac{I_s(x_1)}{I_0(x_1)} \tag{15}$$

In Eq. (8) and Eq. (15) one sees that strangeness content of the particle appears in the suppression factor as the order of Bessel function $I_s(x_1)$. Fig. 1 shows that the suppression factor, for a given value of N_{part}, is smaller for larger values of s. At a given energy (temperature) N_{part} depends on the colliding system. In the small system for p-p collisions $N_{part} = 2$. For p-Be (p-Pb) collisions $N_{part} \approx 2.5$ (≈ 4.75). These values have been determined by the WA97 collaboration from a Wounded Nucleon Model in the framework of the Glauber Model [22]. In particular for small x_1, $F_{CSs} \sim (x_1/2)^s$, and one expects that the larger the strangeness content of the particle the smaller the suppression factor and hence the larger the enhancement. This explains [15] the enhancement hierarchy of the WA97 pattern. Furthermore, one sees on Fig. 2 that for a given strangeness, e.g., $s = 2$, the suppression factor, for any number of participant nucleons, is decreasing with decreasing energy: this means that enhancement increases with decreasing energy. Finally, enhancement saturation appears, as explained above, as the grand canonical limit for large number of participant nucleons (large volume).

IV. STRANGENESS ENHANCEMENT ENERGY DEPENDENCE

We have studied strangeness enhancement at four energies: $\sqrt{s} = 8.73,\ 12.3,\ 17.3$ GeV (NA49 and WA97, SPS) and $\sqrt{s} = 130$ GeV (RHIC). To study the energy and centrality dependence of (multi)strange particle yields in terms of the above model one needs to establish first the variation of thermal parameters with energy and centrality. Temperature is to a good approximation [23] only a function of collision energy and is independent of the number of participating nucleons. Baryonic chemical potential μ_B is weakly dependent on centrality [16]. Thermal parameters are, however sensitive to collision energy. At top SPS energy $\sqrt{s} = 17.3$ GeV we use the parameters obtained [24] in experimental data analysis: $T = 166$ MeV and $\mu_B = 266$ MeV. At RHIC energy we take [25] $T = 175$ MeV and $\mu_B = 51$ MeV. At $\sqrt{s} = 8.73,\ 12.3$ GeV the parameters are fixed according to the method explained in [16]. We have $T = 145$ MeV, $\mu_B = 370$ MeV and $T = 152$ MeV , $\mu_B = 280$ MeV respectively.

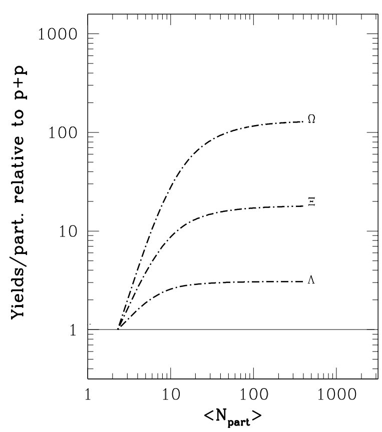

FIG. 3. Centrality dependence of the relative enhancement of particle yields/participant in central Pb-Pb to p-p collisions at fixed energy $\sqrt{s} = 8.73$ GeV.

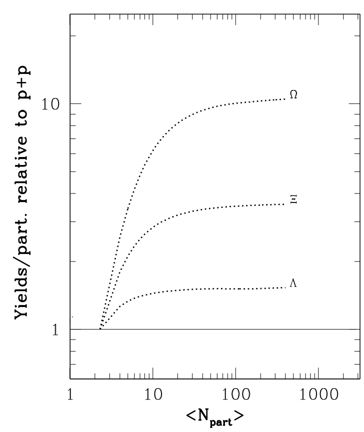

FIG. 4. Centrality dependence of the relative enhancement of particle yields/participant in central Pb-Pb to p-p collisions at fixed energy $\sqrt{s} = 130$ GeV.

In Fig. 3 and Fig. 4 we show the results on relative (multi)strange baryon enhancement from p-p to Pb-Pb collisions at $\sqrt{s} = 8.73$ GeV and at $\sqrt{s} = 130$ GeV respectively. It is clear that the same enhancement pattern as at SPS is expected in SM to appear for all relevant energies. To see the dependence of the strength of the enhancement with energy we show in Fig. 5 and Fig. 6 the relative enhancement of Ξ and $(\Omega + \bar{\Omega})$ for different collision energies. The enhancement is the largest at lowest energy, for Ω it can be even by a factor of almost ten larger at $\sqrt{s} = 8.73$ than observed at SPS top energy. At RHIC enhancement of Ω is smaller than at SPS. These predictions are in contrast with UrQMD [26] which predicts an enhancement at RHIC larger by a factor of four than at SPS. Note that at all energies all figures display the WA97 pattern: hierarchy and saturation, the latter indicating that the grand canonical limit is reached. This pattern was predicted as a signal for quark-gluon plasma formation [7]. In the context of the considered SM the enhancement pattern of (multi)strange baryons should be observed at all SPS energies, with increasing strength toward lower beam energy. Thus, the results of the above SM makes it clear that strangeness enhancement and enhancement pattern is not a unique signal of deconfinement as these features are expected to be also there at energies where QGP is not expected to be formed.

The quantitative results shown in Fig. 3 to Fig. 6 contain some uncertainties. The magnitude of the enhancement is very sensitive to temperature taken at given collision energy. Changing T by 5 MeV, a typical error on T in thermal analysis, can change, e.g., the enhancement of Ω shown in Fig. 3 by a factor of two. The N_{part} dependence of strange baryon enhancement seen in Figs. 3-6 was obtained assuming linear dependence of volume parameter V in Eq. (7) with N_{part}. In general V could have a weaker dependence with centrality which could be reflected in only moderate increase of the enhancement with centrality and saturation appearing at larger volume. In addition, including variation of thermal parameters, in particular the baryon-chemical potential, with centrality or extending the model to canonical description of baryon number conservation [27] or, finally, including a possible asymmetry between strangeness undersaturation factor in pp and AA collisions [27,28], could change our numerical values. However, independently of these uncertainties, the main results: (i) enhancement decreasing with increasing collision energy and (ii) enhancement pattern being preserved at all SPS and RHIC energies, are always valid.

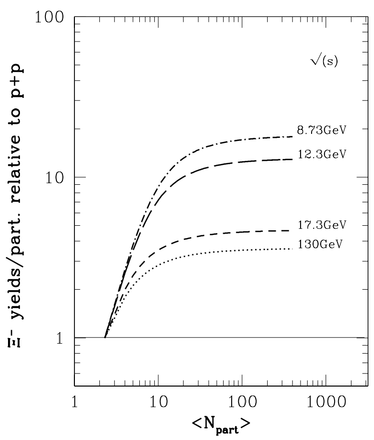

FIG. 5. Centrality dependence of relative enhancement of Ξ^- yields/participant in central Pb-Pb to p-p reactions at different collision energies.

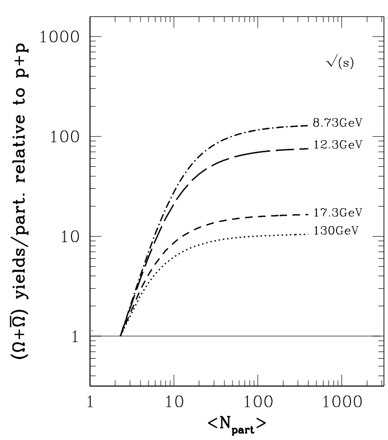

FIG. 6. Centrality dependence of relative enhancement of $(\Omega + \bar{\Omega})$ yields/participant in central Pb-Pb to p-p reactions at different collision energies.

V. CONCLUDING REMARKS

We have shown that in terms of statistical model the relative enhancement of (multi)strange baryons from proton-proton to nucleus-nucleus collisions is a decreasing function of collision energy. Experimentally this fact was already obtained for kaon yields and is shown to be expected for multistrange baryons. In addition, an increase of the enhancement with strangeness content of the particle is a generic feature of our model, independent of collision energy. On the qualitative level the only input being required in the model to make the above predictions is the information that freezeout temperature is increasing with collision energy and that the chemical potential shows the opposite dependence. The above required conditions are well confirmed by a very detailed analysis of particle production at different collision energies.

We have presented the quantitative predictions for relative enhancement of Λ, Ξ and Ω yields in the energy range form $\sqrt{s} = 8.7$ up to $\sqrt{s} = 130$ GeV. We have discussed possible uncertainties of presented results. The relative enhancement at RHIC was found to be lower than at the SPS. This is in contrast with UrQMD [26] predictions and with the previous qualitative predictions of observed enhancement as being entirely due to quark-gluon plasma formation [7]. Even an abrupt change in the enhancement of $\bar{\Xi}/N_{part}$ versus N_{part}, recently reported by the NA57, could be possibly accounted for in terms of canonical model when assuming a very particular centrality dependence of the correlation volume [12]. It would correspond to a sudden jump of the volume, as in a first order phase transition, which can be taken into account in the canonical suppression factor.

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REFERENCES

[1] Proceedings of *Quark Matter '99* , *Nucl. Phys.* A **661** (1999).
[2] H. Satz, *Rept. Prog. Phys.* **63**, 1511 (2000).
[3] R. Stock, *Phys. Lett.* **456**, 277 (1999); *Prog. Part. Nucl. Phys.* **42**, 295 (1999).
[4] J. Stachel, *Nucl. Phys.* A **654**, 119c (1999).
[5] U. Heinz, *Nucl. Phys.* A **685**, 414 (2001); *Nucl. Phys.* A **661**, 349 (1999).
[6] F. Karsch, *Nucl. Phys.* B (Proc. Suppl.) **83-84**, 14 (2000) and Proceedings of *Quark Matter 2001.*
[7] J. Rafelski, and B. Müller, *Phys. Rev. Lett.* **48**, 1066 (1982); P. Koch, B. Müller, and J. Rafelski, *Phys. Rept.* **142**, 167 (1986) ; J. Rafelski, *Phys. Lett.* B **262**, 333 (1991).
[8] J. Rafelski, J. Letessier, and A. Tounsi, *Acta Phys. Pol.* **B 27**.
[9] E. Andersen, *et al.*, WA97 Collaboration, *Phys. Lett.* B **449**, 401 (1999).
[10] N. Carrer, NA57 Collaboration, Proceedings of *Quark Matter 2001*
[11] S. Kabana *et al.*, NA52 Collaboration, *J. Phys.* G **27**, 495 (2001).
[12] B. Müller, private communication.
[13] J. C. Dunlop *et al.*, *Phys. Rev.* C **61**, 031901 (2000); C. A. Ogilvie, Proceedings of *Quark Matter 2001.*
[14] Ch. Blume, NA49 Collaboration, Proceedings of *Quark Matter 2001.*
[15] J. S. Hamieh, K. Redlich, and A. Tounsi, *Phys. Lett.* B **486**, 61 (2000).
[16] K. Redlich, and A. Tounsi, *Preprint PAR/LPTHE/01-39.*
[17] R. Hagedorn, *CERN Rept.* **71** (1971); E.Shuryak, *Phys. Lett.* B **42**, 357 (1972) ; J. Rafelski, and M. Danos, *Phys. Lett.* B **97**, 279 (1980); R. Hagedorn *et al.*,*Z. Phys.* C **27**, 541 (1985).
[18] C.M. Ko, V. Koch, Z. Lin, K. Redlich, M. Stepanov, and X.N. Wang, *Phys. Rev. Lett.* 86 (2001) 5438.
[19] K. Redlich, and L. Turko, *Z. Phys.* C **5**, (1980) 201.
[20] K. Redlich, J. Cleymans, H. Oeschler, and A. Tounsi, hep-th/0110337.
[21] P. Braun-Munzinger, J. Cleymans, H. Oeschler, and K. Redlich, hep-ph/0106066, Nucl. Phys. A in print.
[22] N. Carrer *al.* (W97 and NA57 Collaborations), Proceedings of *Quark Matter'99.*
[23] F. Becattini, M. Gazdzicki, and Sollfrank, *Eur. Phys.* JC **5** (1998) 143.
[24] P. Braun-Munzinger, I. Heppe, and J. Stachel, *Phys. Lett.* B **465**, 15 (1999).
[25] P. Braun-Munzinger, D. Magestro, K. Redlich, and J. Stachel, *Phys. Lett.* B **518**, 41 (2001).
[26] M. Bleicher, W. Greiner, H. Stöcker, and N. Xu, *Phys. Rev.* C **62**, 061901 (2000).
[27] F. Becattini, *Z. Phys.* C **69** (1996) 485; F. Becattini, and U. Heinz, *Z. Phys.* C **76** (1997) 269.
[28] F. Becattini, J. Cleymans, A. Keranen, E. Suhonen, and K. Redlich, *Phys. Rev.* C **64** (2001) 024901.