

# Implementation of Reinforcement-Learning Drift Diffusion Model in rStan

Computational Modeling

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# Introduction

## Suicidal thoughts and behaviors (STB)

: (nonfatal) suicidal ideation or suicidal attempts

: **active** responses to **escape aversive states**

# Research Paper



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0021-843X/19/\$12.00

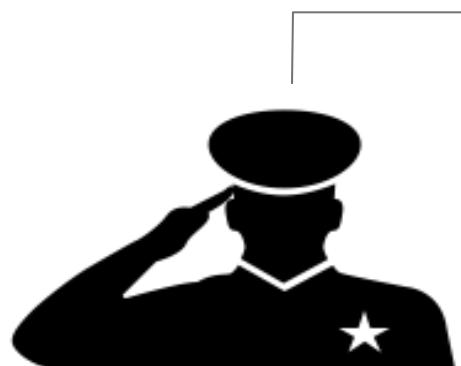
Journal of Abnormal Psychology

2019, Vol. 128, No. 2, 106–118  
<http://dx.doi.org/10.1037/abn0000395>

## Suicidal Thoughts and Behaviors Are Associated With an Increased Decision-Making Bias for Active Responses to Escape Aversive States

Alexander J. Millner  
Harvard University

Hanneke E. M. den Ouden  
Radboud University



**85 military veterans** with Lifetime STB  
**44** with Psychiatric disorders, no STB

# Research Question

Would these two groups of military veterans have different parameter values, which account for response bias, in escape condition?

**Task** Go/No-go

**Model** RL-DDM (Matlab)

Hierarchical Expectation Maximization

1. **Starting points**

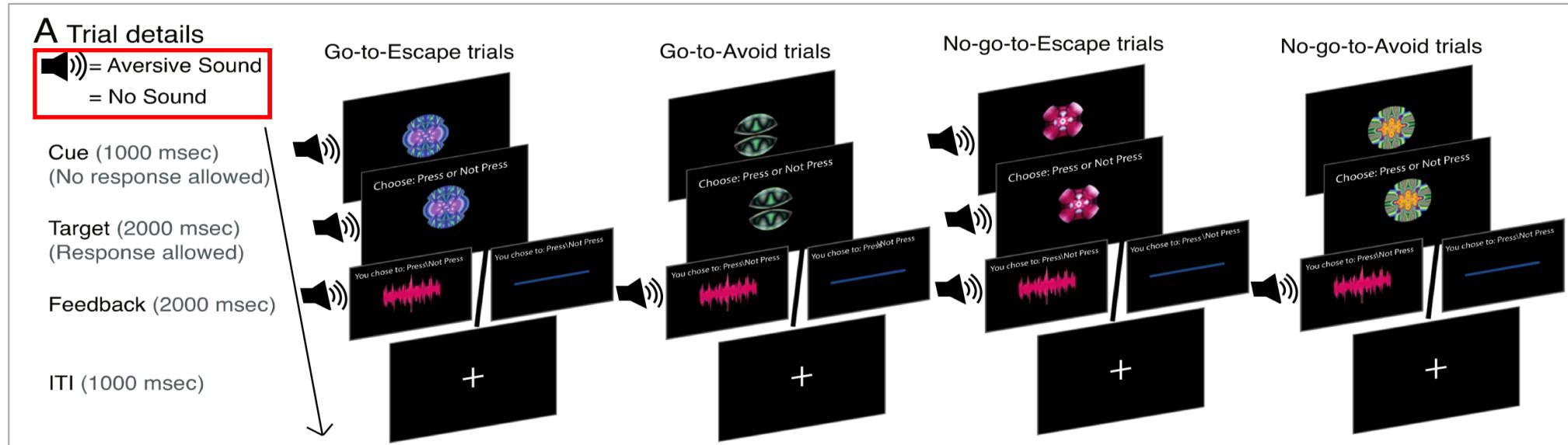
2. Drift rates

|                    | Escape         |            | Avoid          |            |
|--------------------|----------------|------------|----------------|------------|
|                    | Starting point | Drift rate | Starting point | Drift rate |
| Suicidal group     | w1             | $\beta_1$  | w2             | $\beta_2$  |
| Non-suicidal group | w1             | $\beta_1$  | w2             | $\beta_2$  |

**Note** The authors compared two main models: one with different starting points for each condition and one with two drift rates, each for one condition. As they calculated point estimates, BIC was the tool for model comparison.

# Task: Go/No-go

## Go/No-go Task with Aversive Sound Cues



Millner et al. (2019)

**Go/no-go task** with aversive sound cues includes two conditions, escape and avoid, which are distinguished by the existence of aversive sound cues (fork scraping a plate) at the beginning of a trial. A participant makes either go or no-go response for each trial, and the optimal response has to be learned throughout the trials by relating the sound cue and presented image with the corresponding feedback. There are two feedback types: aversive sound or silence.

# Modeling in the Paper

Reinforcement Learning

$$Q_{t+1}(s_t, a_t) = Q_t(s_t, a_t) + \alpha[r_t - Q_t(s_t, a_t)]$$

Drift Rate based on Q-value

$$\mu_t = \beta_0 + \beta_1[Q_t(s_t, go) - Q_t(s_t, nogo)]$$

Drift Diffusion Model

$$Wiener(y|\alpha, \tau, \beta, \delta) = \frac{\alpha^3}{(y-\tau)^{3/2}} \exp\left(-\delta\alpha\beta - \frac{\delta^2(y-\tau)}{2}\right) \sum_{k=-\infty}^{\infty} (2k+\beta)\phi\frac{2k\alpha + \beta}{\sqrt{y-\tau}}$$

$\alpha$ : learning rate,  $s_t$ : stimulus,  $a_t$ : action,  $r_t$ : reward,  $\beta_0$ : constant go bias,  $\beta_1$ : shared go bias

$\alpha$ : boundary separation,  $\tau$ : non – decision time,  $\beta$ : drift rate,  $\delta$ : starting point

# Modeling in the Paper

## Hierarchical Expectation Maximization

Learning rate (alpha)

Starting point for escape condition (w1)

Starting point for avoid condition (w2)

Non-decision time (tau)

Decision boundary (omega)

Constant go bias and shared go bias ( $\beta_0, \beta_1$ )



# Replication

# Implementation

## Model Block

```
// subject loop
for (i in 1:N) {
    // assign individual variables
    real w;
    int c;
    int s;
    matrix[4, 2] Q;

    // initial values for state-action values
    Q = rep_matrix(0.0, 4, 2);

    for (t in 1:T) {
        real v;
        c = C[t, i];
        s = cond[t, i];

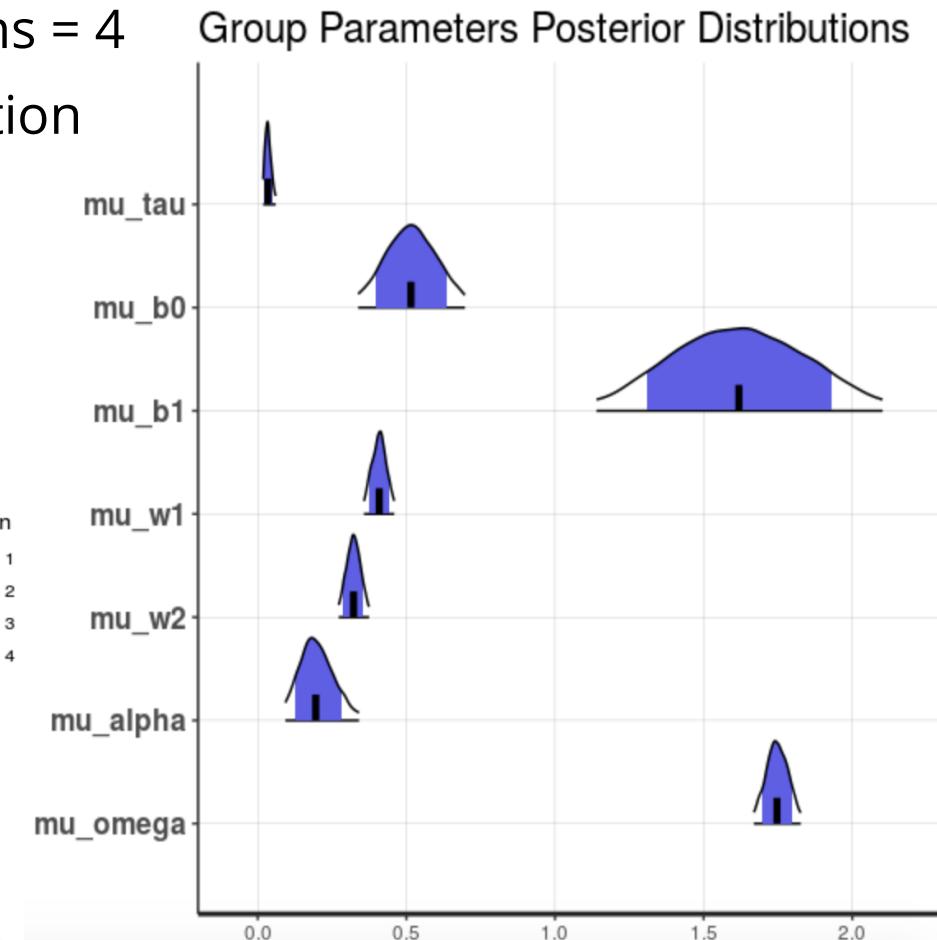
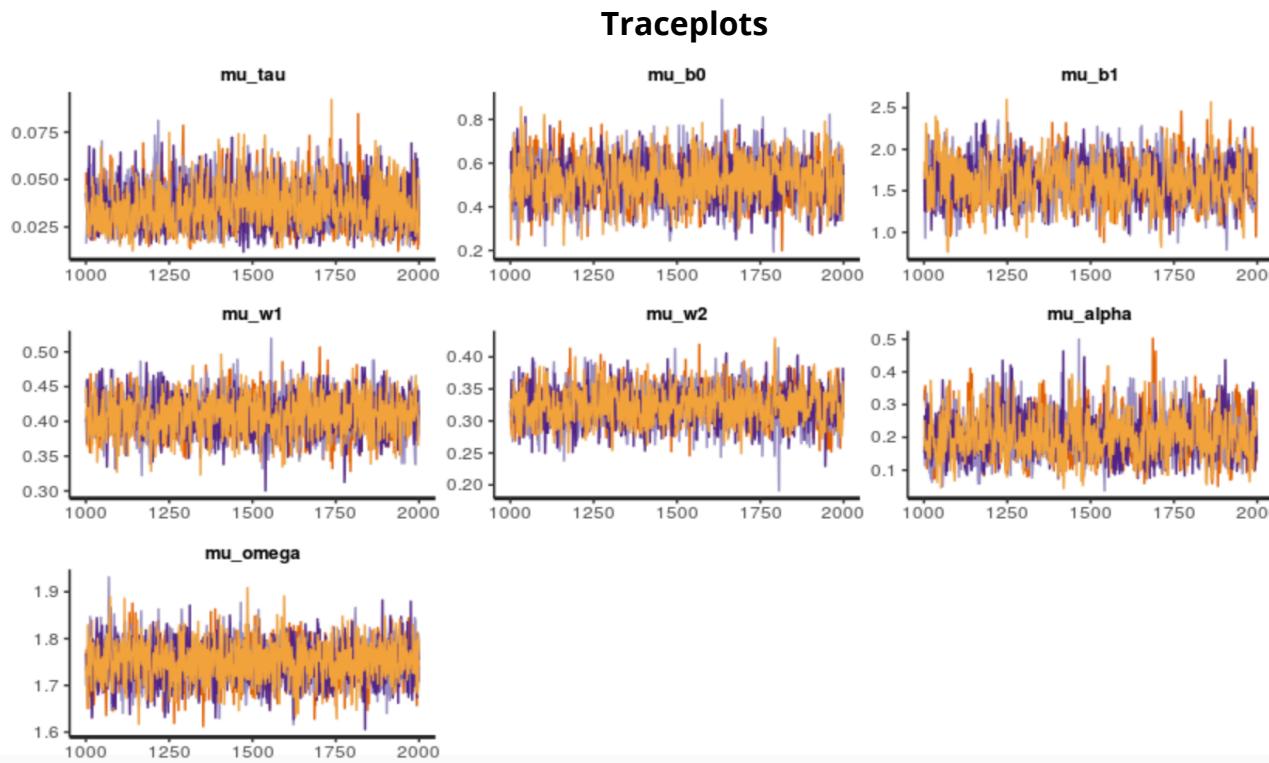
        w = (E[t, i] == 1) ? w1[i] : w2[i];
        v = b0[i] + (b1[i] * (Q[s, 2] - Q[s, 1]));

        // increment log probability density and update posterior
        if (c == 1) {
            rt[t, i] ~ wiener(omega[i], tau[i], w, v);
        } else {
            nogo_lik = 0.0;
            for (n in 1:Steps) {
                step_size = (n * 1.0) / Steps;
                temp_rt = step_size * (RTmax - tau[i]) + tau[i];
                prob_density = exp(wiener_lpdf(temp_rt | omega[i], tau[i], 1-w, -v));
                nogo_lik += ((RTmax - tau[i]) / Steps) * prob_density;
            }
            if (is_nan(nogo_lik) == 1) {
                nogo_lik = 2.220446e-16;
            }
            target += log(nogo_lik);
        }

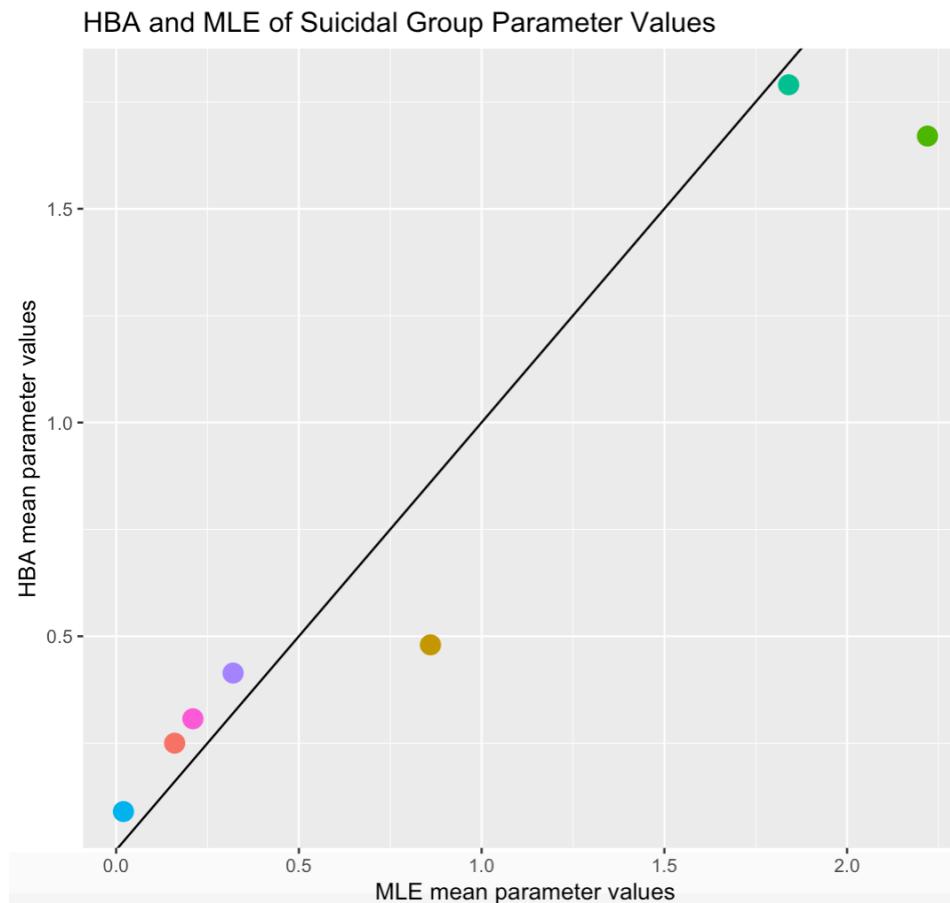
        // update Q-value
        Q[s, c+1] += alpha[i]*(fd[t, i] - Q[s, c+1]);
    }
}
```

# Replication

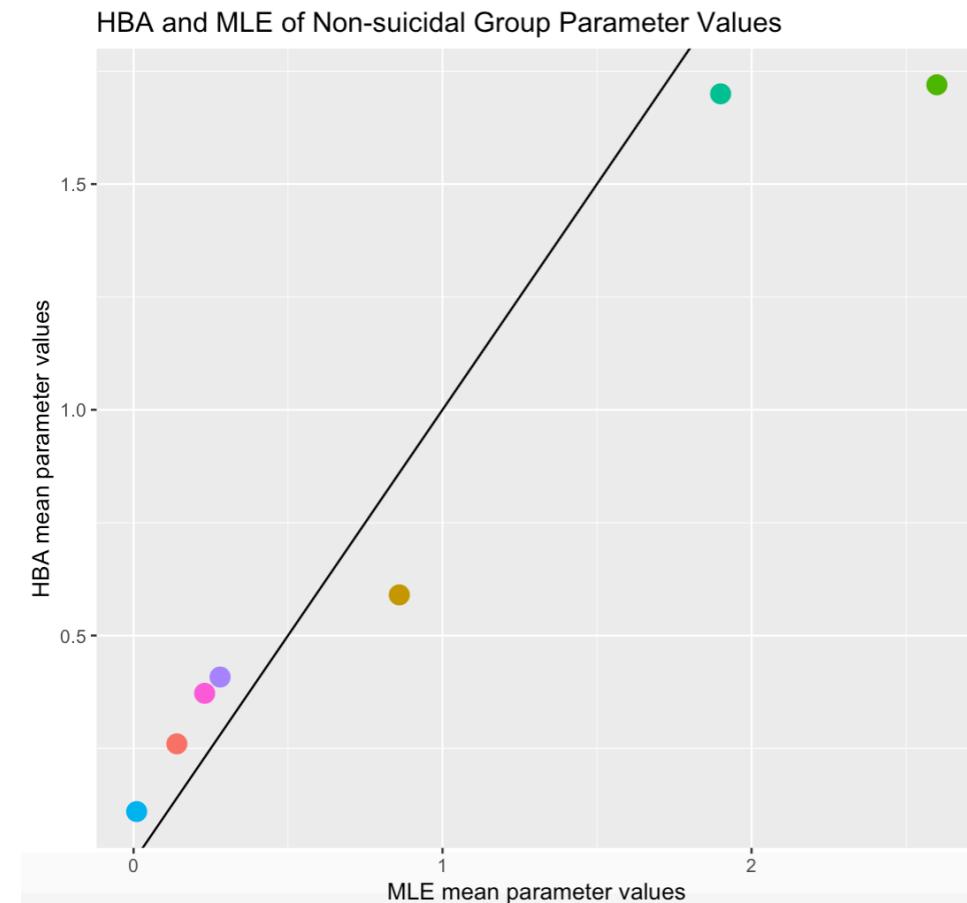
- Number of subjects = **30** (20 suicidal, 10 non-suicidal)
- Step size = 0.04, iters = 2000, warmup = 1000, chains = 4
- Hierarchical Bayesian, two starting points by condition



# Correlation with MLE Estimates

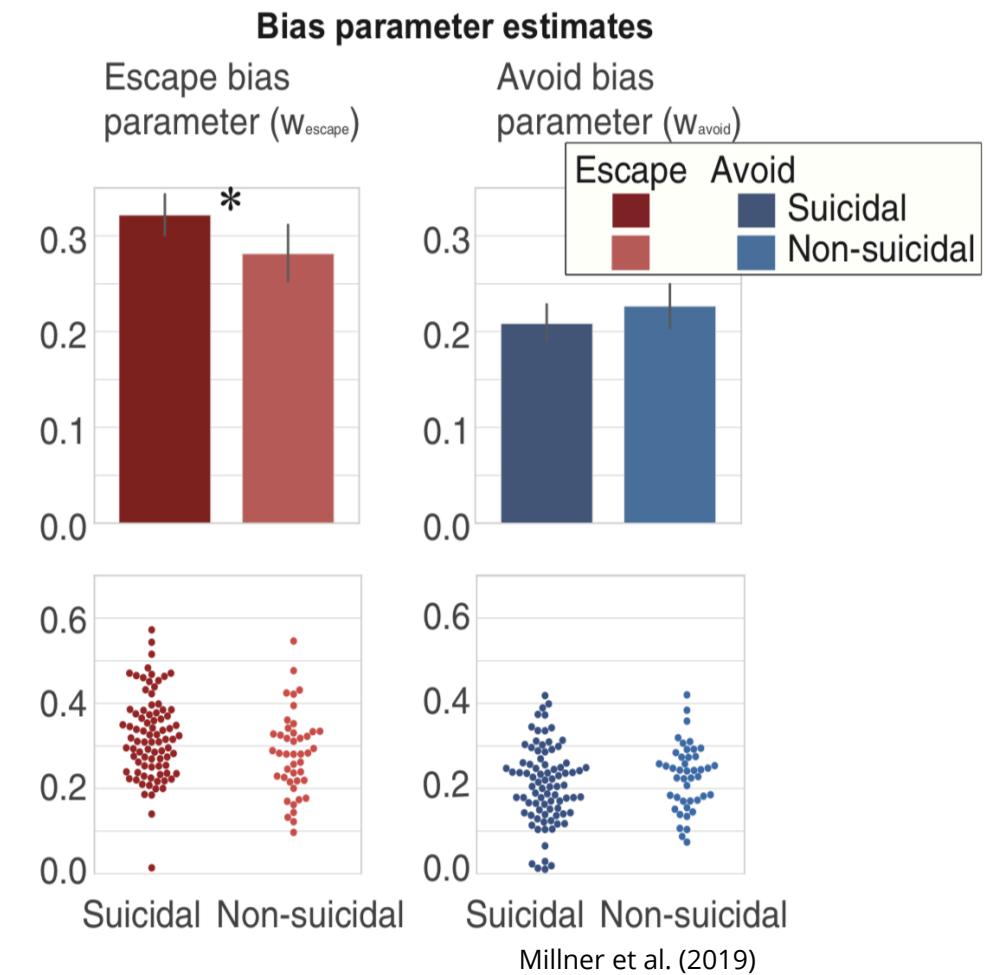
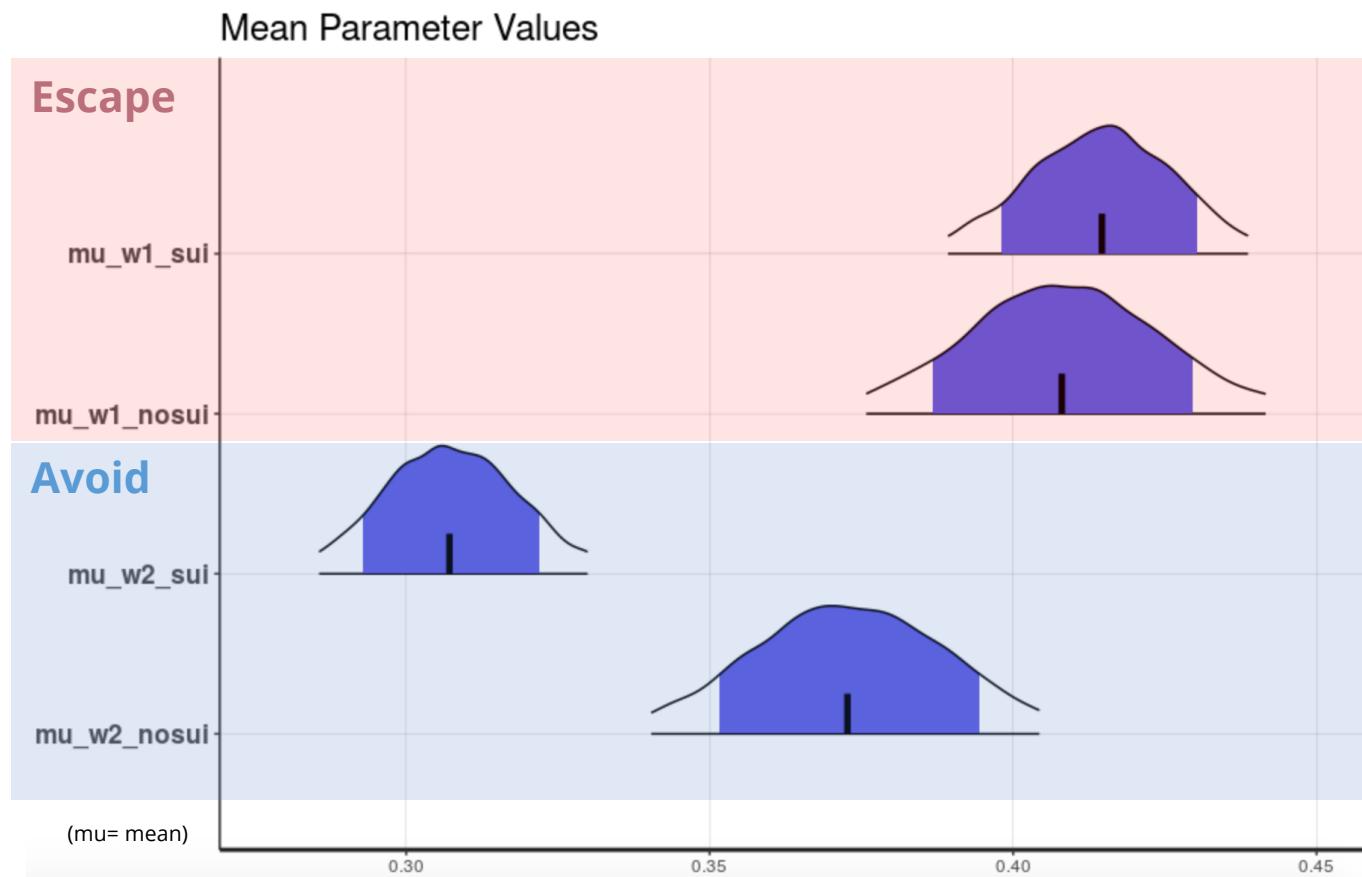


HBA: 20 suicidal participants



HBA: 10 non-suicidal participants

# Results



# Original Model Discussions

1. Sampling using Model 1 has been converged well
2. In escape condition, the suicidal group starts their drifting closer to the go boundary, compared to the non-suicidal group
3. In avoid condition, the suicidal group starts their drifting closer to the no-go boundary, compared to the non-suicidal group
4. The non-suicidal group shows a higher constant go-bias across conditions



# Model Comparison

# Model Comparison

- Number of subjects = **30** (20 suicidal, 10 non-suicidal)
- Step size = 0.04, iters = 2000, warmup = 1000, chains = 4

Shared parameters: non-decision time, decision threshold, learning rate

| Models (hierarchical) | Added parameters             | LOOIC  |   |
|-----------------------|------------------------------|--------|---|
| 1. Original           | $w_1, w_2, \beta_0$          | 6580.8 |   |
| 2. Two drift rates    | $w, \beta_1, \beta_2$        | 6496.0 |   |
| 3. Both               | $w_1, w_2, \beta_1, \beta_2$ | 6404.4 | ● |

# Two Drift Rates Model (Model 2)

Model 2: Two drift rates + One starting point

- For escape trials:

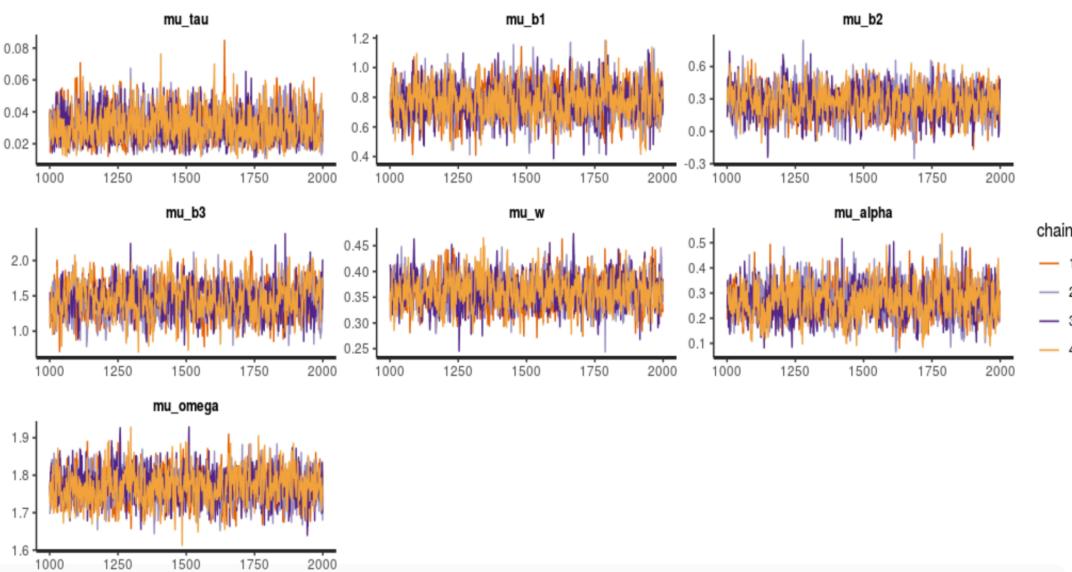
$$\mu_t = \beta_1 + \beta_3 [Q_t(s_t, go) - Q_t(s_t, nogo)]$$

- For avoid trials:

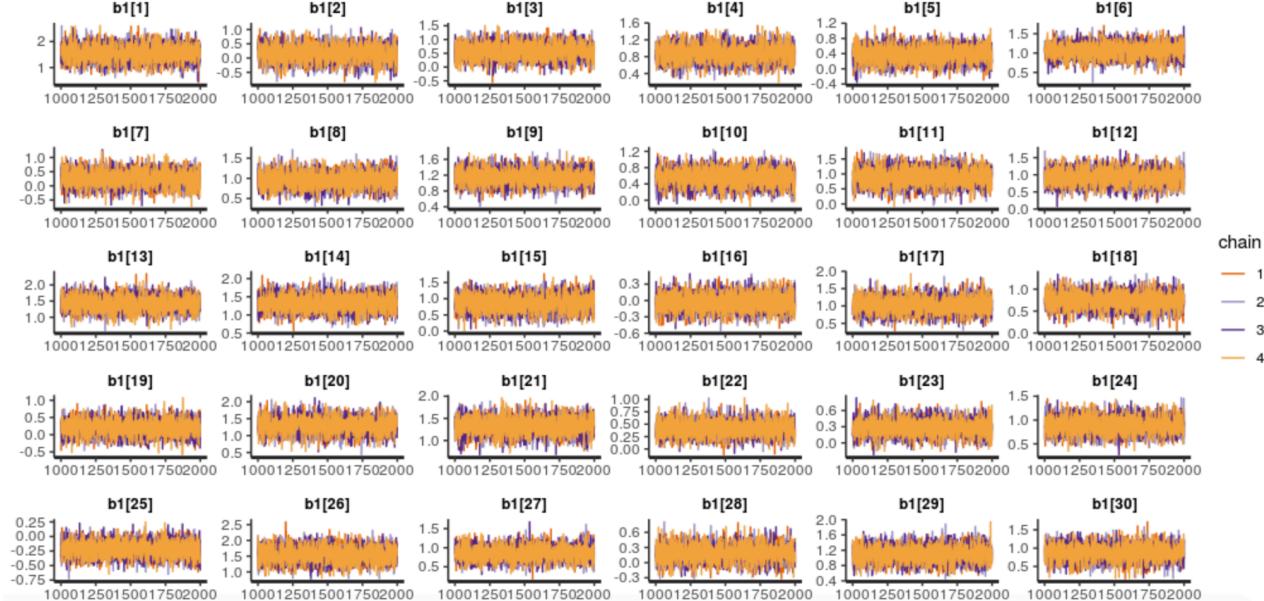
$$\mu_t = \beta_2 + \beta_3 [Q_t(s_t, go) - Q_t(s_t, nogo)]$$

# Model 2 Traceplots

Group Parameters Traceplots



Individual Parameter (b1) Traceplots



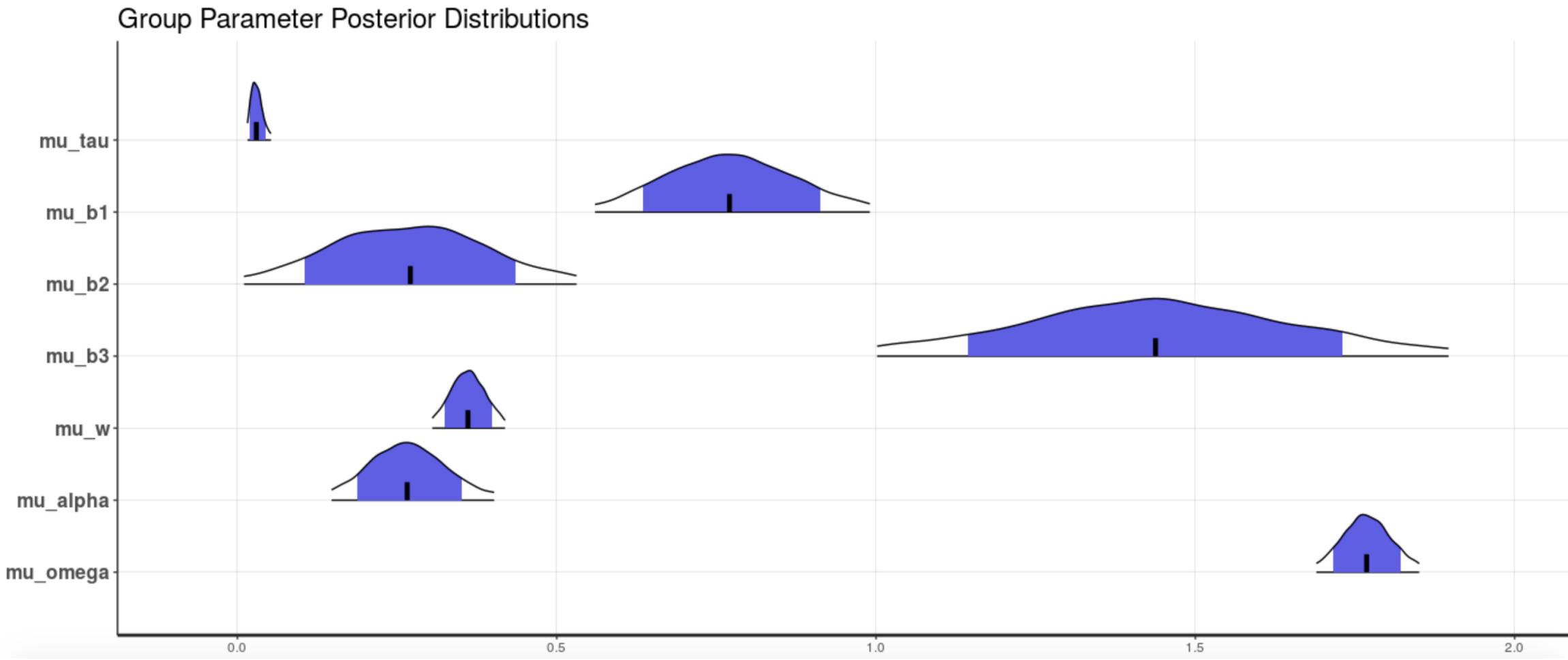
Group Parameters Rhat

| $\mu_{\tau}$ | $\mu_{b1}$ | $\mu_{b2}$ | $\mu_{b3}$ | $\mu_w$  | $\mu_\alpha$ | $\mu_\omega$ |
|--------------|------------|------------|------------|----------|--------------|--------------|
| 1.001166     | 1.001342   | 1.004477   | 1.005206   | 1.002314 | 1.005245     | 1.005193     |

Individual Parameter (b1) Rhat

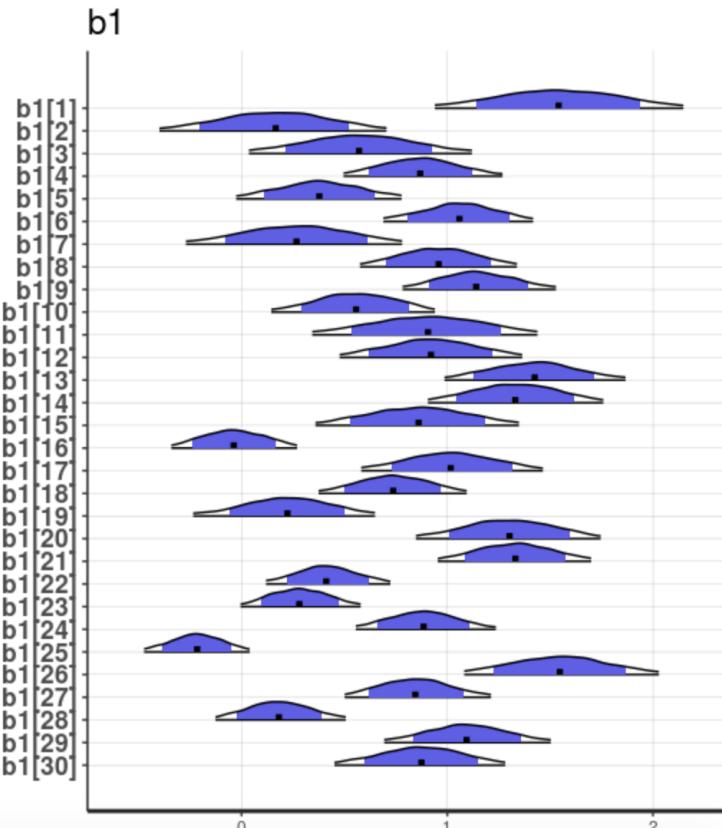
|           |           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.9998846 | 0.9993839 | 0.9991189 | 0.9993771 | 1.0003322 | 1.0001786 |
| 1.0005688 | 0.9996061 | 0.9999021 | 1.0000047 | 0.9998642 | 0.9998884 |
| 0.9996462 | 1.0007733 | 1.0001590 | 0.9995579 | 0.9999377 | 0.9993409 |
| 0.9995668 | 0.9994451 | 0.9993031 | 1.0001534 | 1.0003226 | 0.9996304 |
| 0.9999373 | 0.9997360 | 0.9998385 | 1.0002911 | 0.9994443 | 0.9992494 |

# Model 2 Posterior Distributions

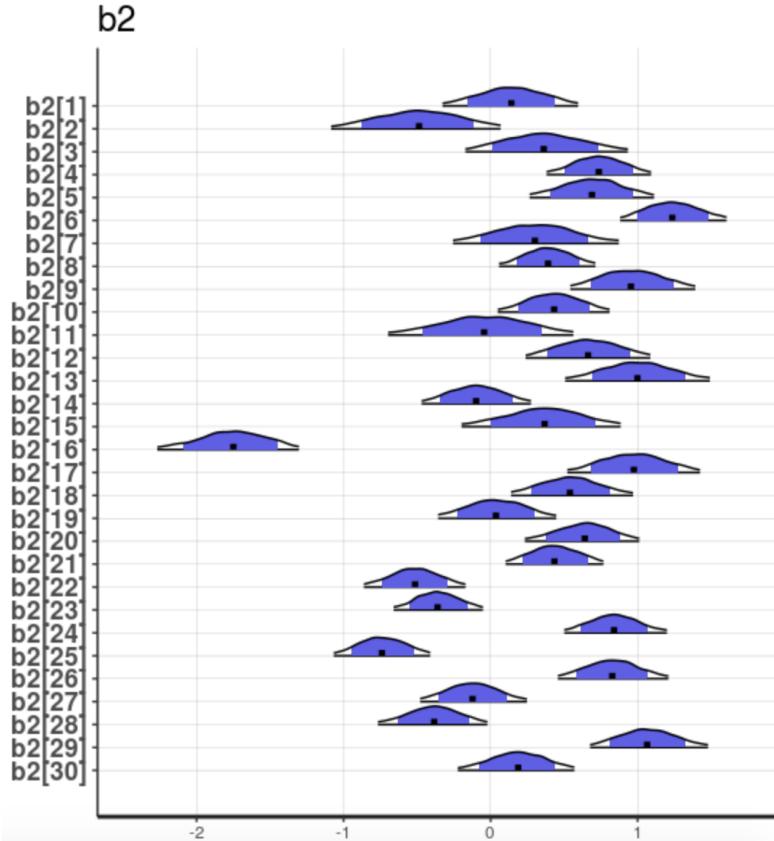


# Model 2 Posterior Distributions

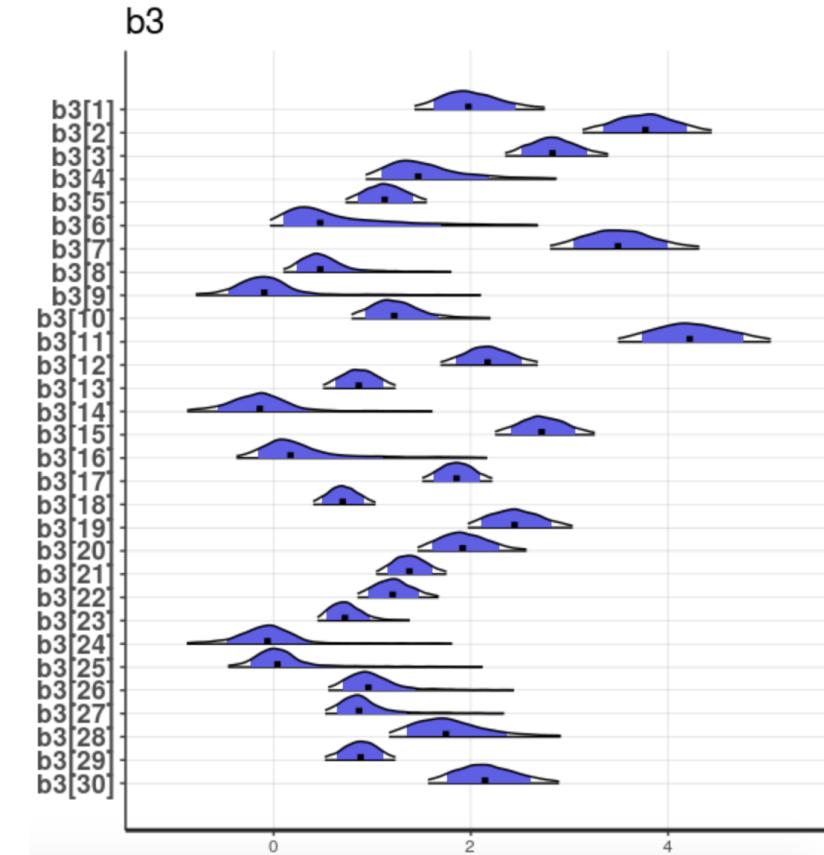
$\beta_1$ : Intercept for drift rate in escape



$\beta_2$ : Intercept for drift rate in avoid

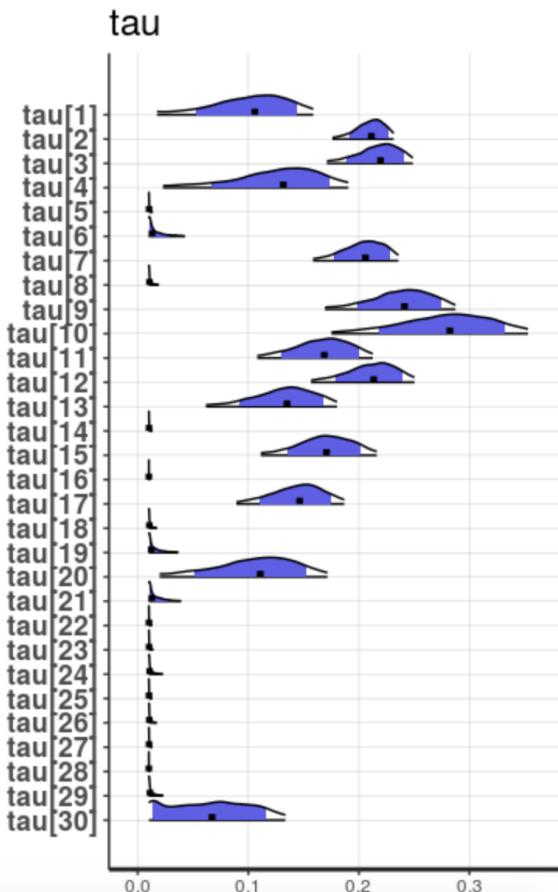


$\beta_3$ : Differential weight

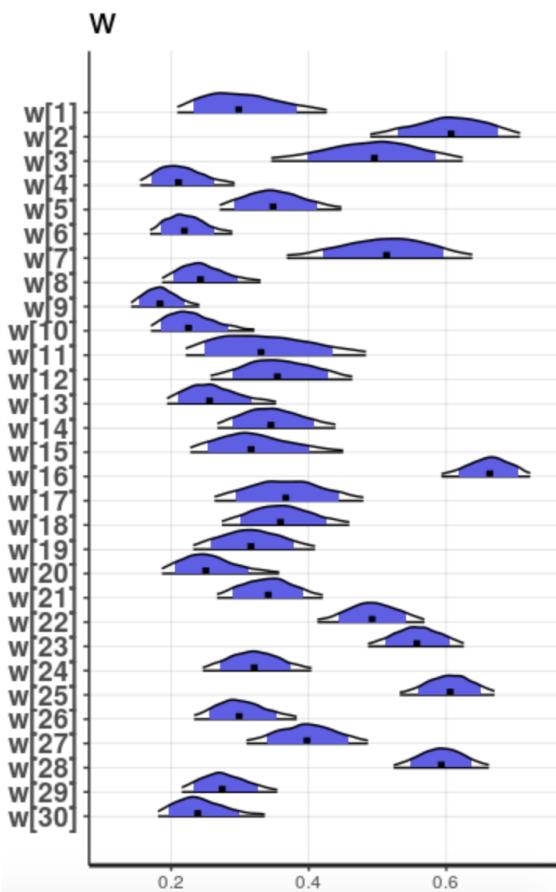


# Model 2 Posterior Distributions

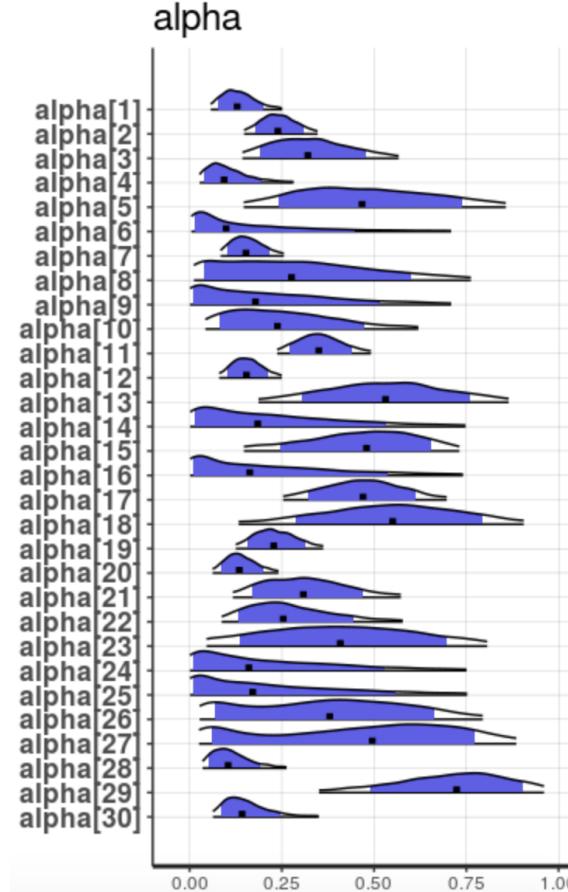
$\tau$ : Non-decision time



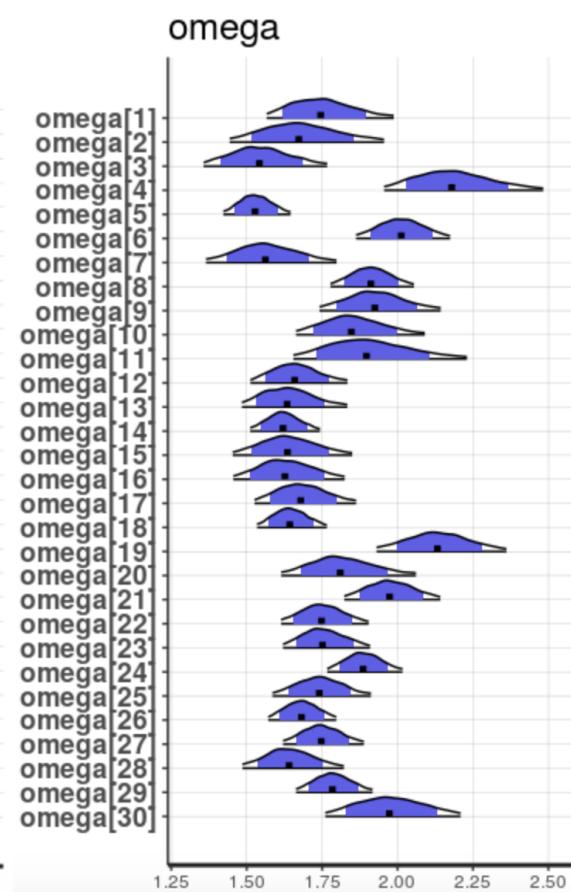
w : Starting point for both conditions



$\alpha$  : Learning rate



$\omega$  : Boundary separation



# Model 2 Suicidal versus Non-suicidal

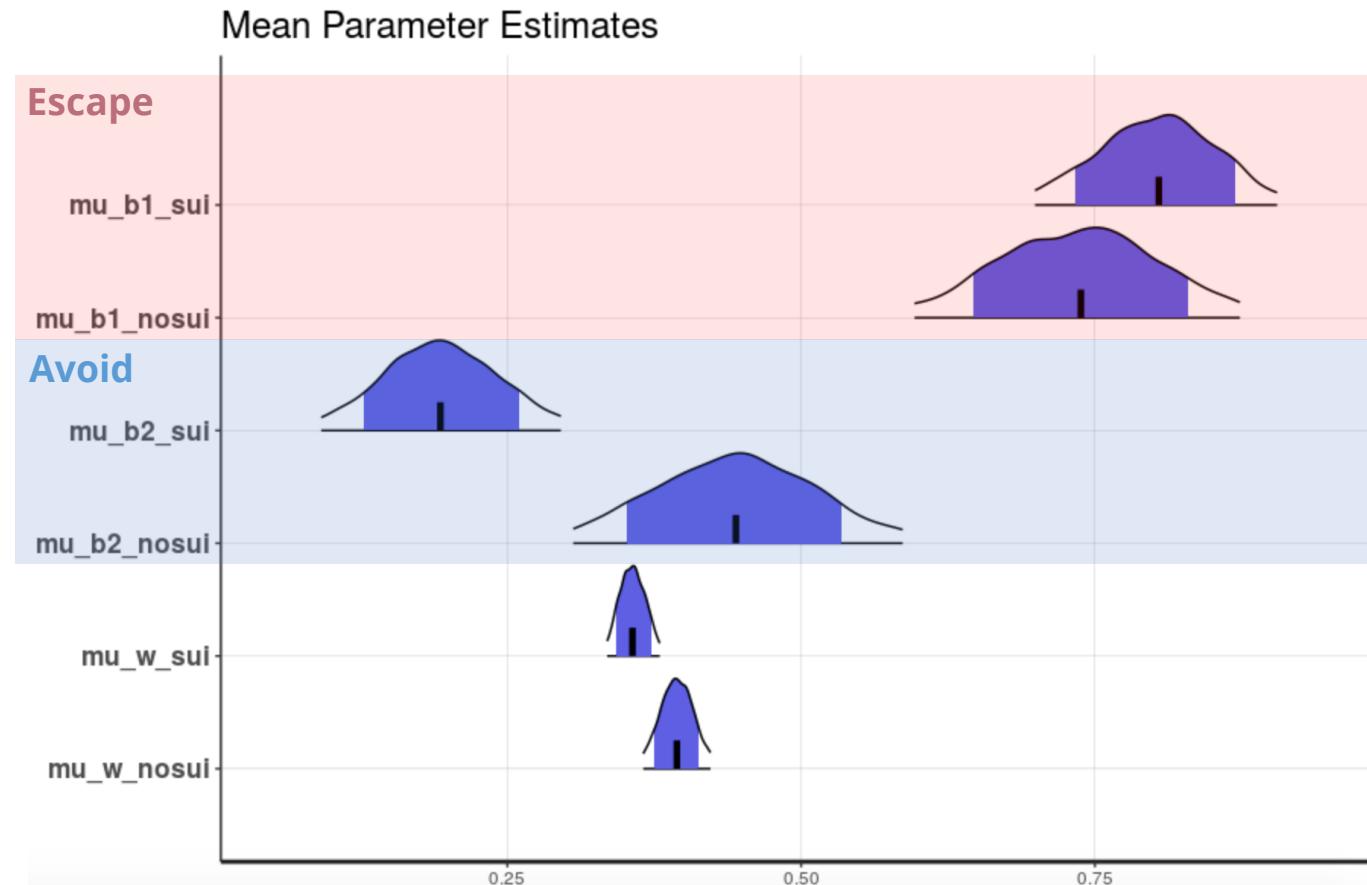


Table 2: Mean Estimated Parameter Values by Group

|             | mean      | sd        | Rhat      |
|-------------|-----------|-----------|-----------|
| mu_b1_sui   | 0.8029664 | 0.0528404 | 0.9994440 |
| mu_b1_nosui | 0.7364864 | 0.0715878 | 0.9999966 |
| mu_b2_sui   | 0.1934772 | 0.0517712 | 0.9994596 |
| mu_b2_nosui | 0.4440975 | 0.0719152 | 1.0005187 |
| mu_b3_sui   | 1.4629532 | 0.1125108 | 1.0049626 |
| mu_b3_nosui | 1.5828252 | 0.1509983 | 1.0060614 |
| mu_w_sui    | 0.3566904 | 0.0115995 | 0.9995703 |
| mu_w_nosui  | 0.3942289 | 0.0146981 | 1.0002459 |

(mu = mean)

# Model 2 Discussions

1. Sampling using Model 2 has been converged well
2. Model 2 is the second best model among three based on LOOIC with 30 subjects
3. In escape condition, the suicidal group has a higher drift rate on average, compared to the non-suicidal group
4. In avoid condition, the average drift rate of the suicidal group is lower than that of the non-suicidal group
5. The non-suicidal group shows a higher constant go-bias across conditions

# Both Model (Model 3)

Model 3: Two drift rates + Two starting points

- For escape trials:

$$\mu_t = \beta_1 + \beta_3 [Q_t(s_t, go) - Q_t(s_t, nogo)]$$

$$w = w1$$

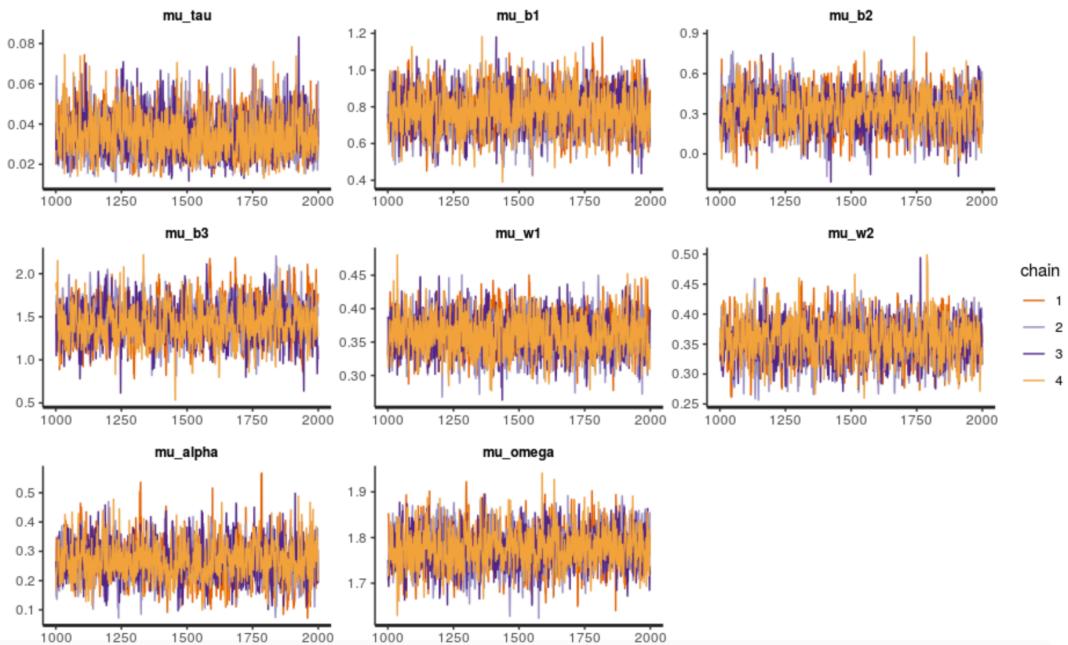
- For avoid trials:

$$\mu_t = \beta_2 + \beta_3 [Q_t(s_t, go) - Q_t(s_t, nogo)]$$

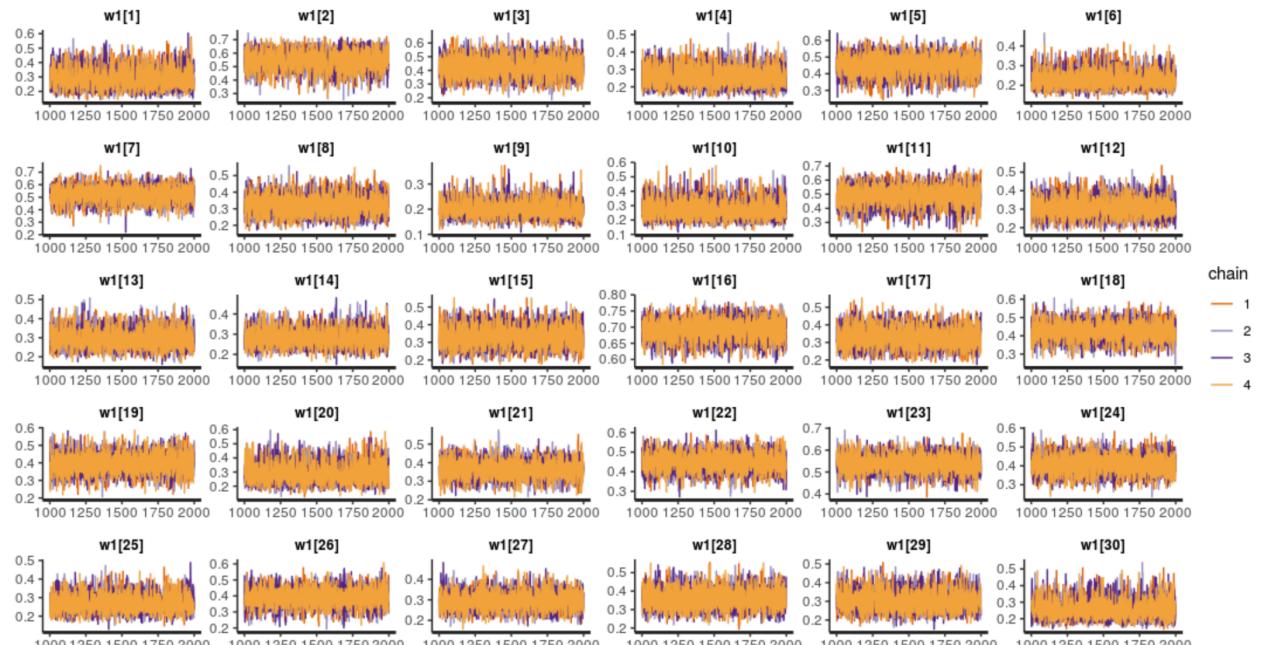
$$w = w2$$

# Model 3 Traceplots

Group Parameters Traceplots



Individual Parameter (w1) Traceplots



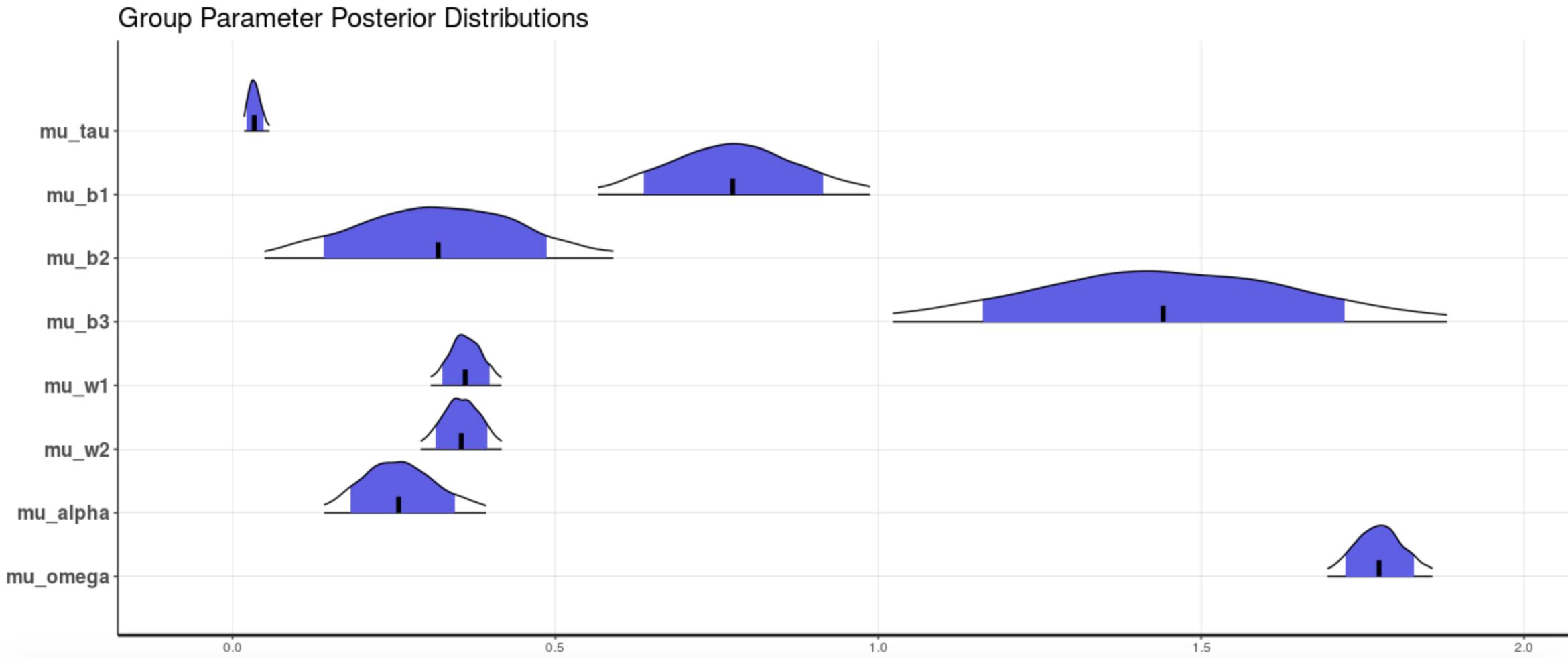
Group Parameters Rhat

| mu_tau   | mu_alpha | mu_omega | mu_b1    | mu_b2    | mu_b3    | mu_w1    | mu_w2    |
|----------|----------|----------|----------|----------|----------|----------|----------|
| 1.004038 | 1.010854 | 1.002062 | 1.001119 | 1.000251 | 1.012955 | 1.003401 | 1.003091 |

Individual Parameter (w1) Rhat

|           |           |          |           |          |           |           |           |           |           |
|-----------|-----------|----------|-----------|----------|-----------|-----------|-----------|-----------|-----------|
| 1.0022604 | 1.0006801 | 1.001479 | 1.0010816 | 1.000591 | 0.9995712 | 0.9996424 | 0.9995426 | 0.9994502 | 0.9996790 |
| 1.0003806 | 0.9995791 | 1.000540 | 0.9999075 | 1.000334 | 0.9998915 | 1.0006931 | 0.9997029 | 0.9992349 | 0.9999627 |
| 0.9993025 | 1.0006932 | 1.000011 | 0.9996544 | 1.000278 | 0.9994667 | 0.9995979 | 0.9998168 | 0.9996939 | 1.0003782 |

# Model 3 Posterior Distributions



# Model 3 Posterior Distributions

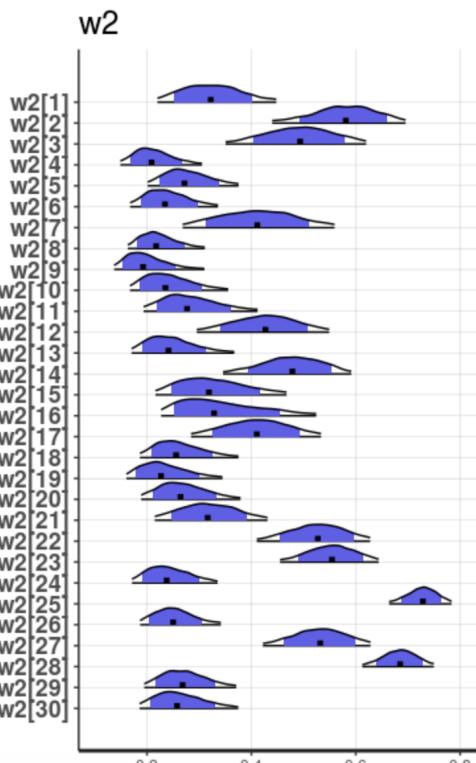
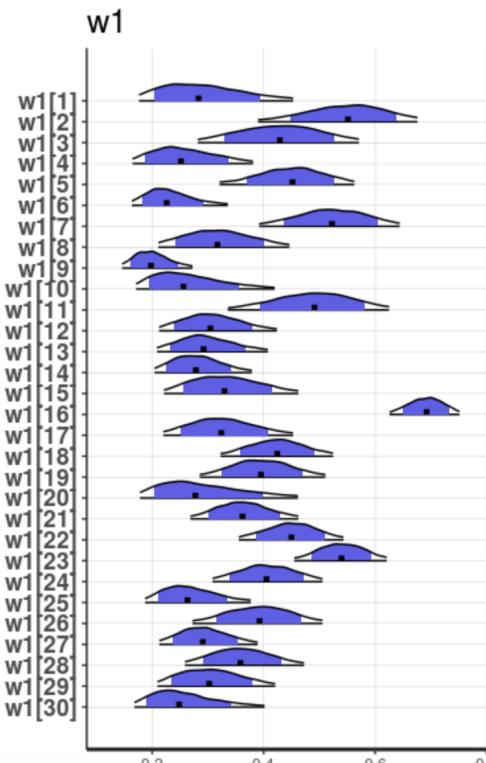
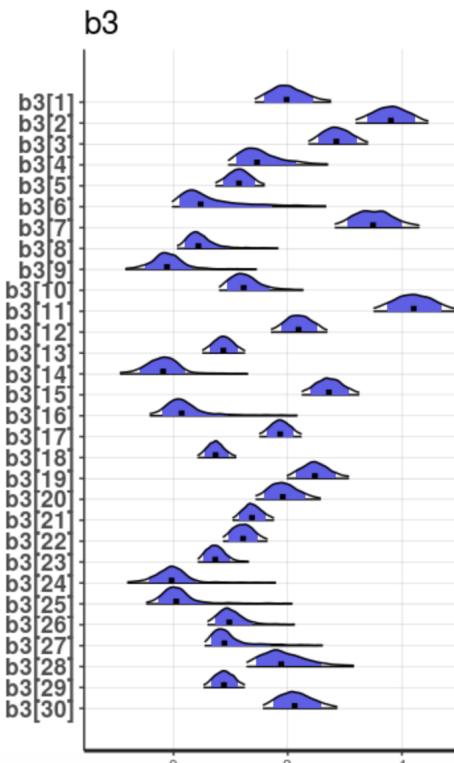
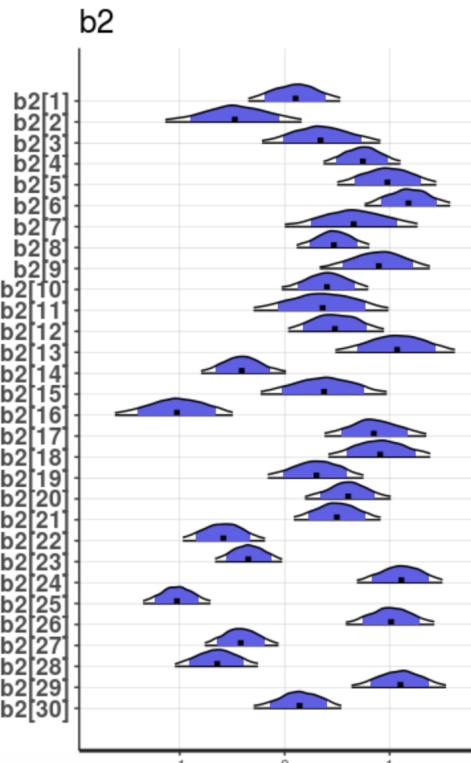
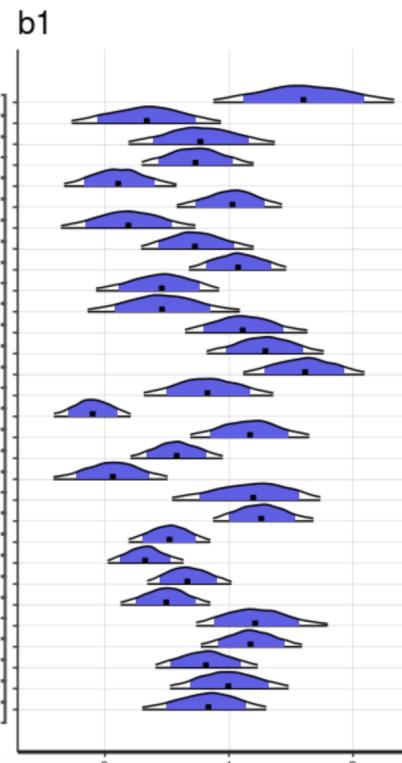
$\beta_1$ : Intercept of drift rate  
in escape condition

$\beta_2$ : Intercept of drift rate  
in avoid condition

$\beta_3$ : Differential weight

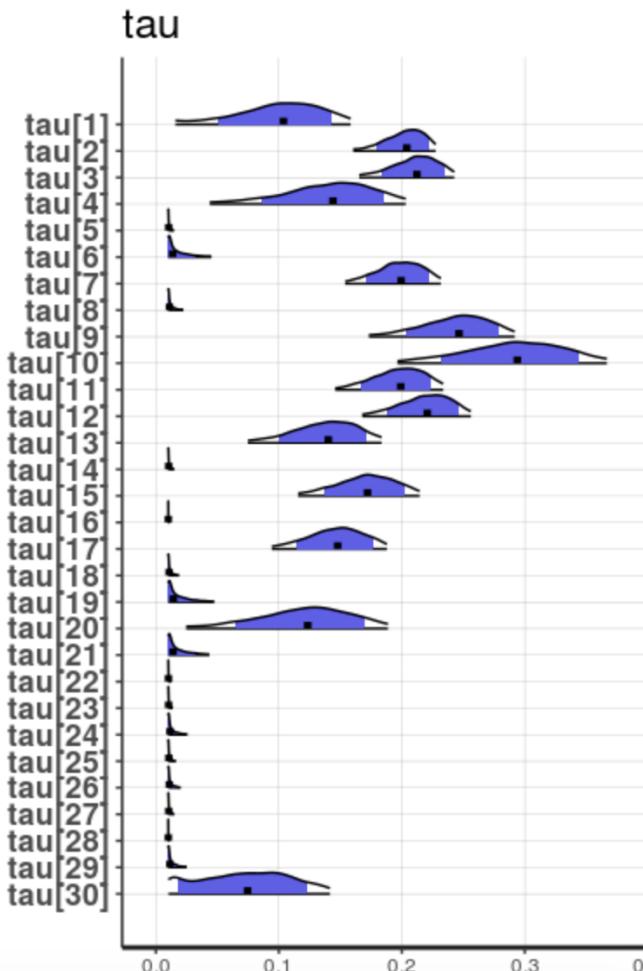
$w_1$  : Starting point  
in escape condition

$w_2$  : Starting point  
in avoid condition

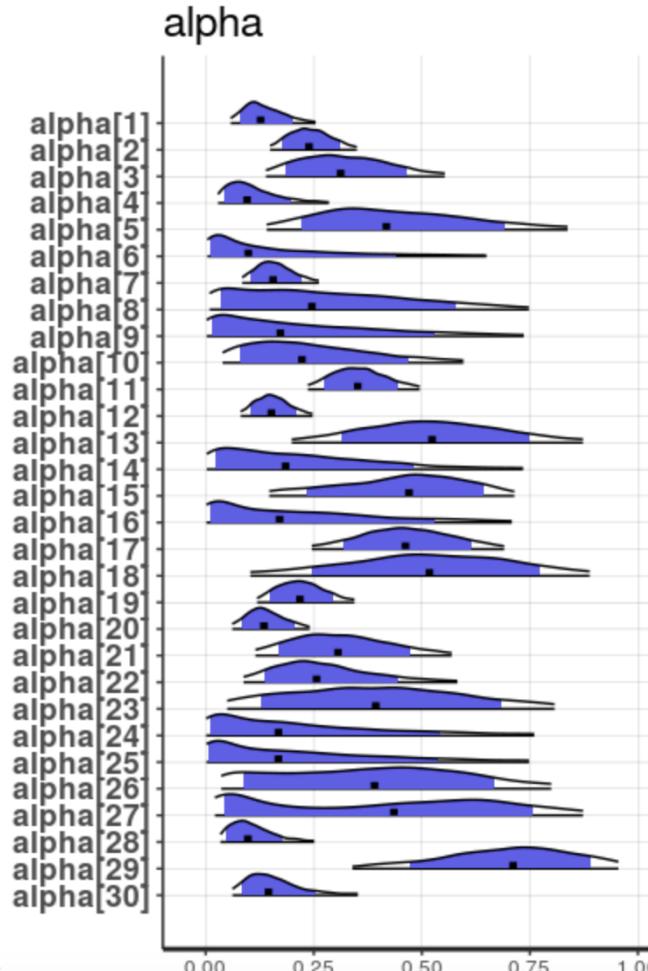


# Model 3 Posterior Distributions

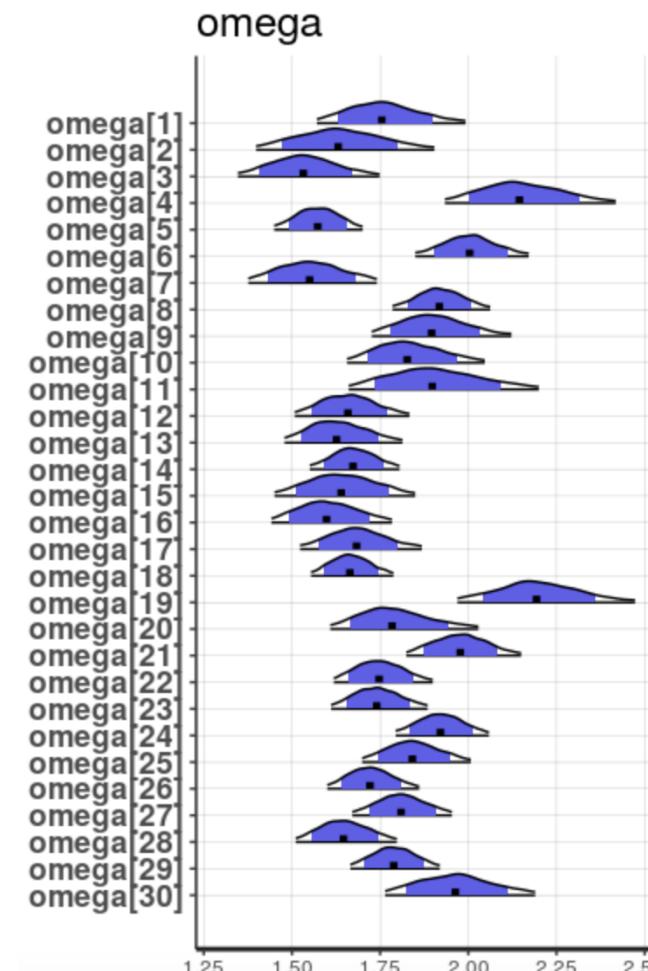
$\tau$  : Non-decision time



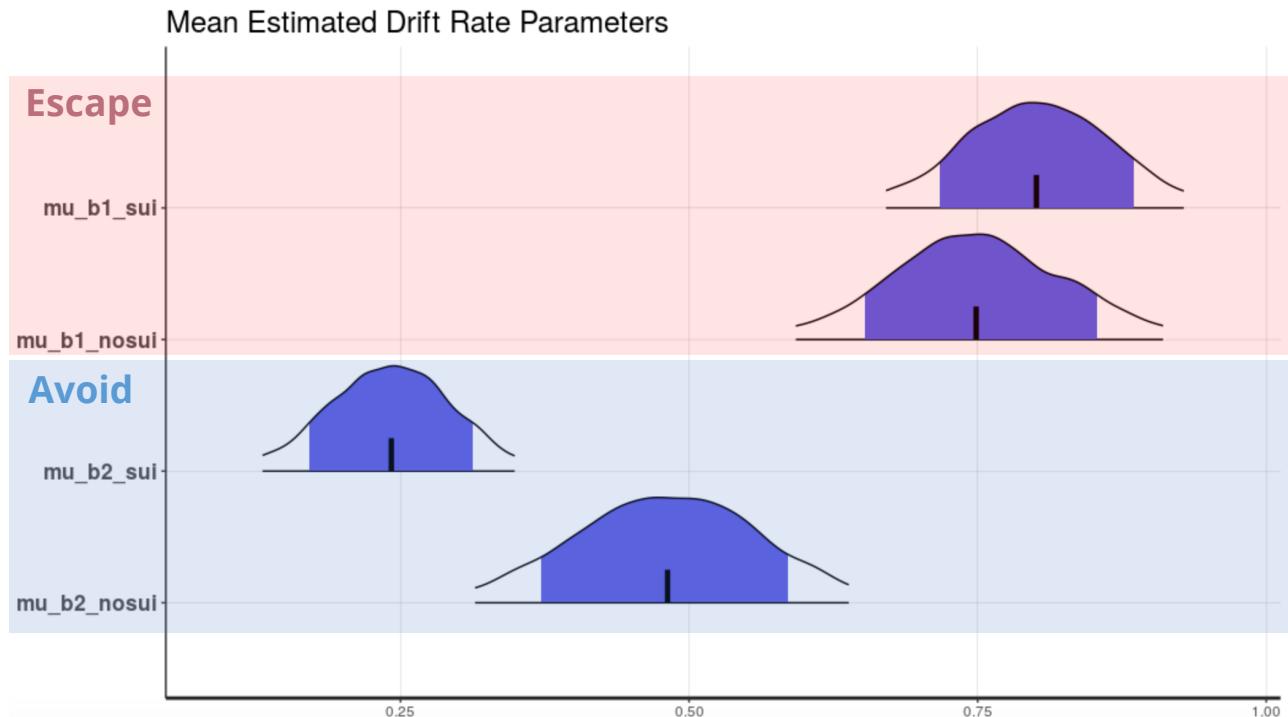
$\alpha$  : Learning rate



$\omega$  : Boundary separation

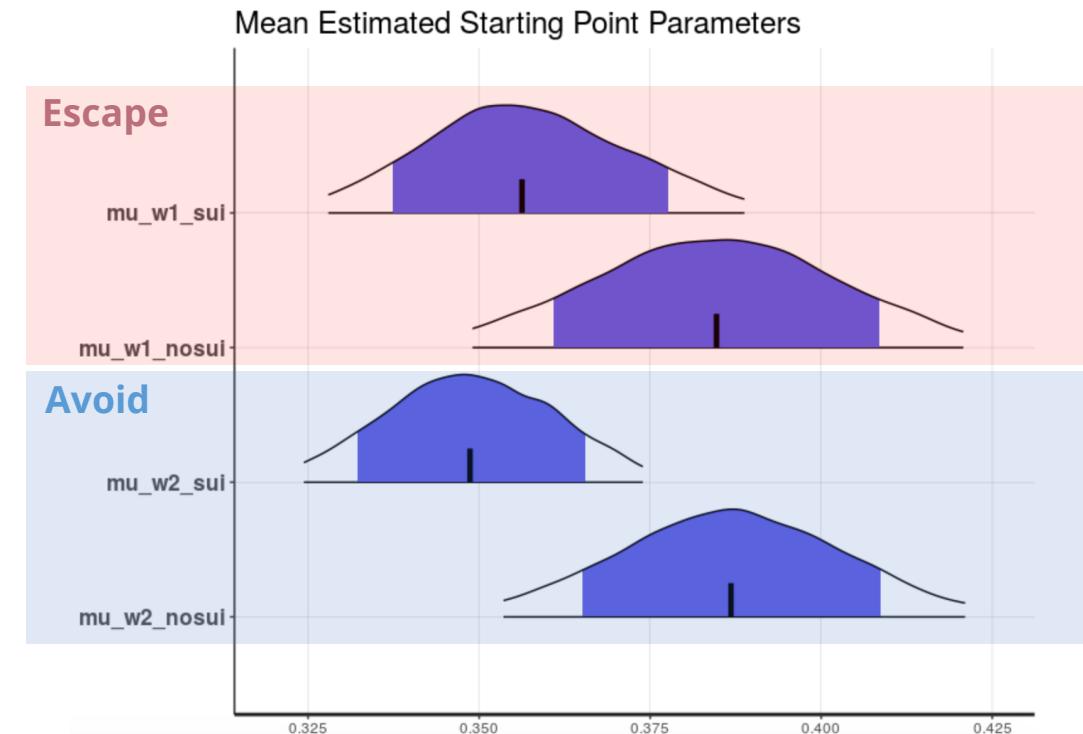


# Model 3 Suicidal versus Non-suicidal



|      | $\mu_{b1\_sui}$ | $\mu_{b1\_nosui}$ | $\mu_{b2\_sui}$ | $\mu_{b2\_nosui}$ | $\mu_{b3\_sui}$ | $\mu_{b3\_nosui}$ |
|------|-----------------|-------------------|-----------------|-------------------|-----------------|-------------------|
| mean | 0.8008688       | 0.7504390         | 0.2413617       | 0.4803858         | 1.4633480       | 1.598060          |
| sd   | 0.0656118       | 0.0791603         | 0.0552296       | 0.0830168         | 0.1052431       | 0.137212          |

(mu = mean)



|      | $\mu_{w1\_sui}$ | $\mu_{w1\_nosui}$ | $\mu_{w2\_sui}$ | $\mu_{w2\_nosui}$ |
|------|-----------------|-------------------|-----------------|-------------------|
| mean | 0.3570474       | 0.3846383         | 0.3488559       | 0.3869334         |
| sd   | 0.0155943       | 0.0184014         | 0.0128842       | 0.0171818         |

(mu = mean)

# Model 3 Discussions

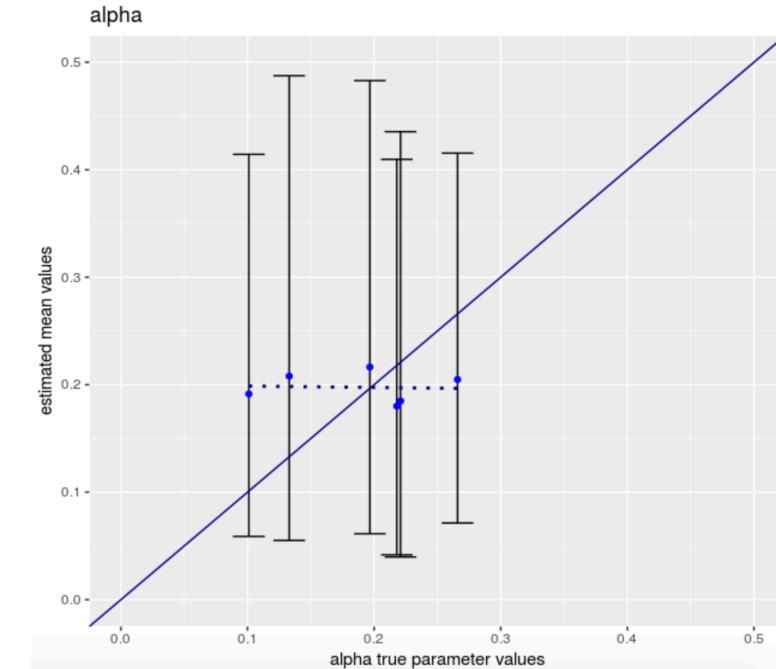
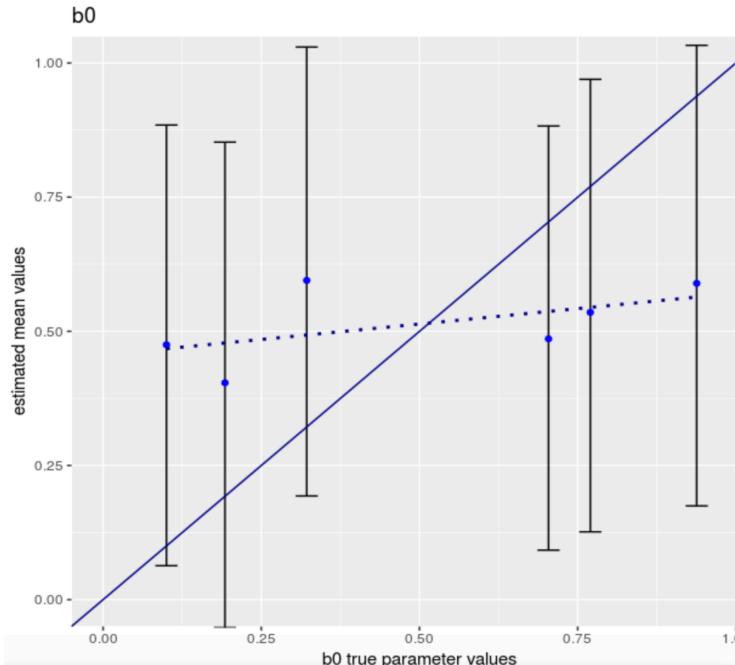
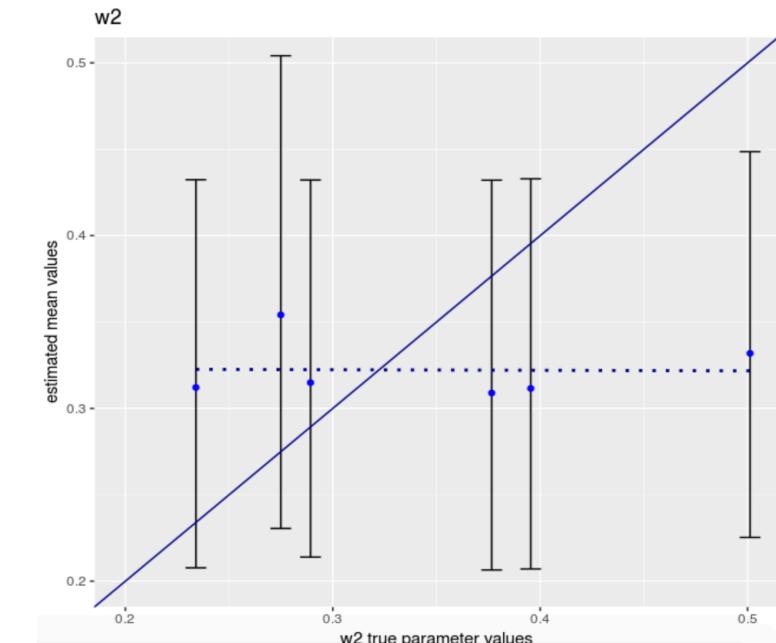
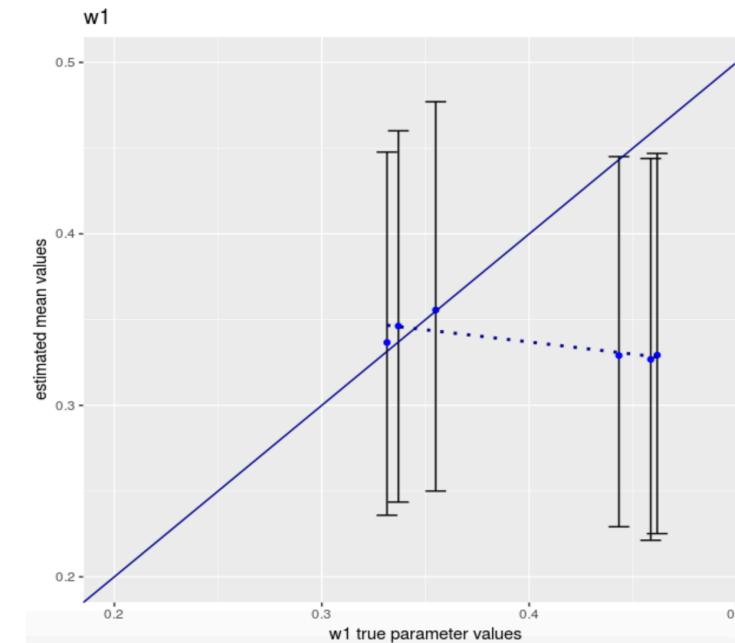
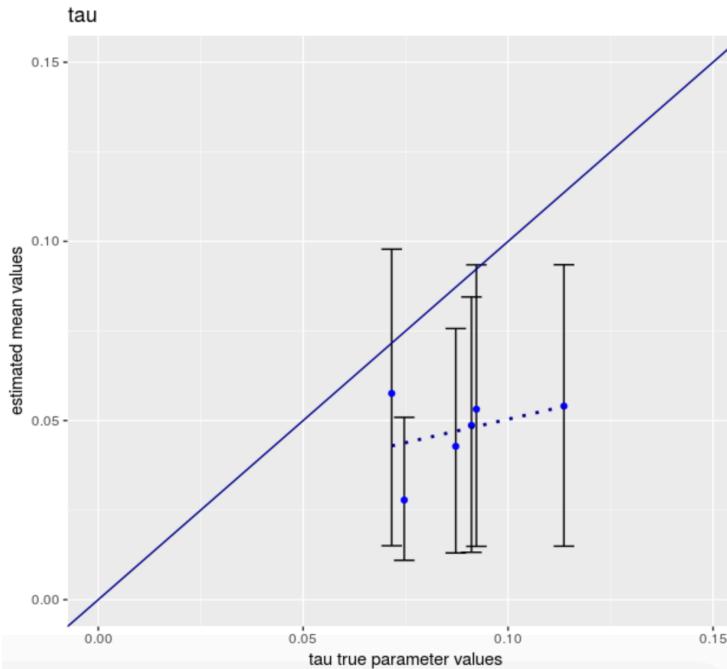
1. Sampling using Model 3 has been **converged well**
2. Model 3 is the **best model** among three based on LOOIC with 30 subjects
3. In **escape** condition, the **drift rate of the suicidal group is higher** than that of the non-suicidal group, whereas the trend is flipped in the avoid condition
- ★ 4. In **both** escape and avoid conditions, the **non-suicidal group** starts its drifting at a point **closer to the go-boundary** than the suicidal group does



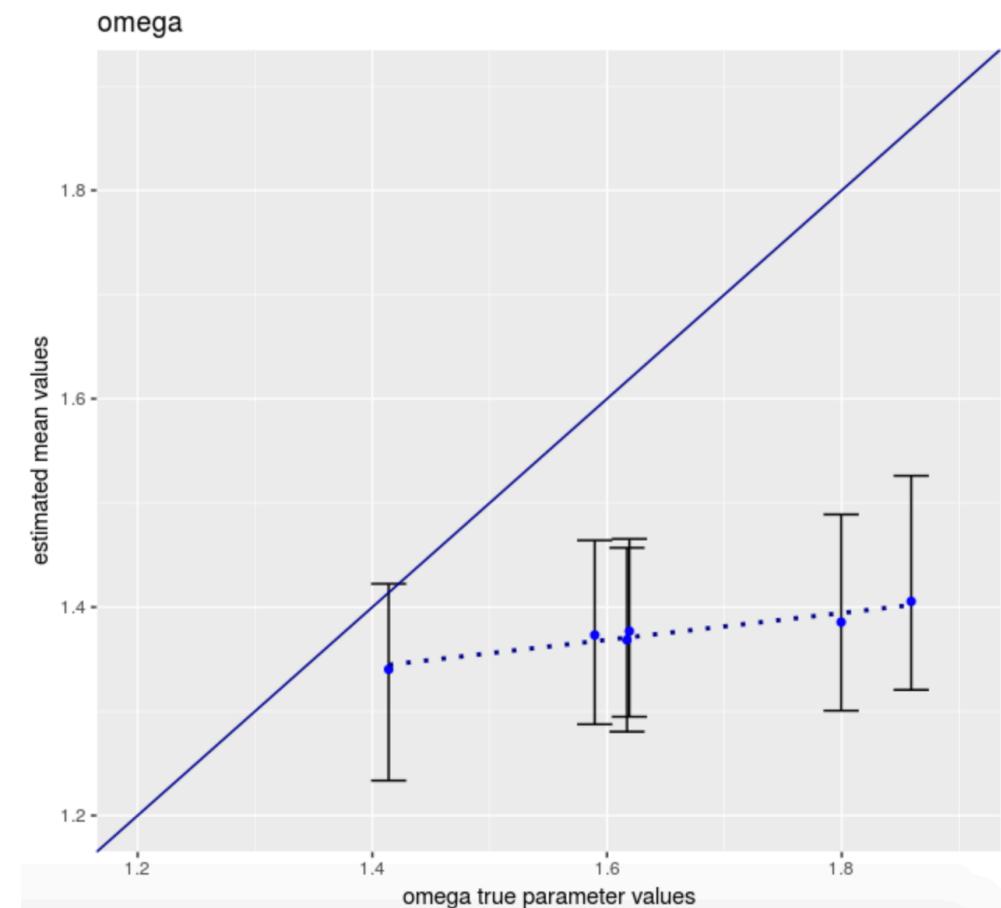
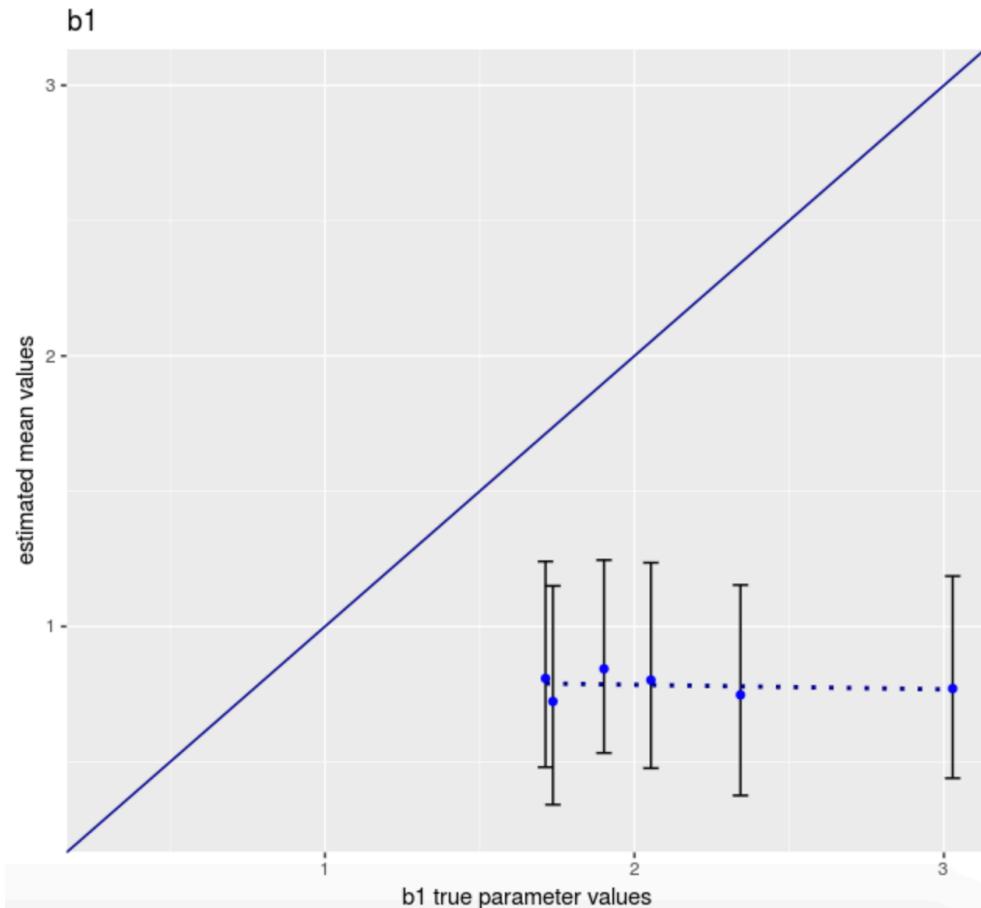
# Parameter Recovery

# Original Model

- Number of subjects = 6
- Step size = 0.02
- Iterations = 2000
- Warmup = 1000
- Chains = 4



# Original Model

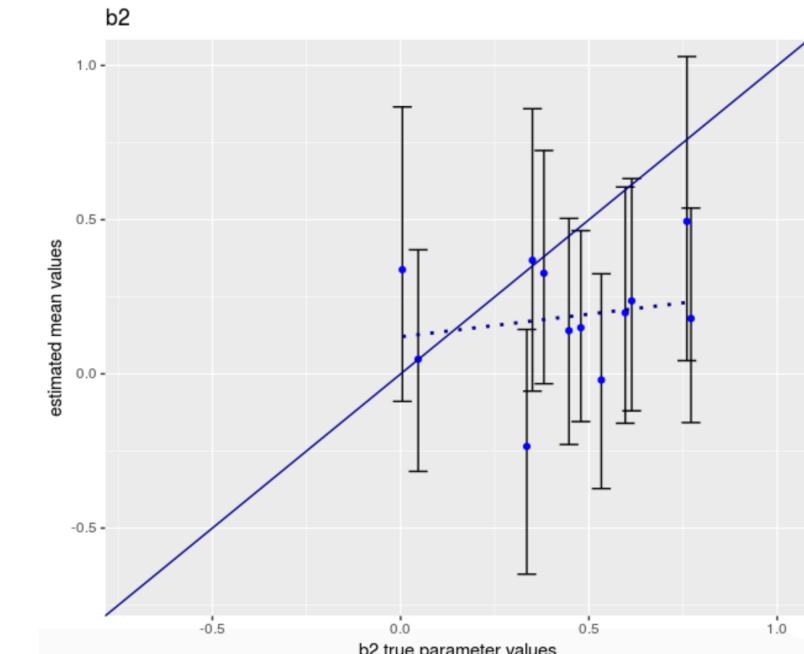
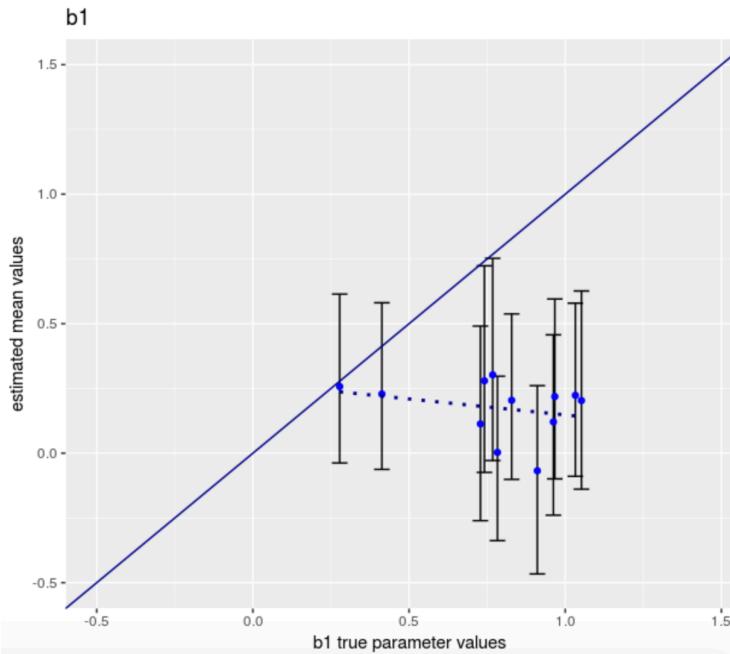
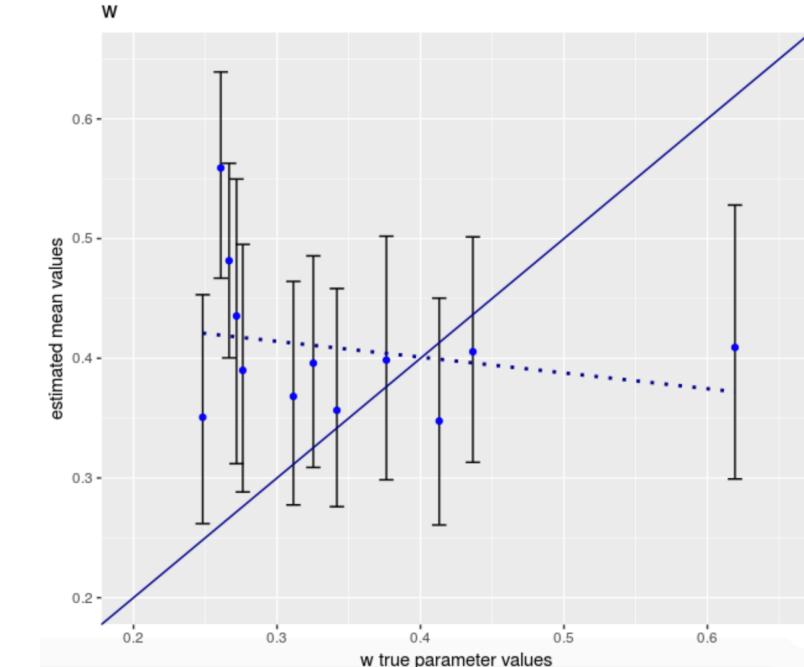
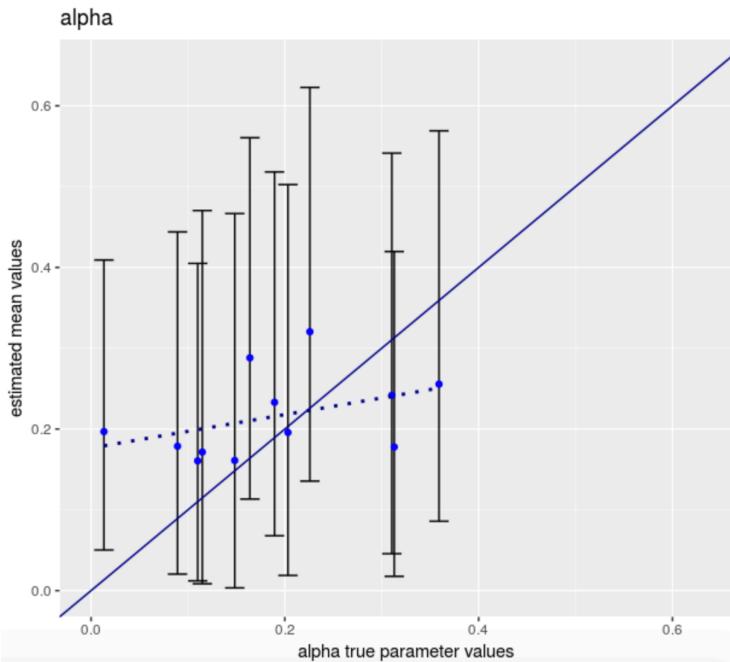
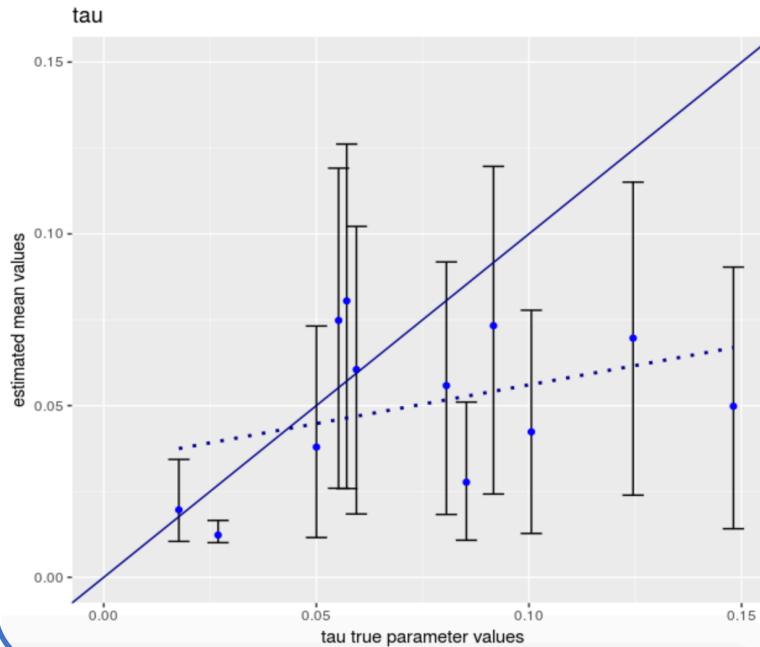


## Possible reasons :

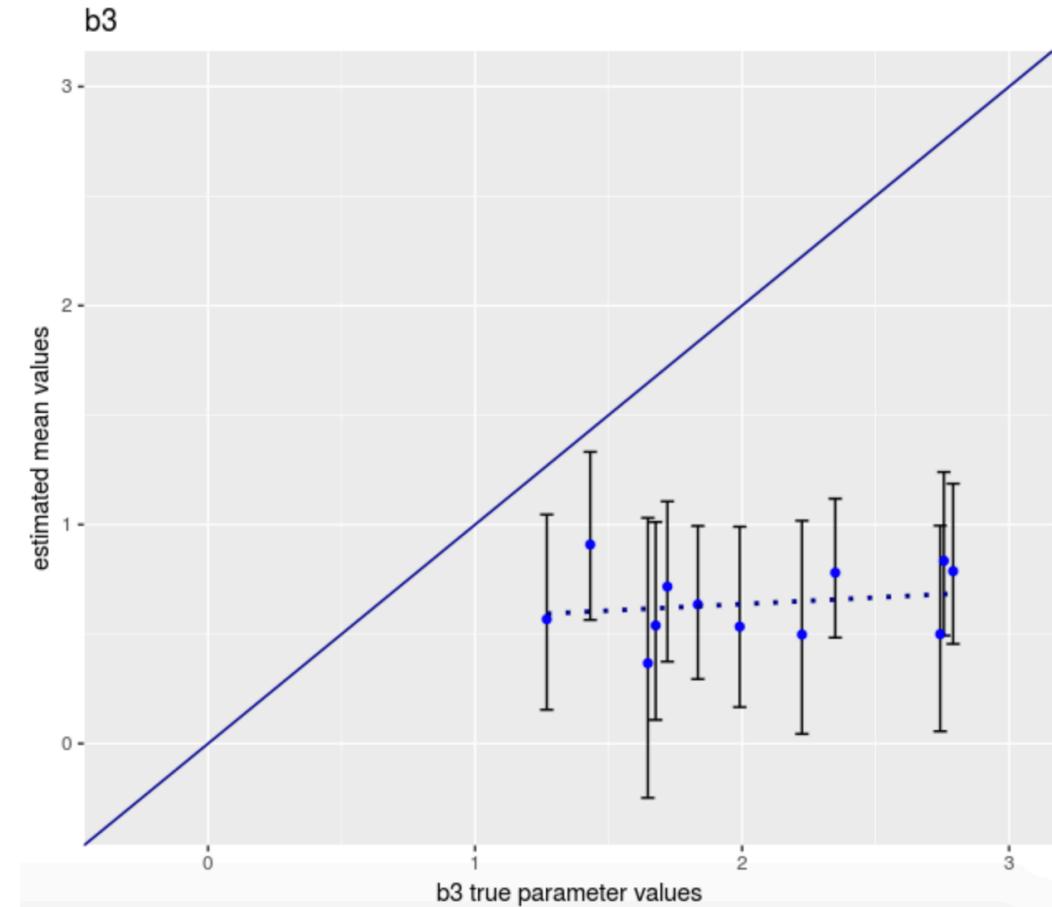
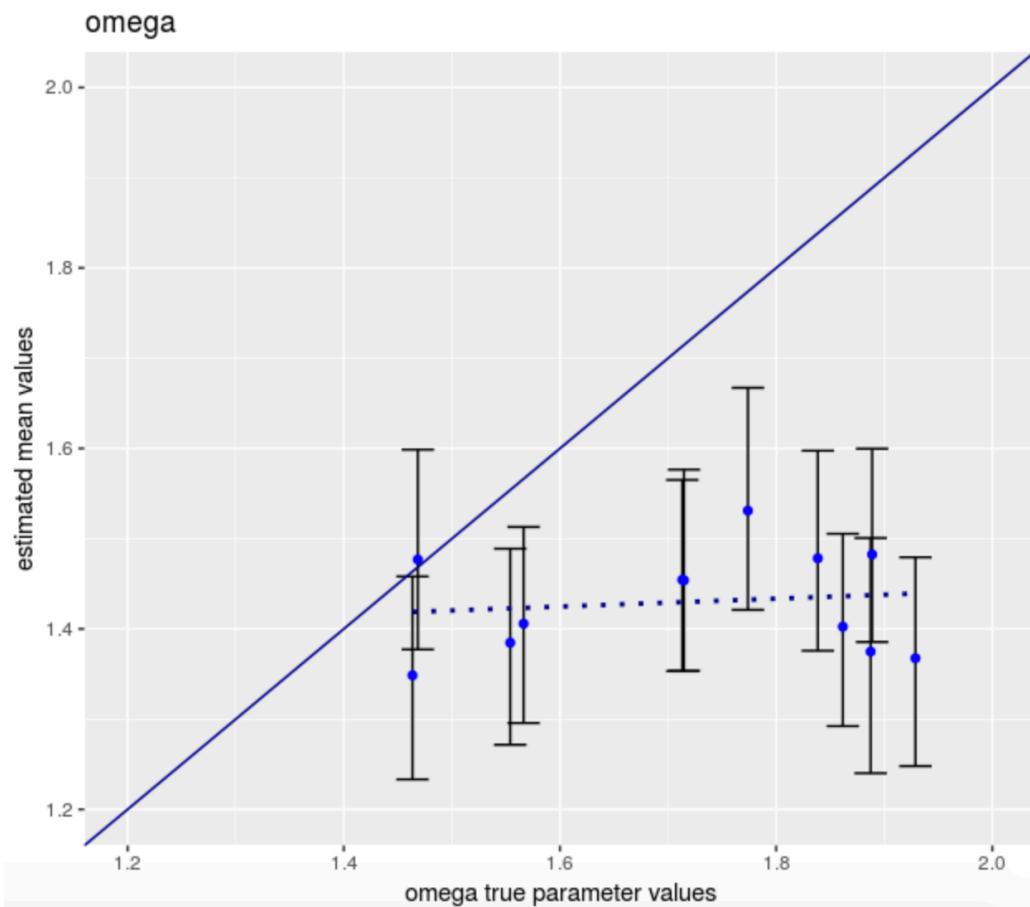
- No inverse temperatures when generating the choice dataset
- Model is too flexible

## Model 2

- Number of subjects = 12
- Step size = 0.04
- Iterations = 2000
- Warmup = 1000
- Chains = 4



## Model 2

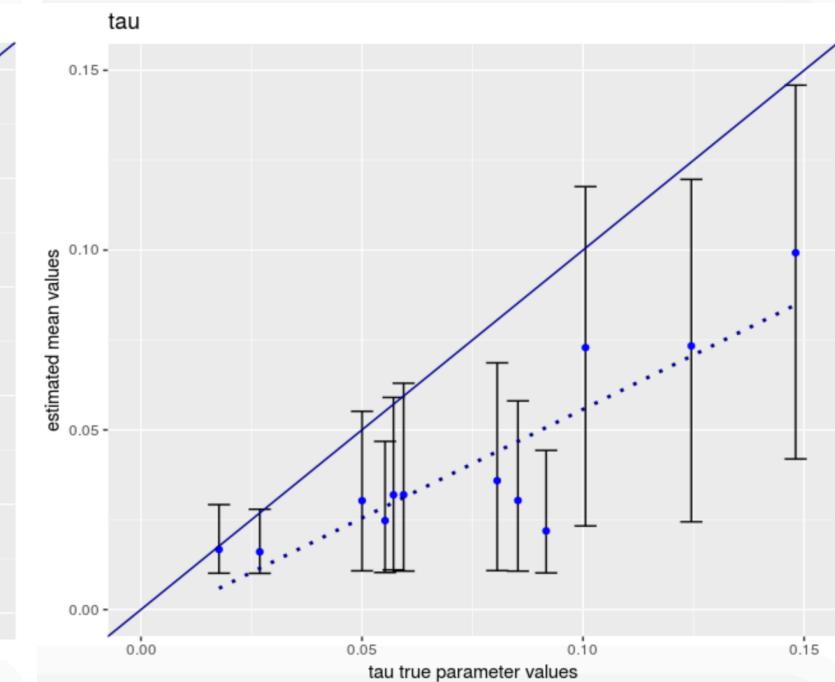
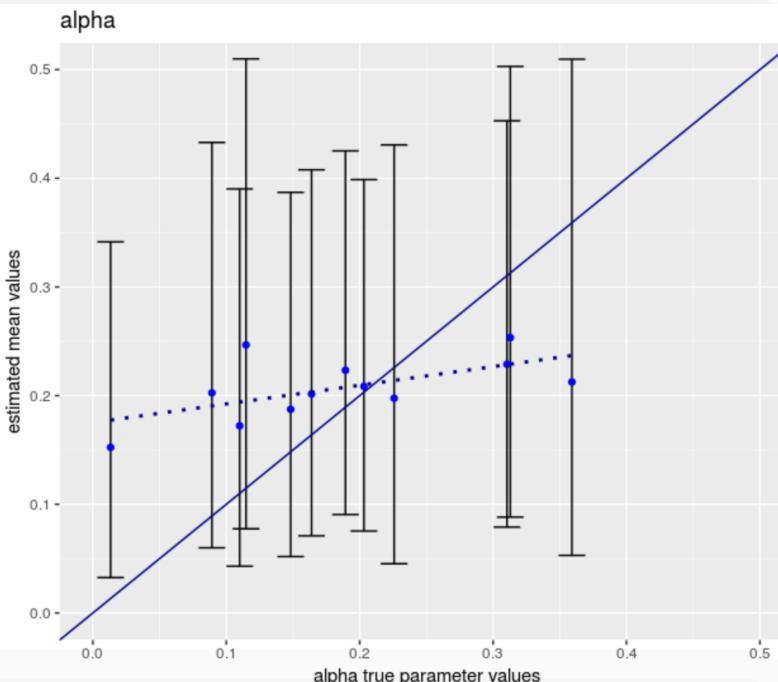
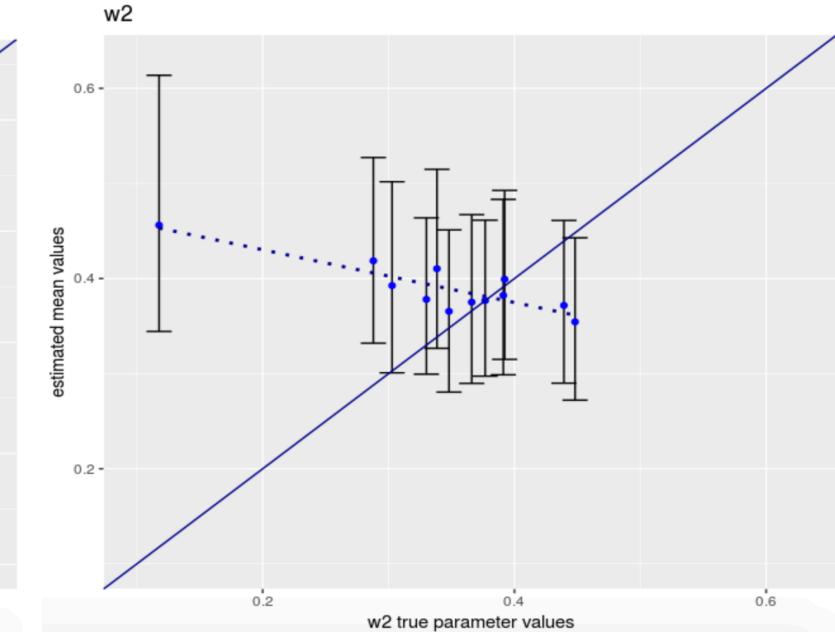
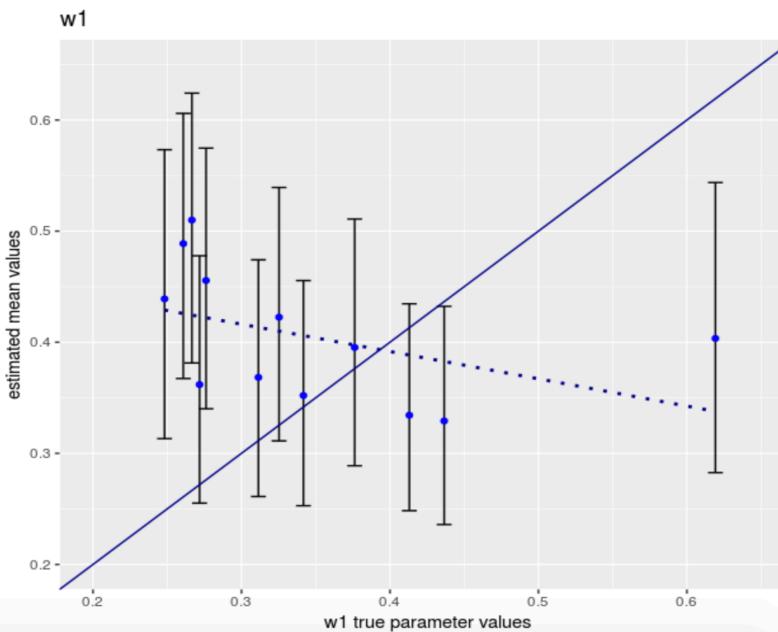
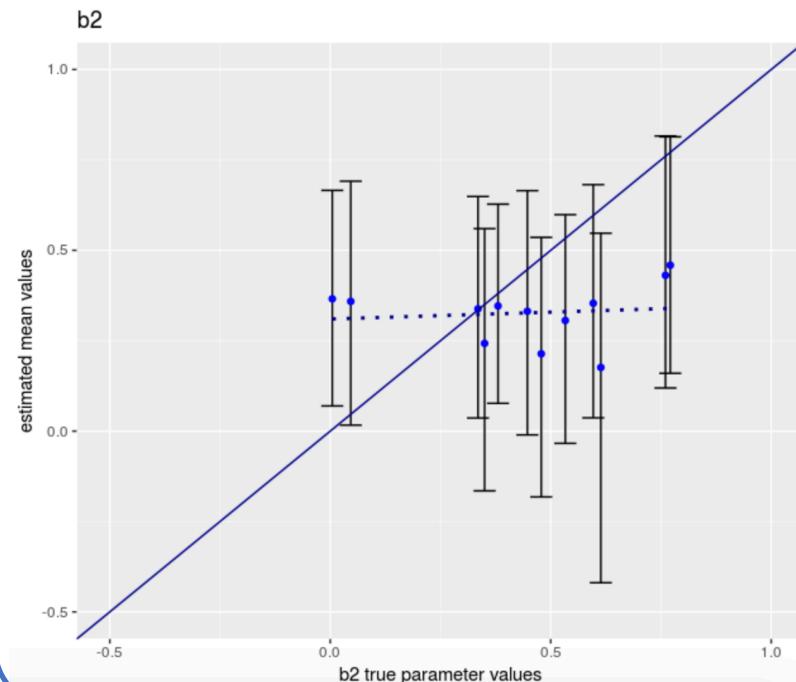


### Possible reasons :

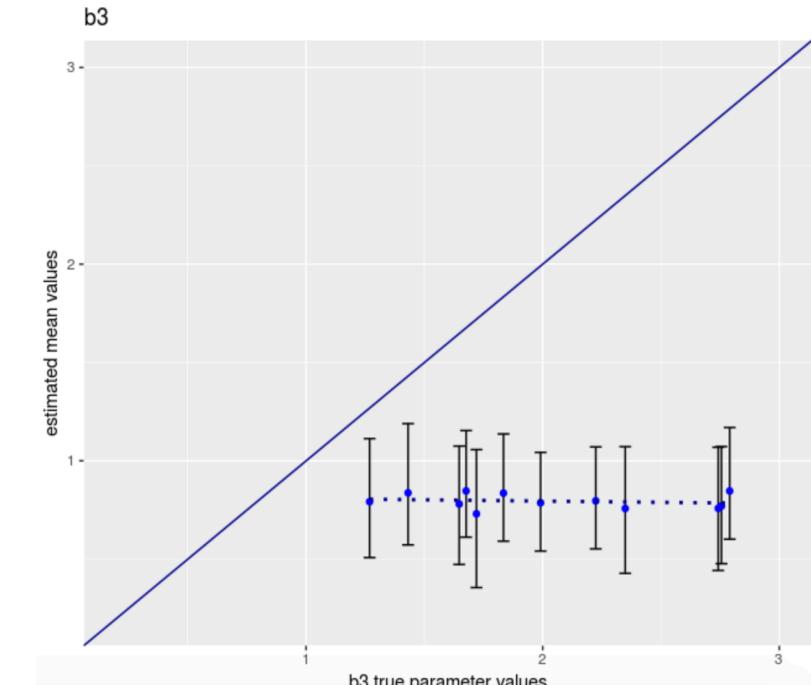
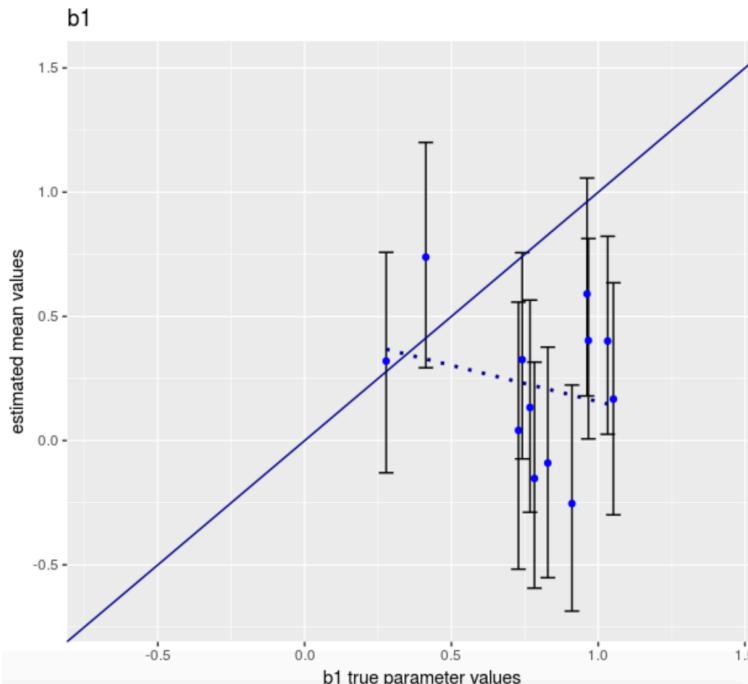
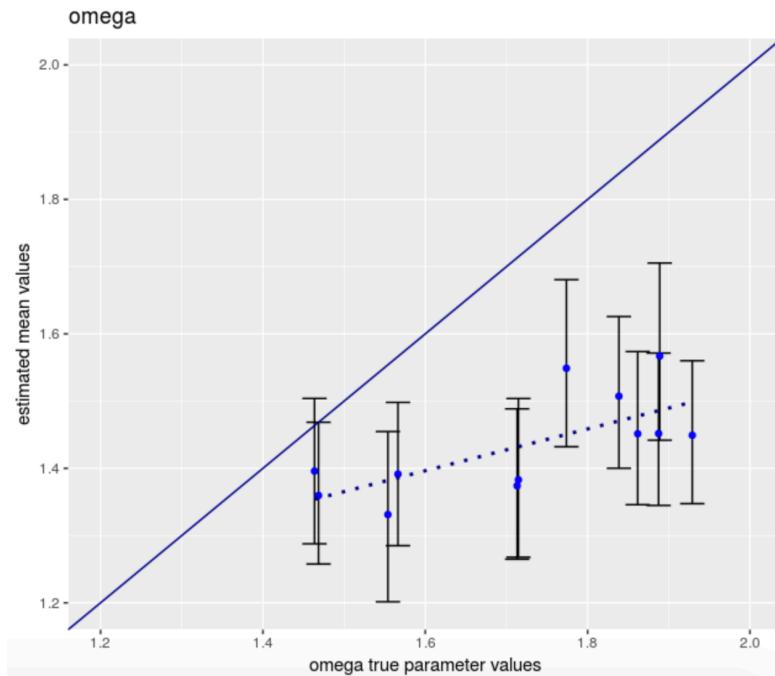
- No inverse temperatures when generating the choice dataset
- Model is too flexible

# Model 3

- Number of subjects = 12
- Step size = 0.04
- Iterations = 2000
- Warmup = 1000
- Chains = 4



# Model 3



## Possible reasons :

- No inverse temperatures when generating the choice dataset
- Model misspecification for drift rate
- Badly simulated data (using an inappropriate seed for random number generator)

# Conclusions

- The suicidal group tends to drift faster to the go-boundary in escape condition, but the non-suicidal group tends to have a constant go bias in both conditions
- ★ • The suicidal group also shows a stronger no-go to avoid response bias in avoid condition
- Model 3 with two drift rates and two starting points is the best among three based on LOOIC with 30 subjects
- Not successful parameter recovery of the differential weight parameter ( $\beta_1$  or  $\beta_3$ ) in all three models
- Fair parameter recovery of the rest of the parameters in all three models

# References

- Millner, A. J., den Ouden, H. E., Gershman, S. J., Glenn, C. R., Kearns, J. C., Bornstein, A. M., Marx, B. P., Keane, T. M., & Nock, M. K. (2019). Suicidal thoughts and behaviors are associated with an increased decision-making bias for active responses to escape aversive states. *Journal of Abnormal Psychology*, 128(2):106-118.
- Millner, A. J., Gershman, S. J., Nock, M. K., & den Ouden, H. E. (2018). Pavlovian control of escape and avoidance. *Journal of Cognitive Neuroscience*, 1-12.