LAB: Linear Regression - Polynomial Regression

Estimated time: 30 minutes

Objectives

After completing this lab, you will be able to:

- Develop **Linear Regression** and **Polynomial Regression** models

Overview

In this section, we will develop two types of models—**Linear Regression** and **Polynomial Regression**—to predict the price of a car using different variables or features. These models will provide an estimate, helping us get an objective idea of how much a car should cost.

Some key questions to consider in this lab:

- Is the dealer offering a fair trade-in value for my car?
- Am I placing a fair value on my car?

In data analytics, **Linear Regression** and **Polynomial Regression** are frequently used to predict future observations based on existing data. These models help us understand the relationships between variables and how these relationships can be used to predict outcomes.

Steps

1. Set up the working environment

- Import the necessary libraries:

```
#you are running the lab in your  browser, so we will install the
libraries using ``piplite``
install.packages("dplyr")
install.packages("ggplot2")
install.packages("scipy")
install.packages("caret")
install.packages("seaborn")

Installing package into '/usr/local/lib/R/site-library'
(as 'lib' is unspecified)

Installing package into '/usr/local/lib/R/site-library'
(as 'lib' is unspecified)

Installing package into '/usr/local/lib/R/site-library'
(as 'lib' is unspecified)
```

```
Warning message:
“package ‘scipy’ is not available for this version of R

A version of this package for your version of R might be available
elsewhere,
see the ideas at
https://cran.r-project.org/doc/manuals/r-patched/R-
admin.html#Installing-packages”
Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Warning message:
“package ‘seaborn’ is not available for this version of R

A version of this package for your version of R might be available
elsewhere,
see the ideas at
https://cran.r-project.org/doc/manuals/r-patched/R-
admin.html#Installing-packages”

library(dplyr)
library(ggplot2)
library(tidyr)          # Hỗ trợ trong việc xử lý dữ liệu như numpy
trong python
library(readr)

# Tắt các cảnh báo
options(warn = -1)


Attaching package: ‘dplyr’


The following objects are masked from ‘package:stats’:

    filter, lag


The following objects are masked from ‘package:base’:

    intersect, setdiff, setequal, union
```

This function will download the dataset into your browser

```r
# Sử dụng thư viện httr để tải file từ URL
library(httr)

#This function will download the dataset into your browser

download <- function(url, filename) {
  # Gửi yêu cầu GET đến URL
  response <- GET(url)

  # Kiểm tra nếu yêu cầu thành công (status code 200)
  if (status_code(response) == 200) {
    # Ghi nội dung vào tệp tin
    writeBin(content(response, "raw"), filename)
    message("Download successful: ", filename)
  } else {
    message("Download failed with status: ", status_code(response))
  }
}
```

This dataset was hosted on IBM Cloud object. Click HERE for free storage.

you will need to download the dataset; if you are running locally, please comment out the following

```r
#you will need to download the dataset; if you are running locally,
please comment out the following
# Sử dụng download.file để tải xuống tập dữ liệu
download.file(
  url = "https://cf-courses-data.s3.us.cloud-object-
storage.appdomain.cloud/IBMDeveloperSkillsNetwork-DA0101EN-
SkillsNetwork/labs/Data%20files/automobileEDA.csv",
  destfile = "automobileEDA.csv",
  mode = "wb"
)

# Kiểm tra nếu tệp đã được tải thành công
if (file.exists("automobileEDA.csv")) {
  message("Download successful: automobileEDA.csv")
} else {
  message("Download failed.")
}
```

```
Download successful: automobileEDA.csv
```

Load the data and store it in dataframe `df`:

```r
# Đọc dữ liệu
df <- read.csv("automobileEDA.csv")
```

```r
# Hiển thị 6 dòng dữ liệu đầu tiên
head(df, 6)
```

```
  symboling normalized.losses make        aspiration num.of.doors
body.style
1 3         122               alfa-romero std        two
convertible
2 3         122               alfa-romero std        two
convertible
3 1         122               alfa-romero std        two
hatchback
4 2         164               audi        std        four
sedan
5 2         164               audi        std        four
sedan
6 2         122               audi        std        two
sedan
  drive.wheels engine.location wheel.base length    ⋯
compression.ratio
1 rwd          front           88.6       0.8111485 ⋯  9.0

2 rwd          front           88.6       0.8111485 ⋯  9.0

3 rwd          front           94.5       0.8226814 ⋯  9.0

4 fwd          front           99.8       0.8486305 ⋯ 10.0

5 4wd          front           99.4       0.8486305 ⋯  8.0

6 fwd          front           99.8       0.8519942 ⋯  8.5

  horsepower peak.rpm city.mpg highway.mpg price city.L.100km
horsepower.binned
1 111        5000     21       27          13495 11.190476    Medium

2 111        5000     21       27          16500 11.190476    Medium

3 154        5000     19       26          16500 12.368421    Medium

4 102        5500     24       30          13950  9.791667    Medium

5 115        5500     18       22          17450 13.055556    Medium

6 110        5500     19       25          15250 12.368421    Medium

  diesel gas
1 0      1
2 0      1
3 0      1
4 0      1
```

```
5 0      1
6 0      1
```

1. Linear Regression and Multiple Linear Regression

Linear Regression

One example of a Data Model that we will be using is:

Simple Linear Regression

Simple Linear Regression is a method to help us understand the relationship between two variables:

- The predictor/independent variable (X)
- The response/dependent variable (Y), which is what we want to predict.

The result of Linear Regression is a linear function that predicts the response (dependent) variable as a function of the predictor (independent) variable.

Notation

$$ Y: \text{Response Variable} \\ X: \text{Predictor Variables} $$

Linear Function

$$\hat{Y} = a + b X$$

- **a** refers to the intercept of the regression line, i.e., the value of **Y** when **X** is 0.
- **b** refers to the slope of the regression line, i.e., the change in **Y** when **X** increases by 1 unit.

Let's load the modules for linear regression and create the linear regression object:

Simple Linear Regression

Simple Linear Regression is a method to help us understand the relationship between two variables: The predictor/independent variable (X) The response/dependent variable (that we want to predict)(Y)

$$ Y: Response \ Variable\\\\\\\\\\ X: Predictor \ Variables $$

Linear Function

$$Y hat = a + b X$$

Building a Linear Regression Model

We will create a linear regression model to predict car price based on highway miles per gallon (mpg). After fitting the model, review the summary of the model, which includes key information such as the coefficients, R-squared value, and p-values.

Questions for Evaluation:

1. What is the coefficient for the `highway.mpg` variable? How does it relate to the price of the car?
2. What is the intercept, and what does it represent in the context of the model?
3. How do you interpret the R-squared value? Does the model explain a significant amount of variance in the price?
4. Based on the p-value, is the `highway.mpg` variable statistically significant in predicting car price?
5. What other factors might you consider adding to improve the model's predictive power?

```r
# Create a linear regression model (price ~ highway.mpg)
lm_model <- lm(price ~ highway.mpg, data = df)

# View the summary of the model
summary(lm_model)
```

```

Call:
lm(formula = price ~ highway.mpg, data = df)

Residuals:
   Min     1Q Median     3Q    Max 
 -8647  -3411  -1102   1092  20970 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 38423.31    1843.39   20.84   <2e-16 ***
highway.mpg  -821.73      58.65  -14.01   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 5653 on 199 degrees of freedom
Multiple R-squared:  0.4966,	Adjusted R-squared:  0.4941 
F-statistic: 196.3 on 1 and 199 DF,  p-value: < 2.2e-16
```


Questions for Evaluation:

1. What is the coefficient for the `highway.mpg` variable? How does it relate to the price of the car?
2. What is the intercept, and what does it represent in the context of the model?
3. How do you interpret the R-squared value? Does the model explain a significant amount of variance in the price?

4. Based on the p-value, is the `highway.mpg` variable statistically significant in predicting car price?
5. What other factors might you consider adding to improve the model's predictive power?
1. The coefficient for highway.mpg is -821.73. This means that for each additional mile per gallon on the highway, the car price decreases by approximately $821.73.

2. The intercept is $38,423.31. This is the estimated car price when highway.mpg is 0. While this value doesn't make practical sense (since a car cannot have 0 mpg), it's required mathematically in the model.

3. The R-squared value is 0.4966, meaning the model explains about 49.66% of the variance in car prices based on highway.mpg. While it explains a fair portion, there's still a large amount of variability not captured by this model.

4. Yes, the p-value for highway.mpg is < 2e-16, which is extremely small and indicates that highway.mpg is statistically significant in predicting car prices.

5. Improve the model by adding other variables like horsepower, engine size, make, or body style, which might better capture the variation in car prices and improve predictive accuracy.

Fit the linear model using highway-mpg:

We can output a prediction:

```
# Generate predictions
yhat <- predict(lm_model, df)

# Display the first 5 predicted values
head(yhat, 5)
```

```
       1        2        3        4        5
16236.50 16236.50 17058.24 13771.30 20345.17
```

```
# Get the intercept (a)
intercept <- coef(lm_model)[1]
print(paste("Intercept:", intercept))
```

```
[1] "Intercept: 38423.3058581574"
```

```
# Get the slope (b)
slope <- coef(lm_model)[2]
print(paste("Slope:", slope))
```

```
[1] "Slope: -821.733378321926"
```

```
# Print the regression equation
cat("Price =", round(intercept, 2), "+", "______", "x highway-mpg\n")
```

```
Price = 38423.31 + ______ x highway-mpg
```

What is the final estimated linear model we get?

After reviewing the model summary, the final estimated linear regression equation can be expressed in the form:

$$\hat{Y} = a + b X$$

Where:

- **a** is the intercept (the predicted price when `highway.mpg` is 0).
- **b** is the coefficient of `highway.mpg` (how much the price changes for each unit increase in `highway.mpg`).

Questions to consider:

- Based on the summary, what are the specific values of **a** (intercept) and **b** (slope)?
- How would you describe the relationship between `highway.mpg` and `price` based on these values?

Plugging in the actual values we get:

Price = 38423.31 + -821.73 x highway-mpg

1. The intercept (a) is 38423.31. The slope (b) is -821.73.

2. There is a negative relationship between highway.mpg and price. For every additional mile per gallon on the highway, the car's price decreases by approximately $821.73.

```
# Based on the summary, find the slope (coefficient for engine.size)
and intercept
# What is the regression equation? Fill in the blanks below:
# Create a linear regression model (price ~ engine.size)
lm_engine_size_model <- lm(price ~ engine.size, data = df)

# Get the intercept (a)
intercept <- coef(lm_engine_size_model)[1]

# Get the slope (b)
slope <- coef(lm_engine_size_model)[2]

# Print the regression equation
cat("Price =", round(intercept, 2), "+", round(slope, 2), "*
engine.size\n")

#Price = intercept + slope × engine.size
#Price
```

```
Price = -7963.34 + 166.86 * engine.size
```

```
# Write your code below and press Shift+Enter to execute
# Find the value of the slope (coefficient for engine.size)
```

```r
slope_engine <- coef(lm_engine_size_model)[2]
slope_engine
```

```
engine.size
     166.86
```

```r
# Write your code below and press Shift+Enter to execute
# Giá trị của hằng số (intercept)

intercept_engine <- coef(lm_engine_size_model)[1]
intercept_engine
```

```
(Intercept)
  -7963.339
```

```r
# Write your code below and press Shift+Enter to execute
# Find the value of the intercept (hint: use coef(lm1)[1])

# Now print the regression equation
cat("Price =", round(intercept_engine, 2), "+",
    round(slope_engine, 2), "x engine-size\n")
```

```
Price = -7963.34 + 166.86 x engine-size
```

```r
# Tính giá dự đoán
price_predicted <- predict(lm_engine_size_model, df)
head(price_predicted)
```

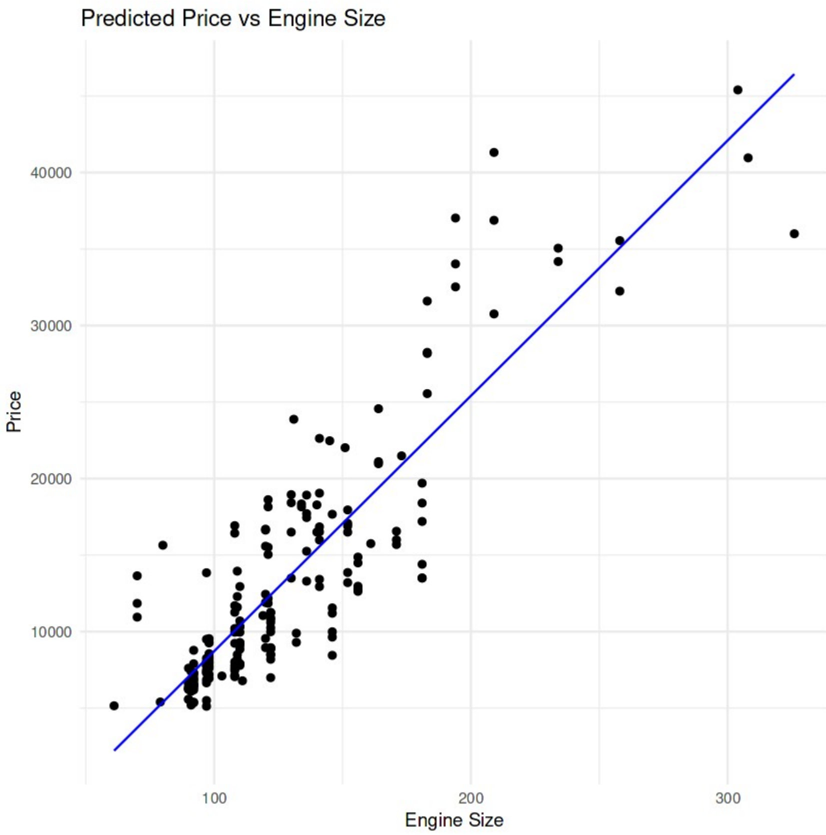
```
       1        2        3        4        5        6
13728.46 13728.46 17399.38 10224.40 14729.62 14729.62
```

```r
# Write your code below and press Shift+Enter to execute

# Define X using df$engine.size
X <- df$engine.size  # Fill in this part

# Calculate predictions using X
Yhat <- intercept + slope * X

# Print the prediction equation
cat("Price =", round(intercept, 2), "+", round(slope, 2), "*
engine.size\n")

# Check the predicted results
head(Yhat)

# Plot to illustrate
ggplot(df, aes(x = engine.size, y = price)) +
  geom_point() +  # Scatter plot of actual data
  geom_line(aes(y = Yhat), color = 'blue') +  # Predicted line
  labs(title = "Predicted Price vs Engine Size",
       x = "Engine Size",
```

```
       y = "Price") +
  theme_minimal()
```

```
Price = -7963.34 + 166.86 * engine.size
```

```
[1] 13728.46 13728.46 17399.38 10224.40 14729.62 14729.62
```




```
# Create a new data frame to display actual and predicted prices
results <- data.frame(
  Engine_Size = df$engine.size,
  Actual_Price = df$price,
  Predicted_Price = Yhat
)

# Display the first few rows of the results
head(results)

# Print the predicted prices
print(results)
```

```
  Engine_Size Actual_Price Predicted_Price
1 130         13495        13728.46
2 130         16500        13728.46
```

3	152	16500	17399.38
4	109	13950	10224.40
5	136	17450	14729.62
6	136	15250	14729.62

	Engine_Size	Actual_Price	Predicted_Price
1	130	13495	13728.463
2	130	16500	13728.463
3	152	16500	17399.383
4	109	13950	10224.403
5	136	17450	14729.623
6	136	15250	14729.623
7	136	17710	14729.623
8	136	18920	14729.623
9	131	23875	13895.323
10	108	16430	10057.543
11	108	16925	10057.543
12	164	20970	19401.704
13	164	21105	19401.704
14	164	24565	19401.704
15	209	30760	26910.404
16	209	41315	26910.404
17	209	36880	26910.404
18	61	5151	2215.122
19	90	6295	7054.063
20	90	6575	7054.063
21	90	5572	7054.063
22	90	6377	7054.063
23	98	7957	8388.943
24	90	6229	7054.063
25	90	6692	7054.063
26	90	7609	7054.063
27	98	8558	8388.943
28	122	8921	12393.583
29	156	12964	18066.824
30	92	6479	7387.783
31	92	6855	7387.783
32	79	5399	5218.602
33	92	6529	7387.783
34	92	7129	7387.783
35	92	7295	7387.783
36	92	7295	7387.783
37	110	7895	10391.263
38	110	9095	10391.263
39	110	8845	10391.263
40	110	10295	10391.263
41	110	12945	10391.263
42	110	10345	10391.263
43	111	6785	10558.123
44	119	11048	11893.003

45	258	32250	35086.545
46	258	35550	35086.545
47	326	36000	46433.026
48	91	5195	7220.923
49	91	6095	7220.923
50	91	6795	7220.923
51	91	6695	7220.923
52	91	7395	7220.923
53	70	10945	3716.862
54	70	11845	3716.862
55	70	13645	3716.862
56	80	15645	5385.462
57	122	8845	12393.583
58	122	8495	12393.583
59	122	10595	12393.583
60	122	10245	12393.583
61	122	10795	12393.583
62	122	11245	12393.583
63	140	18280	15397.063
64	134	18344	14395.903
65	183	25552	22572.044
66	183	28248	22572.044
67	183	28176	22572.044
68	183	31600	22572.044
69	234	34184	31081.905
70	234	35056	31081.905
71	308	40960	43429.546
72	304	45400	42762.106
73	140	16503	15397.063
74	92	5389	7387.783
75	92	6189	7387.783
76	92	6669	7387.783
77	98	7689	8388.943
78	110	9959	10391.263
79	122	8499	12393.583
80	156	12629	18066.824
81	156	14869	18066.824
82	156	14489	18066.824
83	122	6989	12393.583
84	122	8189	12393.583
85	110	9279	10391.263
86	110	9279	10391.263
87	97	5499	8222.083
88	103	7099	9223.243
89	97	6649	8222.083
90	97	6849	8222.083
91	97	7349	8222.083
92	97	7299	8222.083
93	97	7799	8222.083

94	97	7499	8222.083
95	97	7999	8222.083
96	97	8249	8222.083
97	120	8949	12059.863
98	120	9549	12059.863
99	181	13499	22238.324
100	181	14399	22238.324
101	181	13499	22238.324
102	181	17199	22238.324
103	181	19699	22238.324
104	181	18399	22238.324
105	120	11900	12059.863
106	152	13200	17399.383
107	120	12440	12059.863
108	152	13860	17399.383
109	120	15580	12059.863
110	152	16900	17399.383
111	120	16695	12059.863
112	152	17075	17399.383
113	120	16630	12059.863
114	152	17950	17399.383
115	134	18150	14395.903
116	90	5572	7054.063
117	98	7957	8388.943
118	90	6229	7054.063
119	90	6692	7054.063
120	98	7609	8388.943
121	122	8921	12393.583
122	156	12764	18066.824
123	151	22018	17232.523
124	194	32528	24407.504
125	194	34028	24407.504
126	194	37028	24407.504
127	132	9295	14062.183
128	132	9895	14062.183
129	121	11850	12226.723
130	121	12170	12226.723
131	121	15040	12226.723
132	121	15510	12226.723
133	121	18150	12226.723
134	121	18620	12226.723
135	97	5118	8222.083
136	108	7053	10057.543
137	108	7603	10057.543
138	108	7126	10057.543
139	108	7775	10057.543
140	108	9960	10057.543
141	108	9233	10057.543
142	108	11259	10057.543

143	108	7463	10057.543
144	108	10198	10057.543
145	108	8013	10057.543
146	108	11694	10057.543
147	92	5348	7387.783
148	92	6338	7387.783
149	92	6488	7387.783
150	92	6918	7387.783
151	92	7898	7387.783
152	92	8778	7387.783
153	98	6938	8388.943
154	98	7198	8388.943
155	110	7898	10391.263
156	110	7788	10391.263
157	98	7738	8388.943
158	98	8358	8388.943
159	98	9258	8388.943
160	98	8058	8388.943
161	98	8238	8388.943
162	98	9298	8388.943
163	98	9538	8388.943
164	146	8449	16398.223
165	146	9639	16398.223
166	146	9989	16398.223
167	146	11199	16398.223
168	146	11549	16398.223
169	146	17669	16398.223
170	122	8948	12393.583
171	110	10698	10391.263
172	122	9988	12393.583
173	122	10898	12393.583
174	122	11248	12393.583
175	171	16558	20569.724
176	171	15998	20569.724
177	171	15690	20569.724
178	161	15750	18901.124
179	97	7775	8222.083
180	109	7975	10224.403
181	97	7995	8222.083
182	109	8195	10224.403
183	109	8495	10224.403
184	97	9495	8222.083
185	109	9995	10224.403
186	109	11595	10224.403
187	109	9980	10224.403
188	136	13295	14729.623
189	97	13845	8222.083
190	109	12290	10224.403
191	141	12940	15563.923

192	141	13415	15563.923
193	141	15985	15563.923
194	141	16515	15563.923
195	130	18420	13728.463
196	130	18950	13728.463
197	141	16845	15563.923
198	141	19045	15563.923
199	173	21485	20903.444
200	145	22470	16231.363
201	141	22625	15563.923

Multiple Linear Regression

What if we want to predict car price using more than one variable?

If we want to use more variables in our model to predict car price, we can use **Multiple Linear Regression**.

Multiple Linear Regression is similar to Simple Linear Regression, but it explains the relationship between one continuous response (dependent) variable and **two or more** predictor (independent) variables. Most real-world regression models involve multiple predictors. We will demonstrate the structure using four predictor variables, but the results can generalize to any number of predictors:

$$ Y: \text{Response Variable} \\ X_1: \text{Predictor Variable 1} \\ X_2: \text{Predictor Variable 2} \\ X_3: \text{Predictor Variable 3} \\ X_4: \text{Predictor Variable 4} $$

Where:

- **a**: intercept
- $\mathbf{b_1}$: coefficient of Variable 1
- $\mathbf{b_2}$: coefficient of Variable 2
- $\mathbf{b_3}$: coefficient of Variable 3
- $\mathbf{b_4}$: coefficient of Variable 4

The equation for the Multiple Linear Regression model is given by:

$$\hat{Y} = a + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4$$

Predictors for Car Price

From the previous analysis, we know that good predictors of car price could be:

- Horsepower
- Curb-weight
- Engine-size
- Highway-mpg

Let's develop a model using these variables as the predictors.

```r
# Tạo mô hình hồi quy đa biến price ~ horsepower + curb.weight +
engine.size + highway.mpg

multi_model <- lm(price ~ horsepower + curb.weight + engine.size +
highway.mpg, data = df)

# Xem tóm tắt mô hình
summary(multi_model)
```

```

Call:
lm(formula = price ~ horsepower + curb.weight + engine.size +
    highway.mpg, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-8992.6 -1647.2   -70.7  1323.9 13640.3

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -15806.625   4388.993  -3.601 0.000401 ***
horsepower      53.496     14.727   3.632 0.000358 ***
curb.weight      4.708      1.119   4.207 3.94e-05 ***
engine.size     81.530     14.064   5.797 2.66e-08 ***
highway.mpg     36.057     74.167   0.486 0.627390
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3505 on 196 degrees of freedom
Multiple R-squared:  0.8094,	Adjusted R-squared:  0.8055
F-statistic:   208 on 4 and 196 DF,  p-value: < 2.2e-16
```

Fit the linear model using the four above-mentioned variables.

```r
# Tạo dự đoán
y_pred <- predict(multi_model, df)
head(y_pred)
```

```
       1        2        3        4        5        6
13699.11 13699.11 19051.65 10620.36 15521.31 13869.67
```

What is the value of the intercept(a)?

```r
# Lấy giá trị intercept
multi_intercept <- coef(multi_model)[1]  # The first coefficient is
the intercept
print(paste("Intercept:", multi_intercept))
```

```
[1] "Intercept: -15806.6246263292"
```

What are the values of the coefficients (b1, b2, b3, b4)?

```
# Lấy các hệ số
coefficients <- coef(multi_model)  # Get the coefficients from the
model

# Names of the predictors (excluding intercept)
names_predictors <- names(coefficients)[-1]  # Exclude the intercept

# Print the coefficients for each predictor
for (i in 1:length(names_predictors)) {
    print(paste(names_predictors[i], "coefficient:", coefficients[i +
1]))  # i + 1 to skip intercept
}
```

```
[1] "horsepower coefficient: 53.4957442260494"
[1] "curb.weight coefficient: 4.70770099461394"
[1] "engine.size coefficient: 81.5302638212274"
[1] "highway.mpg coefficient: 36.0574888164858"
```

```
# Lấy các hệ số
coefficients <- coef(multi_model)  # Get the coefficients from the
model

# Lấy giá trị intercept
multi_intercept <- coefficients[1]  # The first coefficient is the
intercept

# Names of the predictors
names_predictors <- names(coefficients)[-1]  # Exclude the intercept

# In ra phương trình hồi quy
cat("Phương trình hồi quy: \n")
equation <- paste("Price =", round(multi_intercept, 2),
                  "+", round(coefficients[2], 2), "* horsepower",
                  "+", round(coefficients[3], 2), "* curb.weight",
                  "+", round(coefficients[4], 2), "* engine.size",
                  "+", round(coefficients[5], 2), "* highway.mpg")
cat(equation, "\n\n")

# In ra các hệ số của từng biến
cat("Hệ số của từng biến:\n")
for (i in 1:length(names_predictors)) {
    cat(names_predictors[i], "coefficient:", round(coefficients[i +
1], 2), "\n")  # i + 1 to skip intercept
}
```

```
Phương trình hồi quy:
Price = -15806.62 + 53.5 * horsepower + 4.71 * curb.weight + 81.53 *
engine.size + 36.06 * highway.mpg
```

```
Hệ số của từng biến:
horsepower coefficient: 53.5
curb.weight coefficient: 4.71
engine.size coefficient: 81.53
highway.mpg coefficient: 36.06
```

Final Estimated Linear Model

From the results of the Multiple Linear Regression, we derived the following regression equation for predicting the car price:

Price = -15806.62 + 53.5 * horsepower + 4.71 * curb.weight + 81.53 * engine.size + 36.06 * highway.mpg

Intercept and Coefficients:

- **Intercept (a)** = -15806.62: This is the value of the car price when all predictor variables are 0.
- **horsepower (b_1)** = 53.5: For every 1-unit increase in horsepower, the car price increases by 53.5 units, holding all other variables constant.
- **curb.weight (b_2)** = 4.71: For every 1-unit increase in curb weight, the car price increases by 4.71 units, holding all other variables constant.
- **engine.size (b_3)** = 81.53: For every 1-unit increase in engine size, the car price increases by 81.53 units, holding all other variables constant.
- **highway.mpg (b_4)** = 36.06: For every 1-unit increase in highway miles per gallon (MPG), the car price increases by 36.06 units, holding all other variables constant.

Final Linear Function

The final linear function we obtained in this example is:

$$\hat{Y} = -15806.62 + 53.5 \times X_1 + 4.71 \times X_2 + 81.53 \times X_3 + 36.06 \times X_4$$

Where:

- (X_1): horsepower
- (X_2): curb weight
- (X_3): engine size
- (X_4): highway mpg

This equation allows us to estimate the car price based on these four predictors.

Question #2 a): Creating and Training a Multiple Linear Regression Model

Create and train a Multiple Linear Regression model named **"lm2"** where the response variable is `price`, and the predictor variables are `normalized.losses` and `highway.mpg`.

Here's how we can build the model:

```r
# Write your code below and press Shift+Enter to execute
# Tạo mô hình với normalized-losses và highway-mpg
lm2 <- lm(price ~ normalized.losses + highway.mpg, data = df)

# In hệ số
coefficients_lm2 <- coef(lm2)
print(coefficients_lm2)
```

```
      (Intercept) normalized.losses       highway.mpg
     38201.313272          1.497896       -820.454340
```

Question #2 b): Finding the Coefficients of the Model

After training the model, we can find the coefficients (the intercept and the coefficients for the predictor variables). These coefficients tell us how much the response variable (price) changes with a one-unit change in each predictor variable (normalized.losses and highway.mpg).

```r
# Write your code below and press Shift+Enter to execute
# Lấy giá trị intercept
intercept_lm2 <- coef(lm2)[1]  # The first coefficient is the intercept
print(paste("Intercept:", intercept_lm2))
```

```
[1] "Intercept: 38201.3132724573"
```

```r
# Lấy các hệ số
coefficients_lm2 <- coef(lm2)  # Get all coefficients
names(coefficients_lm2) <- c("Intercept", "Normalized Losses", "Highway MPG")  # Assign names for clarity

# Print each coefficient
for (i in 1:length(coefficients_lm2)) {
  print(paste(names(coefficients_lm2)[i], "coefficient:", round(coefficients_lm2[i], 2)))
}
```

```
[1] "Intercept coefficient: 38201.31"
[1] "Normalized Losses coefficient: 1.5"
[1] "Highway MPG coefficient: -820.45"
```

2. Model Evaluation Using Visualization

Now that we've developed some models, how do we evaluate them and choose the best one? One effective way to assess the performance of a model is through **visualization**.

Visualizations can help us:

- Understand the accuracy of the model.
- Compare the predicted values to the actual values.
- Identify any patterns or inconsistencies in the model's predictions.

Residual Plot

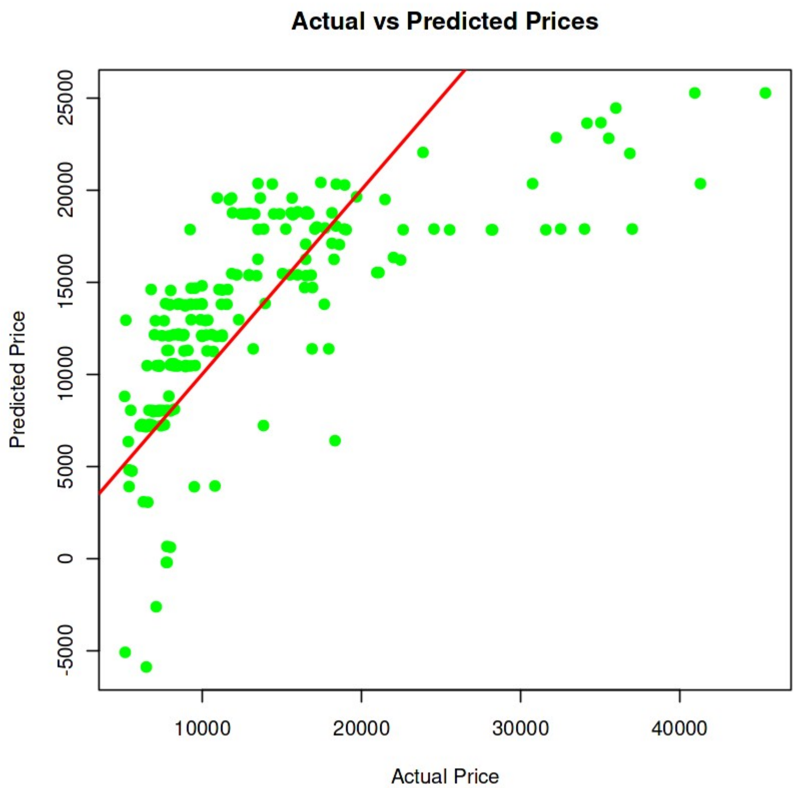
A residual plot is a useful tool for evaluating the fit of a regression model. It shows the residuals (the difference between actual and predicted values) on the y-axis and the predicted values on the x-axis.

- **Good fit**: Residuals are randomly scattered around 0 without any clear pattern.
- **Bad fit**: Residuals show a pattern, indicating that the model may not be capturing all aspects of the relationship.

Here's how we can create a residual plot:

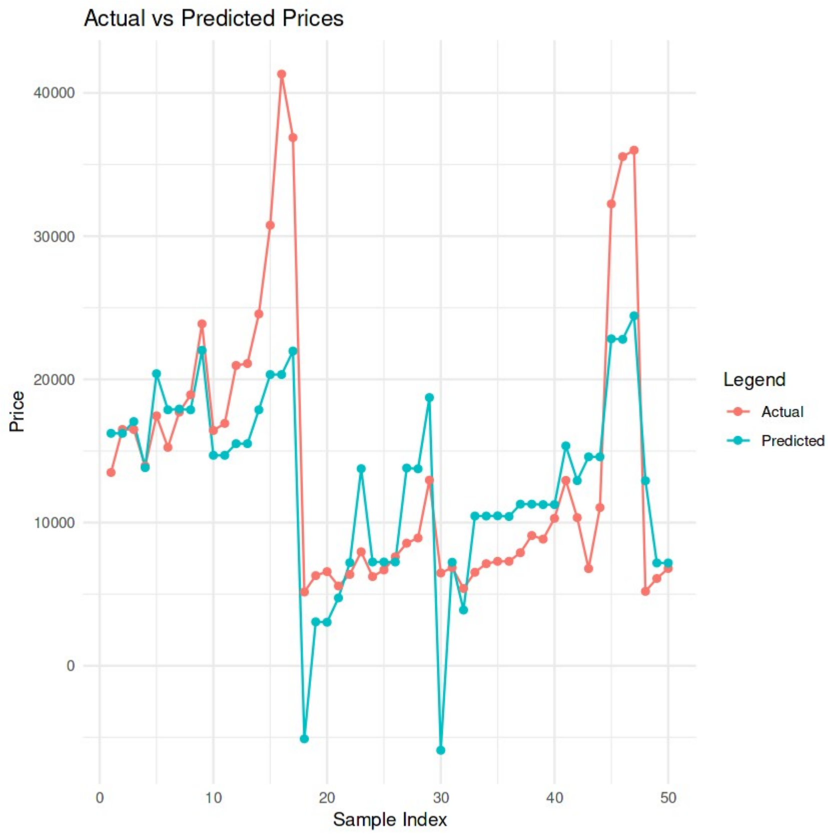
```r

# Create residuals from the model

residuals <- lm2$residuals

# Create a scatter plot of residuals vs fitted values

plot(lm2$fitted.values, residuals, xlab = 'Fitted Values', ylab = 'Residuals', main = 'Residual Plot')

# Add a horizontal line at 0

abline(h = 0, col = "red", lwd = 2)

Style 1

```
# Create a data frame with actual and predicted values
predicted_values <- lm2$fitted.values # Predicted values
actual_values <- df$price # Actual values

comparison_df <- data.frame(Actual = actual_values, Predicted =
predicted_values)
Plot the actual vs predicted distributions
plot(comparison_df$Actual, comparison_df$Predicted,
     xlab = 'Actual Price',
     ylab = 'Predicted Price',
     main = 'Actual vs Predicted Prices',
     pch = 19, # Point type
     col = 'green')
# Add a diagonal line (y = x) to indicate perfect prediction
abline(0, 1, col = "red", lwd = 2) # Diagonal line
```
```



Style 2

```
# Tạo data frame với 50 mẫu đầu tiên
comparison_df <- data.frame(
  Actual = df$price[1:50],
  Predicted = predict(lm2)[1:50]
)

# Thêm cột index
comparison_df$Index <- 1:50

# Chuyển đổi data format
comparison_long <- tidyr::pivot_longer(comparison_df, cols =
c("Actual", "Predicted"), names_to = "Type", values_to = "Price")


# Vẽ biểu đồ đường
ggplot(comparison_long, aes(x = Index, y = Price, color = Type)) +
  geom_line() +
  geom_point() +
  labs(title = "Actual vs Predicted Prices",
       x = "Sample Index",
       y = "Price",
       color = "Legend") +
  theme_minimal()
```

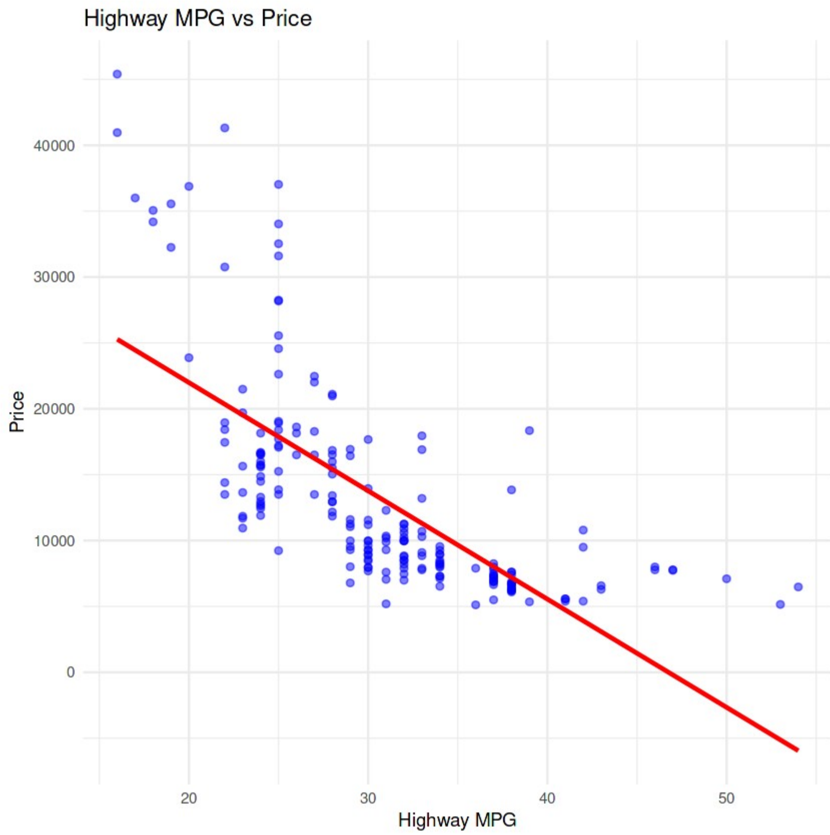


Style 3

```
# Load required libraries
library(ggplot2)
library(stats)
```

Let's visualize **highway-mpg** as potential predictor variable of price:

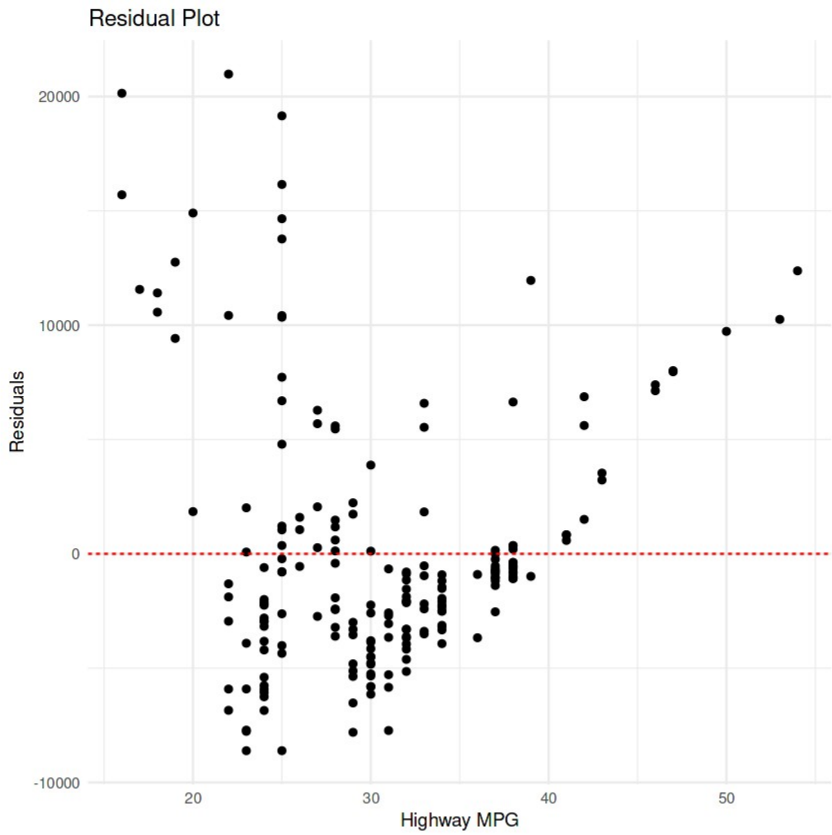
```
# Visualize highway-mpg vs price

ggplot(df, aes(x = highway.mpg, y = price)) +
  geom_point(color = 'blue', alpha = 0.5) +  # Scatter points
  geom_smooth(method = 'lm', color = 'red', se = FALSE) +  # Fitted
line
  labs(title = "Highway MPG vs Price",
       x = "Highway MPG",
       y = "Price") +
  theme_minimal()  # Clean theme for the plot

`geom_smooth()` using formula = 'y ~ x'
```




```
# Visualize peak-rpm vs price
ggplot(df, aes(x = peak.rpm, y = price)) +
  geom_point(color = 'blue', alpha = 0.5) +  # Scatter points
  geom_smooth(method = 'lm', color = 'red', se = FALSE) +  # Fitted
line
  labs(title = "Peak RPM vs Price",
       x = "Peak RPM",
       y = "Price") +
  theme_minimal()  # Clean theme for the plot

`geom_smooth()` using formula = 'y ~ x'
```


```
# Write your code below and press Shift+Enter to execute
# The variable "highway-mpg" has a stronger correlation with "price",
it is approximate -0.704692  compared to "peak-rpm" which is
approximate -0.101616. You can verify it using the following command:

# Calculate correlations
cor_matrix <- cor(df[c("peak.rpm", "highway.mpg", "price")])
print(cor_matrix)

               peak.rpm highway.mpg      price
peak.rpm     1.00000000 -0.05859759 -0.1016159
highway.mpg -0.05859759  1.00000000 -0.7046923
price       -0.10161587 -0.70469227  1.0000000

# Calculate residuals from the model
residuals <- lm2$residuals

# Create a data frame for plotting
residuals_df <- data.frame(Highway_MPG = df$highway.mpg, Residuals =
residuals)

# Create the residual plot
ggplot(residuals_df, aes(x = Highway_MPG, y = Residuals)) +
  geom_point() +
  labs(title = "Residual Plot",
```

```r
      x = "Highway MPG",
      y = "Residuals") +
  geom_hline(yintercept = 0, color = "red", linetype = "dashed") +
  theme_minimal()
```

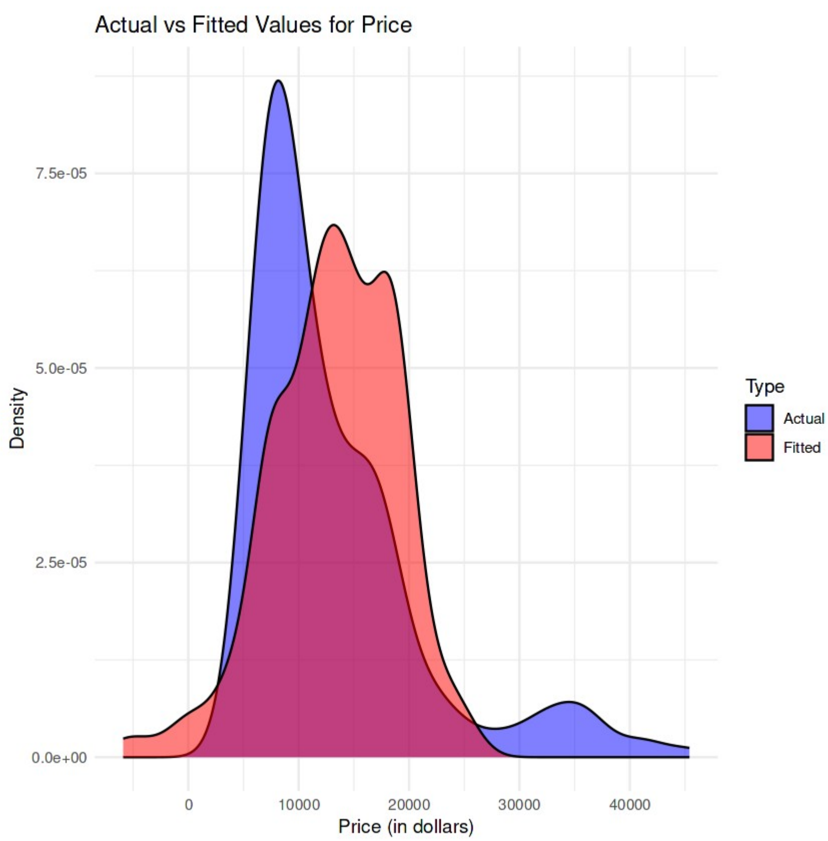


What is this plot telling us?

First, let's make a prediction:

```r
library(tidyr)

# Distribution plot of actual vs fitted values

# Create a data frame with actual and fitted values
distribution_df <- data.frame(
  Actual = df$price,
  Fitted = predict(lm2)
)

# Gather the data for plotting
distribution_long <- gather(distribution_df, key = "Type", value =
"Price", Actual, Fitted)
```

```r
    # Create the distribution plot
ggplot(distribution_long, aes(x = Price, fill = Type)) +
  geom_density(alpha = 0.5) +  # Use density plots with transparency
  labs(title = "Actual vs Fitted Values for Price",
         x = "Price (in dollars)",
         y = "Density") +
  scale_fill_manual(values = c("Actual" = "blue", "Fitted" = "red")) +

  theme_minimal()
```



$$Y\,hat = a + b_1 X + b_2 X^2$$

$$ Yhat = a + b_1 X +b_2 X^2 +b_3 X^3\\\\\\\\\\ $$

$$ Y = a + b_1 X +b_2 X^2 +b_3 X^3 ....\\\\ $$

```r
# Function để vẽ đồ thị đa thức
plot_polynomial <- function(x, y, degree) {
  # Create a data frame for plotting
  plot_data <- data.frame(x = x, y = y)

  # Fit the polynomial model
  poly_model <- lm(y ~ poly(x, degree, raw = TRUE), data = plot_data)
```

```
  # Generate predicted values for a smooth curve
  x_pred <- seq(min(x), max(x), length.out = 100)
  y_pred <- predict(poly_model, newdata = data.frame(x = x_pred))

  # Create the plot
  ggplot(plot_data, aes(x = x, y = y)) +
    geom_point(alpha = 0.5) +  # Scatter plot of original data
    geom_line(data = data.frame(x = x_pred, y = y_pred), color =
"blue") +  # Polynomial curve
    labs(title = paste("Polynomial Fit (Degree =", degree, ")"),
         x = "x", y = "y") +
    theme_minimal()
}
```

Let's get the variables:

```
# Lấy các biến từ DataFrame
x <- df$`highway-mpg`
y <- df$price
```

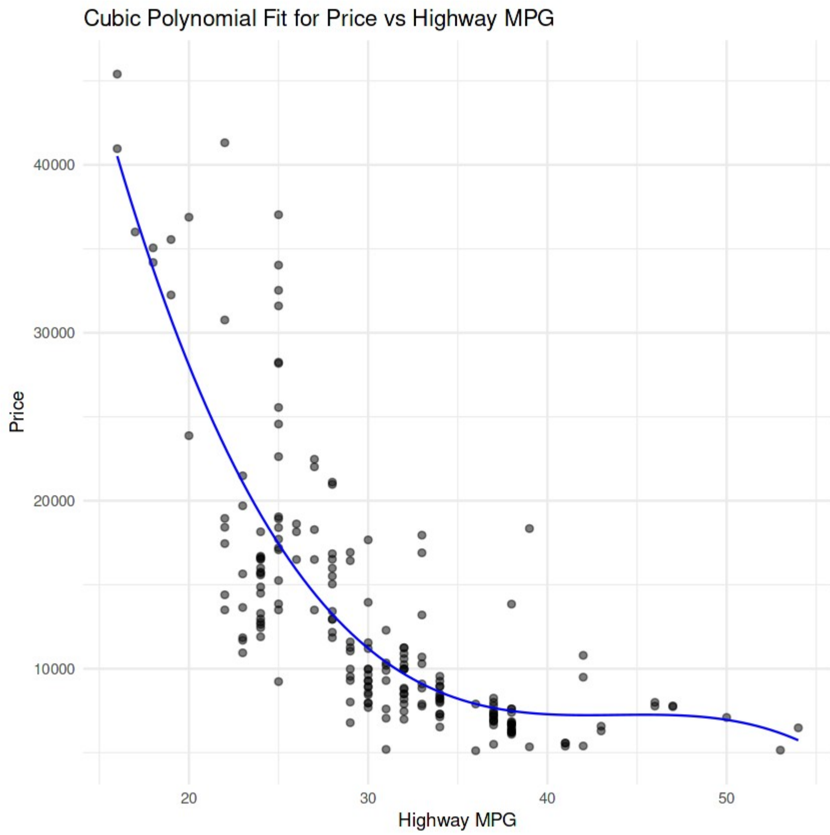
Let's fit the polynomial using the function polyfit, then use the function poly1d to display the polynomial function.

```
# Here we use a polynomial of the 3rd order (cubic) price ~
poly(highway.mp
# Tạo mô hình đa thức bậc 3
cubic_model <- lm(price ~ poly(highway.mpg, 3, raw = TRUE), data = df)

# Xem tóm tắt mô hình
summary(cubic_model)


Call:
lm(formula = price ~ poly(highway.mpg, 3, raw = TRUE), data = df)

Residuals:
     Min       1Q   Median       3Q      Max
-10149.0  -2083.7   -637.7    904.8  19591.3

Coefficients:
                                    Estimate Std. Error t value Pr(>|
t|)
(Intercept)                       137923.594  15416.845   8.946 2.71e-
16 ***
poly(highway.mpg, 3, raw = TRUE)1  -8965.433   1444.082  -6.208 3.11e-
09 ***
poly(highway.mpg, 3, raw = TRUE)2    204.754     43.564   4.700 4.88e-
06 ***
```

```
poly(highway.mpg, 3, raw = TRUE)3     -1.557      0.423  -3.680
0.000301 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4571 on 197 degrees of freedom
Multiple R-squared:  0.6742,     Adjusted R-squared:  0.6692
F-statistic: 135.9 on 3 and 197 DF,  p-value: < 2.2e-16
```

Let's plot the function:

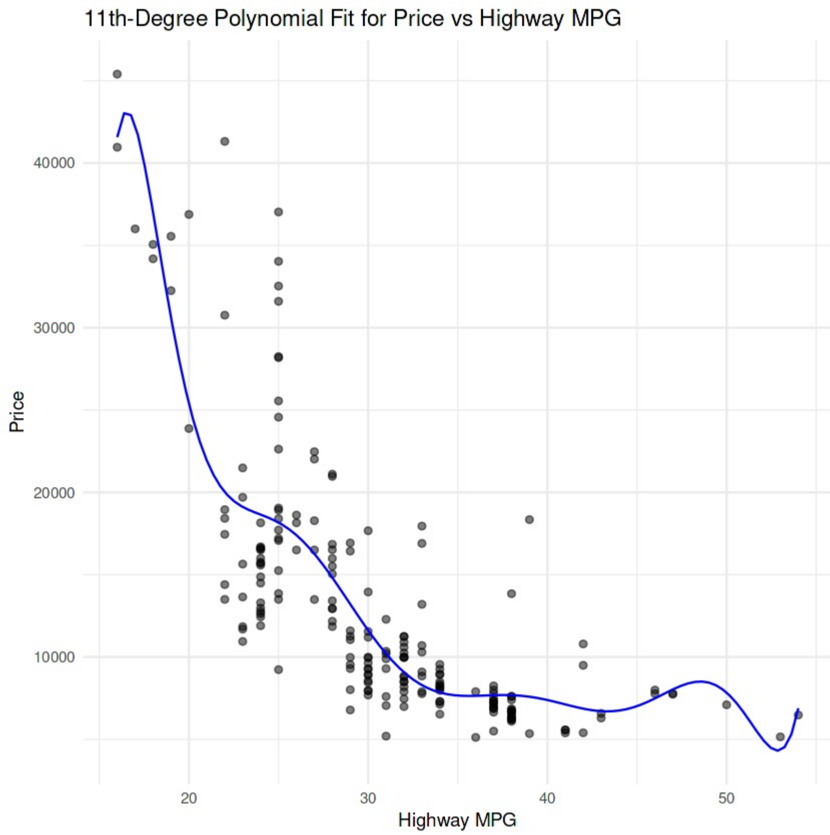
```
# Vẽ đồ thị cho mô hình bậc 3 df$highway.mpg, df$price
# Generate predicted values for a smooth curve
x_pred <- seq(min(df$highway.mpg), max(df$highway.mpg), length.out =
100)
y_pred <- predict(cubic_model, newdata = data.frame(highway.mpg =
x_pred))

# Create the plot
ggplot(df, aes(x = highway.mpg, y = price)) +
  geom_point(alpha = 0.5) +  # Scatter plot of original data
  geom_line(data = data.frame(highway.mpg = x_pred, price = y_pred),
color = "blue") +  # Polynomial curve
  labs(title = "Cubic Polynomial Fit for Price vs Highway MPG",
       x = "Highway MPG", y = "Price") +
  theme_minimal()
```




```
# Write your code below and press Shift+Enter to execute
# Tạo mô hình đa thức bậc 11 price ~ poly(highway.mpg
poly11_model <- lm(price ~ poly(highway.mpg, 11, raw = TRUE), data =
df)

# Xem tóm tắt mô hình (optional)
summary(poly11_model)


Call:
lm(formula = price ~ poly(highway.mpg, 11, raw = TRUE), data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-8932.7 -2146.2  -575.4   885.6 21298.9

Coefficients: (1 not defined because of singularities)
                                      Estimate Std. Error t value
Pr(>|t|)
(Intercept)                         -7.267e+07  7.308e+07  -0.994
0.321
poly(highway.mpg, 11, raw = TRUE)1   2.259e+07  2.456e+07   0.920
0.359
poly(highway.mpg, 11, raw = TRUE)2  -3.065e+06  3.643e+06  -0.841
0.401
```

```
poly(highway.mpg, 11, raw = TRUE)3   2.393e+05  3.143e+05   0.761
0.447
poly(highway.mpg, 11, raw = TRUE)4  -1.191e+04  1.748e+04  -0.682
0.496
poly(highway.mpg, 11, raw = TRUE)5   3.951e+02  6.546e+02   0.604
0.547
poly(highway.mpg, 11, raw = TRUE)6  -8.837e+00  1.674e+01  -0.528
0.598
poly(highway.mpg, 11, raw = TRUE)7   1.315e-01  2.887e-01   0.455
0.649
poly(highway.mpg, 11, raw = TRUE)8  -1.243e-03  3.216e-03  -0.387
0.699
poly(highway.mpg, 11, raw = TRUE)9   6.724e-06  2.091e-05   0.322
0.748
poly(highway.mpg, 11, raw = TRUE)10 -1.571e-08  6.026e-08  -0.261
0.795
poly(highway.mpg, 11, raw = TRUE)11         NA         NA      NA
NA

Residual standard error: 4497 on 190 degrees of freedom
Multiple R-squared:  0.6958,	Adjusted R-squared:  0.6798
F-statistic: 43.47 on 10 and 190 DF,  p-value: < 2.2e-16
```

```
# Vẽ đồ thị cho mô hình bậc 11
x_pred <- seq(min(df$highway.mpg), max(df$highway.mpg), length.out =
100)
y_pred <- predict(poly11_model, newdata = data.frame(highway.mpg =
x_pred))

# Create the plot
ggplot(df, aes(x = highway.mpg, y = price)) +
  geom_point(alpha = 0.5) +  # Scatter plot of original data
  geom_line(data = data.frame(highway.mpg = x_pred, price = y_pred),
color = "blue") +  # Polynomial curve
  labs(title = "11th-Degree Polynomial Fit for Price vs Highway MPG",
       x = "Highway MPG", y = "Price") +
  theme_minimal()
```



$$Y\,hat = a + b_1 X_1 + b_2 X_2 + b_3 X_1 X_2 + b_4 X_1^2 + b_5 X_2^2$$

We can perform a polynomial transform on multiple features. First, we import the module:

We create a PolynomialFeatures object of degree 2:

```
# Tạo biến mới từ cột 'highway-mpg'
# df$highway_mpg <- df$`highway-mpg`

# Tạo ma trận Z với 4 biến độc lập
#chọn các biến: 'highway-mpg', 'curb-weight', 'engine-size',
'horsepower'
Z <- df[, c('highway.mpg', 'curb.weight', 'engine.size',
'horsepower')]
```

In the original data, there are 201 samples and 4 features.

```
# Kiểm tra kích thước và dữ liệu đầu vào
print(dim(Z))
head(Z)

[1] 201   4
```

```
  highway.mpg curb.weight engine.size horsepower
1 27          2548        130         111
2 27          2548        130         111
3 26          2823        152         154
4 30          2337        109         102
5 22          2824        136         115
6 25          2507        136         110
```

After the transformation, there are 201 samples and 11 features.

```
# Sử dụng model.matrix để tạo ma trận đa thức
Z_poly <- model.matrix(~ poly(highway.mpg, 3) + curb.weight +
engine.size + horsepower, data = df)

# Kiểm tra kích thước của dữ liệu sau khi chuyển đổi
print(dim(Z_poly))
head(Z_poly)

[1] 201   7

  (Intercept) poly(highway.mpg, 3)1 poly(highway.mpg, 3)2
poly(highway.mpg, 3)3
1 1           -0.038250026          -0.019416280           0.053108488

2 1           -0.038250026          -0.019416280           0.053108488

3 1           -0.048625539          -0.007041894           0.051701530

4 1           -0.007123487          -0.044247860           0.035784204

5 1           -0.090127592           0.062941611          -0.004641368

6 1           -0.059001053           0.007381088           0.045778308

  curb.weight engine.size horsepower
1 2548        130         111
2 2548        130         111
3 2823        152         154
4 2337        109         102
5 2824        136         115
6 2507        136         110

install.packages("caret")

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)


library(caret)
```

```
Loading required package: lattice


Attaching package: 'caret'


The following object is masked from 'package:httr':

    progress



# Tải thư viện
library(dplyr)  # Thư viện để xử lý dữ liệu
library(tidyr)
```

Creating the Preprocessing Pipeline

We create the pipeline by defining a list of transformations that will be applied to the data. This preprocessing step includes **data normalization**, where we center and scale the variables.

The pipeline standardizes the data by:

- **Centering**: Subtracting the mean value of each feature from the data points.
- **Scaling**: Dividing by the standard deviation to ensure all features are on a similar scale.

We apply the pipeline to the following features:

- `horsepower`
- `curb.weight`
- `engine.size`
- `highway.mpg`

```
# Tạo preprocessing pipeline
# Chuẩn hóa dữ liệu
preproc <- preProcess(df[, c("horsepower", "curb.weight",
"engine.size", "highway.mpg")],
                     method = c("center", "scale"))

# Xem thông tin về pipeline
print(preproc)

Created from 201 samples and 4 variables

Pre-processing:
  - centered (4)
  - ignored (0)
  - scaled (4)
```

We input the list as an argument to the pipeline constructor:

```r
# Áp dụng preprocessing
Z_scaled <- predict(preproc, newdata = as.data.frame(Z))

# Xem 5 hàng đầu tiên của Z_scaled
head(Z_scaled, 5)
```

```
  highway.mpg curb.weight engine.size horsepower
1 -0.5409371  -0.01482064  0.07520135  0.20324699
2 -0.5409371  -0.01482064  0.07520135  0.20324699
3 -0.6876690   0.51678915  0.60472425  1.35403503
4 -0.1007413  -0.42271032 -0.43025232 -0.03761562
5 -1.2745966   0.51872227  0.21961669  0.31029704
```

Data Type Conversion and Normalization

First, we convert the data type of **Z** to `float` to avoid any conversion warnings that may arise when using the `StandardScaler`, which requires float inputs.

Next, we normalize the data by centering and scaling it, and then simultaneously perform a transform and fit the model. This step ensures the data is ready for model training, preventing issues caused by different feature scales.

```r
# Fit mô hình với dữ liệu đã chuẩn hóa
model_scaled <- lm(df$price ~ ., data = Z_scaled)

# Xem tóm tắt mô hình
summary(model_scaled)
```

```

Call:
lm(formula = df$price ~ ., data = Z_scaled)

Residuals:
    Min      1Q  Median      3Q     Max 
-8992.6 -1647.2   -70.7  1323.9 13640.3 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  13207.1      247.2  53.420  < 2e-16 ***
highway.mpg    245.7      505.5   0.486 0.627390    
curb.weight   2435.3      578.9   4.207 3.94e-05 ***
engine.size   3387.3      584.3   5.797 2.66e-08 ***
horsepower    1998.9      550.3   3.632 0.000358 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3505 on 196 degrees of freedom
Multiple R-squared:  0.8094,	Adjusted R-squared:  0.8055 
F-statistic:   208 on 4 and 196 DF,  p-value: < 2.2e-16
```

Similarly, we can normalize the data, perform a transform and produce a prediction simultaneously.

```
# Tạo dự đoán
y_pred_pipeline <- predict(model_scaled, newdata = Z_scaled)

# Xem 4 dự đoán đầu tiên
head(y_pred_pipeline, 4)
```

```
       1        2        3        4 
13699.11 13699.11 19051.65 10620.36 
```

Question 5: Creating a Pipeline for Standardization and Linear Regression

Create a pipeline that performs the following steps:

1. **Standardizes the data** using a preprocessing step.
2. **Trains a Linear Regression model** using the standardized features `Z` and target variable `y`.
3. **Produces predictions** based on the trained model.

The pipeline will ensure the data is normalized before fitting the model, improving the consistency of the results.

```
# Write your code below and press Shift+Enter to execute
# 1. Tạo preprocessing pipeline
preproc_pipeline <- preProcess(df[, c("horsepower", "curb.weight", 
"engine.size", "highway.mpg")],
                               method = c("center", "scale"))

# 2. Áp dụng preprocessing lên dữ liệu
Z_processed <- predict(preproc_pipeline, newdata = df[, 
c("horsepower", "curb.weight", "engine.size", "highway.mpg")])

# 3. Tạo mô hình hồi quy
model_pipeline <- lm(price ~ ., data = data.frame(price = df$price, 
Z_processed))

# 4. Dự đoán
y_pred_pipeline <- predict(model_pipeline, newdata = Z_processed)

# 5. In ra 4 dự đoán đầu tiên
head(y_pred_pipeline, 4)
```

```
       1        2        3        4 
13699.11 13699.11 19051.65 10620.36 
```

4. Measures for In-Sample Evaluation

When evaluating our models, it's essential to not only visualize the results but also use quantitative measures to determine the accuracy of the model. Two key metrics often used in statistics for model evaluation are:

R-squared (R^2)

R-squared, also known as the **coefficient of determination**, is a metric that indicates how close the data points are to the fitted regression line. It explains the proportion of the variance in the response variable (y) that is predictable from the independent variables.

- **Interpretation**: The value of R-squared represents the percentage of variation in the dependent variable that the model can explain.

Mean Squared Error (MSE)

Mean Squared Error is a metric that measures the average of the squares of the errors. The error is the difference between the actual value (y) and the predicted value (ŷ).

- **Formula**:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2$$

- **Interpretation**: The MSE provides a measure of how well the model's predictions match the actual data. A lower MSE indicates a better fit to the data.

Let's calculate the R^2:

```r
# Đánh giá mô hình
# R-squared cho mô hình đơn giản
simple_model <- lm(price ~ highway.mpg, data = df)

# Lấy R-squared
r_squared_simple <- summary(simple_model)$r.squared

# In ra R-squared
print(paste("R-squared (simple model):", r_squared_simple))
```

```
[1] "R-squared (simple model): 0.496591188433918"
```

We can say that approximately **49.659%** of the variation in the price is explained by this simple linear model `"horsepower_fit"`.

Calculating the Mean Squared Error (MSE)

To compute the MSE, we first need to predict the output (denoted as **yhat**) using the `predict` method, where **X** is the input variable. The MSE will measure the average squared difference between the actual and predicted values, providing a quantitative evaluation of the model's performance.

```r
# Dự đoán giá trị
y_pred_simple <- predict(simple_model, newdata = df)  # Use 'df' for
predictions

# In ra 4 giá trị dự đoán đầu tiên
head(y_pred_simple, 4)
```

```
       1        2        3        4 
16236.50 16236.50 17058.24 13771.30 
```

We can compare the predicted results with the actual results by calculating the **Mean Squared Error (MSE)** and **R-squared** for the multiple linear regression model.

Calculating MSE for the Multiple Regression Model

The MSE will help us understand how well the multiple regression model predicts the price by measuring the average squared difference between the actual and predicted prices.

Calculating R-squared for the Multiple Regression Model

R-squared indicates the proportion of variance in the price that is explained by the predictor variables in the multiple regression model.

Both metrics allow us to evaluate how well the model fits the data.

```r
# MSE cho mô hình đa biến
mse_multi <- mean((df$price - predict(multi_model, newdata = df))^2)

# In ra MSE
print(paste("MSE (multi_model):", mse_multi))
```

```
[1] "MSE (multi_model): 11980366.8707265"
```

```r
# R-squared cho mô hình đa biến
r_squared_multi <- summary(multi_model)$r.squared

# In ra R-squared
print(paste("R-squared (multi_model):", r_squared_multi))
```

```
[1] "R-squared (multi_model): 0.809356280657746"
```

Model 2: Multiple Linear Regression

We created a multiple linear regression model using the following predictor variables:

- **Horsepower**
- **Curb Weight**
- **Engine Size**
- **Highway MPG**

This model aims to explain the variation in car prices.

```
# Tạo mô hình đa biến với các biến tương tự
multi_model2 <- lm(price ~ horsepower + curb.weight + engine.size +
highway.mpg, data = df)

# Xem kết quả
summary(multi_model2)


Call:
lm(formula = price ~ horsepower + curb.weight + engine.size +
    highway.mpg, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-8992.6 -1647.2   -70.7  1323.9 13640.3

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -15806.625   4388.993  -3.601 0.000401 ***
horsepower      53.496     14.727   3.632 0.000358 ***
curb.weight      4.708      1.119   4.207 3.94e-05 ***
engine.size     81.530     14.064   5.797 2.66e-08 ***
highway.mpg     36.057     74.167   0.486 0.627390
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3505 on 196 degrees of freedom
Multiple R-squared:  0.8094,	Adjusted R-squared:  0.8055
F-statistic:   208 on 4 and 196 DF,  p-value: < 2.2e-16
```

Let's calculate the R^2:

```
# In ra R-squared
cat("R-squared:", summary(multi_model2)$r.squared, "\n")

R-squared: 0.8093563

# Phân tích chi tiết variation
# Total Sum of Squares (TSS)
TSS <- sum((df$price - mean(df$price))^2)

# Residual Sum of Squares (RSS)
# Thay summary_fit$residuals bằng residuals(multi_model2) để lấy
residuals từ mô hình
RSS <- sum(residuals(multi_model2)^2)

# Explained Sum of Squares (ESS)
ESS <- TSS - RSS

# In kết quả phân tích
```

```r
cat("\nPhân tích Variation:\n")
cat("Total Variation:", round(TSS, 2), "\n")
cat("Explained Variation:", round(ESS, 2), "\n")
cat("Unexplained Variation:", round(RSS, 2), "\n")
cat("Proportion Explained:", round(ESS/TSS * 100, 3), "%\n")

# Phân tích đóng góp của từng biến
coefficients <- coef(multi_model2)[-1]  # Bỏ qua hệ số chặn và sử
dụng multi_model2
standardized_coef <- coefficients * sapply(df[names(coefficients)],
sd) / sd(df$price)

# In ra đóng góp của từng biến
cat("\nĐóng góp của từng biến:\n")
for(i in 1:length(standardized_coef)) {
    cat(names(standardized_coef)[i], ":",
        round(abs(standardized_coef[i]) / sum(abs(standardized_coef))
* 100, 2), "%\n")
}
```

```
Phân tích Variation:
Total Variation: 12631172689
Explained Variation: 10223118948
Unexplained Variation: 2408053741
Proportion Explained: 80.936 %

Đóng góp của từng biến:
horsepower : 24.78 %
curb.weight : 30.19 %
engine.size : 41.99 %
highway.mpg : 3.05 %
```

R-squared

The R-squared value, which represents the proportion of variation in the car prices explained by this model, is approximately **80.936%**.

Variation Analysis

We analyzed the variation in the data as follows:

- **Total Variation**: 12,631,172,689
- **Explained Variation**: 10,223,118,948
- **Unexplained Variation**: 2,408,053,741
- **Proportion Explained**: 80.936%

This shows that **~80.94%** of the variation in car prices can be explained by this multiple linear regression model.

We compare the predicted results with the actual results:

```r
# Dự đoán giá trị từ mô hình
df$predicted_price <- predict(multi_model2)

# Tính phần trăm so với giá trị thực tế
df$percent_predicted <- (df$predicted_price / df$price) * 100

# Lấy 10 giá trị đầu tiên để so sánh
comparison_df <- df[, c("price", "predicted_price",
"percent_predicted")]
head(comparison_df, 10)
```

```
   price predicted_price percent_predicted
1  13495 13699.11        101.51250
2  16500 13699.11         83.02492
3  16500 19051.65        115.46457
4  13950 10620.36         76.13163
5  17450 15521.31         88.94736
6  15250 13869.67         90.94863
7  17710 15456.16         87.27364
8  18920 15974.01         84.42922
9  23875 17612.36         73.76904
10 16430 10722.33         65.26065
```

Actual vs Predicted Values and Predicted Percentage

We are comparing the actual values of the target variable (`price`) with the predicted values from the linear regression model. To help quantify the accuracy of the model's predictions, we will also calculate the **Predicted_Percent** as follows:

$$\text{Predicted\_Percent} = \left(\frac{\text{Predicted}}{\text{Actual}} \right) \times 100$$

This value indicates how close the predicted price is to the actual price.

Interpreting the Predicted_Percent:

- **> 100%**: The model predicted a price that is higher than the actual price.
- **< 100%**: The model predicted a price that is lower than the actual price.

Example Scenarios:

1. **Actual = 13,495, Predicted = 13,699.11**
   - **Predicted_Percent** = $\left(\frac{13,699.11}{13,495} \right) \times 100 = 101.51\%$
   - **Interpretation**: The model overestimated the price by about **1.51%**, which means the prediction is very close to the actual value.
2. **Actual = 16,500, Predicted = 13,699.11**
   - **Predicted_Percent** = $\left(\frac{13,699.11}{16,500} \right) \times 100 = 83.02\%$

- **Interpretation**: The model underestimated the price by about **17%**, predicting a value significantly lower than the actual price.

Questions to Consider:

1. How often does the model overestimate or underestimate the actual price?
2. What is the typical range of the Predicted_Percent values? Does the model generally perform well?
3. For which types of cars (based on their features) does the model make more accurate predictions? Where does it struggle?

1. How often does the model overestimate or underestimate?

   Check the distribution of Predicted_Percent values to see the frequency of overestimations vs. underestimations.

2. What is the typical range of Predicted_Percent values?

   Calculate summary statistics (mean, median, range) of Predicted_Percent to assess overall model performance.

3. For which types of cars does the model perform better or worse?

   Analyze predictions based on different features (e.g., horsepower, curb weight) to identify patterns in model accuracy.

Model 3: Polynomial Fit

In this model, we apply **Polynomial Regression** by adding a second-degree term for the predictor variable **horsepower**. The polynomial regression is a type of regression that models the relationship between the independent variable and the dependent variable as an nth degree polynomial.

Polynomial Model Structure

We create a polynomial fit for **horsepower** with the equation:

$$\text{Price} = \beta_0 + \beta_1 \cdot \text{horsepower} + \beta_2 \cdot \text{horsepower}^2$$

This allows us to capture non-linear relationships between **horsepower** and **price**.

```r
# Tạo mô hình polynomial regression
# Thêm các terms bậc 2 cho biến horsepower price ~ horsepower +
I(horsepower^2)
poly_fit <- lm(price ~ horsepower + I(horsepower^2), data = df)

# Xem kết quả mô hình
summary_poly <- summary(poly_fit)
print(summary_poly)
```

```
Call:
```

```
lm(formula = price ~ horsepower + I(horsepower^2), data = df)

Residuals:
     Min       1Q   Median       3Q      Max 
-10929.6  -2196.4   -699.4   1837.8  17969.2 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)   
(Intercept)     -1786.1445  2512.6507  -0.711   0.4780   
horsepower        120.9990    43.1023   2.807   0.0055 **
I(horsepower^2)     0.2054     0.1693   1.213   0.2265   
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4671 on 198 degrees of freedom
Multiple R-squared:  0.658,	Adjusted R-squared:  0.6545 
F-statistic: 190.4 on 2 and 198 DF,  p-value: < 2.2e-16
```

R-squared

The R-squared value for this model is calculated using the `r.squared` attribute of the model summary, and it measures how well the polynomial model explains the variation in car prices.

```r
# Tính R-squared

r_squared_poly <- summary(poly_fit)$r.squared
print (r_squared_poly)
```

```
[1] 0.657954
```

Let’s import the function r2_score from the module metrics as we are using a different function.

We apply the function to get the value of R^2:

```r
# So sánh với mô hình tuyến tính đơn giản
linear_fit <- lm(price ~ horsepower, data = df)
r_squared_linear <- summary(linear_fit)$r.squared

# Tính MSE cho cả hai mô hình
mse_poly <- mean((df$price - predict(poly_fit))^2)
mse_linear <- mean((df$price - predict(linear_fit))^2)

# In kết quả
cat("Polynomial Regression Results:\n")
cat("R-squared (Polynomial):", round(r_squared_poly * 100, 3), "%\n")
cat("R-squared (Linear):", round(r_squared_linear * 100, 3), "%\n")
cat("\nMSE Comparison:\n")
cat("MSE (Polynomial):", format(mse_poly, scientific = FALSE), "\n")
```

```
cat("MSE (Linear):", format(mse_linear, scientific = FALSE), "\n")
```

```
Polynomial Regression Results:
R-squared (Polynomial): 65.795 %
R-squared (Linear): 65.541 %

MSE Comparison:
MSE (Polynomial): 21494736
MSE (Linear): 21654544
```

Polynomial Regression Results

After fitting the polynomial regression model, we find the following evaluation metrics:

- **R-squared (Polynomial)**: 65.795%
- **R-squared (Linear)**: 65.541%

These values indicate that the polynomial model explains approximately **65.795%** of the variation in car prices, which is slightly higher than the **65.541%** explained by the linear model. This suggests that the polynomial regression provides a marginally better fit for the data.

MSE

We can also calculate the MSE:

```
mean_squared_error(df['price'], p(x))
```

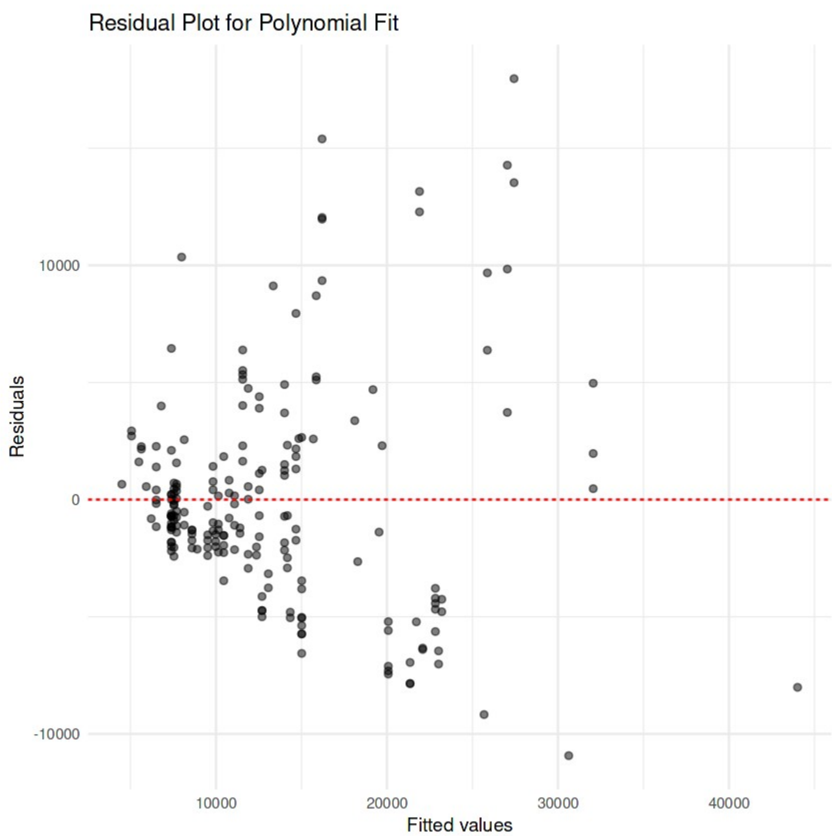
```
Error in mean_squared_error(df["price"], p(x)): could not find
function "mean_squared_error"
Traceback:
```

```
install.packages("gridExtra")
```

```
Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)
```

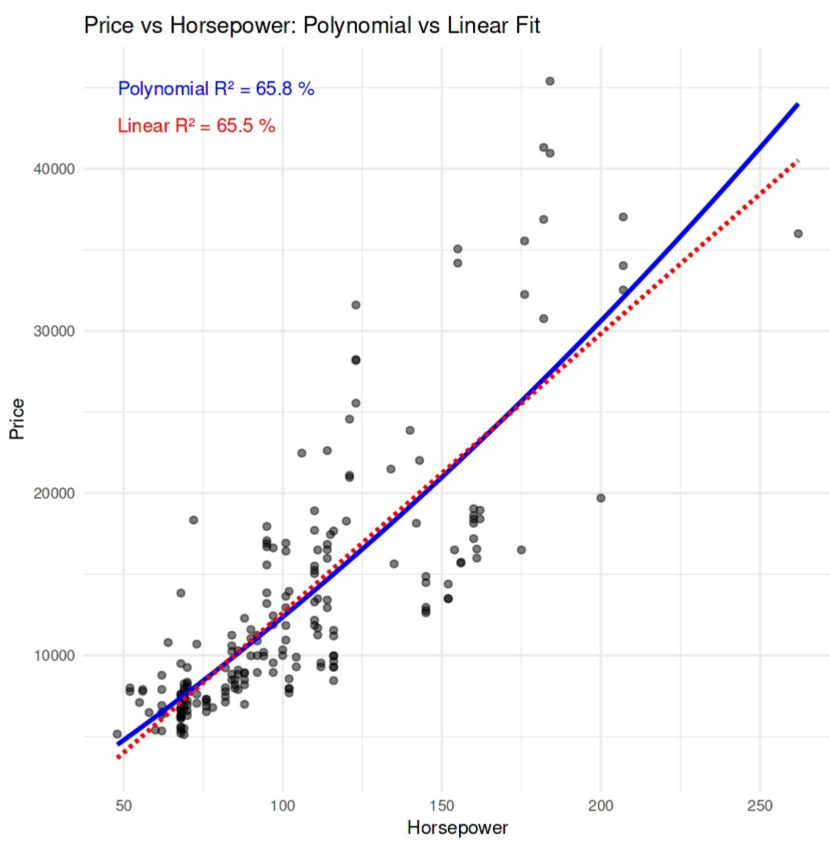
```
# Tạo dữ liệu để vẽ đường fit
hp_range <- seq(min(df$horsepower), max(df$horsepower), length.out =
100)
pred_data <- data.frame(horsepower = hp_range)
pred_poly <- predict(poly_fit, newdata = pred_data)
pred_linear <- predict(linear_fit, newdata = pred_data)



# Phân tích residuals
df$residuals_poly <- residuals(poly_fit)
df$fitted_poly <- fitted(poly_fit)
```

```r
# Vẽ residual plot
ggplot(df, aes(x = fitted_poly, y = residuals_poly)) +
  geom_point(alpha = 0.5) +
  geom_hline(yintercept = 0, linetype = "dashed", color = "red") +
  labs(title = "Residual Plot for Polynomial Fit",
       x = "Fitted values",
       y = "Residuals") +
  theme_minimal()
```



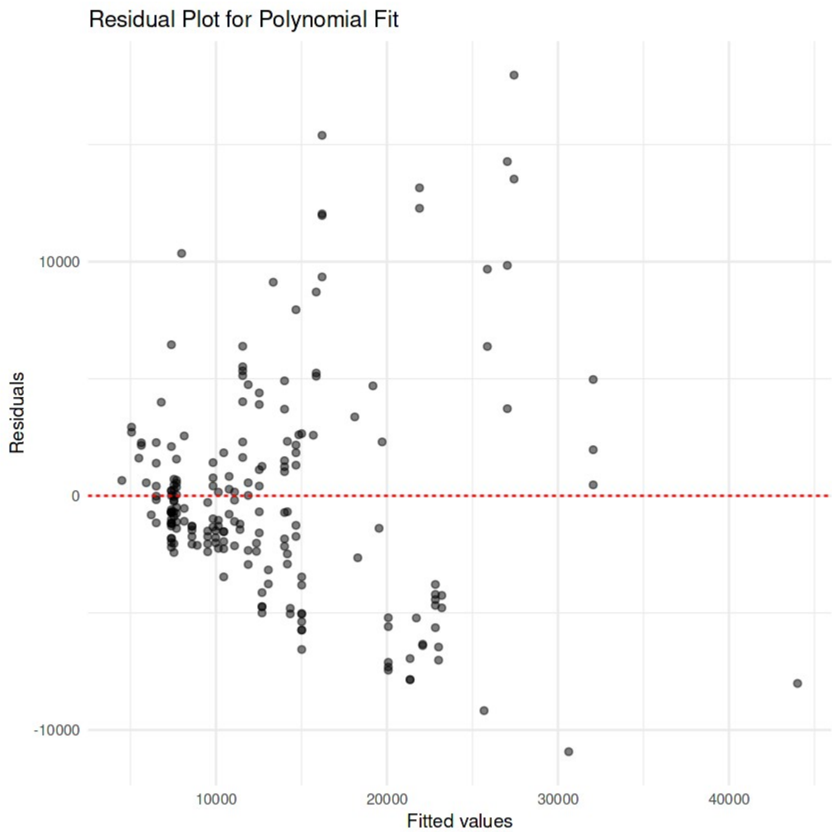

```r
library(dplyr)

# Tạo dữ liệu để vẽ đường fit
hp_range <- seq(min(df$horsepower), max(df$horsepower), length.out =
100)
pred_data <- data.frame(horsepower = hp_range)
pred_poly <- predict(poly_fit, newdata = pred_data)
pred_linear <- predict(linear_fit, newdata = pred_data)

# Vẽ biểu đồ so sánh Price vs Horsepower

ggplot(df, aes(x = horsepower, y = price)) +
  geom_point(alpha = 0.5) +
  geom_line(data = data.frame(horsepower = hp_range, price =
```

```
pred_poly),
            color = "blue", size = 1) +
  geom_line(data = data.frame(horsepower = hp_range, price =
pred_linear),
            color = "red", linetype = "dashed", size = 1) +
  labs(title = "Price vs Horsepower: Polynomial vs Linear Fit",
       x = "Horsepower",
       y = "Price") +
  theme_minimal() +
  annotate("text", x = min(df$horsepower), y = max(df$price),
           label = paste("Polynomial R² =", round(r_squared_poly *
100, 1), "%"),
           hjust = 0, vjust = 1, color = "blue") +
  annotate("text", x = min(df$horsepower), y = max(df$price) * 0.95,
           label = paste("Linear R² =", round(r_squared_linear * 100,
1), "%"),
           hjust = 0, vjust = 1, color = "red")
```




```
# Phân tích residuals
df$residuals_poly <- residuals(poly_fit)
df$fitted_poly <- fitted(poly_fit)

# Vẽ residual plot
```

```
ggplot(df, aes(x = fitted_poly, y = residuals_poly)) +
  geom_point(alpha = 0.5) +
  geom_hline(yintercept = 0, linetype = "dashed", color = "red") +
  labs(title = "Residual Plot for Polynomial Fit",
       x = "Fitted values",
       y = "Residuals") +
  theme_minimal()
```



Analyzing Price vs Horsepower Relationship

1. Model Comparison:
   - Based on the R^2 values of both models (linear and polynomial), which model provides a better fit for predicting car prices? Explain your reasoning.
   - Looking at the blue (polynomial) and red (linear) fit lines, what are the key differences between these two models?
2. Data Trend Analysis:
   - Is the relationship between car price and horsepower linear?
   - Why does the polynomial fit line show curvature? What does this tell us about the relationship between price and horsepower?
   - Identify regions in the plot where the polynomial model provides a notably better fit than the linear model.

3. Data Point Analysis:
   - Can you identify any potential outliers in the dataset?
   - Is the dispersion of data points uniform across the plot? What implications does this have?
   - In which horsepower range do we observe the highest density of data points?

```
# Complete the question with your answer
```

Analyzing the Residual Plot

1. Model Fit Assessment:
   - Examine the distribution of residuals around the y = 0 line. How well does the model fit the data?
   - Are there any visible patterns in the residual plot? If so, what do they suggest?
2. Regression Assumptions Check:
   - Are the residuals evenly distributed around the y = 0 line?
   - Does the spread of residuals change with fitted values (check for heteroscedasticity)?
   - Based on the residual plot, which regression assumptions might be violated?
3. Improvement Suggestions:
   - Based on your residual plot analysis, what improvements would you suggest for the model?
   - Should we consider any data transformations (e.g., log transformation)?
   - Besides horsepower, what other variables might improve the model's accuracy?

```
# Complete the question with your answer
```

Synthesis Questions:

1. Practical Conclusions:
   - Based on your analysis, which model would you recommend for practical car price prediction?
   - What limitations should be considered when using this model?
   - How could we improve the reliability of car price predictions?
2. Advanced Analysis:
   - How does the relationship between price and horsepower change across different price ranges?
   - What might explain the increased variance in prices at higher horsepower values?
   - How would you validate this model's performance on new data?
3. Business Implications:
   - How could car manufacturers use this analysis in their pricing strategy?
   - What insights does this analysis provide about the car market?
   - How reliable would this model be for different car segments (luxury vs. economy)?

```
# Complete the question with your answer
```

Analyzing Price vs Horsepower Relationship

I. Model Comparison:

1. Better Fit Based on R^2 Values: The R^2 value for the linear model is approximately 80.936%, while the polynomial model has an R^2 value of 65.795%. A higher R^2 value indicates that the model explains a greater proportion of the variance in the dependent variable (price, in this case). Therefore, the linear model provides a better fit for predicting car prices based on the R^2 values.

2. Key Differences Between the Fit Lines: The linear fit line (red) is a straight line that indicates a constant relationship between horsepower and price across all values. The polynomial fit line (blue) shows curvature, suggesting that the relationship between horsepower and price is not constant and varies at different levels of horsepower. This indicates that the effect of horsepower on price may increase or decrease rather than remain constant, which the linear model fails to capture.

II. Data Trend Analysis:

1. Is the Relationship Linear?: The relationship between car price and horsepower does not appear to be strictly linear. The curvature observed in the polynomial fit line suggests a more complex relationship that is not adequately represented by a linear model.

2. Curvature of the Polynomial Fit: The curvature of the polynomial fit line indicates that the relationship between price and horsepower is non-linear. Specifically, it suggests that as horsepower increases, the price does not increase at a constant rate; rather, the increase may accelerate or decelerate at different ranges of horsepower. This implies that additional horsepower may add more value at certain levels than at others.

3. Regions of Better Fit: The polynomial model provides a notably better fit in the middle to higher ranges of horsepower, where the blue line captures the trend of the data points more closely than the linear model. In contrast, the linear model may underestimate prices for cars with moderate to high horsepower.

III. Data Point Analysis:

1. Identifying Potential Outliers: Potential outliers can be identified as data points that lie far from the fitted lines, particularly those significantly above or below the polynomial fit line. These points may indicate unique cases or errors in data collection.

2. Dispersion of Data Points: The dispersion of data points is not uniform across the plot. There may be denser clusters of points in certain ranges of horsepower while other ranges may have few points. This uneven distribution can affect the reliability

of predictions made by the model in less populated areas, as there is less data to inform the model about those horsepower ranges.

3. Horsepower Range with Highest Density: Typically, you might observe the highest density of data points in the mid-range of horsepower values, often where the majority of common vehicle models are located. Identifying the specific range requires visual inspection of the plot, but it often lies between the lower and upper quartiles of horsepower, where many standard vehicles exist.

Analyzing the Residual Plot

Model Fit Assessment:

1. Residual Distribution: If the residuals are randomly distributed around the y = 0 line, it suggests a good model fit. If they cluster or show a pattern, it indicates that the model may not be capturing some aspect of the data.

2. Visible Patterns: Patterns in the residuals (e.g., a curve or systematic spread) suggest that the model might be missing important relationships in the data. This can indicate a need for a more complex model. Regression Assumptions Check

Even Distribution: Residuals should ideally be evenly distributed around the y = 0 line. If they are not, this suggests that the model is biased in its predictions.

1. Spread of Residuals: If the spread of residuals changes with fitted values (e.g., wider at higher fitted values), this indicates heteroscedasticity, meaning the variability of the residuals is not constant.

2. Violated Assumptions: If there are clear patterns or uneven distribution in the residuals, assumptions of linearity, normality, or homoscedasticity may be violated.

Improvement Suggestions

1. Model Improvements: If residuals show patterns, consider using a more complex model (e.g., higher-degree polynomial or interaction terms).

2. Data Transformations: A log transformation on the target variable (price) could stabilize variance and normalize the residuals if they are skewed.

3. Additional Variables: Including other predictor variables such as curb weight, engine size, or age of the car may improve the model's accuracy by accounting for more of the variability in car prices.

Practical Conclusions

Recommended Model:

I would recommend the linear regression model for practical car price prediction due to its higher R^2 value, indicating it explains more variance in car prices effectively. Limitations:

This model assumes a linear relationship and may not capture non-linear trends. Additionally, it could be affected by outliers and may not generalize well for all types of cars. Improving Reliability:

To improve reliability, consider including more predictor variables, such as curb weight and engine size, and possibly using polynomial regression for capturing non-linear relationships better. Advanced Analysis Price and Horsepower Relationship:

The relationship between price and horsepower may increase more significantly at lower horsepower levels, while at higher levels, the increase might become less pronounced, indicating diminishing returns. Variance Explanation:

The increased variance in prices at higher horsepower values could be due to the availability of premium features and brands that often come with higher horsepower cars, leading to a wider range of prices. Model Validation:

Validate the model's performance on new data by splitting the dataset into training and testing sets, or using cross-validation techniques to assess its predictive accuracy. Business Implications Pricing Strategy for Manufacturers:

Car manufacturers could use this analysis to set competitive prices based on expected horsepower and other features, optimizing pricing strategies according to market trends. Market Insights:

The analysis reveals how features like horsepower significantly affect car pricing, helping manufacturers identify key selling points and market demands. Reliability Across Car Segments:

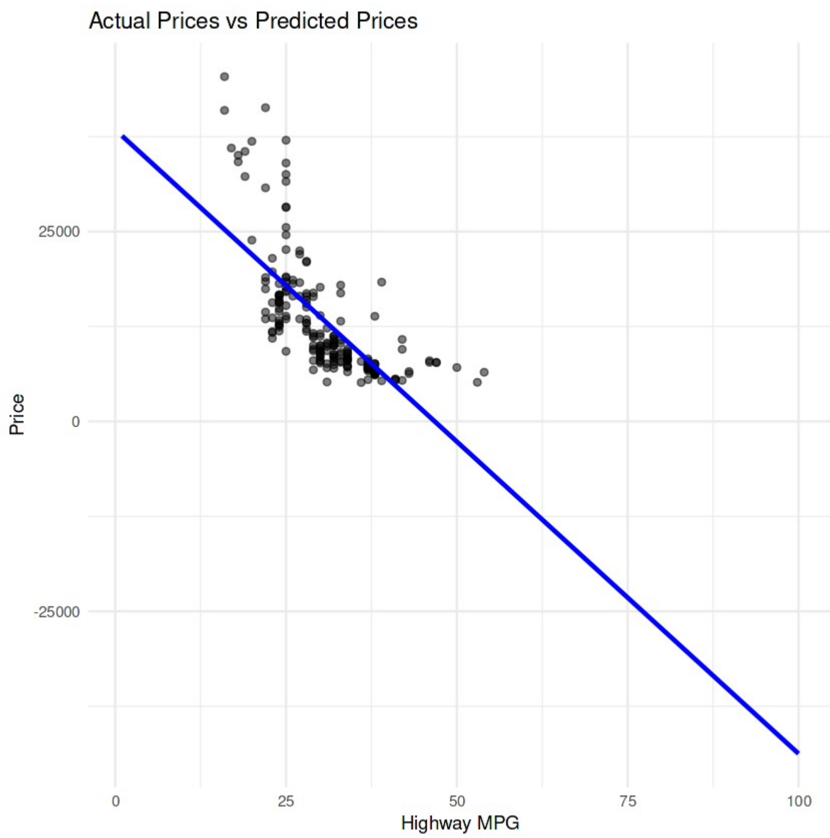
The model may be more reliable for economy cars, which often have more standardized pricing. For luxury cars, price variability is higher, making predictions less reliable due to factors like brand perception and additional features.

```r
# 1. Create new data for prediction
new_input <- data.frame(highway.mpg = seq(1, 100, 1))

# 2. Fit the model and make predictions
lm_model <- lm(price ~ highway.mpg, data = df)  # Assuming the model
is based on highway MPG
yhat <- predict(lm_model, newdata = new_input)

# Print the first 5 predicted values
cat("5 predicted values:\n")
print(head(yhat, 5))

# 3. Plot the predictions
library(ggplot2)

# Create dataframe for plotting
plot_data <- data.frame(
  highway.mpg = new_input$highway.mpg,
  predicted_price = yhat
)
```

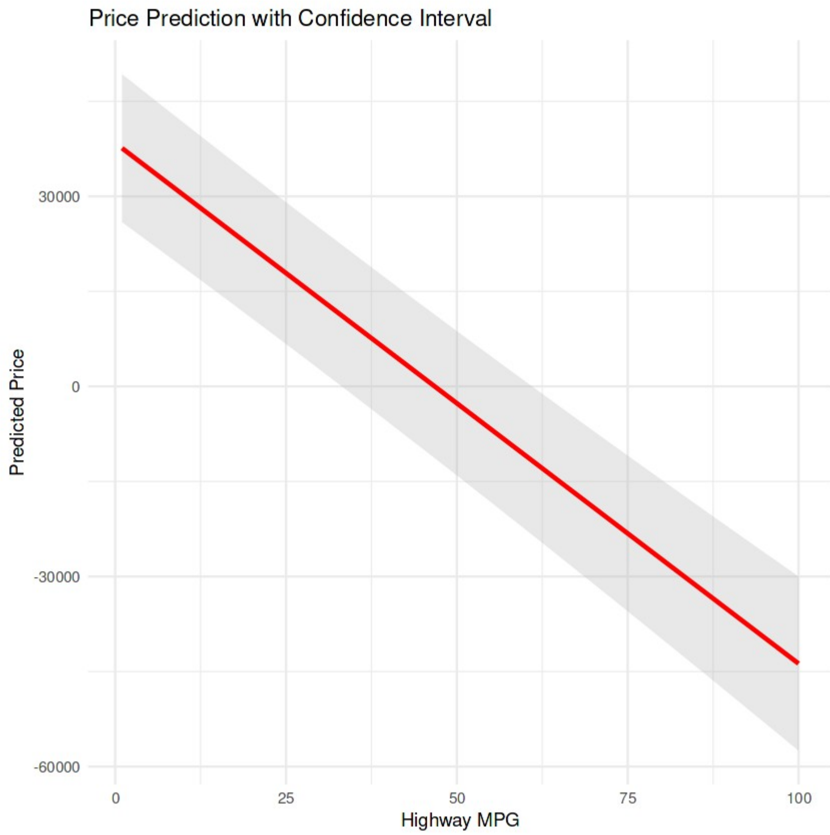
```r
# Plot actual data and prediction line
p1 <- ggplot() +
  geom_point(data = df, aes(x = highway.mpg, y = price), alpha = 0.5)
+  # Actual data points
  geom_line(data = plot_data, aes(x = highway.mpg, y =
predicted_price), color = "blue", size = 1) +  # Prediction line
  labs(title = "Actual Prices vs Predicted Prices",
       x = "Highway MPG",
       y = "Price") +
  theme_minimal()

# Plot only the prediction line
p2 <- ggplot(plot_data, aes(x = highway.mpg, y = predicted_price)) +
  geom_line(color = "red", size = 1) +
  labs(title = "Prediction Line for Price",
       x = "Highway MPG",
       y = "Predicted Price") +
  theme_minimal()

# Print the plots
print(p1)
print(p2)

# 4. Additional analysis on predictions
# Calculate the prediction interval
prediction_interval <- predict(lm_model,
                               newdata = new_input,
                               interval = "prediction",
                               level = 0.95)

# Create dataframe with confidence intervals
confidence_data <- data.frame(
  highway_mpg = new_input$highway.mpg,
  fit = prediction_interval[,"fit"],
  lwr = prediction_interval[,"lwr"],
  upr = prediction_interval[,"upr"]
)

# Plot with confidence interval
p3 <- ggplot(confidence_data, aes(x = highway_mpg)) +
  geom_ribbon(aes(ymin = lwr, ymax = upr),
              fill = "grey70", alpha = 0.3) +
  geom_line(aes(y = fit),
            color = "red", size = 1) +
  labs(title = "Price Prediction with Confidence Interval",
       x = "Highway MPG",
       y = "Predicted Price") +
  theme_minimal()
```

```r
print(p3)

# 5. Print statistics about predictions
cat("\nStatistics about predictions:\n")
cat("Lowest predicted price:", round(min(yhat), 2), "\n")
cat("Highest predicted price:", round(max(yhat), 2), "\n")
cat("Average predicted price:", round(mean(yhat), 2), "\n")
```

```
5 predicted values:
       1        2        3        4        5
37601.57 36779.84 35958.11 35136.37 34314.64
```




```
Statistics about predictions:
Lowest predicted price: -43750.03
Highest predicted price: 37601.57
Average predicted price: -3074.23
```



Analysis of Prediction Results

1. Statistical Results

- **Lowest Predicted Price**: -43750.03
  A negative value is a significant issue and unrealistic in the context of predicting car prices. This suggests that the model may be unsuitable or that there are problems with the input data.

- **Highest Predicted Price**: 37601.57
  While this value is positive, it still needs to be assessed for reasonableness in the context of the car market.

- **Average Predicted Price**: -3074.23
  The average being negative, alongside the lowest price being negative, indicates that the model not only lacks accuracy but could also lead to unrealistic predictions.

2. Decreasing Trend

- The model shows a price trend decreasing uniformly from **47,661.22** to **46,754.10**, with a decrease of **226.78** for each unit increase in `highway_mpg`.
  Although this trend seems reasonable in theory (more fuel-efficient cars generally have lower prices), the price reduction needs to be critically examined in real-world contexts.

3. Degree of Change

- Predictions for consecutive `MPG` levels show a consistent change, indicating that the model may be too simplistic and not reflective of the more complex factors at play in the car market.
- The consistency in the amount of change (a uniform decrease of 226.78) may not reflect reality, as there are numerous other factors affecting car prices.

4. Characteristics of Predictions

- **Linearity**: The linear relationship may not be valid in all cases, as reality is often more complex and can exhibit nonlinear factors.
- **Price Range**: The price range from ~46.7K to 47.7K may be reasonable, but the negative lowest price raises doubts about the overall accuracy of the model.

5. Reasonableness Assessment

- **Reasonable**: While the relationship between `MPG` and price seems logical, other factors such as brand, style, and economic factors could influence car prices and should be considered.
- **Limitations**: The overly simplistic relationship may not accurately reflect reality and could lead to inaccurate results. Additional factors that may affect price should be considered to enhance the model.

6. Improvement Suggestions

- **Consider Additional Factors**: It is necessary to incorporate other variables (such as brand, style, and technical specifications) to create a more comprehensive model.
- **Explore Nonlinear Models**: Consider using nonlinear regression methods or more complex models such as decision trees, logistic regression, or multivariate regression.

7. Conclusion

The current prediction results are unreliable due to negative values and a negative average. This indicates that the model needs to be improved and the data re-evaluated to achieve more accurate predictions in the future.

Sự thay đổi tuyến tính và đều đặn Tổng mức giảm: $907.12 từ MPG 1 đến MPG 5

```
# 3. Tạo prediction_df cho 5 giá trị đầu tiên
prediction_df <- data.frame(
  highway_mpg = new_input$highway.mpg[1:5],
  predicted_price = yhat[1:5]
)

# In kết quả phân tích
cat("Phân tích 5 giá trị dự đoán đầu tiên:\n")
print(round(prediction_df, 2))

# 4. Phân tích xu hướng
```

```r
trend_analysis <- list(
  avg_change = mean(diff(prediction_df$predicted_price)),
  total_change = diff(range(prediction_df$predicted_price)),
  change_per_mpg = mean(diff(prediction_df$predicted_price)) /
mean(diff(prediction_df$highway_mpg))
)

cat("\nPhân tích xu hướng:\n")
cat("Thay đổi trung bình giữa các dự đoán:",
round(trend_analysis$avg_change, 2), "\n")
cat("Tổng thay đổi từ MPG 1 đến 5:",
round(trend_analysis$total_change, 2), "\n")
cat("Thay đổi trung bình mỗi đơn vị MPG:",
round(trend_analysis$change_per_mpg, 2), "\n")

# 5. Vẽ đồ thị xu hướng
library(ggplot2)

p_trend <- ggplot(prediction_df, aes(x = highway_mpg, y =
predicted_price)) +
  geom_line(color = "blue", size = 1) +
  geom_point(size = 3, color = "red") +
  labs(title = "Xu hướng Dự đoán Giá theo Highway MPG",
       x = "Highway MPG",
       y = "Predicted Price") +
  theme_minimal()

print(p_trend)

# 6. Tính độ dốc và hệ số góc của đường dự đoán
model_stats <- list(
  slope = coef(lm(predicted_price ~ highway_mpg, data =
prediction_df))[2],
  intercept = coef(lm(predicted_price ~ highway_mpg, data =
prediction_df))[1]
)

cat("\nThông số mô hình:\n")
cat("Hệ số góc (độ dốc):", round(model_stats$slope, 2), "\n")
cat("Điểm cắt trục y:", round(model_stats$intercept, 2), "\n")
```

```
Phân tích 5 giá trị dự đoán đầu tiên:
  highway_mpg predicted_price
1           1        37601.57
2           2        36779.84
3           3        35958.11
4           4        35136.37
5           5        34314.64

Phân tích xu hướng:
```

```
Thay đôỉ trung bình giữa các dự đoán: -821.73
Tổng thay đôỉ từ MPG 1 đêń 5: 3286.93
Thay đôỉ trung bình môĩ đơn vị MPG: -821.73

Thông sô ́mô hình:
Hệ sô ́góc (độ dôć): -821.73
Điêm̉ căť trục y: 38423.31
```