GPGN 404
1st Midterm Exam
September 30, 2005   **Name:** _____

Question:	1	2	3	4	5	Total
Points:	6	8	14	10	12	50
Score:						

Question 1 . . . . (6 points)

Compute the following sums:

(a) [3 points] $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$

(b) [3 points] $\displaystyle\sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{|n|}$

Question 2 . . . . (8 points)

Let $u[n]$ denote the unit-step sequence, and let $x[n] = u[n] - u[n-4]$. Sketch the following sequences. (In each sketch, label your axes.)

(a) [2 points] $x[n]$

(b) [2 points] $x[-n]$

(c) [2 points] $x[n+2]$

(d) [2 points] $x[2n]$

Question 3 . . . . (14 points)

Let $h[n] = \delta[n+1] - \delta[n-1]$ be the impulse response of an LTI system.

(a) [2 points] Sketch the impulse response $h[n]$.

(b) [4 points] Sketch the sequence $y[n] = h[n] * u[n]$, the convolution of $h[n]$ with the unit-step sequence $u[n]$.

(c) [4 points] What is the frequency response $H(\omega)$ of this system?

(d) [4 points] For input $x[n] = cos(\pi n)$, what is the output $y[n]$ of this system?

Question 4 . . . . (10 points)

Consider an LTI system with frequency response

$$H(\omega) = \frac{1 - a}{1 - ae^{-j\omega}}$$

(a) [4 points] Write the difference equation for this system.

(b) [6 points] Assume that this system is causal and stable. For the input $x[n] = u[n]$, what is the output $y[n]$? (Express your answer in terms of the system parameter a and the sample index n. Check that your answer is correct for $n < 0$.)

Question 5 . . . . (12 points)

For each of the following systems, indicate whether the system is (1) stable, (2) causal, (3) linear, and/or (4) time-invariant. If stable, give the bound B_y on the output $y[n]$, in terms of the bound B_x on the input $x[n]$.

(a) [3 points] $y[n] = x[2n]$

(b) [3 points] $y[n] = (x[n])^2$

(c) [3 points] $y[n] = a + bx[n]$ (where a and b are constants)

(d) [3 points] $y[n] = \sum_{k=-\infty}^{\infty} u[k]x[n-k]$