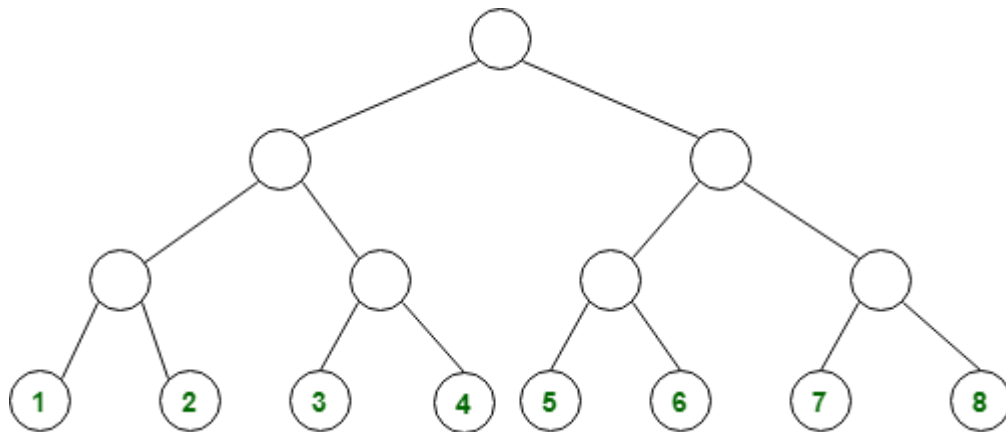


1. Let T be a full binary tree with 8 leaves. (A full binary tree has every level full.) Suppose two leaves a and b of T are chosen uniformly and independently at random. The expected value of the distance between a and b in T (i.e., the number of edges in the unique path between a and b) is (rounded off to 2 decimal places) _____.

Explanation: Full binary tree with 8 leaf nodes,



$$S = \{1, 2, 3, 4, 5, 6, 7, 8\} * \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$= \{(1, 1), (1, 2), \dots, (1, 8), (2, 1), \dots, (8, 8)\}$$

Two leaf nodes can be selected in $8 * 8 = 64$ ways.

Path length (x) between a & b	0	1	2	3	4	5	6
Number of ways	8	0	8	0	16	0	32
P(x)	8 / 64	0	8 / 64	0	16 / 64	0	32 / 64

Where, X is length between two nodes selected.

The expected value of the length between a and b in T ,

$$= E[X]$$

$$= \text{Sigma} (X=x * \text{Pr}[X=x])$$

$$= 0 * (8/64) + 2 * (8/64) + 4 * (16/64) + 6 * (32/64)$$

$$= 272/64$$

$$= 4.25$$

2. An array of 25 distinct elements is to be sorted using quicksort. Assume that the pivot element is chosen uniformly at random. The probability that the pivot element gets placed in the worst possible location in the first round of partitioning (rounded off to 2 decimal places) is _____.

Explanation: Given an array of 25 distinct elements, and pivot element is chosen uniformly randomly. So, there are only 2 worst case position in the pivot element is either first (or) last.

Therefore, required probability is,

$$= 2/25$$

$$= 0.08$$