

Groups and Rings - SF2729

Homework 6

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Exercise 1. Let $H = \langle (12) \rangle \leq S_3$ and $K = \langle (123) \rangle \leq S_3$
Consider the S_3 -set given by $S_3/H \times S_3/K$ Write this S_3 -set as a disjoint union of transitive S_3 -sets.

Solution. $h = (12)$ and $k = (123)$

General rules used:

$$b\langle b \rangle = \langle b \rangle$$

$$b^{-1}\langle b \rangle = \langle b \rangle \text{ (}\langle b \rangle \text{ is cyclic)}$$

$$S_3/H = \{H, (13)H, (23)H\}$$

$$S_3/K = \{K, (12)K\}$$

$$\begin{aligned} S_3/H \times S_3/K = \\ \{(H, K) & \quad, (H, (12)K), \\ ((13)H, K) & \quad, ((13)H, (12)K), \\ ((23)H, K) & \quad, ((23)H, (12)K)\} \end{aligned}$$

Transitive if $\forall x_1, x_2 \in S_3\text{-set} \exists g \in S_3 : gx_1 = x_2$

$$(12)(H, K) = (hH = h\langle h \rangle, hK) = (H, (12)K)$$

$$(13)((13)H, (12)K) = (H, (13)(12)K = (123)K = K) = (H, K)$$

$$(23)((23)H, (12)K) = (H, (23)(12)K = (132)K = k^{-1}K = K) = (H, K)$$

$$(12)((23)H, (12)K) = ((12)(23)H = (123)H = (123)(12)H = (13)H, K) = ((13)H, K)$$

$$(23)((23)H, K) = (H, (23)k^{-1}K) = (23)(132)K = (12)K = (H, (12)K)$$

Since G is closed under the operation one can take any combination of these g_i (or its inverse if you want to go in the other direction) to get from $\forall x_1$ to $\forall x_2$ with some $g = g_2 g_1 \in G$ and all elements are therefore transitive. (One can easily show that these are connected by drawing an arrow from \rightarrow to (and a backarrow with the inverse permutation) the elements in the five equations above in the listing of the set above.)

So the resulting disjoint union of transitive S_3 -sets being simply (without union):

$$\begin{array}{l} S_3/H \times S_3/K = \\ \{(H, K) \quad \quad \quad , (H, (12)K), \\ ((13)H, K) \quad \quad \quad , ((13)H, (12)K), \\ ((23)H, K) \quad \quad \quad , ((23)H, (12)K)\} \end{array}$$