

Groups and Rings - SF2729

Homework 2 (Rings)

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Exercise 1. Let $\sigma_m : \mathbb{Z} \rightarrow \mathbb{Z}_m$ be the natural homomorphism given by $\sigma_m(a) = a \pmod{m}$.

a. Show that $\overline{\sigma_m} : \mathbb{Z}[x] \rightarrow \mathbb{Z}_m[x]$ given by

$$\overline{\sigma_m}(a_0 + a_1x + \dots + a_nx^n) = \sigma_m(a_0) + \sigma_m(a_1)x + \dots + \sigma_m(a_n)x^n \quad (1)$$

is an homomorphism of $\mathbb{Z}[x]$ onto $\mathbb{Z}_m[x]$.

b. Show that $\text{degree}(f(x) \in \mathbb{Z}[x]) = \text{degree}(\overline{\sigma_m}(f(x))) = n \wedge \overline{\sigma_m}(f(x))$ has no nontrivial factors in $\mathbb{Z}_m[x] \Rightarrow f(x)$ is irreducible in $\mathbb{Q}[x]$.

c. Show that $x^3 + 17x + 36$ is irreducible in $\mathbb{Q}[x]$

Solution.

Exercise 2. Let $f(X) = X^4 - X^2 + 1$. Prove that $f(X)$ is irreducible in $\mathbb{Z}[X]$ and show that $f(X)$ is reducible in $\mathbb{Z}_m[X]$ for $m = \{2, 3, 5\}$ by determining the factorization into a product of irreducible polynomials.

Solution.