# Groups and Rings - SF2729

**Homework 2 (Rings)**

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**Exercise 1. Let $\sigma_m : \mathbb{Z} \to \mathbb{Z}_m$ be the natural homomorphism given by $\sigma_m(a) = a\ (mod\ m)$.**

**a. Show that $\overline{\sigma_m} : \mathbb{Z}[x] \to \mathbb{Z}_m[x]$.**

$$\overline{\sigma_m}(a_0 + a_1 x + \ldots + a_n x^n) = \sigma_m(a_0) + \sigma_m(a_1)x + \ldots + \sigma_m(a_n)x^n \qquad (1)$$

**is an homomorphism of $\mathbb{Z}[x]$ onto $\mathbb{Z}_m[x]$.**

**b. Show that $degree(f(x) \in \mathbb{Z}[x]) = degree(\overline{\sigma_m}(f(x))) = n \bigwedge \overline{\sigma_m(f(x))}$ has no nontrivial factors in $\mathbb{Z}_m[x] \Rightarrow f(x)$ is irreducible in $\mathbb{Q}[x]$.**

**c. Show that $x^3 + 17x + 36$ is irreducible in $\mathbb{Q}[x]$**

*Solution.*

**Exercise 2. Let $f(X) = X^4 - X^2 + 1$. Prove that $f(X)$ is irreducible in $\mathbb{Z}[X]$ and show that $f(X)$ is reducible in $\mathbb{Z}_m[X]$ for $m = \{2, 3, 5\}$ by determining the factorization into a product of irreducible polynomials.**

*Solution.*