Groups and Rings - SF2729

Homework 4

Jim Holmström - 890503-7571

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Exercise 1. Let G be a finite group. Show that for any element $a \in G$, we have $a^{|G|} = e$.

Solution. $\forall a \in G$ we have $< a > \leq G$ and Lagrange's theorem $\Rightarrow n = |< a >|$ divides $|G|$
$< a > = \{a, a^2, .., a^n = e\}$
We have that n divides $|G|$ then $m = |G|/n$ is guaranteed to be an integer.
$a^{|G|} = a^{mn} = (a^n)^m = e^m = e$ $\square$

Exercise 2. We say that a G-set is transitive if it has only one orbit. Assume that G is finite and X is a transitive G-set. Show that X also is finite and that $|X|$ divides $|G|$

Solution. X transitive $\Rightarrow X = Gx \forall x \in X$ since G only has $|G|$ elements we have $|X| = |Gx| \leq |G|$ If $G, X < \infty$ then the orbit-stabilizer theorem and Lagrange's theorem gives $|Gx| = [G : G_x] = |G|/|G_x|$ which shows that $|Gx||G_x| = |G| \Rightarrow |X||G_x| = |G|$ which shows that $|X|$ divides $|G|$ $\square$