Groups and Rings - SF2729

Homework 7

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Exercise 1. How many p-Sylow subgroups the group A_5 has for $p = 3, 5, 7$

Solution.

Theorem 0.1 (Third Sylow Teorem). p prime $\bigwedge$ $p \mid |G| \Rightarrow \#\{\text{Sylow } p\text{-subgroups}\} \equiv 1 \; (mod\, p) \bigwedge \#\{\text{Sylow } p\text{-subgroups}\} \mid |G|$

$S = \{a : a \mid |A_5| = 5!/2\} = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$

1 $p = 3$

$\{a \in S : a \equiv 1 \; (mod\, 3)\} = \{1, 4, 10\}$
Subgroups generate by cycles of order 3 in A_5 is Sylow 3-subgroups.
$\langle(123)\rangle, \langle(124)\rangle, \langle(125)\rangle, \langle(134)\rangle, \langle(135)\rangle, \ldots$
Which is more then 4 distinct Sylowgroups which results in:
$\underline{\#\{\text{Sylow 3-subgroups}\} = 10}$

2 $p = 5$

$\{a \in S : a \equiv 1 \; (mod\, 5)\} = \{1, 6\}$
Subgroups generate by cycles of order 5 in A_5 is Sylow 5-subgroups.
$\langle(12345)\rangle, \langle(12354)\rangle, \ldots$
Which is more then 1 distinct Sylowgroups which results in:
$\underline{\#\{\text{Sylow 5-subgroups}\} = 6}$

3 $p = 7$

$\{a \in S : a \equiv 1 \; (mod\, 7)\} = \{1, 15\}$

To fullfill Third Sylow Theorem and Lagranges Theorem we must have a subgroup with $7^0 = 1$ elements which is $\{e\}$

And thus:
$\underline{\#\{\text{Sylow 7-subgroups}\} = 1}$