Groups and Rings - SF2729

Homework 6

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Exercise 1. Let $H = \langle (12) \rangle \leq S_3$ and $K = \langle (123) \rangle \leq S_3$
Consider the S_3-set given by $S_3/H \times S_3/K$ Write this S_3-set as a disjoint union of transitive S_3-sets.

Solution. $h = (12)$ and $k = (123)$

General rules used:
$b\langle b \rangle = \langle b \rangle$
$b^{-1}\langle b \rangle = \langle b \rangle$

$$S_3/H = \{H, (13)H, (23)H\}$$

$$S_3/K = \{K, (12)K\}$$

$$\begin{aligned} S_3/H \times S_3/K = & \\ & \{(H, K) & ,(H, (12)K), \\ & ((13)H, K) & ,((13)H, (12)K), \\ & ((23)H, K) & ,((23)H, (12)K)\} \end{aligned}$$

Transitive if $\forall x_1, x_2 \in S_3{-}\text{set} \exists g \in S_3 : gx_1 = x_2$

$h(aH, K) = (ahH = ah\langle h \rangle, hK) = (aH, hK)$ for $a = \{e, (13), (23)\}$
$(13)((13)H, (12)K) = (H, (13)(12)K = (123)K = K)$
$(23)((23)H, (12)K) = (H, (23)(12)K = (132)K = (132)(123)K = K)$
Since G is closed under the operation one can take any combination of these elements

above to get from $\forall x_1$ to $\forall x_2$ with some $g = g_1 g_2 \in G$ and all elements are therefore transitive.

So the resulting disjoint union of transitive S_3-sets being simply (without union):

$$
\begin{aligned}
S_3/H \times S_3/K = & \\
& \{(H, K) && ,(H, (12)K), \\
& ((13)H, K) && ,((13)H, (12)K), \\
& ((23)H, K) && ,((23)H, (12)K)\}
\end{aligned}
$$